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2007 Volume VI: Keeping the Meaning in Mathematics: The Craft of Word Problems

Crafting Word Problems Even a Child Can Do

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Overview

I teach 3rd grade at Wexler-Grant Community School in New Haven, Connecticut. We are a school that begins with Headstart and ends with 8th grade. We have an interesting history. We were two separate schools at one time, Helene W. Grant and Isadore Wexler. Due to the close proximity of both schools and the remodeling of all of the schools in New Haven, we were scheduled to merge, as many schools in New Haven have, and add 6th - 8th grades. Our population of students is approximately 90% African-American and the remaining 10% is made up of White and Hispanic. Many of our students in our school have never left the state of Connecticut. Many of them have never even been out of the city of New Haven. Teaching in New Haven can certainly be tough because some students seem to come to school for reasons other than learning. You certainly have to have a thick skin to work in our school. You also have to bring enough enthusiasm for the whole class as well as yourself.

Math always was a subject that I had difficulty with while growing up. The teacher would present the problem and I just did not get it. I could not "see" how to do the problem. Everything would be a mess of jumbled numbers that I could not put into the appropriate places. At the same time, I didn't understand why we were doing what we were doing in math. To put it more clearly, I would try to follow the "rules" but I didn't know who made up the "rules" and I didn't understand how and why they worked. For example, if we were learning how to add two digit numbers and regroup, the teacher would typically say, "27 + 19, we start in the one's place and add 7 + 9 which is 16. We then carry the 1 and put it in the ten's place on top of the 2 and then add 1 + 2 + 1 to get a sum of 46." I'm left scratching my head. I carry the 1? Why? What does this all mean? What's going on with the 1 that I have to put it over the 2? I was always told, "That's the rules. Follow the rules and you'll always get the right answer." I learned to follow the "rules," but I never understood how or why the "rules" worked. I realize now that I did not have a good number sense. As I got older and more mature and had a better understanding of numbers, it began to make sense. It actually wasn't until I began the accelerated teachers Masters Program at the University of New Haven that my number sense came in focus. My class in elementary math with Dr. Shirley Wakin was truly an eye-opening experience. Dr. Wakin taught us that there were many different ways to solve math problems. She exposed the class to manipulatives and visual aids that helped me "see" how numbers work and fit together. I realized that math is the science of patterns. Though math has language that tells the reader what must be done, math is also a language unto itself. It is a language that I am trying to become fluent in so that my students will become fluent in turn.

Teaching math and having to explain to my students new math concepts has developed my number sense even more.

In the past, our school was recognized as one of the "good" elementary schools in New Haven. This applied to everything, including math. In 1999 we struck out on our own (against the Math department's wishes) and began a math program that was different from the district's. The New Haven District uses the Saxon math program for Kindergarten through 4th grade. Our school decided to use a program called Investigations developed at TERC (formerly Technical Education Research Centers). At the time, our school was smaller, we only went to 5th grade and we had a dynamic staff that was handpicked by the administration and the staff collaboratively. Investigations seemed to be a breath of fresh air because it presented a whole new way to teach math to our students. Investigations had four major goals. It wanted to offer students meaningful mathematical problems and emphasize depth in mathematical thinking rather than superficial exposure to a series of fragmented topics. Investigations also wanted to communicate mathematics content and pedagogy to teachers and to substantially expand the pool of mathematically literate students. For someone like me, this was better than the invention of sliced bread. The features of Investigations were new and innovative. It was extremely different from Saxon math which focused on learning math from lots of repetition. Students would sit at their desks and the class would do side A together. Side B would be done for homework. Saxon would add new concepts while still providing problems from previous lessons. Although this is a highly effective pedagogical device, there was no creativity from the children to develop their own ways to solve mathematical problems. Investigations provided a curriculum that would ensure that all students are included in mathematics learning. Investigations follows a model of cooperative learning where students move around the classroom exploring math in their environment and talk with their classmates. The students work in groups or in pairs where they try to find various solutions to problems. Students are encouraged to invent their own strategies and approaches to problems rather than depending on memorized procedures. Finally the students express their mathematical thinking through drawing, writing, and discussion.

Investigations also has some very good math games that the students can play to help them develop a good number sense. Though this program seemed like a dream come true, it had its drawbacks. We were always told that Investigations was meant to be a supplemental program, not a stand-alone program. Investigations requires a lot of reading beforehand and a lot of copying for the various games and activities. It required a lot of time to prepare for the lessons. Due to various circumstances such as teacher turnover, loss of supplies, and lack of training, Investigations has not fully met the needs of our students. Many teachers now just rely on the math mini-workbooks that are given to us from the Math department. Many of our staff have watched the math scores start off well and consistently drop as the year progresses. We have met and discussed the results and have attributed it to the fact that we do not have a solid math program in our school.

Most of the resources and teachers' time in New Haven is devoted to literacy. The Reading Department does a great job with providing teachers with training and materials to increase the scores in reading. They push their teachers to become well versed in the various reading strategies so that we can better equip our students with skills to improve their reading. Our students are inundated with various reading strategies to help them become better readers. Those strategies are drilled into our students so that it becomes second nature to them to know the strategies. It doesn't mean that our students always *use* the strategies, but they are at least familiar with them. This unit will take those same reading strategies and apply them to math. Those strategies are making connections with the text. This includes making connections to yourself, making connections to the world, and making connections to other texts. Another strategy is asking questions about what you are reading. Visualizing is a strategy that allows you to "see" the picture the author is trying to paint by imagining in your mind or sketching on paper what is happening in the story. Making predictions allows you to think about what will happen next. Determining what is important is a multi-faceted strategy in that the purpose for reading means that there are number of variables that must be taken into account such as are you reading to learn new information? What is important versus what is interesting? Also, is the author trying to entertain, inform or persuade?(1) Students also have to distinguish in a story the difference between major events and details. They also have to infer which is reading between the lines and possibly determine the author's intent or purpose. The last two strategies are synthesizing and metacognitive monitoring. Synthesizing is taking information you have read that is new and combining it with information that you already know to create something new.(2) Metacognitive monitoring is the monitoring of one's own thought process that they go through based on one's current thoughts and knowledge.(3) Though these strategies have been identified as reading strategies, they are excellent strategies that can be used in all areas of study, especially math. For the purpose of this unit, metacognitive will not be explored further at this time. The emphasis for the final strategy will be synthesis.

Rationale

Many teachers in their eagerness to help kids solve word problems teach them "tricks" to help them figure out what to do. I know that I have certainly done it. We teach them to look for key words that will tell them if they have to add, subtract, multiply, or divide. All together or sum means to add, take away or difference means to subtract. Unfortunately, we are teaching our students not to read the problem, not to imagine the situation, not to think. This method teaches them that they do not have to read the problem or that they don't have to think. All they have to do is use the numbers and compute. This method discourages kids from thinking.

This unit will take strategies that kids are familiar with from reading and apply them to math. Where did such a new and novel idea come from? Mathematics Professor Arthur Hyde of National-Louis University has written an outstanding book titled *Comprehending Math: Adapting Reading Strategies to Teach Mathematics.* The book goes into great detail how to apply reading strategies to math. Here is an example of using a common reading strategy to solve math word problems. My students will use a revised version of a KWL chart which is: What do you **K**now? What do you **W**ant to learn? What did you **L**earn? Instead of KWL, they will use a KWC chart to help them solve math problems. KWC stands for: What do I **K**now for sure? What do I **W**ant to find out? Are there any special **C**onditions, rules or tricks I have to watch out for? This strategy still utilizes the tricks that teachers teach their kids, but the additional two elements helps them to think and visualize the problem.

My goal with this unit is to help my students understand that they have every tool they need to solve math word problems: their brain. They are creative enough to discover different ways that they understand to help them solve word problems. There are strategies and skills that I can teach to help speed up their learning. They can use the same reading strategies used for reading and apply them to solving word problems. These strategies will help my students *understand the problem*. When the unit is finished, I will have a classroom of little mathematicians, and hopefully, they will develop a love of math and have a strong number sense to carry them through high school and beyond. We'll first take a look at some of the reasons that children have difficulty with word problems.

The Problem with Word Problems

A train leaves Santa Fe, New Mexico heading east at 8:00 am. It is traveling at a speed of 70 mph. Another train leaves Wilmington, Delaware at the same time but it is heading west. The train from Delaware is traveling at 60 mph. What time will both trains meet? Most adults at this time will start rolling their eyes or their eyes begin to glaze over as they stare off into space. The next words that utter from their lips usually are, "I hate word problems!" They don't usually say, "Ooh, this sounds like a cool problem! Let me get a pencil and paper and figure this out. This sounds like fun!" Somewhere along the way, word problems have become associated with pain and agony. This reaction is very different for a 1st grader as they begin their foray into the world of word problems. Using various types of manipulatives such as bears, cubes, popsicle sticks, etc., 1st graders have a wide-eyed wonderment as they seek to solve the problem. The book *Children's Mathematics: Cognitively Guided Instruction* by T. Carpenter, et al provides a wonderful diagram as to what encompasses addition and subtraction problems. The following explanation comes directly from this excellent book. The purpose is not for the terms to be used to teach to your students, but to give you the teacher a better insight into the construction of word problems. Addition and subtraction fall into four categories of problems. Those categories are *Join, Separate, Part-Part-Whole and Compare*.(4) These categories help educators identify the different type of word problems that students will encounter. When teachers are better able to identify the type of word problem that they are presenting to their student, the more beneficial to the student as the teacher builds and scaffolds their learning. This also beneficial to the teacher because they can make sure that their students see all the different types, so they will be able to deal with any addition and subtraction problem. Join problems are problems in which a set is increased by a specific amount. The set is increased due to a direct or implied action. In simpler terms, this represents an addition problem. However, sometimes the total is given and one of the summands is asked for. Then it must be found by subtraction. The following example is an example of a Join addition problem:

> 5 birds were sitting on a fence. 4 more birds flew on to the fence. How many birds were on the fence then?

The action in this problem takes place over time. There is a starting amount at Time 1 (5 birds sitting on the fence); a second (or change) amount is joined to the initial amount at Time 2 (4 more birds flew on to the fence); the result is a final amount at Time 3 (the 9 birds then on the fence). T. Carpenter, et al finds that this type of problem spawns three distinct sub-categories. The reason for this is although the resulting set of birds is composed of the birds initially sitting on the fence, another group of birds joined the first group of birds on the fence. The two sets of birds take on very different roles in the problem due to the action which happens in a sequence of time. What does this mean? Well, here is where Carpenter, et al, takes Join problems as well as Separate problems (which we will discuss next) and identifies the sub-categories. The sub-categories are Result Unknown, Change Unknown, and Start Unknown. This is important because children may not realize that 4 birds joining 5 birds is the same as 5 birds joining 4 birds. Let's take a word problem and manipulate the sequence that the numbers are given to illustrate the sub-categories of Join and Separate.

Result Unknown: *Tanya has 2 dolls. Her parents give her 4 more dolls for her birthday. How many dolls did she have then?*

Change Unknown: *Tanya had 2 dolls. Her parents gave her some more dolls for her birthday. Then she had 6 dolls. How many dolls did Tanya's parents give her for her birthday?*

Start Unknown: *Tanya had some dolls. Her parents gave her 4 more dolls for her birthday. Then she had 6 dolls. How many dolls did Tanya have before her birthday?*

Separate problems are similar to Join problems in many ways. Like Join problems, there is an action that takes place over time, but with Separate problems, the initial quantity is decreased rather than increased. There can also be three distinct sub-categories for Separate problems like Join problems. They also have a starting quantity, a change quantity (this is the amount that is removed), and the result (difference). Examples of the different sub-categories that fall under Separate problems are as follows:

Result Unknown: *Valerie had 12 marbles. She gave 7 away to Nancy. How many marbles does Valerie have left?*

Change Unknown: *Valerie had 12 marbles. She gave some to Nancy. Now she has 5 marbles left. How many marbles did Valerie give Nancy?*

Start Unknown: *Valerie had some marbles. She gave 7 to Nancy. Now she has 5 marbles left. How many marbles did Valerie start with?*

The next type of problems that Carpenter, et al have identified are the Part-Part-Whole problems. These problems are different from Join and Separate problems because there is no change over time. There is also no direct or implied action. This means that because one set is not being joined to the other, both sets assume equal roles in the problem. "Part-Part-Whole problems involve static relationships among its two disjoint subsets."(5) Due to the fact that Part-Part-Whole problems are static, there are only two types of sub-categories. The problem will either give the two parts and ask students to find the size of the whole, or, it gives one of the parts and the whole and asks students to find the size of the other part. These two sub-categories can be labeled as Whole Unknown, or Part Unknown. An example of a Whole Unknown is as follows:

Whole Unknown - *3 boys and 5 girls were playing basketball. How many children were playing basketball?*

In this example, we know the two parts (3 boys and 5 girls playing basketball). What we don't know is how many children (boys and girls) were playing basketball all together (the Whole Unknown). Next we will examine the Part Unknown word problem.

Part Unknown - *8 children were playing basketball. 3 were boys and the rest were girls. How many girls were playing basketball?*

As we look at the Part Unknown example we know one of the parts (3 boys playing basketball). We also know the Whole (8 boys and girls all together). What we don't know is the 2nd part (the number of girls playing basketball).

The final type of problem that Carpenter, et al, have distinguished is the class of Compare problems. Compare problems entail relationships between amounts rather than a joining (addition) or separating (subtraction) action. This is similar to Part-Part-Whole problems. Where they both differ is that Compare problems involve comparing two different sets rather than the relationship between a set and its subsets. Given that one set is compared to another set, one set is given a label of the referent set, and the other set is called the compared set. The third component of a compare problem is called the difference. This is the amount by which one set is greater than the other set. Carpenter, et al, does a good job illustrating this concept by using clear examples. Here is an example that illustrates the different elements of Compare problems.

- Nancy has 2 cats. - *Referent set*
- Karli has 6 cats. - *Compared set*
- Karli has 4 more cats than Karli. - *Difference*

With Compare problems, any one of the three components, the difference, the referent set or the compared set, can be the unknown. Using the same problem, let's examine how it is used in a word problem.

- Difference Unknown - *Nancy has 2 cats. Karli has 6 cats. Karli has how many more cats than Nancy?*
- Compared Set - *Nancy has 2 cats. Karli has 4 more cats than Nancy. How many cats does Karli have?*
- Referent Set - *Karli has 6 cats. She has 4 more than Nancy. How many cats does Nancy have?*

Each problem is slightly different but clearly shows the distinctions that Carpenter, et al, have identified as sub-categories of Compared problems.

We will next look at how these distinctions among problems translate into number sentences. Number sentences can be used to represent the Join and Separate problems.

Number sentences are another way to represent the subtleties and differences in certain problem types. Join and Separate problems are the easiest type of problems to be represented by number sentences as seen in *Table 1*

(table 07.06.02.01 available in print form)

Table 1

When teachers are making word problems or even selecting word problems for their children to solve, they should look to make the language as clear as possible. This can be done by making the action or relationship as clear as possible. When working with kindergartners or 1st graders, it is important to make sure that the units are consistent. What do I mean by this? Take this simple word problem:

> Moses has 2 apples. Tyler has 3 oranges. How many fruit do the boys have all together?

As an adult the answer may seem obvious. To a young learner this may be confusing to sort in their mind the fact that each boy has different fruit. The fruit becomes the unit. Combining 2 apples and 3 oranges may be a very foreign concept because they look different, they smell different, they taste different and it may not seem to make sense in the mind of a very young person why they should put them together. The old saying of "comparing apples to oranges" comes to mind. The students also have to make the connection that although apples and oranges are different, they are both fruit. This type of problem involves an implicit agreement to ignore the differences.(6)It can be so easy to see all the ways that apples and oranges are different that a child misses how they are alike. This combining of two very different items can be very perplexing to a six or seven year old because they see each set as two very dissimilar things.

The less abstract a problem is the easier it is for children to make a one to one correspondence. It is important that problems represent easily identifiable, discrete sets of objects. Compare these two sets of problems:

- Angel has a puppy that weighed 3 pounds when her mom brought her home. The puppy now weighs 9 pounds. How many pounds has the puppy gained?
- Paula has 2 stickers to start her collection. She needs 10 to complete the page. How many more stickers does Paula need?

The pounds of puppy are not clearly identifiable objects or "things" that a child can see and touch. Using counters like bears or cubes to represent the puppy's pounds is an abstract representation that may be hard for young children to grasp. Weight is a continuous variable. It takes time to learn to segment it.(7) The second problem deals with items that a young student can easily identify with. It is much easier to use counters to represent something more tangible like stickers.

We will next look at how teachers can use reading strategies to help students with their math. Although the reading strategies are gong to be explored individually, it is important to know that the reading strategies interact and work in conjunction with one another and are not independent of each other. The focus is going to be using a KWC chart to help children solve word problems. The KWC will work in conjunction with the reading strategies to give students an excellent tool to use to solve any word problem.

Asking Questions

When children are presented with a word problem, one of the first reading strategies that they need to employ is to ask questions. One of the first questions to utter from the lips of many students is, "What do I do?" This should not be a question for the teacher, but a question to ask of themselves. Students have learned many defense mechanisms to help them avoid doing the actual work presented before them. By asking the question, "what do I do?" they hope the teacher will come over and explain to them what to do or better yet, the teacher will do the problem for them. Arthur Hyde believes that as students progress in their math education, that as teachers emphasize the one correct answer that is obtained quickly implies to students that if they ask a question, it is a sign that that they do not know. Hyde also believes that students use much of their energy covering up when they do not know an answer or how to solve a problem. It is important for teachers to encourage their students to ask questions in math class. Students should know that mistakes are a natural part of the learning process. The classroom should foster an environment of acceptance where no one laughs or makes fun of another student for asking a "dumb" question or getting an answer wrong. Research in the area of asking questions shows that children who ask questions while working on a task helps in developing their metacognitive awareness by monitoring their own thinking. When students have designed their own study and collected data, or if they have created their own word problems, they are especially good at answering the who, what, when, where, and why. Using the KWC can be an excellent tool to help students to start asking questions about problems or studies that they are not familiar with. Hyde believes the basic kind of questions that students should be asking when solving word problems are as follows(8): What are the conditions, limitations, and constraints of the problem? Is there sufficient information to get an answer to the problem? Is there one answer, more than one answer, or no answer? Are there different ways to represent a problem? Is what I am doing making sense? What have I done so far and where have I been already? In other words, am I making progress with the problem? Finally, are my answers reasonable? As children ask questions about the word problems in front of them, the next step is making a connection to the text in the word problems.

As the child uses the KWC chart, it forces them to ask themselves questions. What do I know about this problem? Next comes, what do I want to find out? These two questions can easily lead to other questions to help the child better understand the problem. As the child analyzes the problem and looks for any special rules or conditions to consider, even more questions can arise to even further help the child with their understanding of the word problem. As the child is asking questions about the problem at hand, they will usually begin making connections to the problem. This represents the final step as a child progresses in their mathematical career. For the purpose of introducing word problems to students, their ultimate question should be do I need to add or subtract?

Making Connections

One key to understanding what you are reading regardless of whether it is a story in a book or a word problem, it is important to make a connection to the words they are reading. Hyde believes that people of all ages are exceptionally equipped to make connections. I would have to agree with his belief. Hyde talks about that people are always looking for patterns in all that they see, do, or hear. Every experience is usually an attempt to classify or organize things into groups and subgroups. People look to see how things come apart and how things can fit together. Hyde talks about how people look for patterns in tea leaves, ashes, chicken bones, and even how people look for animals, faces, and shapes in the clouds. Humans look to find order and structure in most things. People try to connect in some way to the things around them. Even in math and word problems, it is important to connect in some way to the problem before them. Hyde does a very good job of classifying the different types of connections. As kids begin asking questions using the KWC, it begins to stimulate a student's thinking about the situation or the context of the math problem. Hyde categorizes the connections into three distinct categories. The 1st connection is Math to Self. The 2nd connection is Math to World. The 3rd connection is Math to Math. These directly relate to the reading connections of Text to Self, Text to World, and Text to Text. Let's look at these three types of connections a little bit closer.

The connection of Math to Self connects to a student's prior knowledge and experiences. It allows a connection to a student's preconceptions and misconceptions. As students are using the KWC it allows them to ask questions like: What does this remind me of? It also asks the question, have I ever been in a situation like this before? This strategy helps the student to personalize the word problem so that it gives meaning and pertinence to the student.

The connection of Math to World connects to natural or created structures that a student may have seen or experienced, such as, riding a city bus or visiting New Haven's Green. There is also a connection to events, environment, and media. Examples of these are attending a concert, playing a game in a park, and watching a show on TV. This can lead to questions like: Is this something that I've noticed in social studies or science? It can also lead to questions like: Is this related to things I've seen anywhere on television or the movies?

The last connection that Hyde expounds upon is Math to Math. This simply is connecting math concepts to other math concepts. This can be within and across strands of mathematics as well as contexts and representations. Math to Math is also about connecting related mathematical procedures. Again, using the KWC this type of connection leads to questions of: Where have I seen that idea or concept before? What are some other math ideas or concepts that are similar to this one? Can I use those ideas or concepts to help me with this problem?

Visualization

Visualization is probably the most common strategy that children use to help them solve a word problem. They draw pictures to help them figure out a problem or create charts to help them organize the various bits of information. In my opinion, visualization is the most important strategy that a child can use. It helps them to "see" how the problem might fit into a pattern that can then be solved. Visualization is also the strategy that confuses and confounds kids because if they can't see in their heads how things fit together, they will usually have difficulty solving the problem. Visualization is the forming of a mental image when reading text. Good authors use rich text with vivid imagery in their stories. Word problems should be no different. When word problems are not clear in their wording, it is more difficult for children to visualize what is happening in the problem. Using the KWC can help students make a clear picture in their minds about what is being asked of them based on their asking questions about the word problem and making connections to the word problem.

Students create mental images as they read word problems and they also create representations of those mental images.(9) Hyde believes that it is important for students to create sensory images or use mental imagery whenever they read math textbooks or story problems. Teachers can assist in this area by preparing specific passages, sentences, expressions, or words that are going to encourage their students to visualize the ideas in the text of the problem. As the student is preparing to fill in their KWC, they should read each sentence and try to envision what is going on in the problem. When students attack a word problem, they should visualize the story problem as a movie in their head to help them understand what is going on. The more elaborate the visualization that a child can create for mathematical concepts, the easier it will be for them to use those images to solve a word problem.

One thing to be careful about with visualization is to make sure that children don't think or believe that they have to have a rich detailed image of the situation in their mind to solve a word problem. This can be good if it helps the child connect to the problem, but it also may slow down the process of focusing on the issue of: what to do we need to know to answer the question? A problem with older students is that they have difficulty separating relevant from irrelevant information. This step in the process could aggravate or perpetuate that tendency if it is not done well. As visualizing is being talked through with the kids, it is important to ask the question: Do we need to know this to solve the problem?(10)

Inferring and Predicting

Inferring and predicting in mathematics can be a bit tricky. It is different from a traditional story or poem because the text can be rich and flowing and evoke strong images and emotion and passion. Examining a word problem for literal clues as well as inferential clues, can be troublesome for students and requires a lot of practice. Problems that use wording like "week" or "dozen" presume that children know that a week is 7 days and a dozen is twelve. Using a KWC chart helps a child to make those inferences as they are filling in the K of What do I Know? This also allows for the C to come into play. Are there any special Conditions, rules, or tricks I have to watch out for? The C also allows the student to reflect on their notes and determine what is fact and what is inference. Not to be confused, a correct inference should be a fact; it is just not implicitly stated in the word problem. Mathematics is based on this!(11)

Predicting allows the student to make an educated guess, an estimate, as to what the answer may be while incorporating all of the other strategies to solidify what the actual answer is. Students predict when they are asked to determine what shape or number comes next in a pattern. They quantify their prediction using various strategies to help them figure out what comes next. It is important to work with your students to help them build the habit of making predictions using evidence from the story rather than pulling them out of the thin air. It takes a lot of practice for the students to examine a story for literal and inferential clues and to use those clues to make a good prediction. This applies to reading as well as word problems.

Determining Importance

Determining what is important is a critical step in solving word problems. Students are given a variety of information within a word problem that has to be sorted and organized and put into a context that the student can understand. Sometimes, word problems have extraneous information to see if the student really understands what is being asked of them. Students need to decide what is important on several different levels. When reading a word problem they have to consider the text as a whole, each sentence, and even each word.(12) It is important for teachers to help students distinguish irrelevant and interesting information from what is important.

Most word problems follow a three-part format. There is the context of the problem, the information within the problem, followed by the question. The KWC can be very helpful in addressing the three components of a word problem format. When the K (What do I Know for sure?) is being filled out by the student, it helps to lay out the context of the word problem as well as the mathematical information. When the W (What do I want to find out?) is being filled out, children are clearly identifying the second and third components of a word problem. A student working on a word problem cannot figure out what is important amongst the various pieces of information in a word problem until he or she has read the question (done the W) and has thought about what the problem is all about. If a student is doing the KWC carefully, he or she should have a listing of what they know for sure. The student should have thought about and considered what are facts and what are inferences. He or she should also have thought about the question being asked, What do I want to find out? Finally, he or she should have considered the special conditions or constraints.

Synthesizing

As stated earlier, synthesis is the combining of new information with existing information to create something new. Arthur Hyde explains that when new information is encountered, we try to relate that new information to our existing schemata. Sometimes that new information identifies with current thoughts and beliefs. Other times, when that new information does not fit into a student's current schemata of how information is stored and classified, it will form its own new schemata.

Once the KWC is completed, the act of synthesizing all of the information to solve the problem begins. It is believed that synthesis is the most challenging of the comprehension strategies. Synthesis can be seen on a linear scale. At the beginning of the scale is the taking stock of what you reading. Stopping every so often to digest what has been read to make sure you understand what is happening. In the middle of the scale is where you have a better understanding of how to summarize, retell, and even recreate what has been read. At the end of the scale is where authentic synthesis takes place. It is where new thoughts and perspectives are created from the reading. It is through synthesis that students are able to selectively receive a plethora of pieces of information, sensory stimuli, and impressions bombarding them and extract only those to be connected to an idea that they deem important. It is thought that it may even be done only at the subconscious level.(13) Students organize the various pieces to create a tapestry, a portrayal of their new schemata.

The KWC has the ability to incorporate all of the various reading strategies to allow synthesis to take place. The KWC is not the sole method of helping kids solve word problems. Students should be exposed to various different methodologies and strategies. The ultimate goal for this entire process is to help students synthesize the data to achieve a desired result. The KWC is one tool that can provide a good foundation for achieving synthesis. Students who diligently practice the KWC should have a good basis and be better prepared to synthesize word problems successfully.

Let's look at some sample lessons to see how the KWC can be used effectively to solve word problems. These lessons will build upon one another and will be used for approximately 2 weeks per lesson. The third lesson is an assessment lesson to get an idea if the KWC is working.

Lessons

Lesson 1

Purpose: To introduce the students to the KWC chart as a tool to help students solve a variety of word problems. The students will be able to read a word problem and use the various reading strategies to complete a KWC chart

Materials: Chart paper, markers, manipulatives (counters, cubes, etc.) copies of the KWC chart (see Appendix A)

Procedure: The lesson will begin by talking about how a KWL chart works. The concept of the KWC will be written out on chart paper and the students will look and see how the KWL and the KWC are similar and how they are different. For the first few lessons, only the K and the W will be used. The class will use very simple addition and subtraction word problems. It will be done as a whole class activity. Each sentence in the problem will be read and analyzed and each reading strategy will be explored. Here is an example of how a lesson may go.

Jill had 8 Bratz dolls. She received 4 more Bratz dolls for her birthday. How many Bratz dolls does Jill have altogether?

We start by asking questions: Seeing as how Jill had 8 Bratz dolls before her birthday, how old do you think Jill is? Do you think that she likes Bratz dolls? Does she like Barbie dolls also?

We then move to making connections with the word problem. The teacher may ask questions like: How many of you have Bratz dolls? Does anyone in your family have any Bratz dolls? Have any of you ever received a present for your birthday? Have you seen the Bratz television show on Saturday mornings? What are the ages of girls who like Bratz dolls? Do boys play with Bratz dolls?

Next, we move on to visualization. The question can be asked: Can you see the Bratz dolls Sasha, Yasmine, Chloe, or Jade in your mind? Can you draw one of them on your paper?

Now we move on to the more tricky strategy, inference and predicting. Ask questions like: Do you think Jill is happy? Why do you think that she is happy? Do you think that Bratz dolls are her favorite? Why do you think so? Do you think that Jill will want more Bratz dolls for Christmas? Do you think Jill will want accessories to go with her Bratz dolls?

Now, determining what is important for answering the question. To help students figure out what is important, let's eliminate those things which are not important, such as: Is it important to know how old Jill is? Is it important to know when her birthday is? Now what is important? That she had 8 Bratz dolls. She received 4 more Bratz dolls. The other information might be important, but for other purposes.

Finally, let's synthesize this information. What is being asked of you? Do we need to add to answer the question or do we need to subtract to answer the question? After having this discussion about the word problem, we then go back to the KWC and begin to fill in the various components. Starting with the K the teacher writes on the chart paper what do we know for sure? We know that Jill had a birthday. We know that before her birthday she had 8 Bratz dolls. We also know that she received 4 more Bratz dolls for her birthday. Next, we move to the W and ask the question, what are we trying to find out? The teacher writes on the chart paper we are trying to find out how many Bratz dolls she has altogether. Then allow the students to solve the problem by doing the bottom portion of the KWC. Show how you solved the problem using pictures, numbers, and words. I will then have the kids share how they solved the problem.

Lesson 2

Purpose: To introduce the C in the KWC using problems that increase in difficulty. The students will be able to fill out all sections of the KWC chart. This will be probably be done approximately 2-3 weeks after the introduction of the KWC.

Materials: Chart paper, markers, manipulatives (counters, cubes, etc.) copies of the KWC chart (see Appendix A)

Procedure: At this point the students should be more comfortable in running through the various reading strategies to help them complete a KWC chart as well as solve simple word problems. Here is an example of a more difficult word problem that utilizes the C in the KWC.

Trish lives on a farm and gathers 3 eggs from the henhouse every day. How many eggs does Trish gather in a week?

Although this can be a multiplication problem, it will be done as a successive addition problem. This is a great way to prepare the students for multiplication. We then start with questions such as: Is this a chicken farm? Do you think there are other animals on the farm? How many chickens do you think she has?

Next, what connections can you make to this word problem? Have you ever been on a farm before? Have you read about farms or seen them on television? Do you know anyone who has lived or lives currently on a farm? Have you eaten eggs before?

Then, visualize what you see in your mind. Can you see the farm? Maybe it has a big red barn. Can you see the henhouse? Can you see Trish with a basket in her hand walking to the henhouse to gather the eggs?

Next we begin to inference and predict. Trish lives in a rural area. She may not have neighbors who live close by. She probably has other chores that she has to do. A week is 7 days. She will probably have to keep gathering eggs until the chore is passed on to someone else or she leaves home. Each inference should be evaluated for relevance to the problem.

Now let's determine what is important for answering the question. It is important to also recognize that sometimes it is preferable to work backward from the question. What are we being asked? What do we need to know to answer the question? What information in the problem can help us? It is important to provide students with a variety of ways to problem solve. Trish gathers 3 eggs every day. Trish gathers these eggs every day for a week. A week is 7 days. Trish gathers 3 eggs every day for 7 days.

Finally, let's synthesize the information in the word problem. We have to find out how many eggs Trish will gather in one week. We know that she gathers 3 eggs a day for 7 days. Are we going to have to add or subtract? Some kids may know that you can multiply to get the correct answer. Have them show both ways in their work so that they can make the connection that successive addition is another way to multiply.

Now let's fill in the KWC with all of the information that we have compiled. The K, What do we know for sure? We know that Trish gathers 3 eggs everyday. We know that she gathered those eggs for one week. Next, W, What are we trying to find out? We are trying to find out how many eggs Trish will gather in one week. Finally, the C, Are there any special conditions? One week is 7 days. The students can write in the C or in the show your work section of the chart the heart of the question of Trish gathered 3 eggs each day for 7 days.

Again, like the previous lesson, allow the students to solve the problem by doing the bottom portion of the KWC. Show how you solved the problem using pictures, numbers, and words. I will then have the kids share how they solved the problem.

Lesson 3

Purpose: To introduce the students to a new worksheet that can be used also with a KWC, but gives them practice with determining what is being asked of them by having them create the question for the word problem.

Materials: Chart paper, markers, manipulatives (counters, cubes, etc.) copies of the KWC chart (see Appendix A), copies of the Create Your Own Question worksheet (see Appendix B)

Procedure: The Create Your Own Question worksheet is help for my students to practice the skill of determining what is being asked of you. This can also be used as an assessment to determine if the students in my class understand the implications within a word problem. By having all of the details except the final question, this will give me a good insight into my students' understanding. The student will have to use all of the data within the word problem to create a question. The creation of their question shows whether they understand what kind of word problem that they are working with. Is it addition, subtraction, multiplication, or division? The lessons in the beginning will just be addition or subtraction. As the year progresses, multiplication and division will be added. Once students have created their questions, they have to solve them and show their work. More importantly, the students have to explain why their questions good ones for the story. Their explanations gives you the greatest insight into their thinking. Appendix B has a sample problem. You can either type the problem in before-hand or leave that area blank and the students can copy the problem from the board or overhead. The KWC can be used to help the student organize their thoughts and walk them through the reading strategy process.

Notes

1. Comprehending Math, A. Hyde, pg. 126

2. Comprehending Math, A. Hyde, pg. 151

3. http://facultyweb.cortland.edu/ andersmd/metacog.html

4. Children's Mathematics: Cognitively Guided Instruction, T. Carpenter, et al. pg 7

5. Children's Mathematics: Cognitively Guided Instruction, T. Carpenter, et al. pg 8

6. Roger Howe - Yale Seminar 7/2/07

7. Roger Howe - Yale Seminar 7/11/07

8. Comprehending Math, A. Hyde, pg. 38

9. Comprehending Math, A. Hyde, pg. 38

10. Roger Howe - Yale Seminar 8/5/07

11. Roger Howe - Yale Seminar 8/5/07

12. Comprehending Math, A. Hyde, pg. 126

13. Comprehending Math, A. Hyde, pg. 152

Appendix A

K W C

(table 07.06.02.02 available in print form)

Appendix B

Read the following story. Decide what problem-solving question YOU would come up with, based on the details within the story. Your question has to make sense and be solvable.

1. Mrs. Howe's daughter is coming home from Yale University tomorrow. Mrs. Howe is so glad that she bursts into tears at the very thought of it! Every day for a whole week, she used 6 tissues per day. The tissue box had 50 tissues in it.

(Create your question:) ____

____

____

Explain WHY you think this is a good question for this story.

____

____

____

____

Solve your own problem: ____

____

Show your work here.

Appendix C

New Haven Math Standards

The math standards for New Haven that will be followed are as follows:

Content Standard 5.0 - Problem Solving and Mathematical Reasoning:

Problem solving concepts and strategies lie at the heart of mathematics. Students will utilize them in the formulation of problems and the solution of problems. They will appropriately test problem conclusions against conditions. They will reason mathematically.

Content Standard 5.0 - Performance Standard 5.1

Students will solve problems that make significant demands in one or more of these aspects of the solution process: problem formulation, problem implementation, and problem conclusion.

Content Standard 5.0 - Performance Standard 5.2 (Problem formulation)

Students will participate in the formulation of problems.

- a. Students will make decisions about the approach, materials, and strategies to use.
- b. Students will use previously learned strategies, skills, knowledge, and concepts to make decisions.
- c. Students will use strategies, such as using manipulatives or drawing sketches, to model problems.
- d. Students will use charts, manipulatives and problem steps to explain and illustrate information in detail.
- e. Students will extract pertinent information from situations and figure out what additional information is needed.

Content Standard 5.0 - Performance Standard 5.3 (Problem implementation)

Students will make the basic choices involved in planning and carrying out a solution.

- a. Students will make up and use a variety of strategies and approaches to solve problems and learn approaches that other people use.
- b. Students will make connections among concepts in order to solve problems.
- c. Students will solve problems in ways that make sense and explain why these ways make sense, e.g. defend the reasoning, explain the solution.
- d. Students will write problems to a given number sentence.

Content Standard 5.0 - Performance Standard 5.4 (Problem conclusion)

Students will move beyond a particular problem by making connections, extensions, and/or generalizations.

- a. Students will explain a pattern that can be used in similar situations.
- b. Students will explain how the problem is similar to other problems they have solved.
- c. Students will explain how the mathematics used in the problem is like other concepts in mathematics.
- d. Students will explain how the problem solution can be applied to other school subjects and in real world situations.
- e. Students will make the solution into a general rule that applies to other circumstances.

Appendix D

Here is a collection of word problems that can be used to teach this unit.

5 birds were sitting on a fence. 4 more birds flew on to the fence. How many birds were on the fence then?

Tanya has 2 dolls. Her parents give her 4 more dolls for her birthday. How many dolls did she have then?

Tanya had 2 dolls. Her parents gave her some more dolls for her birthday. Then she had 6 dolls. How many *dolls did Tanya's parents give her for her birthday?*

Tanya had some dolls. Her parents gave her 4 more dolls for her birthday. Then she had 6 dolls. How many dolls did Tanya have before her birthday?

Valerie had 12 marbles. She gave 7 away to Nancy. How many marbles does Valerie have left?

Valerie had 12 marbles. She gave some to Nancy. Now she has 5 marbles left. How many marbles did Valerie give Nancy?

Valerie had some marbles. She gave 7 to Nancy. Now she has 5 marbles left. How many marbles did Valerie start with?

3 boys and 5 girls were playing basketball. How many children were playing basketball?

8 children were playing basketball. 3 were boys and the rest were girls. How many girls were playing basketball?

Nancy has 2 cats. Karli has 6 cats. Karli has how many more cats than Nancy?

Nancy has 2 cats. Karli has 4 more cats than Nancy. How many cats does Karli have?

Karli has 6 cats. She has 4 more than Nancy. How many cats does Nancy have?

Moses has 2 apples. Tyler has 3 oranges. How many fruit do the boys have all together?

Angel has a puppy that weighed 3 pounds when her mom brought her home. The puppy now weighs 9 pounds. How many pounds has the puppy gained?

Paula has 2 stickers to start her collection. She needs 10 to complete the page. How many more stickers does Paula need?

Jill had 8 Bratz dolls. She received 4 more Bratz dolls for her birthday. How many Bratz dolls does Jill have altogether?

Trish lives on a farm and gathers 3 eggs from the henhouse every day. How many eggs does Trish gather in a week?

Bibliography

There are a variety of books that I will use to help prepare me to teach this curriculum unit.

Student Resources

Anno, Mitsumasa. (1995). *Anno's Magic Seeds.* New York: Philomel Books.

A wizard gives Jack two golden seeds and directs him to eat one and bury the other. He promises it will grow and give 2 more magic seeds in the fall. Jack does as he is told, and the cycle repeats for a number of years, until Jack decides to bury both seeds. The tale of exponential growth is discovered as Jack buries more and more seeds. The math tale becomes even more rigorous as Jack marries, has a child, begins to store some seeds and sell others... until a hurricane wipes out the crops and Jack must begin all over again.

Axelrod. Amy. (1994). *Pigs will be Pigs: Fun with Math and Money.* New York: Simon

and Schuster Books for Young Readers.

After gobbling up all the groceries, Mr. Pig, Mrs. Pig and their two piglets are hungry again, but the Piggy bank is empty. The Pigs turn their house upside down looking for spare change so that they can go out to dinner. Readers are meant to keep a tally of the dimes and nickels the Pigs locate. Finally, after finding a grand total of $34.67, the Pigs spend almost all of it at a Mexican restaurant and readers can calculate the tab by reading a menu.

Burns, Marilyn. (1997). *Spaghetti and Meatballs for All: A Mathematical Story.* New

York: Scholastic.

Mrs. and Mr. Comfort are throwing a dinner party for their family. They find out there will be 32 people. They arrange 8 tables with 4 chairs each. The problem arises when the families begin to arrive and want to sit together. They begin pushing the tables together to make one big table. As more and more guests arrive, the families continue to rearrange the tables so everyone can sit together. Of course throughout this time Mrs. Comfort is getting very distraught because she knows that if the tables are pushed together there will not be a seat for everyone. The story continues with more arrangements of the tables until eventually the tables start being pulled apart as the rest of the family arrives. In the end the tables again end up as 8 tables with 4 chairs each, just as Mrs. Comfort had originally set.

Demi. (1997). *One Grain of Rice: A Mathematical Folktale.* New York: Scholastic.

It's the story of Rani, a clever girl who outsmarts a very selfish raja and saves her village. When offered a reward for a good deed, she asks only for one grain of rice, doubled each day for 30 days. That's lots of rice: enough to feed a village for a good long time—and to teach a greedy raja a lesson.

Lewis, J. Patrick. (2002). *Arithmetickle: An Even Number of Odd Riddle-Rhymes.* San

Diego: Harcourt.

This book offers a variety of clever math riddles with titles like "Finger Play" (which teaches a nifty trick for multiplying by nine) and "Your Average Cow," which asks kids to compare bovine and human life expectancies. Answers appear (upside-down) below each entry.

Neuschwander, Cindy. (1999). *Sir Cumference and the Dragon of Pi.* Watertown,

MA: Charlesbridge.

Covers the number pi through a story in which Sir Cumference drinks a magic potion that turns him into a dragon and only the magic number pi can transform him back.

_____(1997). *Sir Cumference and the First Round Table.* Watertown,

MA: Charlesbridge.

Explores the topic of taking a rectangular table, which is too long and difficult for everyone seated to see and hear each other, and forming it into various shapes to find the ideal shape for the knights to sit and meet.

_____(2001). *Sir Cumference and the Great Knight of Angleland.* Watertown,

MA: Charlesbridge.

Radius uses a circular family medallion to discover angles and ultimately the use of a protractor.

_____(2006). *Sir Cumference and the Isle of Immeter.* Watertown,

MA: Charlesbridge.

Covers the topics of area and perimeter.

_____(2003). *Sir Cumference and the Sword in the Cone.* Watertown,

MA: Charlesbridge.

Simulates the story of King Arthur and implements elements of geometry to solve the riddle.

Teacher Resources

Burns, Marilyn. (2000). *About Teaching Mathematics.* 2nd ed. Sausalito, CA: Math

Solutions Publications. A good K-8 book that has good lessons and strategies.

Carpenter, Thomas P., Fennema, Elizabeth, Franke, Megan Loef, Levi, Linda, &

Empson, Susan B. (1999). *Children's Mathematics: Cognitively Guided*

Instruction. Portsmouth, NH: Heinemann. An excellent book that provides a wonderful structure as to what encompasses addition and subtraction problems.

Hyde, Arthur. (2006). *Comprehending Math.* Portsmouth, NH: Heinemann. An excellent book that uses reading strategies to teach math.

Lampert, Magdalene. (2001). *Teaching Problems and the Problems of Teaching.* New

Haven: Yale University Press. This book provides a wonderful insight into the problems of teaching word problems and ways to solve those problems.

Ma, Liping. (1999) *Knowing and Teaching Elementary Mathematics*. Mahwah, NJ:

Lawrence Erlbaum Associates, Inc. A very good insight into how the Chinese teach math and why their students score higher on math tests than other countries.

National Research Council. (2005). *Helping Children Learn Mathematics.* Washington

DC: National Academy Press. A well written short booklet that is an easy read that has good strategies to help children learn math.

Russell, Susan Jo and Economopoulos, Karen (1998) *Investigations: Mathematical*

Thinking at Grade 3.Glenview, IL Scott Foresman. A terrific teachers guide that promotes cooperative learning and fun math games that help with number sense.

Vandewalle, John A. (1994). *Elementary School Mathematics: Teaching*

Developmentally. 2nd ed. New York: Longman. An excellent book that is primarily used as a college text book. It has wonderful strategies and lessons.

(2005) *Singapore Math Series: Workbook and Textbook Primary Mathematics 3A and*

Primary Mathematics 6A. Singapore. Times Media Private Limited. Chinese textbooks that are reprinted for the U.S. that scaffolds mathematical concepts.

http://facultyweb.cortland.edu/ andersmd/metacog.html A website that deals with the reading strategy of metacognition.

https://teachers.yale.edu
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