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# Teaching Addition and Subtraction Word Problems to Children

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## Introduction

Many students have difficulties when attempting to solve word problems. Many reasons are given for the students' lack of success in word problems.(Â¹) These reasons include students' lack of exposure to life outside of television and their neighborhoods, minimal reading skills, and difficulty in comprehension skills. I do not dispute these reasons for students' failure, but I propose that there is a way to ensure that problem solving permeates the mathematics classroom while simultaneously maintaining student interest. I have tried to develop a series of lessons that will help teachers develop various strategies to teach problem solving in their classrooms. It is my hope that implementing this curriculum unit will help teachers to teach problem solving in a way that will excite students, assist their connection and application of "real world" scenarios to the problems, aid their use of various strategies, and extend students' abilities to solve math problems in other contexts.

I teach in a school district with approximately 50,000 students. They are 86% African American, and 74% are eligible for free or reduced lunch. For the past 12 years, I have taught in under-resourced, urban areas. My students come with various academic deficiencies. Some of these deficiencies can be positively affected in the classroom, some cannot. Since NCLB has come into existence, when we speak about student outcomes there is not much that matters except for their standardized test scores at the end of each school year. If students meet standards according to our state test, schools are viewed as successful and the stress in "passing the test" is put off for another year. Sometimes, students can pass the test while simultaneously doing poorly on specific domains. One of the domains that my students have consistently performed poorly on is problem solving.

For the past few years I have focused on mathematics, being a middle school math coach at one time, and most recently being the Early Intervention Program (EIP) math teacher for grades three, four, and five in my school. According to my state, EIP is designed to serve students who are at risk of not reaching or maintaining academic grade level. The purpose of EIP is to provide additional instructional resources to help students who are performing below grade level obtain the necessary academic skills to reach grade level performance in the shortest possible time.(2) Whether in the classroom teaching all subjects, or whether focusing on one subject area, namely math, I have noticed that problem solving has always been a weak area for my students.

Each year, students come into my classroom who are supposed to possess skills that are prerequisites for the

math activities that I teach. Usually, they don't. More often than not, there is a great disparity between what the students need to know to "get started" and what they actually know. Of course, I have to begin my instruction "where they are." This means that I will not have the privilege of merely working on word problems and the strategies with which to solve them. I will have to teach my students how to add two-digit numbers and how to regroup. If they master these two skills, I will need to teach them how to add three-digit addends with regrouping as well. This will be an added issue as I work with my students on nuances involved in word problems.

Ultimately, I would like for teachers to view this unit as one that assists them in helping students to master the complex skills of critical thinking and solving word problems in the primary grades. As the math portions of our standardized tests become more difficult for our students to master because of the plethora of word problems they contain, it is my hope that understanding the different dimensions of word problems will help teachers to become more thoughtful of their content as they present them to their students.

## Rationale

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Young children are naturally curious and have a desire to make sense of their world.(3) Even before they realize what they are doing, they are involved with quantities and different relationships involving quantities. They come to school with intuitive ways of thinking about and doing mathematics. They use these early skills that they have acquired to solve problems. It is evident from the lack of positive results on standardized test performance that the ways in which we have been teaching children about word

problems - and possibly even mathematics as a whole - have been disconnected from their understanding of what math is and the relationships between and among numbers.

Learning mathematics involves learning ways of thinking. It involves learning powerful mathematical ideas rather than a collection of disconnected procedures for carrying out calculations.(4) I believe that all children were born with the ability to do math. One thing that children must do for themselves is maintain the positive attitude and belief that they can do math. It is the teachers' jobs to make sure that this attitude and belief of the students is nurtured. Math must be nurtured with a supportive learning environment that promotes risk taking and creativity. It must also be one that focuses on problem solving.

A classroom that focuses on increasing students' proficiency with problem solving will also be increasing the critical thinking skills of those students. Word problems can serve as a context in which to learn mathematical concepts. Experiences with word problems can provide a meaningful bridge for connecting classroom mathematics with real-world mathematics. Word problems can be used as a basis for application and a basis of integrating the real world in mathematics education. They can provide practice with real life problem situations, motivate students to understand the importance of mathematics concepts, and help students to develop their creative, critical, and problem solving abilities.

The rationale for creating this unit is to help my students become more proficient word problem solvers. I have seen students become intimidated and frustrated with word problems because they are not able to "break them down" and understand what is being asked. As I expose students to a variety of addition and subtraction word problems, I expect them to become fluent in solving word problems. I also expect them to

become able to create their own word problems.

I want to clarify to readers and implementers of this unit that there are some lessons that I am certain that I will have to conduct to prepare for the ones that will be mentioned in this curriculum unit. I may have to conduct some of these lessons over the course of the entire school year based on the achievement level of individual students. I have had several students enter my classroom lacking the necessary technical skills to complete operations such as addition and subtraction with regrouping. There are some students who will even need additional assistance and clarification with adding and subtracting numbers without regrouping. I will need to make sure that these students know about place value, and its role when we are adding two and three digit numbers without regrouping.

## Overview

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According to Ron Aharoni, the author of *Arithmetic for Parents: A Book for Grownups about Children's Mathematics* (6), the meaning of numbers and arithmetical operations is students' link to reality.

**What is Addition?**

At its most basic, addition is putting things together or joining. There are subtle differences with joining as described by Aharoni. Those differences include dynamic addition and static addition. In dynamic addition, the situation changes over time: one group is added to another. In static addition, a large group is made of two subgroups, but there is no action of combination - the two parts simply coexist in the whole. An example of each is listed below:

*Dynamic Addition*

3 butterflies were sitting on a limb. 2 butterflies joined them. How many butterflies are there now?

*Static Addition*

A vase contains 3 red flowers and 2 yellow flowers. How many flowers are there altogether?

The Random House Unabridged Dictionary defines addition as "the process of uniting two or more numbers into one sum, represented by the symbol +." The numbers added are called the addends. As stated earlier, addition focuses on joining. According to *Children's Mathematics: Cognitively Guided Instruction* (Carpenter et al., 1999), in dynamic addition, three different types of join problems can be created by changing the quantity that is the unknown. The following chart demonstrates the join problem types as described in Carpenter's book.

Unknown

> Result Unknown. Example: Robin had 5 toy cars. Her parents gave her 2 more toys cars for her birthday. How many toy cars did she have then?
> Change Unknown. Example: Robin had 5 to cars. Her parents gave her some more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Robin's parents give her for her birthday?

Start Unknown. Example: Robin had some toy cars. Her parents gave her 2 more toy cars for her birthday. Then she had 7 toy cars. How many toy cars did Robin have before her birthday?

**What Is Subtraction?**

According to Aharoni, subtraction means removal. Very similar to addition, there is a dynamic subtraction which means that the situation changes over time. An example of dynamic subtraction is listed below.

*Dynamic Subtraction*

There are 7 girls playing in the park. Four of the girls leave the park and go home. How many girls are left in the park?

The number sentence to solve the above problem is 7 - 4 = 3. In this number sentence, the 7 is called the **minuend**. The 4 s called the **subtrahend**. The result of the action is called the **difference**.

In addition to dynamic subtraction or "take away", *Children's Mathematics* describes two other meanings to subtraction. One is identified as "part-part-whole" or "whole-part." The third meaning of subtraction is comparing. An example of each is listed below.

*Whole-Part Subtraction*

In a group of 5 children, 2 are girls. How many boys are there?

*Comparing*

Joseph has 7 cats and Travis has 4 dogs. How many more cats does Joseph have than Travis has dogs?

**Relationship between Addition and Subtraction**

Children need to understand that there is an inverse relationship between addition and subtraction. Before teaching this unit (and very early in the year), I will ensure that students have mastered number facts and fact families. Students will learn that a number fact is made up of three numbers. Those three numbers can be used to make up other number facts. Knowing one fact can help children with other facts. Look at the number facts that we make with 2, 5, and 7.

Addition Facts | Subtraction Facts

2 + 5 = 7 | 7 - 5 = 2

5 + 2 = 7 | 7 - 2 = 5

Generally, subtraction facts are harder for children to learn than addition facts. If a child knows that 6 + 9 = 15, and he or she sees the subtraction sentence 15 - 9 = __, the child can think, 9 and what are 15? I plan to encourage students to think of the related addition fact when they encounter a subtraction fact they don't know. Children often find themselves either counting up or counting back to solve subtraction. That is inefficient. If children learn the important inverse relationship between addition and subtraction, subtraction facts will become much easier. As I work with the children, I plan to use questions that encourage this strategy of the inverse relationship between addition and subtraction.

## Strategies

In *Children's Mathematics*, it is stated that of the various ways that word problems can be distinguished from one another, one of the most useful ways of classifying them is by focusing on the types of action or relationships described in the problem. Within addition and subtraction problems, there are four classes of problems that can be identified, and ultimately described and taught to children. The four basic classes of problems are join, separate, part-part-whole, and compare. The size of the numbers can vary, as well as the theme or context of the problems; however, the basic structure involving the actions and relationships remains the same. Helping my students to understand this "four class" framework will be essential in assisting them in becoming better word problem solvers. As we progress through addition and subtraction word problems, I will name these four classes, I will have students identify the four-classes, and I will work to help them understand the relationships between the classes.

**Joining - Class One**

Join problems involve action; they take place over time. In Join problems, elements are added to a given set. They involve a direct or implied action in which a set in increased by a given amount. The following are examples of join problems with the result unknown. I think that it will be important for students to realize the differences within the Join class because it will better equip them with the cognitive tools that they need in order to solve the word problems.

*Join Problems with Result Unknown*

> Todd cooks 26 hotdogs. Will cooks 14 hotdogs after Todd. How many hotdogs did the boys cook altogether?
> Mrs. Kelsey baked 13 pies last week. She baked 3 more pies this week. How many pies did Mrs. Kelsey bake in all?

The word problems all require students to find the unknown result. This is probably the most basic type of addition word problem, although there are a few subtleties that may prove to be important based on the achievement level of the students that are in the class. One possible variation is to use number words rather than numerals. I have included such problems in my problem set because my students have had difficulty with reading word problems. I believe that the more that they see certain words, the easier it will be for them to recognize them. This will take away the stress that they feel with reading the problems and allow them an opportunity to concentrate on the math component of the problems.

Another pair of similar type problems is the differentiation between problems that require the students to regroup and those that do not. Problem numbers one, three, and four in the appendix do not require students to regroup. I may introduce these to my students first. I say this because concurrently, I may have to teach students adding and subtracting, with and without regrouping. Though this will not be included in the discussion of this curriculum unit, I plan to utilize the strategies discussed by Liping Ma in her book *Knowing and Teaching Elementary Mathematics* (7). The problems in the appendix that require regrouping are numbers two and five.

An added dimension to this problem set is extraneous information. Problem number five in the appendix shows a number of cones being produced in a certain amount of time - specifically, five minutes. Although this

problem seems very simple, I believe that this is a type of problem that would cause several of my students to focus on the terms "five minutes" instead of the numbers of cones being produced, which is what the problem asks for. I will include many types of problems with extraneous information in my problem sets because there are always several of them on the standardized test that we take in Georgia, the Criterion Referenced Competency Test (CRCT).

*Join Problems with Change Unknown*

Although the problems presented in this set may seem familiar or even identical to the problem set in the previous section, they are very different. Each of these problems represents a different problem to young children because children use different strategies to solve them. Changes in the wording of the problems and the situations they depict can make a problem more or less difficult for children to solve.

> Todd cooks 26 hotdogs. Will cooks some more hotdogs. How many hotdogs did Will cook if the boys ended up with 38 hotdogs altogether?
> Mrs. Kelsey baked 13 pies last week. She baked some more pies this week. How many pies did she bake this week if she ended up with a total of 16 pies?

Since I know that these problems will challenge many of my children. As they work to solve problems, I will keep in mind that problems are easier for children to solve when the action or the relationships in the problems are as clear as possible. This will be something that I check for as I create sets of word problems for my students. It is evident that the largest group of these problems includes word problems one through seven in the appendix because they are all Join problems in which the change is unknown. A second dimension of this set is problems that do not require regrouping to find the answer. Problem numbers one, three, and six in the appendix are representative of this group. A final similar set of this group is problems that require regrouping to find the answer. These types of problems include numbers two, four, and seven in the appendix.

As I created several of the problems in this word problem set, I realize that these problems are difficult for my students to solve as addition problems. I realize that I have to find alternate ways for my students to "set up" the problems. For example, if I use problem number one to demonstrate, it will look like this:

26 + ___ = 38

One of the important practices in the Singapore Math(5)collection is to make sure that students are knowledgeable about adding numbers to ten. If I had students organize the problem differently, I believe that my students would see things much differently (See below.)

26

+___

38

I may even have my students draw a line all the way down the middle of the problem to focus on the ones and tens column separately. When students are able to look at the problem in this manner, it becomes easier for them to see that in the ones column, six plus a number equals eight. They are also able to see that in the tens column, two plus a number equals three. If they are not able to mentally use what they have learned about adding to ten, then they can use the *Joining To* method as described in *Children's Mathematics: Cognitive Guided Instruction*. In this method, a child would use six counters (to show the number in the ones column)

and "count on" from six in order to find how many they need to get to eight. In this case, the child would add two more counters to the six. This would help the student to know that six plus two is eight. They would do the same for the number two in the tens column. One should make sure that the student realizes that s/he is adding tens when dealing with the tens column.

*Join Problems with Start Unknown*

The third type of Join problem is created when the start (minuend) of the problem is unknown. Several of these types of problems are listed below.

> Todd had some hotdogs. Will gave him 12 more hotdogs. Now Todd has 27 hotdogs. How many hotdogs did Todd start with?
> Mrs. Kelsey some pies on Monday. She baked 12 pies on Wednesday. She ended up with 21 pies in all. How many pies did she bake on Monday?

The dimensions of this problem include the overall dimension of being problems that all have the first addend, or the start, missing. Another dimension includes the problems that call for students to utilize regrouping. Examples of problems in this dimension include numbers one, three, and five. Problem numbers two and four are examples of problems that do not require regrouping to solve.

These types of problems will likely cause students the same trouble as those in the "Join with Change Unknown" section of this unit. I would use this opportunity to use the prior knowledge that the students have acquired with setting the problems up differently, utilizing their knowledge of place value, and adding to ten to solve these in a way similar to the previous ones. Using problem number four as an example, I would initially like to see the students set up problem number three in the following way:

___ + 14 = 28

After this, I would have the students set the problem up again vertically. It would look like this:

___

+ 14

28

In a way that is not very different from before, I would probably have the students draw a line down the middle of the problem in a way that would divide the ones column from the tens column. If necessary, I would encourage them to use the *Joining To* method in order to solve the problem. There is an additional issue with this problem that allows me to incorporate the students' prior knowledge of fact families. The fact that the students need to start "counting on" with the bottom number will allow me to have the conversation with them that shows them more about the commutative property. It will be interesting to see if students will explore the possibilities of listing the addend on the bottom first in order to find the addend on the top. Some students will need to review commutative basics such as:

23 + 15 = 38 and 15 + 23 = 38

Students could even write the problems; once again, in a vertical form so that they can see that they are completing the same type of problem as word problem number four.

Separating - Class Two

Separate problems are closely related to Join problems. Very similar to Join, Separate problems require an action that takes place over time. The difference is that the initial quantity is decreased over time versus increased with Join problems. Again, very similar to Join problems, Separate problems involve three quantities. There is a starting quantity (minuend), a change quantity, or the amount taken away (subtrahend), and the result (difference). Any of these quantities can be the unknown.

*Separate Problems with Result Unknown*

> Todd cooks 27 hotdogs. Will took 14 of Todd's hotdogs. How many hotdogs does Todd have left?
> Mrs. Kelsey baked 20 pies. She gave 8 pies to her neighbor. How many pies did Mrs. Kelsey have left?

The above word problem set include the overall dimension of being subtraction problems that require the students to find the result. Another dimension includes problem number one solely because it uses number words instead of numbers in the problem. Problem numbers one and two are of the dimension that requires subtraction without regrouping. There is also a dimension that requires regrouping with subtraction and those include problems number three, four, and five.

*Separate Problems with Change Unknown*

> Todd cooks 27 hotdogs. Todd then gives some hotdogs to his neighbor Will. Todd is left with 13 hotdogs. How many hotdogs did Todd give to Will?
> Mrs. Kelsey had 20 pies. She gave some pies to Ms. Kelsey. If Mrs. Kelsey was left with 12 pies, how many pies did she give to Ms. Kelsey?

There is a dimension that includes both of the above word problems because they are subtraction problems that include a missing subtrahend. There is also another dimension of this problem that does not require regrouping in order to find the answer. Ironically enough, all five of the problems are a part of this category. I would encourage the students to find the answer to these types of word problems in a way that is very similar to the way in which we solve the *Join with Change Unknown* problems. Initially, I would have the students set up the word problem as follows:

27 - ___ = 15

Again, students would need to realize that if they are not able to rely on their mental math skills, they will need to recall what was learned about the commutative property and how it can be utilized in this circumstance. They would also need to rely upon what they have learned about fact families. I would guide the students to arrange the problem vertically as follows:

27

-__

15

As done earlier in the unit, students requiring more assistance than others would need to draw a line down the middle of the problem which would divide the tens column from the ones column. From there, if necessary, they would need to use the *Join To* method and figure out "five plus what number equals 7" or "what number

plus five equals 7", depending on the direction that they chose to work.

*Separate Problems with Start Unknown*

The last type of separation problem that students will need to understand is one that is missing the first number, or the minuend. Some examples of those types of problems follow:

> Todd had some hotdogs. He then gave Will12 hotdogs. Now Todd has 27 hotdogs. How many hotdogs did Todd start with?
> Mrs. Kelsey baked some pies. She gave 8 pies to her neighbor. She was left with 24 pies. How many pies did Mrs. Kelsey start with?

The largest dimension of these problems includes all of them and that is the fact that they are all missing the starting number, or the minuend. Another dimension is when a problem requires regrouping and those are problems one and three. A third dimension includes problem numbers two, four, and five. This dimension includes the problems that do not require regrouping. The last dimension includes problems numbers one and four. These problems utilize number words instead of numbers in the problems. As stated before, this is important for my students who experience literacy issues with the problem solving activities. In other words, they are not reading on grade level and I want to incorporate some type of practice every chance I get.

**Part-Part-Whole Problems - Class Three**

With *Part-Part-Whole* problems, there is no direct or implied action and no change over a period of time. *Children's Mathematics: Cognitively Guided Instruction* defines these types of problems as "static relationships among a particular set and its two disjoint (separate) subsets." Unlike the three types of *Join* and *Separate* problems, there are only two types of Part-Part-Whole problems.

*Whole Unknown*

This type of problem gives the two disjointed parts and asks the problem solver to give the size of the whole. A few of these problems are listed below.

> There are 18 girls in the class and 15 boys in the class. How many students are in the class?
> Four blue fish are in the tank with eighteen green fish. How many blue and green fish are in the tank?

The overall dimension which includes all five of the aforementioned problems is that they are all requiring the problem solver to give the size of the whole. The next dimension (although in no particular order) includes the problems where you are required to regroup. These problems include numbers one, two, three, and five. Another dimension does not require regrouping and problem number four is the sole problem in this dimension. A further look into the dimensions of these problems shows us a group of problems that use number words instead of numbers. This dimension consists of problems number two and five.

*Part Unknown*

In these types of word problems, the students are given one of the parts along with the whole, and they are to find the size of the other part. Unlike the *Join* and Separate problems, it is not obviously clear to the students as to how they are to go about solving these problems. For example, if you look at number one below, you can either subtract 15 - 7 to solve the answer, or you can say, "7 plus a number equals 15" and this will cause you to use the count on or *Join To* strategies. For the purpose of this curriculum unit, I will guide my students to

use subtraction to solve these problems. As an aside, if students used an alternative (addition) method to solve this type of problem, I would not count it against them or tell them that they were wrong. Several of the problems in this category are identified below.

> 15 children were playing baseball. 7 were boys and the rest were girls. How many girls were playing baseball?
> Deedre has 28 marbles. 17 are pink and the rest are green. How many marbles are green?

The most obvious of the dimensions is that they are all giving the whole and one part and they are asking for the missing part. There is a dimension that asks for students to regroup with the subtraction and those are question numbers one and four. A dimension that does not require regrouping with subtraction in the word problems are numbers two, three, and five. An added dimension to this set of word problems is a problem that contains extraneous information. This is problem number four. Students will need to realize that the eight bones that T.J. lost is not important as we work to solve this problem.

**Compare Problems - Class Four**

*Compare* problems are more like *Part-Part-Whole* problems versus *Join* and *Separate* problems. Where the *Part-Part-Whole* problems "involve relationships between quantities rather than a joining or a separating action", *Compare* problems "involve the comparison of two distinct, disjoint sets rather than the relationship between a set and its subsets." There are three elements of a *Compare* problem. The chart below describes them.

Mark has 3 mice. | **Referent set**

Joy has 7 mice. | **Compared set**

Joy has 4 more mice than Mark. | **Difference | *Amount by which one set exceeds the other***

Very similar to the *Part-Part-Whole* problem set, it is difficult to absolutely specify how I would prefer my students to solve *Compare* problems. As you will see in the set of problems that follow, students will be able to draw on their prior knowledge of *Join To* or counting on skills as well as *Separate* strategies in order to find the answers.

*Difference Unknown*

Some of the most common *Compare* problems are listed below.

> Mrs. Kelsey had 19 pies. Mrs. Polite had 7 pies. How many more pies did Mrs. Kelsey have than Mrs. Polite?
> If Veronica has 17 sandwiches and Jill has 11 sandwiches, how many more sandwiches does Veronica have than Jill?

In order to identify the dimensions of this problem, you have to realize the most evident dimension of all five of these problems. This dimension is that all of the questions are missing the difference. Another dimension is one that includes the problems that require regrouping. The regrouping problems in this set are numbers three and five. An added dimension is the set of problems that do not require regrouping with subtraction and those problems are numbers one, two, and four. The last dimension for this set includes a problem with extraneous information. The only problem in this category is problem number five.

*Compared Set Unknown*

These word problems are a little more difficult than the previous problems and call for students to pay more attention to the way in which the problems are worded. Although the students are comparing, these are not subtraction problems. They are actually problems in which they have to add in order to solve. Samples of *Compared Set Unknown* problems are listed below.

> Lacie had 9 marbles. Laila has 8 more marbles than Lacie. How many marbles does Laila have?
> Langston has 24 gumballs. Lamar has 19 more gumballs than Langston. How many gumballs does Lamar have?

One dimension in the problem set listed above requires students to regroup when they add. These problems are two, three, four, and five. There is only one problem in the dimension that does not require regrouping when adding and that is problem number one. Another dimension that I may explore when I continue to create word problems for my student is increasing complexity. Problems number one and two are basic addition problems and should not be very difficult for students to solve. Problem number one - although it is a two-digit plus one-digit problem - is simple because there is no regrouping required. Problem number two should be very easy to solve as well because it consists of two numbers that students are exposed to early on when they learn "adding doubles" and "adding doubles plus one." A final dimension is word problems that use number words as opposed to numbers within the context of the sentence. In this set, problem number four is the only one that is in this dimension.

*Referent Unknown*

In this *Compare* problem category, students are at a point where they need to subtract to solve the problems. Examples of *Referent Unknown* problems are listed below.

> Joy has 9 toy cars. She has 3 more cars than Mike. How many toy cars does Mike have?
> Tanya has 27 pens. She has 16 more than Travis. How many pens does Travis have?

As stated before, students must know that they have to subtract to solve these word problems. One dimension of these problems is that they all require subtraction to find the answer. Another dimension includes the problems that require regrouping. These problems are numbers three and five. An added dimension which include problems that do not require regrouping are numbers one, two, and four.

Although the wording in the problems in the preceding "four classes" of problems are similar, the basic structure of each problem is unique. The structures of the problems are related to how the students solve the problems.

## Classroom Activities

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**Lesson One**

*Identify the Strategy*

Time Frame: This lesson can be completed in one to two days.

Preparation: The teacher will need to make sure that they create several examples of each class of word problems.

Activity Flow: The first thing that the teacher will do is ensure that students are aware of the different types of word problems. Although you may not teach the problems using the vocabulary presented in this unit (i.e., join problems with start unknown), students must have been exposed to several samples of that type. This can be accomplished in one of several ways. Teachers can choose to review sample problems written on chart paper. Another way can be to create a PowerPoint presentation. Other choices include putting individual problems on index cards for small groups to review or even putting the problems on a sheet for each student to have for himself or herself.

After about ten to fifteen minutes of reviewing various problems, teachers will present the students with at least ten word problems that are comprised of a mixture of samples from each class. The best grouping for this activity will be small, heterogeneous groups consisting of no more than four children each. The students will decide what operation they will use to solve the problem. Along with identifying the operation, students will write the number sentence that they will create to solve. These are the only things that they will do. Although this activity is seemingly very simple, I believe that this is a crucial activity because this will help the teacher to learn which students have mastered the concepts and those who have not.

An alternative to this activity could be to discuss several problems of one type, and then introduce a new type while using a strategic set of questions for students. When introducing a second set, teachers may ask, "How is this problem different from the problems we have been looking at?" When it is solved, a new problem could be introduced, ask whether it was like the last one, or the ones we had been doing earlier. Questions to ask could ask students which problems are related to which, how they are similar, and how they are different. It is not important for students to learn the whole taxonomy. What is important is that they learn to read the problem carefully and think about what it says. The comparison activities are to help them make distinctions in language.

Assessment: As stated before, students will work in small, heterogeneous groups for this activity. The teacher will circulate among the groups listening to discussion. This is how you will be able to assess whether or not individual students understand the previous lessons that have been describing the methods for solving word problems. Of course, teachers will use the results of this activity to decide whether to reteach or continue to expand the word problems lessons.

**Lesson Two**

*What, How, Why Activity*

Time Frame: This activity can be completed in one to two days.

Preparation: The teacher will need to make sure that there are several examples of each class of word problems that could be taken from the appendix.

Activity Flow: Once again, teachers will need a variety of word problems from the four classes of problems presented in this curriculum unit. The way that teachers choose to present the problems should be in a manner that best fits their personalities and the class' needs and learning styles. Teachers will arrange students in pairs for this activity.

The name of this activity explains exactly what the students will be doing. After more exposure to a variety of word problems, teachers will use this opportunity to verify that students are indeed becoming more proficient at solving word problems. While in their pairs, students will decide whether they will use addition or subtraction. This will satisfy the "what" of this activity. After specifying the operation, students will show the number sentence that they will use to solve the word problem. This is the "how" of the activity. In order to satisfy, the "why" portion of the activity, the students will explain to their partners why they choose to solve the problems in that manner. It is during this time that the teacher will listen for reasoning that will help them to solve various problems *consistently*. There will be times when students will be able to "guess" the correct answer, but the discussion piece will clarify whether or not they really understand what they are doing.

**Lesson Three**

*Creating Word Problems*

Time Frame: This activity can be completed in one to three days.

Preparation: In order to complete this activity, students will need to be placed in small heterogeneous groups of no more than four students. They will need paper and pencils to complete the activities and then chart paper and markers or crayons to display their word problems and share them with the class.

Activity Flow: This activity will be completed at a point when the teacher is confident that the majority of students are able to complete any of the word problems within the four classes of addition and subtraction word problems. Students will work in their groups to create four different word problems. Students can decide whether or not they will use addition or subtraction as long as they use different types of word problems. For example, they can create two addition word problems as long as they are not both "join problems with start unknown".

After students have created the problems in their groups, the teacher should review their work. Once students have followed the guidelines, the teacher should give them approval to create a poster that they will share with the class. Once all groups have created their problems, had them approved, and rewritten them on chart paper, the class should go through each chart, discuss, and solve the problems.

**Implementing District Standards**

*Georgia Performance Standards*

This curriculum unit satisfies several sections of the Georgia Performance Standards for second grade students. The second Number and Operations domain states that "students will build fluency with multi-digit addition and subtraction". The elements of the standard include correctly adding and subtracting two whole numbers up to three digits with regrouping, understanding and using the inverse relationship between addition and subtraction to solve problems and check solutions. Two additional elements of importance in this unit include students using mental math such as benchmark numbers to solve problems and using basic properties of addition (commutative, associative, and identity) to simplify problems.

In addition to the Numbers and Operations domain, this curriculum satisfies several elements that fall under the Process Standards that are required for the second grade. Some of these standards include building new mathematical knowledge through problem solving, applying and adapting a variety of appropriate strategies to solve problems, recognizing and using connections among mathematical ideas, and understanding how

mathematical ideas interconnect and build on one another to produce a coherent whole.

## Notes


1. Bluman, A. (2005). *Math word problems demystified*. New York, NY: The McGraw-Hill Companies.

2. Georgia Department of Education (n.d.) *Georgia department of education, early intervention program (EIP) guidelines, 2006-2007*, Retrieved July 2, 2007, from Georgia Department of Education website: http://public.doe.k12.ga.us/DMGetDocument.aspx/ FY2007_BT2_EIPGuidelines.pdf? p=6CC6799F8C1371F65533EE3A 4A7A828051D50E7B57607D83CD0C006D73ADC423&Type=D

Â³Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann, xiii.

4. Carpenter, T., Franke, M., & Levi, L. (2003) *Thinking mathematically: Integrating arithmetic & algebra in elementary school*. Portsmouth, NH: Heinemann, 1.

5. Georgia Department of Education (n.d.) *K-2 mathematics georgia performance standards*. Retrieved July 3, 2007, from http://public.doe.k12.ga.us/DMGetDocument.aspx/ Standards%20K%20-%202%20July%202006.pdf? p=6CC6799F8C1371F61C554E6689B 2EED97A50D88D28116A0806905B1EE582EBE3&Type=D

6. Aharoni, R. (2007) *Arithmetic for parents: A book for grownups about children's mathematics.* El Cerrito, CA: Sumizdat

7. Ma, L. (1999) *Knowing and teaching elementary mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates, Publishers


## Appendix

### Join Problems with Result Unknown

The ice cream store sold twenty-five popsicles this morning. They sold three popsicles this afternoon. How many popsicles were sold altogether?
Todd cooks 20 hotdogs. Will cooks 10 hotdogs. How many hotdogs did the boys cook altogether?
Mrs. Kelsey baked 26 pies last week. She baked 13 more pies this week. How many pies did Mrs. Kelsey bake in all?
Veronica made 3 sandwiches for the picnic. Jill made 25 sandwiches for the picnic. How many sandwiches did the girls make together?
Jason can make 35 cones in five minutes. Erin can make 7 cones in five minutes. How many cones can they make in five minutes together?
Jeremy used 35 different crayons to make his drawing. Later, he used 56 more crayons. How many crayons did Jeremy use altogether?
Andy had 19 videotapes. He bought 24 more at the store. How many videotapes does Andy have now?
Toni has 17 blue beads for her bracelet. Her mother gave her 19 green beads. How many beads does Toni have in all?

Leon put 37 cans of fly soup in the cart and Ken put 13 cans of cream spider soup in the cart. How many cans of soup went into the cart?
You have saved $2.36 last month and $.35 this month. How much money have you saved in the last two months?

**Join Problems with Change Unknown**

Todd cooks 26 hotdogs. Will cooks some more hotdogs. If they end up with 40 hotdogs altogether, how many hotdogs did Will cook?
Mrs. Kelsey baked 13 pies last week. She baked some more pies this week. How many pies did she bake this week if she ended up with a total of 20 pies?
Veronica made 13 sandwiches for the picnic. Jill made some sandwiches for the picnic. If they ended up with a total of 28 sandwiches, how many sandwiches did Jill make?
Jeremy bought 18 crayons. How many more crayons will he have to buy in order to have a total of 37 crayons?
Jeremy bought 14 crayons. His mother gave him some crayons. If Jeremy ended up with 32 crayons, how many crayons did his mother give him?
Andy has 23 videotapes. He buys some more. If Andy ends up with 48 videotapes total, how many videotapes did he buy?
Toni has 17 blue beads. Her mother gave her some more beads. How many beads did her mother give her if Toni ended up with 30 beads?
Leon put 37 cans of fly soup in the cart and Ken put some more cans of cream spider soup in the cart. If they ended up with 50 total cans of soup altogether, how many cans of soup went into the cart?
You have saved $2.30 last month. You saved some more money this month. If you ended up with a total of $3.10, how much money did you save this month?
The ice cream store sold twenty-five popsicles this morning. They sold some more popsicles in the afternoon. If the ice cream store sold a total of thirty-eight popsicles, how many did they sell in the afternoon?

**Join Problems with Start Unknown**

Todd had some hotdogs. Will gave him 12 more hotdogs. Now Todd has 27 hotdogs. How many hotdogs did Todd start with?
Mrs. Kelsey baked some pies on Monday. Tuesday she took a break from baking, but she baked 12 pies on Wednesday. She ended up with 21 pies. How many pies did she bake on Monday?
Veronica made some sandwiches for the picnic. Jill made 14 sandwiches for the picnic. Veronica and Jill had a total of 28 sandwiches. How many sandwiches did Veronica make?
Jeremy bought some crayons. His mother gave him 27 crayons. Jeremy ended up with a total of 40 crayons. How many crayons did Jeremy buy?
Andy has some videotapes. He bought 15 more at the store. If Andy ends up with 48 videotapes, how many tapes did Andy begin with?
Toni has some beads. Her friend Zora has 19 beads. Together the girls have 37 beads. How many beads does Toni have?
Joseph had some stamps in his collection. He bought 42 more. Now he has 60 stamps in his collection. How many stamps did he have to begin with?
The museum sold some tickets on Monday and 72 on Wednesday. If a total of 87 tickets were sold on Monday and Wednesday altogether, how many tickets were sold on Monday?
Some people that got on the bus in Atlanta. 12 people got on the bus in Decatur. If there are now 28

people on the bus altogether, how many people got on the bus in Atlanta?

**Separate Problems with Result Unknown**

The ice cream store had twenty-five popsicles this morning. They sold three popsicles this afternoon. How many popsicles were left?
Todd cooked 27 hotdogs. Will took 14 of Todd's hotdogs. How many hotdogs does Todd have left?
Mrs. Kelsey baked 20 pies. She gave 8 pies to her neighbor. How many pies did Mrs. Kelsey have left?
Jill had 35 sandwiches and gave 19 of them away. How many sandwiches did Jill have left?
Katie had 23 stickers. She gave 14 stickers to Drew. How many stickers did Katie have left?
Andy has 48 videotapes and he sells 29 of them. How many videotapes does Andy have left?
Toni has 21 beads. She gave her friend Zora 6 beads. How many beads does Toni have left?
Alexis loves collecting dolls. She had 48 dolls in her collection. She gave away 29 dolls to the orphanage. How many dolls does Alexis have left?
There were 74 dogs at the shelter. 27 of the dogs were adopted and taken to new homes. How many dogs were left at the shelter?
There were 28 combs in the drawer. 19 of the combs fell on the floor and we had to throw them away. How many combs were left?

**Separate Problems with Change Unknown**

The ice cream store had twenty-five popsicles this morning. If they had 19 popsicles left at the end of the day, how many popsicles did they sell?
Todd cooks 27 hotdogs. Will took some of Todd's hotdogs. If Todd had 13 hotdogs left, how many did Todd take?
Mrs. Kelsey baked 20 pies. She gave some pies to her neighbor. If Mrs. Kelsey ended up with 12 pies, how many did she give to her neighbor?
Veronica made 28 sandwiches for the picnic. She gave some sandwiches to Jill to take home. If Veronica had 15 sandwiches left, how many did she give to Jill?
Katie had 23 stickers. She gave some stickers to Drew. If Katie was left with 12 stickers, how many stickers did she give Drew?
Jeremy bought 32 crayons. He gave his brother some crayons. If Jeremy ended up with 14 crayons, how many crayons did he give his brother?
Andy has 48 videotapes and he sells some of them. If he is left with 29 tapes, how many videotapes did Andy sell?
Toni has 21 beads. She gives some to her friend Zora. If Toni is left with 9 beads, how many did she give to Zora?
Al collects baseball cards. He has 29 cards. He used to have 68, but he had to sell some to raise money for his mother's birthday present. How many cards did Al sell?

**Separate Problems with Start Unknown**

We had some popsicles in the freezer this morning. We sold 8 popsicles this afternoon. Now we have 17 popsicles. How many popsicles did we start with?
Todd cooks some hotdogs. Will took 14 of Todd's hotdogs. How many hotdogs did Todd start with if he now has 25 hotdogs left?
Mrs. Kelsey baked some pies. She gave 8 pies to her neighbor. How many pies did Mrs. Kelsey bake if she has 4 pies left?
Jill had some sandwiches and gave 19 of them away. How many sandwiches did Jill begin with if she has

16 sandwiches left?
Katie had some stickers. She gave 14 stickers to Drew. How many stickers did Katie start with if she has 54 left?
Jeremy bought some crayons. He gave his mother 27 crayons. How many crayons did Jeremy start with if he had 50 of them left?
Andy has some videotapes and he sells 29 of them. How many videotapes did Andy start with if he has 17 left?
Toni has some beads. She gives 26 to her friend Zora. How many beads did Toni have in the beginning if she has 45 beads left?
Al had some baseball cards. He sold 14 to another collector. How many cards did Al begin with if he had 23 left?
Mrs. Kennedy had some fruit. She have away 14 pieces to her neighbor. If Mrs. Kennedy had 18 pieces of fruit left, how many pieces did she begin with?

**Part-Part-Whole Problems - Whole Unknown**

There are 12 strawberry popsicles and 23 grape popsicles in the freezer. How many grape and strawberry popsicles are in the freezer altogether?
Todd cooked 16 beef hotdogs and 10 pork hotdogs. How many beef and pork hotdogs did Todd cook?
Mrs. Kelsey has 12 boys and 15 girls in her class. How many students does Mrs. Kelsey have in her class altogether?
Veronica has 23 ham sandwiches and 48 turkey sandwiches in her picnic basket. How many sandwiches does Veronica have?
There are 25 sharks and 15 turtles in the tank. How many sharks and turtles are in the tank altogether?
Katie drew 34 blue squares and 16 red squares. How many red and blue squares did Katie draw altogether?
Katie drew 12 butterfly pictures. She then drew 14 ladybug pictures. How many pictures did Katie draw?
Toni has 21 blue beads and 19 red beads. How many red and blue beads does Toni have altogether?
Al has 67 old baseball cards and 14 new baseball cards. How many baseball cards does he have in all?
Mrs. Kennedy has 14 tall glasses and 41 short glasses. How many glasses does Mrs. Kennedy have in all?

**Part-Part-Whole Problems - Part Unknown**

The store sold 20 popsicles today. 7 were grape and the rest were strawberry. How many popsicles were strawberry?
Todd had 43 hotdogs. Twelve of them were pork and the rest were beef. How many were beef?
Mrs. Kelsey baked pecan pies and key lime pies. She baked 23 pies in all, and 12 of them were pecan. How many pies were key lime?
Veronica made 36 sandwiches. 19 were turkey and the rest were roast beef. How many were roast beef?
Katie drew 23 pictures. 11 are colored blue. The rest are colored red. How many of Katie's pictures are colored red?
Jeremy bought 56 crayons. 23 were green and the rest were yellow. How many were yellow?
Andy bought 28 videotapes. 12 were cartoons and the rest were westerns. How many were westerns?
Toni has 42 beads. 23 are square shaped and the rest are round. How many are round?
Al has 34 t-shirts. 18 of them are white and the rest are red. How many of Al's t-shirts are red?
Mrs. Kennedy baked 13 pies. 4 of them are blueberry and the rest are apple. How many of the pies are

apple?

**Compare Problems - Difference Unknown**

The ice cream shop sold 23 popsicles on Monday and 35 popsicles on Tuesday. How many more popsicles did they sell on Tuesday?
Todd cooks 27 hotdogs. Will cooks 14 hotdogs. How many more hotdogs did Todd cook than Will?
Mrs. Stacey had 19 pies. Ms. Kelsey had 7 pies. How many more pies did Mrs. Stacey have than Ms. Kelsey?
If Veronica has 17 sandwiches and Jill has 11 sandwiches, how many more sandwiches does Veronica have than Jill?
Jason can make 38 cones in five minutes. Erin can make 29 cones in five minutes. How many more cones can Jason make than Erin in five minutes?
Katie had 14 stickers and Drew had 19 stickers. How many more stickers did Drew have than Katie?
Jeremy has 47 crayons. His brother has 28 crayons. How many more crayons does Jeremy have than his brother?
Andy has 38 videotapes. His friend Roger has 27 videotapes. How many more videotapes does Andy have than Roger?
Toni has 21 beads. Her friend Zora has 6 beads. How many more beads does Toni have than Zora?
Lara scored 27 goals this season. Her sister scored 15 goals this season. How many more goals did Lara score than her sister?

**Compare Problems - Compared Set Unknown**

Tony has 12 popsicles. Lara has 8 more popsicles than Tony. How many popsicles does Lara have?
Gwen has 15 skirts. Selina has 17 more skirts than Gwen. How many skirts does Selina have?
Todd has 23 hotdogs. Shane has 14 more hotdogs than Todd. How many hotdogs does Shane have?
Mrs. Kelsey has 23 pies. Mrs. Stacey has 7 more pies than Mrs. Kelsey. How many pies does Mrs. Stacey have?
Veronica has 34 rings. Shana has 16 more rings than Veronica. How many rings does Shana have?
Katie has 68 stickers. Drew has 14 more stickers than Katie. How many stickers does Drew have?

**Compare Problems - Referent Unknown**

101. Loni has 27 popsicles. She has 14 more than Mike. How many popsicles does Mike have?
102. Tony has 83 pencils. He has 17 more than Andrew. How many pencils does Andrew have?
103. Gwen has 65 shirts. She has 28 more than Selina. How many shirts does Selina have?
104. Todd cooked 42 hotdogs. He has 26 more than Shane. How many hotdogs does Shane have?
105. Mrs. Kelsey has 23 pies. This is 9 more than Mrs. Stacey has. How many pies does Mrs. Stacey have?
106. Veronica has 43 rings. This is 34 more than Shana has. How many rings does Shana have?

# Annotated Bibliography for Teachers

"Math Word Problems Demystified" - Word problems are the most difficult part of any math course - and the most important to both the SATs and other standardized tests. This book teaches proven methods for analyzing and solving any type of math word problem.

"Children's Mathematics: Cognitively Guided Instruction" - One of the best things about this book are the cds that comes along with it. They provide a remarkable inside look at students and teachers in real classrooms implementing the teaching and learning strategies described in the text. *Children's Mathematics* was written to help you understand children's intuitive mathematical thinking and use that knowledge to help children learn mathematics with understanding.

"Thinking Mathematically: Integrating Arithmetic and Algebra in Elementary School" - In this book the same author team of "Children's Mathematics" takes teaching and learning mathematics to the next level, revealing how children's developing knowledge of the powerful unifying ideas of mathematics can deepen their understanding of arithmetic and provide a solid foundation for learning algebra. This book also shows how teachers can increase their own knowledge of mathematics in the process of interacting with their children and reflecting about their practice.

"Arithmetic for Parents: A Book for Grownups about Children's Mathematics" - Aharoni illuminates the inner workings of arithmetical algorithms, making the logic clearer even to someone who is well practiced in math. He offers a glimpse into the minds of children, explaining how to break down arithmetic into bite-size pieces appropriate for students who need or want to see the exact process in order to make sense of it.

"Knowing and Teaching Elementary Mathematics" - This book describes the nature and development of the "profound understanding of fundamental mathematics" that elementary teachers need to become accomplished mathematics teachers, and suggests why such teaching knowledge is much more common in China than the United States, despite the fact that Chinese teachers have less formal education than their U.S. counterparts.