



Curriculum Units by Fellows of the National Initiative

2007 Volume VI: Keeping the Meaning in Mathematics: The Craft of Word Problems

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## **A = C - B: Subtraction is the Cousin of Addition**

Curriculum Unit 07.06.03, published September 2007

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### **Introduction**

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For the past few years, I have literally stood on my head to teach my fourth grade bilingual students that one is unable to subtract a larger number from a smaller number in the positive number system. Many of my students, when given a problem such as the following, will simply arrange the numbers to fit a pattern and arithmetic operation with which they are comfortable: *Siggy has 54 crayons. His brother Ziggy used some of the crayons for a school art project. Now Siggy has 29 crayons. How many crayons did Ziggy use for his art project?* This is a *Separate* problem where the result is known and the change is unknown, frequently referred to as a *Missing Addend* problem.

The students are unsure of what they are trying to find out. It seems they consider 29 and 54 as interchangeable in terms of minuend and subtrahend. Therefore, some children will make this problem:  $29-54=35$ .

They are having at least two difficulties here. First of all, they do not know how to decompose tens, so they subtract the four ones from the nine ones and get five, and then, realizing perhaps that they can't subtract five tens from two tens, they simply flip the numbers around, subtract two from five and call it a day. It was this baffling (to me) maneuver that inspired me to stand on my head.

Secondly, the children don't understand the idea of missing addends, or *change unknown*. They do not realize that they are working with a problem where the result is known; hence the decision to make one number the minuend and another number the subtrahend, in a sort of "whatever works" spirit. On the positive side, they do understand that the problem is best solved by subtraction, although some students will make the problem  $54+29= 83$ , because *carrying* or composing a ten is an easier operation than decomposing a ten.

For all of the above reasons, I chose this very basic and fundamental mathematical operation of subtraction to be the focus of this curriculum unit. I want to explore the "cousin" relationship between addition and subtraction because it is the foundation for understanding inverse relationships in arithmetic, and because it is a metaphor my students well understand.

Many of my students are Mexican immigrants and their schooling has been uneven and choppy with gaps of time when families are moving around and the children do not attend school. Therefore, there are also gaps in

their mathematical knowledge. It is my desire to fill in those gaps, and subtraction is a good place to start. With a solid background in understanding the process of subtraction, students will be able to move forward into the more complicated operations of multiplication and division. They will attain this understanding by thoroughly exploring the subtraction process and by beginning this unit with an in-depth study of the decimal, i.e. base-ten system.

## Objectives/Strategies

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My overarching objective for this unit is to thoroughly explore the process of subtraction through the study of word problems. Students will gain an understanding of the decimal number system and base-ten operations which will provide them with the tools to be able to *compose* and *decompose* tens in addition and subtraction problems. Thus, they will have an understanding of what is traditionally called *borrowing* and *carrying*, rather than just memorizing the procedures. As well, students will understand subtraction as an inverse process of addition. This unit assumes that, by fourth grade, students have a fairly solid background in addition, so although I include some problems in addition in the unit, it is mainly to strengthen the understanding of the relationship between addition and subtraction. I have emphasized subtraction because typically students have a much more difficult time with subtraction problems than with those of addition. By the end of the unit students will be able to identify and solve various subtraction situations, including take-away, difference, and comparison and part-whole relations, especially missing addend problems.

Because of understanding subtraction as a process of decomposing tens, when a digit in the minuend is smaller than the corresponding digit in the subtrahend, by the end of the unit students will be able to add and subtract across zeros with ease. The difficulty of the numbers will increase as students gain facility working with word problems and using base-ten strategies to solve them. Students will also learn techniques for subtracting mentally by breaking numbers into decimal units. Mental math increases brain agility and helps children understand addition and subtraction processes.

Another goal is for students to be able to identify what it is they are solving for in a word problem, particularly part-whole relations problems, and often comparison problems, where they are looking for a missing addend. Students will be able to identify whether they are solving for one of two parts, or for the whole. More about part-part-whole identification strategies follows in the Rationale section.

In addition to thoroughly knowing the process of subtraction and being able to identify various types of subtraction situations, students will gain skills in formulating mathematical conjectures, illustrating word problems, writing about their solution processes and revising their conjectures. They will improve their abilities to work interactively in small groups and will gain an understanding of group process.

I am writing this unit for fourth graders in a Dual Immersion (English/Spanish) class. As a bilingual teacher, I have frequently encountered difficulty in obtaining word problems in Spanish. Another goal therefore is to provide a body of word problems in both languages so that I have a pool of problems from which to draw and so that other bilingual teachers reading and perhaps adapting this unit for their students will also have access to them. You will find these word problems in an appendix at the end of the unit. Because I am a 50/50 Dual Immersion language teacher, which means I teach alternate days in English and in Spanish, the word problems are not translated, but simply switch back and forth from English to Spanish, so that I may choose

problems in the language I need for any given day. Bilingual teachers may easily cull problems in either language and adapt the unit presentation to suit their individual teaching needs.

The beginning problems are at a third grade level in order to go back and re-establish a cognitive basis for understanding the subtraction process. A few of the problems attain a fifth grade arithmetic achievement level, and some will have two parts so that students of various achievement levels will be engaged. This strategy allows me to encompass the vast array of ability levels I always have in my class, as well as serves to revisit concepts previously learned and stimulate and motivate students to move forward in their understanding of mathematical concepts. Although the unit is written for a dual immersion Spanish/English class, it could be easily adapted for a monolingual English fourth grade class.

My underlying goal for this unit is to promote students' confidence in their own basic arithmetic understanding thereby instilling a confidence in their continuing mathematical abilities throughout their lives.

## Structure

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I am structuring this teaching unit using a template called *Problem of the Day*. I will thoroughly explore and study at least one word problem a day, although I will usually use more than one problem so that students have a number of examples and so that they may practice the operation(s). As well, additional word problems will serve as extension activities. However, the focus of the day will be on one word problem, and the unit presents the material in this manner. It is assumed that teachers will draw additional problems of the same type from the appendix or from their own reservoir of word problems. I have structured the unit in the form of lessons that present mathematical concepts and problem-solving techniques.

I introduce the problem of the day, explore its concepts and procedures and lead, model and encourage inquiry and discussion. When I feel students are ready, they will break into groups to work on the problem or set of problems. Students will make conjectures about their problems in their groups, and later come together whole-class for a reflection time during which time students will discuss their solutions and revise their conjectures if need be. I'll say more about the conjecture process later. Once some clarity is achieved, practice sessions will follow the whole class meeting, and students will sometimes work individually, sometimes in pairs and sometimes back in their groups.

This Problem of the Day format will take up a forty-five minute to a one-hour session, and the unit as a whole will continue for about a month. Most days there will be new word problems; however, sometimes problems may be carried over from the previous day, depending on their difficulty. I include instructional lessons in the unit to introduce, model and cover the mathematical processes. Additional practice and homework are not covered in the unit; however, I will give both and assume that other teachers using the unit will too.

A key component of the instructional guidance will be in teaching students the process of inquiry along with learning the rules governing arithmetic. They will learn how to investigate a problem and be able to discuss their problem solution. I hope that by using a word problem format to teach the process of inquiry, and by group discussion and group work to find solutions, that students will develop the self-confidence to know they are capable of reasoning out and through a math problem.

I plan to use a format for the problem of the day which requires the children to fold a blank piece of paper in

fourths and then reopen it with the folds intact. They will use the first quadrant of the paper to copy the problem of the day from the board. The second quadrant will be used to illustrate the problem; the third to actually solve it and the fourth to write about their thinking processes. Most of the time the children will be working in small groups of three or four; however, each child is responsible for recording her or his own work. I won't always require students to perform all four of these tasks, nor will they always work in groups, but for the most part, this format will serve to organize our classroom study and work.

To begin, students will formulate and record on the back of their papers a *conjecture* or *hypothesis* about the problem under study. These conjectures may take many forms and serve many functions. They can be a way of describing the solution that allows many children to participate in the solution process without feeling intimidated as to whether or not they have obtained the "right" answer. Conjectures can be a collection of descriptions of patterns across a mathematical domain that children have discovered. They can be an agreement about a mathematical rule or algorithm. Children will write down their conjectures, and I or they will write them on the board when we come together for whole class discussion.

Often children will be asked to illustrate their conjectures or solutions on the second quadrant of their paper by making drawings, charts or other problem specific illustrations. For example, students may be asked to write the number 5,432 in expanded form and they might show the numbers in place value columns in the second quadrant of their work page.

A third section of the page will be devoted to "experiments." This is where students put their conjectures or hypotheses to work to try and figure out the problem. I leave lots of room here for different solution strategies. Our first priority is not finding the right answer but finding a solution strategy that students can articulate and defend.

Finally, in the last quadrant of the page, the children write down their thinking or reasoning about the problem. This is often the most difficult work for them, because they have difficulty articulating their mathematical processes. However, I have seen students become more and more adept at this with practice and with class discussion and teacher modeling.

After the children have worked and solved the problem(s) in their groups, their work will be used as a basis for whole-class discussion. The groups report out and conjectures are written on the board or on a flip chart. As mentioned above, here is where we look at a revision process to see which of the conjectures we think worked best, and students decide whether or not they would like to change anything about their conjectures or any facet of their problem solving regimen. Here I do a great deal of modeling of listening to many conjectures. Rather than judging them, I use them for a basis for discussion as to what is reasonable. If necessary, students then revise their conjecture and record it below their original hypothesis. The last part of the class period is used for practice, and homework is often whatever practice problems were not finished in class.

I will assess individual student's progress through my observation and note-taking in class while the children are working in groups, periodic quizzes which will be in the form of the types of word problems we have been studying, individual student explanations and rationales presented in class and assessment of the progress, processes and solutions of the working groups.

## Rationale

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My rationale for emphasizing the decimal number system is based on the work of Liping Ma in her book *Knowing and Teaching Elementary Mathematics*. Ma compared elementary school math teachers in the United States and China and found that the Chinese teachers were more successful in teaching addition and subtraction because they taught the regrouping (borrowing and carrying) as aspects of these operations, as opposed to U.S. teachers who often teach these subjects as procedures and facts. This was the missing link I had been seeking in my years of teaching this subject in elementary school. I knew that simply teaching the procedure of carrying and borrowing by crossing out numbers and moving a "one" to the needed column so that addition or subtraction could take place was not enough information. Clearly, my students did not understand what they were doing; hence many were lost, especially in the mystical (to them) realm of subtraction.

When I began to talk about "tens" and carrying and borrowing them from, for example, the tens column to the hundreds column and vice versa, students began to perk up. I was beginning to help them demystify the process. We used manipulatives bundled in tens and worked simple one-digit addition and subtraction problems by adding to a "tens bundle" or subtracting from it. I was on the right track.

However, it wasn't until reading Liping Ma that I understood the teaching of these "simple" mathematical operations was predicated on children understanding some basic ideas about the base ten system. What follows is taken directly from Ma's account (Ma 1999) of her investigation and her subsequent abovementioned book. I reference the subtraction process because that is the focus of this unit.

One of the key concepts that many Chinese elementary school math teachers teach is that of "decomposing a higher value unit." This phrase describes the "taking" step in the subtraction algorithm. For example, rather than teach students that you borrow one from the ten's place, Chinese teachers use the catchphrase, "you decompose 1 ten." Ma goes on to explain that "in the decimal system numbers are composed according to the rate of ten." Therefore ten units are organized or *composed* into "1 unit of the next higher place value" in addition. In subtraction a unit is decomposed into 10 units of the lower value. (Ma, 1999, p.8). Hence, the fundamental idea of regrouping in subtraction is that of decomposing higher value units into lower value units.

An advantage to teaching addition and subtraction by the process of composing and decomposing higher value units is that subtraction as the inverse operation to addition is implied. And when it is then articulated, students more easily grasp the *unmaking* or *undoing* process of subtraction. Therefore, addition involves a process of making or building tens, and subtraction involves the inverse or opposite process of unmaking or taking apart tens.

As well as teaching the standard algorithm for subtraction by decomposing a higher value unit, many Chinese teachers also teach other ways of regrouping. For example in the problem  $62 - 45$ , 40 can be subtracted from 62 and 5 can be subtracted from 22. Or, the 62 can be regrouped as 50 and 12. Then the 5 can be subtracted from 12, 40 from 50, and we get 17.

Or, we can regroup the 62 as 50, 10, and 2. Then we subtract 5 from 10 and get 5, add it to the 2 and get 7, subtract 40 from 50 and get 10 which we would then add to the 7 to get 17.

Although the standard algorithm is usually the preferred method, offering alternatives to students serves to

reinforce the decomposing process, shows students that there is often more than one way to solve a problem, and helps strengthen mental math processes.

A taxonomy that is helpful in classifying addition and subtraction problem types is that found in the book *Children's Mathematics: Cognitively Guided Instruction*. The authors employ a schema that "focuses on the types of action or relationships described in the problems. The taxonomy is as follows:

*Join Problems* are problems where elements are added to a given set. The action takes

place over time. For example: 4 children brought candy to school. The next day 2 more children brought candy. How many children brought candy?

*Separate Problems* are those in which elements are removed. They also take place

over time. They are the inverse process of Join Problems.

In both Join and Separate Problems the problems can be one of three types: Result

Unknown, Change Unknown or Start Unknown—the Result Unknown being the answer,

Change Unknown is the second summand in addition or the subtrahend in subtraction, and Start Unknown is either the first summand or the minuend.

Thus, the abovementioned candy problem would be a Result Unknown Problem.

To make it a Change Unknown Problem we could say: 4 children brought candy to school. The next day 6 children altogether had brought candy. How many children brought candy the next day? A Start Unknown Problem could be: The first day some children brought candy. The next day 2 children brought some more candy. Altogether 6 children brought candy. How many children brought candy the first day?

*Part-Part-Whole Problems* are problems that are static. There is no change over

time and no action. The relationship is among a set and two subsets.

These problems are of two types: Whole Unknown or Part Unknown. For example:

4 boys and 6 girls sold cookies. How many children sold cookies? This is Whole Unknown Problem. A Part Unknown Problem could be: 10 children sold cookies.

6 of the children were girls. How many boys sold cookies?

*Compare Problems* are problems that treat relationships between quantities. They

contain a Referent Set, a Compared Set and the Difference. Any one of the three can be the unknown. For example: Jake has 3 pretzels. Zach has 8 pretzels. How many more pretzels does Zach have than Jake? This is an example of a Difference Unknown problem. To make it a Compared Set Unknown problem: Jake has 3 pretzels. Zach has 5 more pretzels than Jake. How many pretzels does Zach have? And a Referent Unknown problem using the same data could be: Zach has 8 pretzels. He has 5 more pretzels than Jake. How many pretzels does Jake have? (Carpenter et al., 1999, pp. 7-10).

This taxonomy is helpful in that it demonstrates how to make the action or the relationships in word problems as clear as possible. It shows ways to vary problems and it gives children descriptors for talking about problems and for targeting the part for which they need to solve. In fact, eleven different situations are described above that represent various interpretations of addition and subtraction (Carpenter et al., 1999, pp. 10-12). Although this taxonomy is presented for primary grades, I think it is useful throughout the elementary school years as a way of helping children target what parts of a problem are given and what part needs to be found. It is especially useful when solving missing addend problems. This taxonomy and the Chinese teachers' use of decimal operations will be discussed and used throughout this unit.

Another taxonomy that deserves mention is that of Larry Sowder in an article entitled "Addressing the Story-Problem Problem," from the book *Providing A Foundation for Teaching Mathematics in the Middle Grades*. Sowder divides the arithmetic world into two types: "Real World" and "Math World". He links "situations in which groups or amounts are put together, either literally or conceptually," as Real World situations, and addition as the Math World situation that handles these real world settings. "Take-away situations" and "Comparison situations" belong to the Real World, and the subtraction process belongs to the Math World. (Sowder, 1995, pp.129-130).

However, missing addends can be linked to both addition and subtraction, because the problems can encompass joining groups or amounts or take-away and comparison situations (Sowder, 1995, p. 131). This is where the confusion often sets in for students. Thus, articulating this taxonomy to them and pointing out that missing addends are the anomalies in the cousin family of addition and subtraction can be helpful in aiding students to choose the best of the two processes to solve a given problem.

## Processes and Problems

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### Unit Focus 1: Base Ten Operations

Key concepts I want to explore initially with the students are first of all writing numbers in *expanded form*. Following that we will move on to the concept of making and unmaking tens, also known as *composing and decomposing* tens. I particularly want to address the addition and subtraction process within 20 as the pivotal process or key understanding that students need in order to understand subtraction with regrouping. (A corollary understanding here is that a two-digit number is not two numbers, but one number with two digits: it is the *sum* of so many tens and so many ones, the *many* being specified by the digits. Although it is a subtle difference, in order to make and unmake tens it is important for children to comprehend numbers as being able to have more than one digit. Indeed, this is the whole notion behind place value.) The thinking here is that if students can compose and decompose numbers within 20, it should reasonably follow that, with instruction and guidance, they will be able to perform these operations with 100's, 1000's, etc. The goal is for students to understand that exchanges can take place between higher value units without changing the value of the number. I will introduce the concept of *rounding* as it relates implicitly to writing numbers in expanded form, and explicitly to higher value units of tens. Lastly, I will introduce a *mental subtraction* exercise that gets children thinking about place value unit exchanges that don't change the value of the minuend and subtrahend and that also facilitate a kind of mental gymnastics that children can build on and use to increase their arithmetic prowess.

Lesson 1: I begin teaching base ten operations by introducing a problem in expanded form. My reasoning is that this process gets children thinking about place value, it lays groundwork for adding, and later subtracting, across zeros and my experience has shown that children enjoy performing this operation, so they tend to think positively about their own abilities to handle place value with increasingly large numbers. Perhaps most importantly, this reasonably simple operation affords me the opportunity to lay the groundwork for the classroom format and group dynamics we will use for this unit.

I begin by writing a four digit number on the board, for example: 8,765. Employing the dialectic method (question and answer format) students identify the place values of each digit and I rewrite the number as  $8,765=8,000+700+60+5$ . Then I introduce the Problem of the Day: Write the number 5,678 in expanded form. Students work individually, or in pairs if they are more comfortable and rewrite the problem in expanded form as per the paper-folded model.

As students work, I walk around the room and note who understands the process, who doesn't and why. A rule I will employ, borrowed from Magdalene Lampert as described in her wonderful narrative description of her own mathematics teaching process in her book, *Teaching Problems and the Problems of Teaching*, is that no student gets to ask the teacher a question unless she or he has asked everyone in the problem-solving group and nobody knows the answer (Lampert, 2001, p.82). However, at this time, I allow and encourage questions as I circumambulate the classroom.

Bringing the children back to whole-class attention we discuss student solutions and some are written on the board. We come to consensus as to which processes and solutions are reasonable. Here I begin modeling how to make conjectures as students discuss and continue to work the problem. For example a conjecture might be that a number in the thousands place, in expanded form, is a number from 1 to 9 with three zeroes behind it. What conjecture could we make about a number in the hundreds place, using the expanded form schema? I continue to ask questions about each place value unit until we get to ones. What conjecture can we make about the number in the ones' place? The answer I am seeking is that it will be a number from 0 to 9 that stands alone. Or some version of the above. The emphasis here is on how to formulate a conjecture or conjectures.

Next, I have students draw four columns on their papers and label the columns from right to left: ones, tens, hundreds and thousands as I do the same on the board. Then I plug in the numbers from the original number example, 8,765, and we discuss the place value positions of each digit and its place value name. Students repeat the exercise on their four-quadrant paper this time using their Problem of the Day, 5,678. Again we come together for a whole-class discussion, find reasonable solutions and come to an agreement about our conjectures.

After this I will break the children into groups of three or four and they will tackle some more challenging problems: Write 50,678 and 500,678 in expanded form. Students will store their work in their Word Problems Notebook, and I will check them for any misunderstandings that need to be cleared up the following day.

The next day I will follow up by having a conversation about the relative size of numbers. For example: What is the relationship between 8,000 and 800? How about that between 8,000 and 80? What about 8,000 and 8? The object is to have the children be thinking about tens and higher value units of tens.

After discussion and whole class practice with other examples, I will introduce the concept of rounding as a strategy to help determine the reasonableness of answers to whole number computations. Again, much practice will follow, in not only rounding of numbers, but using rounding as a computational strategy. A note



here is that I am not strictly attached to a time frame, and if it takes students longer than two days to absorb these base-ten concepts, I will extend the modeling, discussion and practice as need be.

Lesson 2: This is a lesson in *Adding Base Ten Numbers by Place Value Components*. The objectives in this lesson are to teach the commutative rule of addition, to introduce the concept of decomposing numbers of the same place value without changing their value, and in the process, to learn a non-standard method of addition. This method of adding was taught to me by Professor Roger Howe of Yale University as a way to teach the commutative and associative rules for addition under the child-friendly name of the *Any Which Way Rule*. I name the procedure as I introduce and discuss the problem of the day. The procedure demonstrates to children how they can break up numbers into their place value components in order to perform an arithmetic operation.

I write a problem on the board such as  $24+35 = (20+30) + (4+5) = 50+9 = 59$ . Again using a question and answer format we discuss the concepts presented. First of all, the Any Which Way Rule allows us to *decompose* and regroup the numbers into bundles of tens and ones, add the numbers in their respective place value positions together and finally recombine the place value parts into a familiar base ten number we all recognize, know and love. I will encourage the children to come up with their own versions of why this works, i.e. make conjectures. Ultimately, the conjecture I'm looking for is some version that recognizes that we are breaking the numbers up into tens and ones and then adding them up. Next I write the problem on the board vertically and we solve it using the standard algorithm and compare the two processes. Then I give students a number of problems to solve using both methods and they break into their working groups to solve them. We reassemble whole-class after they have worked the problems and discuss their processes and revise our conjectures if necessary. Here a conjecture might be a definition of *decomposing* numbers, for example. Again students store their work to be collated later.

Lesson 3 will be an exploration of a process called *equal additions* which is a method demonstrating the principle of maintaining the values of numbers by adding the same number to each number in a subtraction problem in order to mentally subtract it. The problem:  $369 - 199$ . Working on the board, I add 1 to 369 and 1 to 199 giving me the new problem  $370 - 200$ , pointing out I haven't changed the value of the difference between the two numbers. This is called a *method of compensation*. Students check the answer, 170, by using "traditional" means. Some fourth graders will be able to do this and I will guide the regrouping process. After a number of demonstration problems, I assign problems for children to work on their own. Later we will make some conjectures about why this works and post some examples around the room.

By now students should be ready to tackle some word problems in base-ten operations, using the four-quadrant page format they have been practicing.

### *Sample Problems of the Day Reinforcing Operating in Base Ten*

In this section of the unit I will lead and encourage students in an exploration of place value to 1000's, identifying and using even and odd numbers and rounding numbers to nearest ten, hundred or thousand in order to estimate the answer to an addition or subtraction problem. Please keep in mind that these are only sample problems. I will determine when to move on in light of the speed of comprehension of individual students and the class as a whole. (Note: I have not delineated a specific lesson teaching odd and even number recognition due to the fact it is out of the scope of this unit; however, this concept will need to be introduced and practiced before doing the following problems.)

The first three problems are a related suite of numerical word problems designed not only for their arithmetic

content but to familiarize students with reading arithmetic problems and determining what to do. I will probably introduce all three in the same class period, depending on the level of comprehension. The problems would also work well as variants on a theme to be presented to different working groups who would then report out to the whole class at the end of the period. I will model the first one as a whole-class exercise. It is important to articulate the clues that are given in each problem; therefore, students draw a clue chart on their papers (in quadrant two), as I model it on the board. When working in their groups I again require them to make a clue chart for the second step of their procedure.

1A. The ones' digit is five. The number is greater than 640 and less than 650. What is the number?

1B. Escribe el número par lo más grande que tiene un seis en el lugar de centenas y un 4 en el lugar de decenas.

1C. ¿Cuáles son los cinco números pares que tienen un seis en el lugar de centenas y un cuatro en la posición de decenas?

In the next suite of problems students practice writing numbers in expanded form. Again I model one or two or more of the first type (2A), then students will tackle 2B as a Problem of the Day. More problems and practice follow as time permits and comprehension requires.

2A. Write the number one thousand, six hundred and fifty in expanded form.

2B. Se vendió Pablo 1000 billetes para el juego de fútbol. Se vendió Marta 900 billetes. Jaime se ha vendido 80 billetes y Hermanita Lupita se vendió 2. ¿Cuántos billetes en total se vendieron toda la familia López? Usa la forma extendida a resolver.

Parte dos: ¿Quién ha vendido más, Pablo solo, o Marta, Jaime y Lupita entre si? ¿Cuántos más?

The second part of the above problem can be an extension activity for those students who finish quickly and need more of a challenge, or it can be a required problem for all students.

The last problem in the Base Ten section is a sample problem in rounding. I will reintroduce the rounding operation as a whole class demonstration first. A key understanding is that a given number is sandwiched between two numbers that differ only in the last non-zero digit place.

We will be rounding to nearest thousands, so I'll begin by having students round off to nearest tens, hundreds and then thousands by representing the sample numbers I give them as numbers in expanded form: for example: write 278 in expanded form. If we want to round this number to nearest hundreds, what place value digit will we need to use to round off? What place value digit gives us our clue?

2C. There were 1,289 tickets sold for a baseball game. The following week 1, 982 tickets were sold to attend another game by the winning team. Round off the numbers to give an estimated number of how many more people attended the second game than the first. Hint: What place value will you use for rounding? What do you need to do after you round off the numbers in order to get an estimate of how many people attended the second game.

### **Unit Focus Two: Take Away/Finding Difference—Subtracting Across Zeroes**

The next set of problems is designed to help students recall and reinforce what they have learned about

addition and subtraction in third grade, i.e. problems that don't require regrouping. We will practice more two digit additions and subtraction using mental math. I will begin this subunit with a discussion of regrouping by composing and decomposing tens using simple one digit problems in order to formally introduce the concept and give them the new vocabulary. The objectives are for students to understand regrouping by making and unmaking tens; to identify and solve problems of finding the difference and to begin learning how to subtract across zeroes. I will conduct a review lesson on fact families, again to reinforce a concept the children should have learned in third grade, and give a diagnostic quiz on addition and subtraction facts to determine where the children are in memorization of the facts.

Lesson 1: I write the following problem on the board:  $9 + 6 = 15$ . I lead students in an inquiry to discover that 9 is really nine ones, 6 represents six ones and the 15 means one ten and five ones. I'll use tokens to demonstrate this and bundle them into one bundle of tens with five ones. At the same time I tell them that I'm *composing* or *making* a ten. Then, as per the Commutative Rule of Addition, I'll switch the addends and have students verify that we get the same answer. From here I'll have students work with their own pile of tokens to bundle simple practice problems I give them into tens and ones.

Next I'll go to the inverse operation using the above problem as  $15 - 6 = 9$  and  $15 - 9 = 6$ . Again we will use tokens, this time to *decompose* or *unmake* a ten and five ones by taking the five ones and one token from the ten bundle away. And so forth. Again the students practice the inverse practice problems with their own pile of tokens.

It is important to give the students the new vocabulary, i.e. *composing* and *decomposing* so that they have a way to talk and write about the process. Also, it helps them to concretize the new concepts and to remember them. So the key learning here is that when we perform addition and subtraction we are really making and unmaking tens.

Lesson 2: Students meet in their groups to work a number of simple problems, arranged in quartets of two addition and two subtraction problems that I give them on a printed sheet. These quartets are *fact families* or *cousins*. A sample is:  $5 + 4 = 9$ ;  $4 + 5 = 9$ ;  $9 - 5 = 4$  and  $9 - 4 = 5$ . The objective is for them to figure out what the pattern is and if it will always happen. They will create other quartets of fact families. When I bring them back to whole-class discussion, groups will report out and post their fact families on the board.

The following day we are ready to begin the word problem(s) of the day.

#### *Sample Problems of the Day Demonstrating Inverse Relationships/Finding Difference—Subtracting Across Zeroes: An Introduction*

The first two problems are a review. They are followed by a suite of two problems that move to three-digit addition and its inverse cousin, subtraction, and then two more that, by changing one digit, begin the process of adding and subtracting across zeroes. In this suite, our follow up discussion will talk about "successive composition and decomposition" as described by Liping Ma in her book *Knowing and Teaching Elementary Mathematics*. She points out that students need to learn that "when the next higher place in a summand or a minuend is zero, one has to compose or decompose a unit from further than the next higher place" (Ma, 1999, p.15). For example:  $203 - 15$  requires that we decompose 100 into 10 tens, and one ten into ten ones. (Ma, 1999, p.15). I will pay particular attention to students' conjectures before and after working these problems.

I teach each suite of the following as I have them grouped, i.e. 1A and 1B on one day and 2A, 2B, 2C on another.

1A. Sarah has 30 baseball cards. Tricia has 48 cards. How many cards do they have together?

This is a great mental math problem that I will model: If we round 48 to 50, we can simply add 30 and 50 to get 80 then subtract the two we rounded off and get 78. Another way to compute this problem mentally is to add  $30 + 40 + 8 = 70 + 8 = 78$ . Now I'll have groups write two or three problems that can be solved the same way. I'd like them to solve 1B by writing it down and solving it in the traditional manner to judge whether or not they are able to regroup the tens, because the next trio will contain regrouping problems. I check individual work to determine how much additional instruction children need before moving to the next trio.

1B. Rogelio tiene 53 conchas. Juanita tiene 95 conchas. ¿Cuántas conchas tiene Juanita más que Rogelio?

In this trio of problems students move from solving a simple one column addition to regrouping with zeroes and then subtracting across zeroes.

2A. Maggie collected 543 beads last year. This year she collected 456 beads. How many beads did Maggie collect?

2B. Margarita recogió 544 cuentas de collar de su abuelita. También recogió 456 cuentas de collar de su tía. ¿Cuántas cuentas de collar ha recogido en total?

2C. Margarita ha recogido 1000 cuentas de collar. Ha recogido 456 más cuentas de collar que su amiga Catarina. ¿Cuántas cuentas de collar ha recogido Catarina?

After coming together whole class to post conjectures and discuss solutions, children will be given similar trios to practice the regrouping with zeros skill. Then the groups are asked to write and solve their own trios and present them to the class. This may take another day to complete.

### **Unit Focus Three: Building Skills in Composing and Decomposing in Base Ten—Introducing Subtraction of Decimals and Some Comparison Problems.**

The next set of problems will expand on the composing and decomposing of tens while at the same time introducing addition and subtraction of decimals through the means of money by exploring comparison problems. The last problem in the set introduces students to the infamous *missing addend* problems.

Lesson 1: Each child receives a stack of ten dollar bills, one dollar bills and a few dollars change in play money. I ask them to keep the tens, ones and change in separate piles until they need them to work the following sample problem. How much money do you have if you add \$7.69 and \$9.67? Arrange your money to show the correct number of tens, ones and change you will have? Remember that 100 cents equals one dollar. How many cents do you have when you add up the change part of this problem? What are you going to do with that \$1.36? What is the decimal point telling us in this problem? How do we read it? We will solve these problems together, one at a time, as I model them in front of the class. Children are working individually now, but they may work in pairs if they are more comfortable. I am circumambulating the room, guiding and answering questions. When we are finished children should have a ten, seven ones and thirty-six cents in change.

Again I will ask the question: What is the decimal point telling us in this problem? We discuss that it means less than one whole dollar; however, the relationship between the places is the same as for whole numbers. We will keep working problems until I'm satisfied that all children understand how to represent addition of money problems. Then, I'll switch to subtraction of money problems and model decomposing tens in order to

solve the problem. Again, students will work with their own stacks of tens, ones and change in order to solve some decimal subtraction problems. Throughout the lesson I will introduce and employ the word *hundredths*, explaining the suffix "th" to signify less than one whole.

Using the money manipulatives we discover, and I articulate, that dimes are  $\frac{1}{10}$  of a dollar since a dollar equals ten dimes. Likewise, a penny is  $\frac{1}{100}$  of a dollar and  $\frac{1}{10}$  of a dime. I write the decimal representations on the board. We are ready for our problems of the day.

1A. Cuando fue al pueblo Juana tenía \$14.87. Cuando regresaba, tenía \$6.39. ¿Cuánto dinero gastaba Juana?

I will use this problem to observe whether or not students know the rule: *the largest decimal place rules*. The next more difficult problem will point the way.

2A. Juana tiene \$140.87. Su amiga Flora tiene \$210.26. ¿Cuánto dinero más tiene Flora que Juana?

Problem 2A seems a simple comparison problem where the amount of money Juana has is compared to the amount Flora has. However; students must first be able to determine who has more money. Thus, I articulate the rule, *largest decimal place rules*, and we discuss it and explore more examples. Mathematically the problem is challenging, because it requires the student to decompose across every place value, while at the same time keeping track of the decimal points.

Lesson 2: I take some time to explain a missing addend problem as a subtraction problem that looks like addition and can, in fact, be solved by addition. Professor Roger Howe told me another way to teach this is to "think of  $a-b$  as the number you add to  $b$  to get  $a$ ".

I introduce students to some terminology and patterns that will help them to identify a missing addend problem. At this point I will discuss the Carpenter et al taxonomy referred to in the *Strategies* section. Addition and subtraction problems are named "Join" and "Separate" problems. And these can be further identified as Join/Separate problems with "Result Unknown, Change Unknown or Start Unknown" (Carpenter et al, 1999, p.10). Thus, for Join problems:  $5 + 7 = X$ ,  $5 + X = 12$  and  $X + 7 = 12$ . Similarly, for Separate problems:  $12 - 7 = X$ ,  $12 - X = 5$  and  $X - 7 = 5$ . I model and discuss some sample problems that treat action situations which happen over time.

The Unknown, or what we want to find out, is  $X$ . I'll have students meet in their groups to solve some Join and Separate problems, identifying whether  $X$  represents Result Unknown, Change Unknown or Start Unknown. I also want them to identify the action in the problem and its time frame. When they report out to the class, we will review the new terms again, articulate the action and its time frame, check their problems and try a few more. Then, in groups, students will formulate their own set of four problems. This time the groups will post their new problems, and we will review them to discern if they are reasonable.

Lesson 3: A sample problem is  $X + 6 = 10$ . What is  $X$ ? From what we learned yesterday, what kind of problem is this? When the Start or the Change is unknown, children often have difficulty. These are the missing addend problems. After the students identify the above as a "Start Unknown" problem, I give them a simple word problem that fits the formula pattern. For example, Alice found some marbles on the playground. After school Ahmad gave her 6 marbles. Then she had 10 marbles. How many marbles did Alice find?

Here is where I give them the term *missing addend* and explain how to identify the problem as such by setting it up as  $X + 6 = 10$ , to use the above example, and by identifying it as a Start Unknown problem. A missing

addend can also be a Change Unknown problem, so I reverse the above problem to make it fit the pattern  $4 + X = 10$ . I also discuss the time factor and the action (finding and receiving marbles) in the problem.

Sometimes, it's easier to ask a subtraction question than an addition question. So, could we ask the question, "What is  $10 - 6 = X$ ?" This is changing the Start Unknown problem to a Result Unknown problem. Let's try it. After I work the problem and we check it together, I'll give students a number of sample problems to explore to see if the algorithm works. Then I'll give them some time to practice the algorithm with other problems in their groups. Later, we'll come together to discuss solutions and make some conjectures. The goal is for the children to discover that these problems are most easily solved by subtraction. Now we are ready to continue our word problems.

1B. Habían muchos gatos en la calle al mediodía. Setenta y cinco gatos se fueron. Cuarenta y cinco gatos se quedan. ¿Cuántos gatos habían en la calle al mediodía?

2B. Mickey had 275 rocks in his rock collection. His little brother *borrowed* a number of them for his school science project. Now Mickey has only 170 rocks left. How many did his brother borrow?

3B. Mickey had some rocks in his rock collection. His little brother took 125 of them to build a fort. Now Mickey has only 375 rocks in his collection. How many did he have?

In the above problems although they could easily be mental math problems, the challenge is in identifying the problems as missing addend problems. I would like the children to identify first what component is missing in each one—the Start Unknown or the Change Unknown. Can we make a conjecture here that when either one is missing the problem is most easily solved by subtraction?

Now it is time to look at missing addend problems that are classified as Part-Part-Whole and Comparison Problems, according to the Carpenter et al taxonomy. Again whole class, I give students some sample problems that involve static relationships where there is no action and no change over time. This time I use two-digit numbers in the problems. For example, John has some oranges and Sally has 29 oranges. Together John and Sally have 54 oranges, how many oranges does John have? I articulate that there is no time frame here and no action. And I set up a formula:  $X + 29 = 54$ . What is  $X$ ? Is it a Part Unknown problem or a Whole Unknown problem?

A Compare problem might be: John has 25 oranges. Sally has 4 more oranges than John. How many oranges does Sally have? In this case, students are choosing whether the unknown to be solved for is Compare Quantity [subtrahend Unknown, or Referent (minuend) Unknown]. These are fine, and to fourth-graders, subtle distinctions that will require lots of examples of each type and lots of practice. Again, I identify the problems as missing addend problems, and I make the connection that the Sowder taxonomy makes that missing addend problems can be Take-away (Separate) situations or Comparison situations.

After much practice of both types of problems over several days, we are ready to begin our Word Problems of the Day. Two examples follow.

1C. There were 4750 apples on the big tree. There were a total of 8295 apples on the big tree and the little tree together. How many apples were on the little tree?

This is a Change Unknown subtraction problem that also requires decomposing one thousand.

2C. The library has some books. In Ms. Wasser's fourth grade class of 21 students, each student checked out 2

books. Now the library has only 958 books. How many books does the library have altogether?

This is a multi-step, Start Unknown addition problem. Students need lots of practice with these types of problems to determine when to add and when to subtract in order to find the missing part.

3C. Flora tiene \$2100.26 en el banco. Entre si, Flora y su amiga Juana tiene \$3600.13 en el banco. ¿Cuánto dinero tiene Juana?

Problem 3C is a fairly substantial problem in that it deals with a number of mathematical dimensions, notably subtraction across zeroes and subtraction of decimals as well as finding the missing addend.

Please see the appendix for more problems of all the various types covered in this unit.

In sum, students will work individually and in small groups to solve word problems using their knowledge of base ten operations to explore subtraction problems involving finding difference, subtracting across zeros, using mental math, comparing numbers, finding the missing addend, subtracting with decimals and subtracting with fractions. They will learn to identify key elements of the problem, formulate conjectures and revise them if need be. It is to be hoped that the byproduct will be a lively spirit of inquiry and enthusiasm that will serve to inform student work and invigorate their mathematical thinking and problem-solving abilities.

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## Implementing District Standards

This curriculum unit meets the following New Mexico Fourth Grade Standards from which all material is directly quoted:

Strand: Numbers and Operations, Content Standard I: Students will understand numerical concepts and mathematical operations.

*Benchmark A. Understand numbers, ways of representing numbers, relationships among numbers, and number systems.*

Performance Standard

1. Exhibit an understanding of the place-value structure of the base-ten number system by reading, modeling, writing and interpreting whole numbers up to 100,000; compare and order numbers.

*Benchmark B. Understand the meaning of operations and how they relate to one another.*

Performance Standard

Demonstrate an understanding of and the ability to use standard algorithms for the addition and subtraction of multi-digit numbers.

Select and use appropriate operations to solve problems.

Extend the uses of whole numbers to the addition and subtraction of simple decimals (positive numbers to two places).

*Benchmark C. Compute fluently and make reasonable estimates.*

Performance Standard

Add; subtract...up to two double-digits accurately and efficiently.

Use a variety of strategies (e.g., rounding and regrouping) to estimate the results of whole number computations and judge the reasonableness of the answers.

Strand: Algebra, Content Standard II; Students will understand algebraic concepts and applications.

*Benchmark B. Represent and analyze mathematical situations and structures using algebraic symbols.*

Performance Standard

Identify symbols and letters that represent the concept of a variable as an unknown quantity.

Explore the uses of properties (commutative ...associative) in the computation of whole numbers.

Express mathematical relationships using equations.

Determine the value of variables in simple equations.

Strand: Data Analysis and Probability, Content Standard V; Students will understand how to formulate questions, analyze data, and determine probabilities.

*Benchmark A. Formulate questions that can be addressed with data, and collect, organize and display data to answer questions.*

Performance Standard

Organize, represent, interpret numerical...data and clearly communicate findings: choose and construct representations that are appropriate for the data set.

*Benchmark C. Develop and evaluate inferences and predictions that are based on data.*

Performance Standard

Propose and justify conclusions and predictions based on data.

Develop convincing arguments from data displayed in a variety of formats.



## Annotated Bibliography

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## Appendix of Word Problems

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### Place Value

- 1A. Escribe el número lo más grande de tres cifras que tienen seis centenas y cuatro decenas.
- 2A. Listan los cinco números impares que tienen un nueve en el lugar de millares, un ocho en la posición de centenas e un siete en el lugar de decenas.

Once children learn how to read the clues for the above type of problems, the teacher can make them up and continue to increase the place value positions, as above. It's important for students to differentiate between even numbers (números pares) and odd numbers (números impares). This is an important clue. My students come to really enjoy these problems and their increasing difficulty across the dimension of place value.

### Take Away/Finding Difference

- 1A. Jorge pesaba 140 libras. Se puso a régimen y perdió 15 libras. ¿Que pesa Jorge ahora?
- 2A. The little league baseball teams in Littleton have 102 more members than the basketball teams. If 45 members switch from baseball to basketball, then what will the difference be?
- 1B. An album can hold 400 stickers. Lily has 199 stickers. How many more stickers does she need to fill the album?
- 1C. A worker needs 3606 bricks to build a house. She has 2679 bricks now. How many more bricks must she get?

2C.The Declaration of Independence was signed in 1776. How many years was this before you were born? (note:This years crop of fourth graders was born in 1998.)

I have arranged these problems in order of increasing place value difficulty. As well the number of zeroes that need to be subtracted across increases in problem 1B. Problem 2C requires that children know the year in which they were born and know which number to use as the minuend and which to use as the subtrahend.

### **Subtracting Across Zeroes**

1A.Luisa e Raúl teinen entre si \$15.00. La parte de Luisa es \$8.42. ¿Que parte de ese dinero es de Raúl?

2A.Luz se compró un bocadillo a \$0.39 e un zumo de manzana a \$0.55. Si pagó con un billete de \$1.00, ¿cuánto vuelto recibió?

1B.La senora Bazán tenía 46 en 2007. ¿Cuántos años tenía en 1993?

1C.Las clases de la Escuela Azteca está recogiendo periódicos para la esfuerzo de recogida de periódicos. La meta es recoger 10,000 periódicos. Recogimos 768 periódicos la primera semana, 3,456 la segunda semana, 2,987 periódicos la tercera semana. ¿Cuántos periódicos más necesitamos recoger?

I grouped problems one and two together because they both require subtraction across two zeroes. Problem 2A has the added dimension of being a multi-step problem that requires adding first and then subtracting. It also uses decimals.

1B is an interesting problem because the problem solver needs to first subtract years to find the number of years between 1993 and 2007. Then he needs to subtract that answer from 46 to find out how old Sra. Bazán is in 2007.

In problem 1C another dimension has been added—the place value has increased to 10,000. Like problem 1B, it is a multi-step problem requiring addition first and then subtracting the total from 10,000 to see how many newspapers still need to be collected to reach the goal.

### **Comparison**

1A.Cesi tiene 42 cacahuates. Nestor tiene 22 cacahuates. ¿Cuántos cacahuates tiene Nestor menos que Cesi?

2A.¿Por cuánto es más grande ciento cincuenta que veintitres?

3A.Se vendió Lupita 312 cajas de galletas. A Marta se vendió 228. ¿Cuántas cajas de galletas más se vendió Lupita que Marta?

1B. Juana, Marucha y Rita fueron a jugar á los bolos. ¿Cuántos puntos más que Juana marcó Marucha?

Jugador Total

Puntos de Juana 105

Puntos de Rita 129

Puntos de Marucha 181

1C. Tony pagó \$920 por una moto y \$245 por una bicicleta. (a) ¿Cuánto pagó Tony para las dos? (b) ¿Que tanto más era la moto que la bicicleta?

Problems 1A, 2A and 3A are both simple comparison problems; however 2A requires a subtraction in the ones place and also requires that students read the numbers in script. 3A requires decomposing across three place value units. Another important feature of the Spanish word problems is that native speakers do not have to translate them back into their mother tongue in order to solve them, so they are able to go directly to the language of mathematics. I have found that this increases their mathematical learning curve. Then, when the students do reading problems in English, they are able to more easily understand the mathematical similarities in structure.

Problem 1B sets up a little chart where students need to choose the relevant data in order to solve the problem.

1C is a multi-step problem requiring first addition and then subtraction.

### **Decimals**

1A. Fred's time in a race was 14.5 seconds. Jordan's time was 15.3 seconds. Who ran faster and how much faster?

1B. Betty earned \$16.00 selling lemonade at a lemonade stand. Marvin earned \$13.50. How much more lemonade money did Betty earn than Marvin?

2A. Nathan's weight was 82.5 pounds three years ago. Now he weighs 76 pounds. How much weight did he lose?

3A. Betty and Marvin want to buy a C.D. that costs \$19.95. Betty earned \$16.00 selling lemonade and Marvin earned \$13.50. If they put their money together, will they be able to purchase the C.D.? Will they have any money left over?

Problems 1A and 1B are straight subtraction with decimals except that B has two decimal places and also subtracting across zeroes. Problem 2A, however, is introducing a new dimension of subtraction requiring the problem solver to add a decimal point and a zero to hold the tenths place. This problem may require some modeling, but I will first see if students can come up with a conjecture in their groups. 3A is a multi-step problem combining addition and subtraction.

### **Fractions**

1A.  $\frac{4}{5}$  de los jóvenes en un coro son muchachas. ¿Que fracción de los jóvenes son muchachos?

1B. Minghua spent  $\frac{3}{7}$  of his money on a book and the rest on a tennis racket. What fraction of his money was spent on a racket?

2A. Elena tiene  $\frac{3}{4}$  de un litro de jugo de naranja. Tomé  $\frac{1}{2}$  litro del jugo. ¿Cuánto jugo resta?

3A. A container can hold 3 cups of liquid. It contains  $1\frac{3}{4}$  cups of water. How much more water is needed to fill the container?

Problems 1A and 1B require subtracting the given fraction from the whole fraction which is not given. The challenge for the students, therefore, is to come up with the unnamed fraction. 2A requires finding a common denominator. After students meet in group to make their conjectures, we will probably come together whole-class in order to solve the problem. 3A combines adds the dimension of decomposing a whole number into a fraction. It will be interesting to see how much of the process children can figure out in their groups.

### Multi-Step Problems

The following problems are arranged, as much as possible, to increase in technical difficulty, and/or number of steps needed to solve the problems.

1. En la mesa hay 10 manzanas. Navier se comió tres, Laura se comió dos y Raquel se comió cuatro. ¿Cuántas manzanas quedaron?
2. There are 260 members in a chess club. 136 of them are boys. (a) How many girls are there in the chess club? (b) How many more boys than girls are there in the club?
- 3 .Ms. Montoya has 13 boys and 15 girls in her class. On Friday, 7 students were absent. Four of the absent students were girls. How many boys were in class on Friday?
4. Eusenia tiene \$20.53 que ahorré de cuidar niños. Quiere comprar una blusa que cuesta \$15.99. (a) Cuánto dinero restará? (b) Tiene suficiente dinero a comprar un pedazo de torta que cuesta \$3.25?
5. Martín participó en dos de estas actividades. Pagó con un billete de \$10.00. Recibió de vuelto \$3.75. ¿ En qué dos actividades participó Martín?

Actividad Costo

Cine \$3.50

Minigolf \$3.00

Patinaje \$2.00

Cochecitos \$2.75

6. La clase se propuso como meta 1,250 latas de aluminio para su proyecto de recoger latas. Hasta ahora han recogido 735 latas. ¿Cuántas latas más necesitan para alcanzar su meta?
7. Cuando tres muchachas se subieron a una báscula esta marcó 166 libras como su peso total. Uno de los muchachos bajó y la báscula marcó 106 libras. Otra muchacha bajó y la báscula marcó 57 libras. ¿Cuánto pesaba cada muchacho? (Sugerencia: Utilizando los hechos dados, comienza con 166 libras y resuélvelo al revés.)
8. A painter mixed 1.46 liters of black paint with 0.8 liters of white paint to get gray paint. Then he used 0.96 liters of the gray paint. How much gray paint did he have left?
9. One football stadium, built in 1982 has 64,035 seats. Another stadium, built in 1987, has 74,916 seats. How many more seats does the newer stadium have?

10. Ms. Wasser is ordering supplies for her class for the next school year. She has three tally sheets. The first tally sheet totals \$28.46, the second totals \$17.83 and the third totals \$30.00. She has \$100.00 to spend. How much money has she spent altogether? How much money does she have left?

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