Curriculum Units by Fellows of the National Initiative
2007 Volume VI: Keeping the Meaning in Mathematics: The Craft of Word Problems

Percents in Real Life Situations

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by Moses B. Jackson

Overview

This unit is intended to scaffold students in the thinking process of solving word problems that involve percents. Its specific focus is to present a simple template that students can use to translate and solve percent word or story problems of varying dimensions. The goal of this template is to develop a robust understanding of the language used in solving word problems, so that many variations of problems can be correctly read.

While the intent is to alleviate the negative stereotypes and fear about math problems, the unit will assist math teachers to cultivate a tangible trend of thought, in translating and resolving word problems involving the use of percents. I hope to accomplish two things: first, the unit will set out to create a thought process that will respond to questions such as: What makes certain problems alike? Are they alike in mathematical structures, or are there also some differences? How do the problems differ when placed in groups referred to as bins? Second, the unit will utilize general principles to describe the bins by considering which bins have similar or different characteristics. This grouping process will present a structure that categorizes the nature of word problems concerning percents.

From a logical standpoint, word problems are the starting point and ending point since they embody the meaning of operations. The meaning of arithmetic namely, is how to translate real life situations into arithmetic exercises. As a starting point, word problems should be an integral part of mathematics instruction, but instead, many textbooks include word problems as a separate chapter making it appear as if it is one of the arithmetic fields or an application (Aharoni, 2007). The traditional method of teaching mathematics- progressing from computation to word problems- is a pedagogical version of putting the cart before the horse (Burns, 1984, p. 228).

From time immemorial, teaching math in grade school, especially word problems, has been a venture marked by positive challenges. In recent times, these challenges have been so profound that it is now a battle between teachers and students. Experience has shown that misbehavior and poor classroom management are prevalent in math classrooms.

On one hand, every teacher soon becomes aware when students do not understand a topic, their interests diminish, they withdraw, and they become bored and negative behavior sets in. On the other hand, when students are motivated and can relate to the topic being taught, they develop interest, become engaged and participate more.

The concern about withdrawal and lack of interest in math is due not only to poor math foundations that students bring from lower grade schools to high school, but also due to insufficient strategic methods in math. When they reach high school, students' expectations become overwhelming. The coddling that occurs in elementary and middle schools disappear and an unnecessary state of anxiety develops among new students. This state of anxiety is engendered by students' need for self esteem and fear of having their ignorance exposed in front of their peers.

The other concern that affects the teaching of math is the negative groundswell of opinion that "math is hard." As a result, students develop a dislike for numbers and analytical thinking. Although some of the apprehensions students have about numbers are based on poor foundations, much is intrinsic and due to cognitive dissonance; Psychoanalyst Jean Piaget's theory of cognitive dissonance occurs when the schemata or knowledge reservoir of an individual encounters new information that it cannot easily absorb resulting to disequilibrium. (Berger 2006). Parents demoralize their children by reinforcing their cognitive dissonance that math was not a subject for them and their siblings during their own school days and therefore, failing math could be a family trait. While this type of conditioning continues in the core unit of the community, kids continue to get passed on without meeting minimum promotion requirements.

In spite of the belief that math is hard, it still remains the queen of the sciences, and arithmetic the queen of mathematics; hence, for students to acquire appropriate math skills, a study of arithmetic is necessary. In this age and time, students who come to our classrooms face extreme difficulties in algebra and science courses because they lack basic skills in dealing with arithmetic problems to enable a smooth transition to algebra. There are also students who come to our classes with excellent computational skills but have never been taught how to translate and structure story problems into mathematical statements. Because of this situation, as a high school math teacher, I have included in this four-week curriculum unit, simple pedagogical strategies and hands-on activities related to the teaching of percent word problems to 9th graders in my intensive math class. The strategies that will be frequently used are the KWL and T-Charts; the KWC chart which is a modified version of the KWL and more applicable to science topics will be introduced. Cooperative learning and differentiated instruction methodologies of teaching will form the core of my classroom activities.

The word problems that I have included in this unit involve calculating percents of quantities, finding changes such as the decreases and increases of percents, finding missing information in a percent problem, expressing percents in several real life situations such as finding discounts and sales commissions. A more complex application of percent stories involving the calculation of ratios is presented later on in the unit to accommodate advanced students and induce critical thinking.

Rationale

The main reason for which I write this unit is to contribute to the nationwide efforts being exerted by all educators to curtail, if not eradicate, poor academic performance in public schools. Statistical data from the School District of Philadelphia indicate that academic performance among public school students on state, college and school district tests, specifically in math, decreases every year. Students who do not score the required points on public tests are categorized as being below basic while performance serves as one of the yardsticks which determine the status of schools. Schools that remain in low categories for three consecutive years become eligible to either be assigned special management teams, have enrollment reduced, or teachers and administrators replaced. This situation makes it compelling for educators to exert much more effort to increase student performance.

First, in my view, although the issue of poor academic performance can be explained from several perspectives, economic, political and social factors play the most significant role. For example, from the economic standpoint, in many public schools, which are categorized as "comprehensive neighborhood schools", classrooms are crammed with minorities and kids from disadvantaged low income neighborhoods. Considering the tender age at which these kids are, it can never be overemphasized how much more attention and guidance they need in order to remain focused on activities that would help them To the contrary, experience has shown that parents in these low income homes do not spend much time with their kids. Few parents who make the effort to help their kids do not have much time because they either must work at more than one job or spend much of their time trying to bring home income to sustain the family or as is locally said, "Bring home the bacon".

Second, the economic perspective is intertwined with the social perspective because it serves as a precursor to various social ills in the communities. Current day society is so modernized and intricate that kids spend more time learning new video game strategies than reading. When children are left alone by themselves, they tend to take control of their own lives and utilize their time in trivial ways. For example, in many low income neighborhoods where parents are hardly engaged with heir kids, there is high tendency for promiscuous sexual activities among youths, teen age pregnancy, high drop out rates, gang activities, drug abuse, and gross lack of interest in school attendance. In their fantasy state, these kids still day dream of being successful celebrities, and famous athletes but refuse to accept the fact that school is not an alternative route to success but the major conduit to self actualization and success.

Lastly, I would argue that many political policies affect academic performance negatively. Due to political objectives, the goal of grade school education in recent times has shifted from the production of functional individuals in the community to attracting support for education administrations. For example, teachers hardly teach to prepare students for their future careers anymore, but instead teach them to pass standardized tests in order for schools to meet district standards and become eligible to receive more support. As a consequence, administrators insist that lesson plans be designed to prepare students for standardized state tests such as the Terra Nova, Benchmarks and the PSSA (Pennsylvania System of School Assessment). Straight jacket curriculum timelines are enforced by consistent monitoring in schools by administrators. When I consider the trend of events in public school districts, it becomes apparent that politicians who propagate these policies hardly take into consideration the social diversity among schools in different communities. There are two areas where politicians need to make adjustments regarding the eligibility factor which determines how much support goes to particular schools. They should consider existing school culture as well as academic performance, and the NCLB (No Child Left Behind) Act, which has been misconstrued so much that the classroom is approaching the situation when it will become ever more difficult to teach, needs to be revisited. A major issue is that there is no effective policy for managing students who have committed serious violations and who return to the classroom without counseling.

Classroom Demography and Students' background

School climate and students' academic performance

I am a math teacher assigned to the Overbrook High School in urban West Philadelphia, Pennsylvania. I teach five regular 9th grade intensive math and algebra-I classes with a total of approximately 150 students. I have also taught two terms of EOP (Educational Option Program) geometry, statistics, and algebra II classes with a total of 84 students. The EOP is a special program for adults and older students who are pursuing high school diplomas. The age range of students in my regular morning classes is 14 to 16 years and for the EOP program, 17 to 42 years. The school opened in 1924 and currently has an enrollment of approximately 2000 students.

Overbrook Community consists primarily of old row homes and a few apartment buildings that house its population of nearly 50,000, according to 2004 statistics. The average household income is approximately $45,000, with a median disposable income of $29,598. Besides a small shopping center, several corner stores have emerged over the years, an indication that low income inhabitants are thriving, and moving towards self reliance and middle income level (*Overbrook Community Profile*: 2005).

Overbrook High school is ethnically homogenous: 90% of the students are African Americans and by and large, come from the same neighborhoods. The lesser percentage comprises Jamaicans and Africans who have assimilated into the culture of the community so much that it is difficult to determine their original identity from their way of talking or lifestyle; nevertheless, there is high diversity among the staff which comprises, besides African Americans, other ethnicities and nationalities, such as Hispanics and Africans.

The low motivation among students at my school results in a prevalence of poor academic performance, especially in math. The scores of my students on standardized math tests such as the benchmark and Terra Nova have long been low and have further declined within the past two years. Every time I administer these tests, my students show no sign of motivation to complete the math packages. One area in which scores are extremely demoralizing is Critical Response section which requires dealing with word problems. Because my students have not been taught how to translate word problems to mathematical statements, they usually become overwhelmed by the very thought of having to deal with them, resulting in low scores. Despite this consistent lack luster attitude on the part of students towards standardized tests, school district officials still continue to insist on administering the tests every year and to use the scores as yardsticks that categorize students as being below basic, basic, proficient or advanced. My students are usually below basic.

My experience in the classroom has revealed that most of the students who come from middle school to 9th grade are not prepared to do pre-algebra, especially word problems, because they lack analytical and basic arithmetic skills. Although by grade level, they should have acquired basic foundational skills in arithmetic computations at the elementary and middle school levels, they still cannot perform basic operations with fractions, decimals, percents, ratios, proportions and signed numbers when they enter 9th grade.

To this end, I have chosen to design a curriculum unit on percent to assist my students in understanding word problems and be able to translate them into mathematical statements. My choice of percents is due to the fact that a bulk of my students carries out daily financial transactions at corner stores, malls, groceries, and diners where percent calculations are usually commonplace and they can draw on this knowledge.

Knowledge presented

The topics that I will teach in this curriculum unit are based on the concepts of translating word problems, but I will use the concept of percents to articulate this. I chose this problem territory as a strategy to make the learning process intrinsic and more proactive since the percents territory is vast and impacts nearly every aspect of daily life.

Percentage is inseparable from real life situations. In his book "Arithmetic for Parents, Ron Aharoni declares that percentage is the universal language for fractions. He further points out that percentage are tools among other things that enable us to compare fractions as parts of wholes.

The headings under percents that I will utilize in this unit are grouped into three categories or dimensions: Basic knowledge which deals with the manipulation of the three-piece percent formula to find percent when the whole and part are given, to find the whole when percent and part are given, and to find part when percent and whole are given. The topic on percent change or increase and decrease will be addressed. Secondly, I will teach students how to find complementary percents which entails determining remaining or corresponding quantities or values when percent is the given and finding percent when a certain quantity is missing from a whole. Finally, the unit will present a collection of more complex problems such as ratios and mixture problems.

Academic Standards

Pennsylvania Department of Education Academic Standard for mathematics

The Mathematics Standards describe what students should know and be able to do at four grade levels (third, fifth, eighth and eleventh). They reflect the increasing complexity and sophistication that students are expected to achieve as they progress through school.

This document avoids repetition of learned skills, making an obvious progression across grade levels less explicit. Teachers shall expect that students know and can apply the concepts and skills expressed at the preceding level. Consequently, previous learning is reinforced but not re-taught.

Students who achieve these mathematical standards will be able to communicate mathematically. Although it is an interesting and enjoyable study for its own sake, mathematics is most appropriately used as a tool to help organize and understand information from other academic disciplines.

School District of Philadelphia Mathematics Standards

The unit fulfills the requirement of the School District of Philadelphia's Math Core Curriculum for 9th grade and will utilize concepts of modeling and problem solving strategies as a basic teaching tool. The National Council for Teachers of Mathematics Standard (NCTM 1998) sets the purpose of patterns, functions and algebra in mathematics education at all grade levels. Mathematics instructional programs should include attention to functions, symbols, and models so that all students can have various options in problem solving.

These standard objectives set the purpose of patterns, functions and algebra in mathematics education at all grade levels. They use symbolic forms to represent and analyze mathematical situations and structures and they use mathematical models and analyze change in both real and abstract contexts.

Objectives

The primary focus of this unit is to help students internalize the concepts of percents, and to utilize the percent formula in order for them to translate various types of word problems into mathematical statements, and determine their solutions.

The secondary objective of my curriculum unit is to help demystify the issues concerning solving word problems, and to teach students how to translate grammatical sentences into mathematical statements.

Strategies

I will adopt several adjustable strategies to help my students construct the percent problems in the unit as they progress from easy, to not too difficult, to difficult, to very difficult.

Since word problems present challenges beyond the straightforward computation problems, the key feature of my strategies will be constructivism which utilizes students' prior knowledge to help the structure learning through discovery. Constructivism is a set of assumptions about the nature of human learning that guide constructivist learning theories and teaching methods of education. Constructivism values developmentally appropriate teacher-supported learning that is initiated and directed by the student (Ormod, 2006).

According to Ma (1999), U.S. teachers and Chinese teachers do not approach the construction of a topic in the same way. Ma maintains the goal of U.S. teachers is to teach what students are expected to know, especially regarding computational skills, on the one hand. The goal of Chinese teachers, on the other hand, is to help students "know how and know why".

A common technique applied in constructivism is concept mapping. Concept mapping is a technique for visualizing relationships among different concepts. A concept map is a diagram showing the relationships among concepts. In particular, constructivists hold that learners actively construct knowledge. Concepts are connected with labeled arrows, in a downward-branching hierarchical structure. The relationship between concepts is articulated in linking phrases, e.g., "gives rise to", "results in", "is required by," or "contributes to".

Expository teaching of word problems using concept maps

Before the start of every activity, I will teach the whole class as a means of engagement and motivation. This will be in the form of reviewing previous homework problems and providing solutions to warm up exercises. The new concept of analyzing word problems will then be introduced through the use of a concept map which the whole class will construct together.

By expository teaching I will command the attention of the whole class and engage them with mind boggling problems, explain steps and thought processes involved each day before we commence the activities. The teaching of math to these specific students requires teachers to initiate a process by articulating the concepts to be taught with real life examples before they grasp the procedure.

Additional Teaching Strategies

Think-pair-share strategy is a collaborative method which pairs a weak student and a strong student to solve a given problem and share their solutions with the whole class. During the class activities at least two word problems will be assigned pairs of students for them to translate into mathematical statements. The same problem in mathematical form will be assigned another pair of students for them to translate into word problems. The Singapore math books will serve as the major workbook for student activities (Cavendish, 1982).

In applying Cooperative Learning, I will combine the think-pairs into teams of four students each with a specific role and assign the group at least 3 odd number problems. Each group will translate the problems and present or submit them.

Differentiated Instruction will entail the utilization of tier lesson planning. Taking cue from previous years' experience that there are always inclusions or special education students in my classes, I will do differentiated instruction by assigning specific problems and tasks to ensure that my students perform at their appropriate levels of cognition.

Hands on applications such as the use of tables and charts to help them structure word problems will be introduced. Students participating in each of the listed strategies will be required to use charts and hands-on materials such as the KWL and KWC, which are two auxiliaries of concept mapping.

General Exposition

The beginning of every math lesson is usually marked by teacher to student exposition. This is the stage at which the teacher introduces the topic and initiates the dialog on the procedure involved in dealing with the problems. Before engaging the students I usually give them at least three problems which they are require to solve as warm ups. These are intended to be the engaging or motivating activity. One of these problems always serves as a transition to the new concept. For these lessons, I will write three problems, one of which will require converting a fraction to a decimal and a decimal to a percent and the others will be a simple word problem that asks for a part and whole solution. I will then give my students a list of vocabulary of main concept words on the board. Volunteers will give a wild card definition of what they think the words mean or how they are applied. We will proceed to look them up and determine how close our definitions were to the words. We will then discuss the words and I will write their symbolic meanings on the board as an entry point to teaching the concepts.

Explanation of the three-piece-percent formula

Mathematicians and authors of math textbooks concur that the term percent means parts out of one hundred, and usually represents a fractional part of some whole that can be used for comparison. Steddin (2004) discussed percent in the following manner:

> "Percent means 'per 100', so a percent can be thought of as a fraction with a denominator of 100.

> In other words, 15% is the same as 15/100, which can be reduced to the fraction 3/20. Percents can also be written as decimals. The decimal equivalent of 15% is 0.15" (p.99).

According to Stein (1964), percent is the language of business and we use it extensively in our daily affairs. Percent problems usually come as word problems, a topic that is dreaded the most by students in math and arithmetic. I will utilize the three-piece-formula as a template to help students organize the problems by giving an explanation of the template itself.

In applying the template the following rules must be followed:

Given the percent and the whole, multiply them to find the part

Given the whole and the part, divide the part by the whole then multiply by 100 to get percent. Given the part and the percent, divide the part by the percent to get the whole.

a. (Percent x whole)/100 = part
b. Percent = part /whole times 100
c. Whole = (part / percent) x 100

As long as you have two of the three pieces you can plug them into the formula to solve for the third. The key is to properly identify from the formula which piece is which. The key pointer to look out for in applying the template is that percent is the number with the word "percent" or the percent symbol (%) after it. The whole is often referred to as the total and is the number before or after the word "of". The words "what" or "how many" often indicate the unknown value that must be found.

For clarity, I shall label operational signs in the template in their order from left to right, such as the percent (%), the multiplication sign (x), the equal sign (=) and the part as 'n' or just call it 'part'. When I receive feedback that the students understand the formula, I shall re-arrange it in the other forms: whole equals part divided by percent times one hundred, and percent equals part divided by whole times one hundred.

For example, I will begin by writing the problem "what is 20% of 80?" to demonstrate to the whole class that this word problem can be solved using the basic three-piece-percent-formula. We will together solve the first problem to get 16 as an answer. From that answer, we will together solve the next problem; "16 is what percent of 80?", and the third problem will be "16 is 20% of what number?"

Problem Territory and Dimension Discussion

The unit discusses three dimensions of the percent problem territory which includes basic operations that involves manipulating the formula to find part, when the whole and percent are given, to find whole when the percent and part are given, and to find percent when the whole and part are given.

The manner in which the problems will be classified is by progression from easy to very difficult. The groupings shall begin with the basic concepts and understanding of the three-piece-formula (percent times whole = part). The objective of this group is for students to apply basic skills by directly applying the three-piece-percent formula to straight forward word problems. They will demonstrate basic skills involved in manipulating the three-piece-template in order to set up word problems.

The second group of problems which also relate to the first objective involves percent of change, percent increase and percent decrease. The objective of this group is for students to apply the three-piece-percent-template in order to find percents and solutions to word problems that involve change in percents as well as increases and decreases in percents.

The third group of problems progresses to a more complex and challenging means of applying the percent formula. It deals with complementary percents, that is, whereby a percent of a quantity is given and the remainder in fractional portion is required to be calculated. It also requires students to give the percent when the fractional portion of a quantity is given. The objective of this group is for students to apply the basic concepts of percents to solve more challenging word problems involving complementary percents. Students will solve percent word problems with one step and also with multiple steps.

Analysis of problems A1 to A8: (Basic Concept Problems)

This dimension of problems involves finding percents of numbers by applying the three- piece-template. The key skill involved, as with all word problems, is to identify the various known and unknown parts of the problem that can fit into the logical trend of thought, a formula or a template before carrying out the computation. For problems A1 to A8, three basic skills are required; the most fundamental procedure is to find the part of a whole quantity when the percent is given. This is straight forward because it involves the application of prior knowledge from converting simple percent to decimal first, then multiplying the result by the whole, to get the part. Specific examples of the application of this skill are problems such as:

a. What is 20% of 400?"
b. Linda treats Winston to lunch at Nushboy's Pizza and receives a bill of $35. She decides to leave a tip that is 18% of the bill they receive. How much did she leave for tip?"

These two examples have two given quantities, the percents and the wholes, and one unknown, respectively, and can be resolved by multiplication. Also, they are both of the same mathematical form: given the whole and a percent, find the part.

The second basic skill requires more thinking as it is not straight forward but necessitates some manipulation of the percent template and division. This objective is to isolate the quantity that is not given or required to be determined, which can be accomplished by substituting the given quantity in the template, then dividing it by what it is equal to. It is customary for the unknown or isolated quantity to remain on the left side of the template.

This skill is applicable in the case of problems:

A2:80 is what percent of 400? And

A5: Yarvoe takes Togar for a treat at Martin Luther King convention Hall. After the program, they pay a total amount of $65 and tip of $4.00 to the parking lot security, what is the percent of the total amount that was tipped?

Each of these two problems gives part and whole, but percent is not given. In this case, you use section 'b' of the template by dividing the whole by the part and multiplying by 100.

The third basic skill requires much more thinking and some manipulation as in the case of the second skill since the percent and part are given and the whole is the unknown. Two example problems to which this applicable are, A3: 80 is 20% of what number?

20% is changed to a decimal to read .20 and used to divide 80 to get 400.

Problem A6 is similar:

Moses treats Neekoyu to lunch at Six Flags Diner. He decides to leave a tip that is 15% of the bill they receive. If Moses leaves a tip of $7.50, what was the total amount of the bill they received?

The same method applied to A3 can be used to resolve this problem.

Analysis of problems B1 to B10 (Percent change; increase or decrease)

In the case of problems B1 to B10, the three-piece template is applicable. It must be noted that these problems either have a percent given with another quantity or two quantities given with the percent to be determined. This dimension is different from the first in several ways and must not be mistaken. Although these problems have "given" values they are rather asking for operations to be done on the values which are "not given" first, before applying the template. Traditionally in percent change, subtraction is the necessary operation that must be carried out before the percent template is applied. The operations involved in solving problem B8 to B10 are similar to the problems in B1 to B7. The difference is that only numbers are given with no percents, also, it is not possible to directly insert the given numbers into the percent formula.

Set A

1. The average attendance at a basketball game this year was 35,000 people. This was an increase (percent change) of 12% from last year. What was the average attendance at a basketball game last year?

First we must recognize that there are two given values, a percent and a number; we must also consider the size of the answer. Since the attendance after an increase is 35,000, the original attendance must be less than 35,000. Also since the attendance of 35,000 represents a 12% increase from the previous year, it represents 100%+12% which gives 112% of last year's attendance. So it is this new percent or increase that is applied to the percent template and not the 12% representation.

Given: Increase this year

Given: Attendance last year

Not given: Percent increase

Using the template

112 % x last year's attendance = 35,000

1.12 x last year's attendance = 35000

Last year's attendance (lya) = 35,000\1.12 = 31250 people

Solution: Since the attendance of 35,000 represents 12% increase from last years, it represents 100% + 12% = 112% of last year's attendance. Therefore, percent and the whole are given and we must find the part by division.

Using the template (Part A),

112 % x last year's attendance = 35,000

1.12 x last year's attendance = 35000

Last year's attendance (lya) = 35,000\1.12 = 31250 people

2. The average attendance at a basketball game this year was increased from 31,250 to 35,000 people this year. What was the percent of change?

To get the number of increase, we subtract 31250 from 35,000, to get 3750.

Given: Increase this year

Given: Attendance last year

Not given: Percent increase

Template (part B):

% x whole (last year's average) =part; hence,

% x 31,250 = 3750

Percent of change = 3750\31,250 = .12 = 12%

For example, the following three problems show the variants in the use of the template:

a. The average attendance at a basketball game this year was 35,000 people. This was an increase (percent change) of 12% from last year. What was the average attendance at a basketball game last year?
b. The average attendance at a basketball game this year was increased from 31,250 to 35,000 people this year. What was the percent of change?
c. The average attendance at a basketball game this year increased by 3750 people. This is 12% of last year's average attendance. What was last year's average attendance?

The main concept involved with the problems in this dimension is that with some of them, you add or subtract the percents before applying the percents template, and with others, you add or subtract the quantities before applying the percents template. Symbolically, the formula can be represented as:

(Qnew - Qold)/ Qold = percent change/100

The other type of problems in this group is to subtract a given percent from 100% before applying it to the template.

Example: Sixty percent of the students in a class are boys. If 12 of the students in the class are girls, what is the total number of students in the class?

In this problem the missing portion of the template is the whole. Since according to the problem, 60 percent the students are boys, we need to find the other 40 percent to make a total of 100 percent. The given information is that 12 of the students in the class are girls; hence, that is the part. The next step is to find the percent; since 60% of the students are boys, the task is to find out how much percent makes up the 12 girls. Subtracting 100% - 60% gives 40%. It is this new percent (40%) percent that will be applied to the percent template and not the 60%.

Analysis of problems C1 to C4 (Complex and complementary percent change problems)

The key word involved in dealing problems C1 to C4 is critical thinking, because these problems require extra care before applying the percent template. Most of the process involves applying all of the four basic operations of arithmetic; a common characteristic of these problems is that they may have four given values comprising two numbers and two percents. With this data, the problem either asks for calculation of a percent increase or decrease. The way to go about this is to apply the necessary addition and subtraction to obtain the refined data that can be inserted in the (compound) template.

An example is problem C1, which has two given numbers and will give percents.

There were 24 boys and 20 girls in a chess club last year. This year the number of boys in the club was increased by 25% but the number of girls decreased by 20%. How many boys represent an increase of 25% and how many girls represent a decrease of 20%? Find the overall increase or decrease in the membership of the club.

To analyze this data, requires the introduction of an extension of the percents template where you first calculate the overall increase or decrease that is required: Overall increase or decrease = % (whole of first) + % (whole of second). Because there was an increase in the percent of boys, 25% must be added to the original percent of 100% to give 125% or 1.25, and because there was a decrease in the number of girls, 20% must be subtracted from the original percent of 100% to give 80% or .80. 100 - 20 = 80. These values can subsequently be inserted into the template to yield 1.25(24 boys) + .80 (20 girls) =30 + 16 =46 (boys and girls). Finally to get the overall increase, you subtract 44 from 46 to get 2 members.

The other form in which complex percent change problems appear is when the problem asks for a percent change when only four numbers are given with no percent value given. These types of problems require the application of the percent equals part divided by whole portion of the template (% = part / whole), but must be in a compound form or applied two times. Since the four numbers are usually part and wholes, it is only logical to divide the corresponding parts by their wholes, multiply by 100 to get the percent. The next step is you may have to add respective calculated percents to 100% if it is an increase and subtract it from 100% if it is a decrease.

An example of this group of problems that have this form is C2. This problem suggests the application of the % increase =part/whole portion of the template but has to be compounded or applied twice. An increase in the number of boys by 6 implies the form Percent increase =6\24 = .25 = 25%=1.25(24) = 30, and percent decrease in the number of girls by 4 suggests the form % decrease =part\whole or percent decrease =4\20 =.20=20%, but 100-20 = 80% so, .80(20) =16.

Analysis of problems C5 to C10 (finding percents of percents: percents without a whole)

This group of percent problems involves much more critical thinking because they are questions that involve situations where you are not given a whole. The major key about this group is that you are told about a percent change or changes and asked what effect the change or changes will have on the whole. To solve these problems where the "wholes" are not given presents a high level of complexity. This verbal description is not too helpful. For example:

Increasing a number by 20% and then increasing this result by 10% is the same as increasing the original number by what percent?

The practical approach is to make up a number for the whole since you do not have one to meet the criteria for the percent template whose basic format is % x (whole) = part. The number that you make up will be used to apply the percent change(s), and see what result you get. A convenient choice for the whole often is 100. If the question has multiple choices, use this same number in each answer and select the one that matches your result. It doesn't matter what number you use as long as you use the same number through out the problem, your result will be correct.

To demonstrate this concept, a step by step analysis of the example problem would help. First, here is a high tendency to make the common mistake that this problem requires you to think that an increase of 20% followed by an increase of 10% increase is the same as a 30% percent increase. This is not the case because only percents of the same whole can be added. In this problem, the 20% increase is applied to the original whole but the 10% increase is applied to the larger whole that is applied to the first increase.

The problem can now be solved by making up a whole and seeing what happens; since this is a percent problem, the whole that should be made up is 100. Since 20% of 100 is 20, increasing 100 by 20% results in a new whole, 100 + 20 = 120. Similarly, since 10% of 120 is 12, increasing 120 by 10% results in the new whole 120 + 12=132. Since the original whole was 100, this represents an increase of 132-100=32. Since 32 is 32% of 100, this is the same as increasing the original number by 32%.

Analysis of problems D1 to D6

This group of problems deals with simple application of the percents template. The distinguishing common feature is that they calculate percents of earnings, purchases and sales taxes. The problems in this group, similar to the other groups, progress from the simple to the difficult and usually require addition and subtraction operations to be performed in some cases. Sometimes your commission rate increases as your sales increase, resulting in a graduated commission. This is a more difficult approach which offers a different rate of commission for each of several levels of sales. According to Lange et all (1989), "The graduated commission provides an extra incentive to sell more. It is determined by adding the sums of commissions for levels of sales." The template is Total graduated commission = Sum of commissions for all levels of sales.

An analysis of problem D1 will give a better understanding of this concept: "Eric Jackson sells appliances at Four City Sales. He receives a graduated commission of 14% on his first $1000 of sales, 16% of the next $1500, and 18% on sales over $2500. Eric's sales for the past month totaled $4578. What is his commission for the month?"

The major feature about this group of problems is to initially apply the first level of the regular percents template (% times whole = part) to all of the various commissions, then simply add the results. Where subtraction is necessary is when the sum of the levels of previous sales is less than the current sales; hence, in the sample problem, 14% commission of $1000 is $140; 16% commission of $1500 is $240; For the 18% commission portion, you have to subtract $2500 from $4578 before applying the percent to the difference of

$2078, to get $374.04, because $4578 is greater than the sum of Eric's last two commissions. The total graduated commission is hence, $140+$240+374.04 = $754.04.

Note: Another way to compute the commission, which is more informative, is to calculate the 14% commission on the full amount: $640.92. Then compute the additional 2% on the $1500: $30. Then compute the additional 4% on the amount above 2500: $83.12. Then add. This shows that the graduated structure earns Eric an extra 113.12 over a straight 14%.

Classroom Activities/Lesson Plans

Lesson Plan 1 (Concept Map Construction)

Objective: The objective of this lesson is to introduce students to the basic concepts of percents in order for them to utilize it in solving word problems. It shall help students exercise the craft of translating word problems to mathematical statements. It will prepare students on the lesson involving the application of basic skills three-piece-percent-formula to straight forward word problems.

Materials: Overhead projector, transparency paper, markers, poster sheets, handouts, math journals, and scotch tapes.

Strategies: Constructivism (Concept mapping): Tapping on the available knowledge students already have to build new knowledge profile by using a concept map as hands-on.

Motivation: I will begin the lesson by engaging students in doing computational warm-up exercises that involve a review of changing fractions to decimal and decimals to percents. The problems will be placed on the overhead projector for the whole class to solve. After at most 10 minutes, I will ask volunteer students to share their answers with the whole class. I will award points and guide the class in solving the trouble problems before moving on to the new concept.

Engagement: I will ask around the class if any body can write any of the percent problems and relate it to their own real life situation. For example, 50% of 100 is what? After receiving the answers, I will write Mom gave my little brother and me $100. She said 50% should be given to my little brother. What is 50% of $100?

Elaboration/Teaching: I will first teach the class about percents and how to use the percent template to set percent problems. I will break the class up into groups of three and provide several words that relate to percents, decimals, and fractions. I will then place an empty concept map on blackboard and ask a volunteer to pass out hand outs with the same concept map that is on the blackboard.

Cooperative instruction: Each group will connect the concepts and fill out the empty spaces in any logical form they think is right. For instance, a word problem or mathematical problem will be given. The group will be required to direct a path or connect steps that need to be taken to set up the problem by using the concept map. Each group will post their work on the board for the class to see and make comments.

Assessment (Reading inn the content area): At the end of the lesson each student will write in their math journals and answer the following question in two good grammatical paragraphs: What did you learn today?

Homework: Write two word problems that relate to yours or somebody else's real life transaction.

Lesson Plan 2 (Snowball Effect Activity)

Objective: The objective of this lesson is to provide students with basic hands on and step-by-step skills involved in solving percent word problems.

Materials: Overhead projector, transparency paper, markers, handouts and math journals.

Strategies: Constructivism: Tapping on the available knowledge students already posses to build new knowledge profile by engagement.

Motivation: I will begin the lesson by engagement. I will review previous day's homework with the students. The homework problems will be placed on the overhead projector for the volunteers to demonstrate how they solved the problems. After at most 10 minutes, I will take over the class and make the necessary corrections and award points for participation.

Engagement: I will introduce the "snowball effect" as the motivating activity. I will write the three forms of the percent template on the black board or overhead. Then I will pass out plain sheets of papers and ask each student to write a word problem that can be solved by utilizing at least one of the template forms. I will ask all of the students to crumble or roll up their problems. They will toss the rolled up problems to the front of the class like the snow falls.

I will then ask everyone to get out of their seats and pick up take a rolled up problem from the lot and solve it. Each student should be prepared to come up and solve their problems because I will call upon them at random.

Elaboration/Teaching: I will observe the difficulty level and solve some sample problems but will not solve problems the students have picked.

Assessment (Reading in the content area): At the end of the lesson each student will write in their math journals and answer the following question in two good grammatical paragraphs: What did you learn today?

Homework: Write three word problems, one that can be solved by utilizing each of the template forms.

Lesson Plan 2 (Blind Pick Competition)

Objective: The objective of this lesson is to provide students the opportunity to demonstrate their skills in dealing with percent word problems in order to reinforce and assess their understanding of how to apply the percent template.

Materials: Overhead projector, transparency paper, markers, handouts, stick-on pads and math journals.

Strategies: I will use cooperative group instruction. I will place students in groups of fours by combining weak students with the strong.

Motivation: I will begin the lesson by engagement. I will review previous day's homework with the students. The homework problems will be placed on the overhead projector for volunteers to demonstrate how they solved them. After 10 minutes, I will take over the class and make the necessary corrections and award points for participation.

Engagement: I will then place at least 50 stick-on pads on the blackboard with word problems that are assigned points ranging from 5 to 20 according to difficulty levels. The stick-on pads will have color codes such as green for easy, yellow for difficult, and red for very difficult. Each group will take turns and select a problem worth any point from the blackboard. Each group is expected to solve a number of problems worth at least 70 points within the 30 minutes that that we will conduct the exercise, to get a C; no score of D or 60 points will be accepted as passing. If a group acquires more than 100 points they can choose to reserve at least 30 for the next duel date.

Elaboration/Teaching: I will observe the difficulty level and solve similar sample problems including some of the problems the students picked.

Assessment (Reading in the content area): At the end of the lesson each student will write in their math journals and answer the following question in two good grammatical paragraphs: What did you learn today?

Homework: Write six word problems that can be solved by utilizing each of the template forms. Two problems for each form.

Annotated Bibliography

Works Cited

Abramson, Marcie, Painless math Word Problems, Barron's Educational Services, Inc. (2001)

This book shows you how to see patterns in math word problems and presents various strategies in tackling them.

Aharoni, Ron, Arithmetic for Parents: A Book for Grownups about Children's Mathematics, Sumizdat, (2007)

This book presents a workable design for teaching word problems from an early stage where parents' involvement is involved.

Burns, Marilyn, The Math Solution: Teaching Mastery through Problem Solving, Marilyn Burns Education Associates (1984)

Burns provides justifications and strategies on teaching word problems before computation math as the best pedagogical practical.

Fearon, Globe, Success in Math: Consumer Math, Global Fearon Educational Publisher, (1996)

This book presents a huge number of consumer math problems that involve daily business transactions in which percents are used.

Get Wise! Mastering Math Word Problems, Steddin, Maureen, The Thomson Corporation and Peterson's, (2003)

A simple math made easy book that solves math word problems in clear and simple steps that are ideal for all levels.

Lange, Walter, H., Robert D. Mason, Temoleon G. Rousos, Consumer Mathematics, Glencoe Division of McMillan/McGraw-Hill School Publishing Co. (1992)

This book presents a huge number of percent problems that involve calculating commissions and discounts.

Ma, Liping, Knowing and Teaching Elementary Mathematics, Lawrence Erlbaum Associates, Inc (1999)

This book deals with word problems from the conceptual standpoint and provides explanation of the trend of thought structuring arithmetic problems.

Marshall Cavendish Education, Primary Mathematics 6A Textbook, SingaporeMath.com Inc, (2003)

This is a practical hands-activity book which provides practical procedures in dealing with percent word problems.

Overbrook Community Profile: 2005 House Values, Inc. Aerial Imagery 2005 Air Photos USA, LLC.

A brief statistical information on the demography of Overbrook community in West Philadelphia. This viable statistics helps explain the lifestyle and culture of the schools around West Philadelphia area.

Annotated Teachers' and Students' Reading List

Bailey, Rhonda et all, Glencoe Mathematics: Applications and Concepts-Course 2, The McGraw -Hill Companies, Inc. (2004)

This is the prescribed textbook for 7th graders of the P. G. County Schools in Maryland.

Percents are addressed extensively in this text which serves as a valuable resource for

teachers and students.

Dudeney, H. E., The Canterbury Puzzles and other Curious Problems, Dover Publications, Inc., (1958)

This book is Consists of numerous math mind boggling problems that would prepare

students for more analytical thinking.

Ellis, Hollowell, Kennedy, and Schultz, Holt Algebra1, From Patterns to Algebra, Holt,

Rinehart, Winston (2004)

This is the prescribed textbook for 9th graders in the School district of Philadelphia. It does not detail percents but contains challenge problems that help students analyze their thoughts.

Entertaining Mathematical Puzzles, Dover Publication, Inc, (1961)

This is a compilation of word problems that introduce students to the craft of math

translation.

Gardner, Martin, Mathematics Magic and Mystery, Dover Publications, Inc., (1956)

This is a compilation of word problems that introduce students to the craft of math

translation.

Marshall Cavendish Education, Primary Mathematics 6A Textbook, SingaporeMath.com Inc, (2003)

This book is a simple but thorough compilation of basic strategies that can be applied in solving word problems in all forms.

APPENDIX A - ACADEMIC STANDARDS

Pennsylvania Department of Education Academic Standard for Mathematics

. Numbers, Number Systems and Number Relationships
. Computation and Estimation
. Measurement and Estimation
. Mathematical Reasoning and Connections
. Mathematical Problem Solving and Communication
. Statistics and Data Analysis
. Probability and Predictions
. Algebra and Functions
. Geometry
0. Trigonometry
1. Concepts of Calculus

National Council of Teachers of mathematics Standard 2 (1998)

Equations: Patterns and Functions
Algebra and Functions

APPENDIX B

Percent Word Problems

A1 Basic Percents Template Concepts Problem
a. What is 20% of 400?
b. 80 is what percent of 400?
c. 80 is 20% of what number
A2
a. Linda treats Winston to lunch at Bassa Pizza and receives a bill of $35. She decides to leave a tip that is 18% of the bill they receive. How much did she leave for tip?
b. Linda treats Winston to lunch at Bassa Pizza and receives a bill of $35. If she left $6.30 as tip, what was the percent?
c. Linda treats Winston to lunch at Bassa Pizza and left a tip of $6.30. If this was 18% of her bill, what was her bill?
A3
a. Tanya takes Huwerl for a treat at Martin Luther King convention Hall. After the program, they pay a total amount of $65 and tip of $4.00 to the parking lot security. What is the percent of the total amount that was tipped?

b. Moses treats Angel to lunch at Six Flags Diner. He decides to leave a tip that is 15% of the bill they receive. If Marie leaves a tip of $7.50, what was the total amount of the bill they received?
A4
a. If 18 of the 45 students in the band play wood string instruments, what percent of the students do NOT play wood string instruments?
b. If 40% of the 45 students in the band play wood string instruments, how many boys played wood string?
c. If 18 students of a total number of students play wood string, and this constitutes 40%, what was the total number of students?
A5
a. Moses needs to get at least 85% of the questions on her final exam correct in order to be noted honor roll. If there are a total of 60 questions on her final exam, at least how many questions must she answer correctly?
b. Moses has solved 51 problems out of 60 problems on his final exam. What percent has he solved so far?
c. Moses solved 51 problems out of a set of problems he has to solve on his final exam. If this constitutes 85% what is the total number of problems?
A6 Doreen Steele sells used cars for Bonded Auto. She receives a straight commission of 5% of the selling price of each car. What commission will she receive for selling a $2495 compact?

Percent change: Increase or Decrease Problems

B1
a. The average attendance at a basketball game this year was 35,000 people. This was an increase (percent change) of 12% from last year. What was the average attendance at a basketball game last year?
b. The average attendance at a basketball game this year was increased from 31,250 to 35,000 people this year. What was the percent of change?
c. The average attendance at a basketball game this year increased by 3750 people. This is 12% of last year's average attendance. What was last year's average attendance?
B2
a. In a class of 20 students, 60% are girls. How many students are girls?
b. There are 20 students in a class. Of the 20 students, 12 of them are girls. What percent does this represent?
B3
a. 5000 people visited a book fair in the first week. The number of visitors increased by 10% in the second week. How many people visited the book fair in the second week?
b. The number of people visited a book fair increased by 500 people. If this represents a 10% percent change, how many people visited the fair during the first week?
c The number of people that visited a book fair increased from 5000 in the first week to 5500 in the second week. What percent change does this represent?
B4 On the first day of school this year, 435 students reported to Howard Middle School. Last year on the first day, 460 students attended. Find the percent of change for the first day attendance. Round it to the nearest whole percent if necessary.
B5
a. The usual price of a washing machine was $400. At a sale, the price was reduced by 25%. What was the price of the washing machine at the sale?

b. The usual price of a washing machine was reduced by $100 during a sale. If this is 25%, what was the usual price?
c. The usual price of a washing machine that cost $400 was reduced to a sale price of $300. What was the percent of change?
B6 Julie plans to buy a new computer with a flay screen that costs $1,299. She lives in Florida where there is a sales tax of 6%. Calculate the sales tax by finding 6% of $1,299. What will be the total cost including the sales tax?

Complex and complementary percent problems

There are 25% more girls than boys in a club. If there are 36 more girls than boys, how many boys are there?
There were 24 boys and 20 girls in a chess club last year. This year the number of boys in the club was increased by 25% but the number of girls decreased by 20%. How many boys represent an increase of 25% and how many girls represent a decrease of 20%? Find the overall increase or decrease in the membership of the club.
mplate A
x whole = part
5 x 24 = 6 boys
0 x 20 = 4 girls
mplate A compounded
erall increase or decrease =% X whole of first + % X whole of second
increase in boys membership = 100% +25% = 125% or 1.25
decrease in girls membership =100% - 10% = 80% or .80
ys + girls membership=24 + 20 =44
5(24) + .80 (20) =30 + 16 =46
Last year a chess club comprising boys and girls had a total membership of 44. If there are 16 girls this year representing a decrease of 20% in the number of girls, how many girls were in the club last year? (Whole = 16/.80).
Last year a chess club comprising boys and girls had a total membership of 44. If there are 30 boys this year representing an increase of 25% in the number of boys, how many boys were in the club last year? (Whole = 30/1.25).
There were 24 boys and 20 girls in a chess club last year. This year the number of boys in the club was increased by 6 but the number of girls decreased by 4. What percents of change do the increase and decrease represent?
whole/part) or % = 6\24 x100 = 25% increase
= whole/ part or % = 4\20 x 100 = 20% decrease
This year the number of boys in the club was increased by 6 new members which is 25% of the boys membership for last year, but the number of girls decreased by 6 which represents a 10% decrease in last years membership. What were the respective memberships for boys and girls last year?
60% of the students in a class are boys. If 12 of the students in the class are girls, what is the total number of students in the class?"

Percents of Percents: Percents without Whole Problems

C5 Increasing a number by 20% a then increasing this result by 10% is the same as increasing the original number by what percent?
C6 The price of a stereo is reduced by 20% for a sale. By what percent must the sale price be increased

to return the stereo to its original price?

C7 The price of a share of stock increases by 20% on Monday. On Tuesday, the price of the share of stock increases by further 15%. This is equivalent to the original price of the stock being increased by which percent?

C8 A number is increased by 10% and the result is then decreased by 25%. This is the same as decreasing the original number by what percent?

C9 At a certain store, the price of a handbag is reduced by 25% for a sale. A week later this sale price is reduced by an additional 25% for an end-of-season clearance. What percent of the original price is the clearance price?

C10 Decreasing a number n by 10% and then increasing the result by 30% is equivalent to:

(A) Decreasing n by 40 percent

(B) Decreasing n by 17 percent

(C) Increasing n by 10 percent

(D) Increasing n by 17 percent

C11 If you buy a shirt at $50, plus tax of 8%, and get 20% off the total bill, do you do better or worse than if you buy a shirt at $50, get 20% off, and then pay 8% tax on the net bill?

Graduated Commission Problems

D1 Eric Jackson sells appliances at Four City Sales. She receives a graduated commission of 14% on her first $1000 of sales, 16% o the next $1500, and 18% on sales over $2500. Eric's sales for the past month totaled $4578. What is his commission for the month?

D4 Irene Tomas sells office machines. She is paid a commission of 7% of her first $2000 in sales during the week and 8.5% on all sales over $2000. What is her commission in a week in which she sells $2925 worth of office machines?

D5 Charles Westerly sells computer hardware for a computer firm. He is paid a 4% commission on the first $6000 of sales, 6% on the next $4000, and 8% on sales over $10,000. What is his commission on $13,550 in sales?

D6 Steve Borden sells home appliances at Fletcher Sales. The rate of commission is 8% on the first $1000, 10% on the next $1500 and 15% on all sales over $2500. Last week, Steve's total sales were $1975.00. What was his commission for the week?