Curriculum Units by Fellows of the National Initiative
2007 Volume VI: Keeping the Meaning in Mathematics: The Craft of Word Problems

Word Problems: Looking for the Similarities in Problems to Help Categorize and Solve Them

Curriculum Unit 07.06.09, published September 2007
by Patricia Marasco

Objectives

The purpose of this unit is to supple students with the basic tools and ideas they will need to begin their study of linear equations. I will help students go from simple to more complex problems by having them recognize the similarities and differences between problems. I believe that once a student can recognize and categorize the given problems, then will they feel confident enough to attempt to solve theses and the more complex problems we will study in the latter part of the year. This is just another way of saying we are going to categorize or group problems. It is important for students to recognize what "red flags" there are in each problem that categorizes a problem as such, by identifying the subtle differences or similarities. This is a good exercise for students of any age. I benefit from the idea of having to categorize problems because I am forced to stop and think about the problem. Then I think about what I am trying to accomplish before I even begin. I hope to get my students to stop and think about the problem and the process. This will hopefully lead them to a better understanding of math, as well as more success at solving word problems.

We have all met the student who looks at a problem and instantly determines he/she either can or can not do the problem - end of story! There is not much thought or questioning in this response - it is a knee jerk reaction to the problem. This reaction happens more in mathematics than in other classes. If a student is asked a question in a history class and they do not know the answer, their first reaction is to go back in the book and look for the answer. The same is true in literature and world language classes. In math students have a very different response. They determine that they can or can not do the problem. That is it. There is no going back in the book to look for the answer. I can sympathize with this issue as many math texts are poorly written. I am a math teacher, and I have a hard time finding different examples of problems I am discussing in the poorly organized book. I am hoping to change that for my student. This unit is designed to help my students recognize various problems and use several different tools to derive at an answer that makes sense. In addition, I hope to add some fun to their math experience.

It is fun to do things that you are good at and you only get good at something by doing it. If my students find success with word problems from the beginning they will be more inclined to attempt them, thereby strengthening their skills. Success breeds success.

I will be presenting three types of problems and each could be solved several different ways. Applying variables, using substitution to eliminate one of the variables, combining variables, solving for one variable and plugging it into one of the equations to solve for the other variable, are several methods that we will work our way towards. The students will find this sm"rgÃ¥sbord of options for solving confusing at first, so I will be introducing a less abstract way of looking at the information first and then progress to these method.

My target students are freshman, ages 13 to 14. They will be coming from various feeder schools with a wide range of skill levels. It is unwise of me to assume that they have a solid understanding of linear equations. I have found that it is worth my time to check skill levels early and often. I spend the beginning of every school year going over the basic skills needed to move on in the Algebra 1 class. This lesson is designed to be used at the beginning of my Algebra 1 class, the first week/weeks of class. I hope to build a solid foundation for students for their present and future math classes. Students are really learning a new language, the language of mathematics. There are many different dialects and different ways to say or ask the same question. I can not expect my students to answer a question if they do not understand what I am asking them. For this reason, I will spend time on language and math jargon. For example, I want my students to know that "increased by" and "more than" are just different ways of saying add and that when someone says double they are just multiplying by two.

After clarifying the language we will be using, I will then focus on another valuable tool for beginning word problems and that is sketching a picture of the problem. This simplistic visual image of the problem will enable my students to organize the information of the problem better in their minds. Drawing a simple line can enhance their comprehension level immensely. It is amazing what a line can represent. I will demonstrate problems when a line represents a log cut into sections or that same line can represent the amount of apples and oranges in a box. The use of this visual will be helpful in understanding, writing, and solving the equations.

I also plan on teaching the students to use "mental math" as a tool. Instead of concentrating on the actual numbers in the problems, which may be confusing, I sometimes simplify the numbers so I can do the math in my head. This trick helps me check if I am headed in the right direction because I can easily figure out if my answer makes sense. I have a few other tricks up my sleeve which I will go into more detail later. However, the whole point of this unit is to give students the tools, the confidence and the success needed to become life long learners. Like I said before, success breeds success.

Strategies

My first strategy requires that the students learn basic *math terminology*. So I will begin my lesson by going over the language of math. I think that there are too many assumptions by math teachers. It would be unwise of me to assume that everyone has been introduced to linear equations and the language associated with them. That is why I think is important to clarify the obvious sometimes, I will go over all the various ways to describe the operations we will be using to solve our problems. I look at this lesson as one of those "stitch in time, saves nine" ideas. It will also send a very powerful message to the students and that is that there is no question that is to simple to ask. This quick review will make it easier for my students to understand the questions being asked in word problems. I will increase my students ability to recognize the mathematical operation by increasing their familiarity with the language.

Students will list all the ways they know of to say add. The list should include the mathematical terms along with a mathematical phrase to show understanding. The first mathematical term on the list will be "plus", and the mathematical phrase will be something like "three plus two". The mathematical term "increased by" will have a mathematical phrase something like "the number is increased by 7". The mathematical term "more than" can be accompanied with the mathematical phrase "Thomas has 8 more than John". The mathematical term "combined" may be accompanied by the mathematical phrase "Trish and Jes combined their money". The mathematical term "together" and a mathematical phrase such as "together we have 7 books" will demonstrate understanding, as would the mathematical term " total" combined with the mathematical phrase "the total is 14". The list should also include the mathematical term "sum" with a mathematical phrase "the sum is". The mathematical term "added to" along with the mathematical phrase "five added to nine" will give us a rather complete list of mathematical term/phases to helps students recognize addition in a word problem. Students will be encouraged to add to the list at any time in the future we encounter another example.

Student will make a second list that connects the mathematical terms for subtraction to a mathematical phrase that demonstrate understanding of both concepts. The mathematical term "minus" along with the mathematical phrase "three minus nine" demonstrates understanding. The mathematical term "decreased by" and the mathematical phrase "The class was decreased by 14 leaving only 31 remaining" also demonstrates understanding. The mathematical term "less" and the mathematical phrase "my pay is less than yours" demonstrates not only understanding but disappointment. Finally the mathematical terms "fewer than" and "take away" when paired with the mathematical phrases "there are fewer flowers than plants" and "if I take away 7 from the original group I will only have 91 remaining", should give the students a beginning list of terms for subtraction.

Students will make a third list once again connecting mathematical terms with mathematical phrases but this time we will be covering the mathematical operation of multiplication. The mathematical term "times" and the mathematical phrase "two times" demonstrates understanding. Pairing the mathematical term "multiplied by" with the mathematical phase "multiply three by" along with the mathematical terms "double, triple and quadruple and the mathematical phrases "twice as many, three times as many and four times as many" will also show understanding. The final and often the most overlooked or misunderstood mathematical term "of" will be strongly emphasized because of its importance in fractions. If my students can understand that 1/2 of 1/2 is 1/2 times 1/2, or 1/4, and understand the math that they need to arrive at the answer, then they will have a much easier time in the future when we are doing a/b. Once again, this list may be added as the lesson progresses, but for now I think we have enough information to continue on with the final operation and that is division. We will use division to "undo" multiplication because they are inverses of each other so this connection must be noted here.

Just like before, students will compile a list of a the mathematical terms that define division and connect them with a mathematical phrase to demonstrate understanding. The mathematical term "quotient" and the mathematical phrase "the quotient of ten and five" demonstrates understanding as does the mathematical terms "shared/split" and the mathematical phrase "we split or shared the pizza".The mathematical term "percent" can be represented by the mathematical phrase "piece of 100". This list will be a good start for students understanding of language and can be added to as the lesson progresses.

The next tool or strategy I will introduce is the *power of a sketch*. This is second only to understanding the vocabulary in its importance as a tool for solving math problems. When you were in first grade you were given visuals to help you understand the idea of addition and subtraction. It helped you then, why wouldn't it help now? The drawing or sketch can be as simple as the ones you drew in first grade too. These visuals will give us an image of the problem. I believe that we have learned all of the basic math skills we need (addition, subtraction, multiplication, division) by the time we are in third grade. So when in doubt go back to basics. I will use simple line sketches to demonstrate something like, Kristen's book and Paula's book together are 36 books, or ____ + ____ = 36. The same simplistic idea can be used for many problems and students can use this to scaffold information as they gather it. The idea is to start with a simple true representation of the problem. When I learn more about who has more books or if the books are evenly divided then I can alter my sketch.

My third strategy is to help students to *identifying the variable,* and also stress the importance of clarifying what the variable really represents. We will become the detectives in the story and eliminate the suspects. This narrowing down of information will ultimately make our job easier so that we can "solve the case". To get my students to do this we will begin by identifying what they do not know. I know this seems backwards but just trust me on this. The usual response will be something like, "I don't know anything about the second side," or "I don't know how many apples there are." Great, now we all know that in algebra we call the unknown X. Once they have identified the unknown they have their entry into the problem and we can begin scaffolding. If I know that the first side is two times longer than the second side, or I have twice as many oranges as apples - all I have to do is take the second side or the number of apples (which we are calling X) and double it. This we will call 2X. Using this process, my students will have an easy way to approach the problem and be less likely to give in to the knee jerk reaction of just making no attempt.

I will give my students one final tool - *success!* I know as adults we find it hard to believe that students would not want to see if their answer is correct, but trust me, students are sometimes just so happy to have come up with any answer, that they gladly accept it, right or wrong. Let us have a little sympathy for them. Instead of getting upset with them, show them that checking answers is like getting paid at the end of a long work week. The satisfaction in getting the correct answer and knowing it is correct may not be as rewarding as a paycheck, but it is a mental pat on the back when a student checks his/her work and knows it is correct. It just feels good. I want to make sure that my students feel good about the hard work that they do. I want them to experience success.

Clearly, I plan on using very basic tools to assist my students in the process of unraveling word problems. Know the language, draw a sketch, organize the facts, tell me what you do not know, and reward yourself!

Let us put this plan to the test. It is time to take it to the classroom and check out the strategies. I will begin with three simple problems in which the students can use the tools outlined above. I will refer to the first sample problem as a **Picture Problem** because we will draw a sketch or picture to make solving it easy. I will have another problem that I will refer to as an **Equal Problem** because we will be setting it equal to each other to solve. Finally, I will refer to my last problem as a **Two TypesProblem.** For this problem we will be marrying two different types of information together before we can solve it. Below are examples of each type of problem. I have attached many more examples in the appendix that I will use and refer to through out this lesson.

Picture Problems

Students should look for the "red flag" to help them identify Picture Problems. Picture problem are problems that have several pieces that are combined together to get one total. In the case below we are trying to find the lengths of two pieces of string that together total 126 centimeters. This type of problem is easy to solve with the assistance of a simple sketch. A more extensive list of Picture Problems is listed in the appendix.

Amy cut a string that was 126 centimeters long into two pieces so that one piece is twice as long as the other.

How long is each piece?

Students will be asked to find any of the mathematical terms or phrases that we have discussed. Relying on their mathematical term/phrase list (*stratagie1- language*), "twice as long" is the intended response.

Students will be instructed to sketch a simple line to represent the two pieces of string (*strategy 2 - sketch)*. The sketch should have this information _______/______=126, and represent a true statement. It is not complete, but it is true, because it shows the two pieces of string and they are equal to 126. We will adjust the sketch as we learn more information.

Stratagie3 or identifying the variable, is asking the student to become a detective. I will ask them what they don't know. If they reread the problem they will realize that they know nothing about the one piece of string, but they do know something about the other piece. They know that the one piece is "twice as long" as the other piece. We can scaffold this information and add it to our simple sketch, resulting in the equation $2x + x = 126$. Combining like terms is the mathematical term for the next step but I can not assume that my students know it. I will discuss the meaning and have students add that to the list of mathematical term for addition. This is also an example of the distributive property because we can also look at $2X + X$ as $X(1 + 2)$. We have now created a new equation or $3x = 126$. I would now have my students focus in on the x, we have in front of us three x's, but we are only interested in one x. What is the three doing to the x? We can refer bask to our mathematical term/phrase list again and see that the three is multiplying the x. What undoes multiplication? Division since they are inverses of each other. If I divide both sides by three, I will have the x by itself, resulting in the new equation $x = 42$.

There is to much math here to be ignored though. I need to make sure that my students understand why they did what they just did and why it is correct. We just used math magic to turn 3x into x by dividing both sides of the equation by 3. We have already discussed the idea about changing the look of something by doing the same operation to everything. Now I want to emphasize another point. What is the secret that allows us to change 3X into X? Since the symbols or saying three divided by three, or how many three's are there in three, our answer is 1. Therefore 5/5, 6/6, x/x and 1/2/1/2 or ANY number divided by itself, are all just another representation of 1 because equals divided by equals are equal. This is just one more opportunity to explain the obvious and increase understanding and not just having students do the steps. This will be helpful when we begin using fractions in our problems because, yes a fraction divided by the same fraction is still one. Fractions are numbers too, so they follow the same rules.

Success (strategy 4) is the final step. I can not stress enough the importance of teaching your students the satisfaction of knowing you are correct. It is a great confidence building skill and that confidence produces students who are more willing to step out of the box and look at math problems and other problems from different view points. The original question was about the length of two pieces of string whose total was 126 centimeters. We found that $x = 42$, so 2x (one piece of string) + x (second piece of string) = 126, or $2(42) + 42 = 126$, when we distribute the 2, results in $84 + 42 = 126$, or $126 = 126$. Since all our statements are correct, we are must be correct. Congratulations!

Equal problems

This next type of word problem once again relies on the student's understanding of language to help make setting up and solving the problem easy. Equal problems can be identified by the term "*the same*" appearing somewhere in the problem. Students will first be asked what does $X = 2$ mean. The symbols do not look at all the same but they are equal in value, so "the same value" is represented by the equal sign (=). This subtle reminder will make solving the **Equal Problems** a simple process of setting the equations equal to each other. Students will already have experience making and solving linear equation. The following is an example of an Equal Problem along with a path to follow for solving the problem. The appendix has a more examples of Equal Problems that I will refer to and use through out this lesson.

Sam and Hector are gaining weight for the football season. Sam weighs 205 pounds and is gaining two pounds per week. Hector weighs 195 but is gaining three pounds per week. In how many week will they both weight *the same* amount?

We can identify this problem as an Equal Problem because we are being asked when Hector's weight will be equal to ("the same as") Sam's weight. The language once again is important because students must understand that "the same" is the mathematical phrase for the mathematical symbol "=". Once this is understood we can concentrate on defining the meaning of the X. In this case the X represents the weeks. Finally we can think about the question we are being asked. When is Sam's weight the same as Hector's weight? When we translate the questions into symbols the result is: Sam's weight (2x +205) the same (=) as Hector's weight (3x + 195) We can now just write the question in symbols and the result is (2x + 205) = (3x + 105).

The next step of getting the variables together is what I have termed the "bully problem". I have lovingly named it this because that is what you do, you pick on, or start with the smaller of the two variables. This will eliminate a negative variable. If I start with 2x and simply eliminate it by adding -2x to both side, the result will be 205 = x + 195. I will show that I could have started with the 3x but if I did I would have had to deal with negative numbers so that is why whenever possible starting with the smaller of the two numbers is the easiest way. The same idea of elimination by adding the opposite number is used to eliminate 195. This will give us the final answer of x = 10. The x represent the number of weeks, so the final answer is in ten weeks Hector and Sam will be the same weight.

Two Type Problems

The **Equal Problems** above will show the students that understanding the question and being able to translate words to symbols makes solving the problem easy. This lays the foundation for moving on to the type of equation the I have called **Two Type Problems.** The reason I have named them this is because, as you will see, we will need to satisfy two different conditions. The trick to recognizing these problems is counting the actual numbers given in the problem, there will be a total of four numbers given. Three of the numbers will pertain to the same thing and one will represent a total. The question will be asking about the total or quantity that we are give the least amount of information. The following is an example of a Two Type Problem.

Lilly sold 75 tickets worth $ 462.50 for the school play. The cost was $ 7.50 for adults tickets and $ 3.50 for student tickets. How many of each kind of ticket did she sell?

We need to organize the many numbers that we are given and identify what their values represent. It is important to note the differences between the numbers, not only because they must be noted in the problem but because this distinction will help students identify Two Type Problems in the future.

The number 75 represent the total number of tickets sold, and the remaining numbers represent money. The $ 462.50 represents the total dollars collected. The 7.50 represent the cost of an adult ticket and the 3.50 represents the cost of the student ticket. We have identified the two different conditions(tickets/dollars), so we can make two separate equations now that represent our problem.

The total number of adult and the total number of student tickets sold was 75, or A + S = 75. The money earned from the adult tickets and the students tickets was $ 462.50, and we know the price of each ticket, so 7.5A + 3.5S = 462.50. Lets make sure that our symbols match our sentences. A + S = 75, that is true, and 7.5A + 3.5S = 462.50 is also true. Now that we have two accurate equations, we can make a system of equations and either use substation or multiplication/distribution to eliminate one of the variables. The equations will be 7.5A + 3.5S = 462.50

A + S = 75

Rewriting the equation to A = 75 - S, I would stop here and ask my students if the statement A = 75 - S is still a true statement, since it is still true, we can substitute this new equation into the first equation which would result in 7.5(75-S) + 3.5S = 462.5. Distribution of 7.5 and combining like terms will result in a new equation (562.50 - 4S = 462.50) by subtracting 562.50 to both sides of the equation we would get -4S = -100 and finally divide both side by -4 to solve for S. The number of students (S) would be 25.

I would also show students that they could have distributed either a -7.5 or a -3.5 to the first equation and eliminated a variable, using this opportunity to go over the idea that if you do something to one side of an equation you can do the same to the other side and not change the value. You may change the look but not the value stressing the idea that 1/2 is the same value as 5/10 even though they do not look the same. I hate to pass up an opportunity to reinforce basic ideas that students may not have thought about for a while.

Students will have mastered the necessary math skills needed to solve this problem because of the progression of problems presented. There were two ways to solve this problem, substitution or distribution both resulting in correct answers, the choice is left to the students.

The examples shown above are a natural progression from a simplistic approach to a more complex approach to solving word problems. The student will have mastered identification of problems which will give them a direction to begin and ultimately solve the problem. This solid foundation will make categorizing and solving more complex word problem an achievable goal for students.

I have added several detailed examples of each type of problem at the end of this lesson along with many unsolved examples of the three types of problems. It is beneficial to have students determine what type of problem it is before they begin working. It is necessary to give the students the "red flags" in order for them to be able to complete this task, this is just another way to help students be successful.

Classroom Activities

Lesson 1:(language)

We will begin by asking the students to list all the names that they can think of for saying addition. Together we will compile the above list, but if not I will add any omissions to the list. We will do the same for subtraction, multiplication and finally division. Students will be expected to save this list and refer back to it when needed.

I will give several examples of word phrases that use the same language and have a class discussion about why they are the same or why they are different. Here are a few examples of the phrases I will use.

5 more than x. (addition)

The number is increased by 5 (addition)

The sum of the 2 numbers equals 34 (addition)

The first number is 3 less than the second (subtraction)

What is the difference between the two numbers (subtraction)

If I double a number what happens (multiplication)

What is the product of the two numbers (multiplication)

If I split something between 7 people (division)

2/5 of a number (multiplication)

I will have the student just think about the operation. The list that we compiled at the beginning of class will be very useful and students will be encouraged to refer to it.

Lesson 2:(symbols)

The students will change the above phrase into symbols.

The words "5 more than x" will become "x + 5".

The number is increased by five can be shown as "x + 5".

The sum of the two equal number is 34 can be shown as 2x = 34 because if the numbers are equal you are just doubling the two numbers

The first number is 3 less than the second number can be show as (x-3) and (x).

I will continue to go through our list of phrase and help students convert them into symbols. I will continuously stop to ask students if our statements are true and if the symbols match the words.

Lesson 3:(sketch/picture)

Students can connect the word to the symbols, now we will begin making simple sketches to represent the symbols. The scaffolding is evident because we have gone from words to symbols and now to pictures. We will take the mathematical statements and turn them into mathematical sentences. This is just a "picture problem" but I have not introduced them as such to my students yet. An example of a mathematical sentence is "Trish has twice as many books as Paula, together they have 5 five books". To translate this into a sketch we would focus on the fact that two thing together (+), is (=) five. The sketch would be ____ + ____ = 5. Make sure that the sketch is a true representation of the symbols. I would very slowly say the sentence and point to each piece of the sketch to see if there is a representation of each word in my sketch. The sketch must be a true statement.

The statement "three consecutive number whose sum is 138" can be sketched ___ + ___ + ___ = 138.

I will do a few more example to emphasize the connection between the words, symbols and the sketch. I will always ask students to check to make sure that the sketch is representing a true depiction of the statement.

Lesson 4: (identifying the unknown)

Students will be asked to read the following problem.

A full bucket of water weighs eight kilograms. If the water weighs five times as much as the bucket empty, how much does the water weigh?

The students will be instructed to draw a sketch that represents the problem, ___ + ___ = 8 will be a good representation. The problem is talking about water and a bucket and we are given some information about one of them. The problems tells us something about the relationship between the water and the bucket, but we know nothing about the bucket. We will call our unknown (bucket) x. Students will be instructed to put that in the sketch, __X__ + ____ = 8. We now can build on the problem. If the water weighs 5 times as much as the bucket, translating that into symbols would give us "5X", and if we transfer that to the sketch, we would have X + 5X = 8.

Lesson 5: (brushing up on algebra skills)

Students will be called to the board to complete the problem (X + 5X = 8) but only if they are able to explain to the class what and why they are doing to the problem. I will be hoping for "Combining like term" to explain the transition to 6X = 8. Students may have an understanding of this term as just a collection of like terms but may not recognize it as an application of the distributive property so I will demonstrate by showing them that (X + 5X) is X(1 +5) and that is why we can say 6X = 8.

Teacher: "Should "Combining like terms" be included with your list of terms meaning add?"

Student: "Yes"

Teacher: "Is that the only way to think about it?"

Student: "No, we could add it to the list of multiplication terms."

Teacher: "Great, now what do we want to do?"

Student: "Divide both sides by 6."

Teacher: "Why are you allowed to do that?"

Student: "Because what you do to one side you have to do to the other."

Teacher: "Why?"

Student: "Because the statements are equal or balanced, and if I change one of the sides without changing the other, it will be out of balance."

Teacher: "Why did you divide by 6 though?"

Student: " Because any number divided by itself just becomes one and since I wanted only one x, I can turn six into one by dividing by six."

Teacher: "Great, you really understand what you are doing."

Student: "Yes, I had a great teacher."

This conversation may be far fetched but it is important to check for understanding. The concept that should be stressed is the idea of doing the same thing to both sides and the math magic that allows you to turn 6x's into one x, and why you are allowed to do it.

Lesson 6 (Identifying an Equal Problem)

Students will read the problem.

The number of wild horses at the Lazy Z Dude Ranch could be found by counting them, but when Hank visits the ranch, he suggests that 23 fewer than five times the number of horses at the ranch is the same as 58 more than twice the number of horses on the property. If Hank is right, how many horses does the Lazy Z have?

Teacher: "Can you draw a simple sketch for this story?"

Students: "No."

Teacher: "Do you see any math terms than can be changed to math symbols?"

Students: "Yes, 23 fewer than 5 times the number of horses."

Teacher: "How would you change that to symbols?"

Students: "$5x - 23$"

Teacher: "How do you represent the same as in symbols?"

Students: "Equal sign (=)."

Teacher: "Are there any more math terms we can change to math symbols?"

Students: "58 more than twice the number of horses, and we can change that to $2x + 58$."

Teacher: "Do you see where we are going with this?"

Students: "Yes, we can now just make an equation that tells our story. The equation will be $5x - 23 = 2x + 58$.

The algebra steps that are needed to solve the equation are very important and should be emphasized. It is another opportunity to not only stress the steps but to check for understanding. I will ask my students why they are doing what they are doing and why they are allowed to do it. The following steps are used to solve the problem.

We will add -2x to both equations because whatever you do to one side of the equation you must do to the other, we chose -2x because we are trying to get all the x's on one side of the equation and -2x will make the +2x disappear on the one side. I like to call this "math magic" because it makes one of the variables disappear and creates a new equation (3x -23 = 58). Students can use the same step from the Picture Problem to finish the problem, and will discover that x = 21 or that there are 21 horses at the ranch. Students will be expected to check the answer by plugging the value of x into the original equations to check to see if our statements are true.

I will continue this lesson by doing several of the same problems and emphasizing why I have named them **Equal Problems.** I will stress the skills needed to not only solve these types of problems but to also recognize one when we see it. I will finish this lesson by pointing out the "red flags" to look for to spot a equal problem and talk about the differences between a equal problem and a sketch problem.

Lesson 7 (recognizing Two Step Problems)

The students will be asked to read the problem aloud.

Fifty more "couples" tickets than "singles" tickets were sold for the dance. "Singles" tickets cost $ 10 and "couples" tickets cost $ 15. If $ 400 was collected, how many of each kind of ticket was sold?

Students will be asked if this problem has any of the "red flags" from the first two example problems. It will be hard to draw a picture of this problem so we can not call it a Picture Problem, and since there is no equal sign any where in the problem we can not call it an Equal Problem. Students will asked what two things we are talking about in the problem (tickets/money). I have already give a very detailed example in my Strategies portion so I will not go into details for solving this problem. The lesson is recognition first and the solving will just fall into place with some teacher instruction.

Lesson 8 (recognizing different problems)

Students will be given one example of the above types of problems on a sheet of paper and asked to name the type of problem. They will them be asked to go to the group that has the same problems and talk with the other students to see if they are all similar. Once the group is in agreement that they are all the same, they will be asked to explain what helped them decide. They will determine the "red flags" from each problem and share their finding with the rest of the class. Students will state the mathematical terms or symbols that helped them decide what category their problem fell into. I will use this bank of problem for future lessons and will have them as warm-ups to use through out the year. I will add problems and titles as the year progress and will continue to expect students to identify the problems by name. The complete list of problems is attached.

This lesson has been broken down into eight mini lesson because of the different scheduling throughout the country. I think that several of these lessons can be taught in one day depending on the skill level of the students but each idea addressed is an important point that should not be overlooked. Having a solid base will assure success for the more complex problems that students will face as the year progresses. The students will cover Age Problems, Distance Problems, Mixture Problems, Work Problems, Finance Problems and Quadratic Problems throughout the course of this class. I have designed this first lesson of the year in the hopes of creating success and understanding. I will compare and contrast the problems through out the year and continue to assist students in recognizing problems and giving them the tools they need become problem solvers.

Appendix

Picture Problem

The perimeter of a rectangle is 100ft. If the length is five feet more than twice the width, find the length and width.

- What language is telling you what to do? (five feet more that twice the width)
- Can you draw a picture, sketch, line to show this problem situation? (____ +____ + __________ + __________ = 100)
- Tell me which one of the pieces do you know nothing about? (I know something about the length, but I know nothing about the width - lets call the width X)
- Therefore X + X + (2X + 5) + (2X + 5) = 100 and by combining like terms we can now say 6X + 10 = 100. Since we are only interested in the value of the X we can eliminate the positive 10 by adding a negative 10 to both sides of the equation or 6X = 90. Now the X is being multiplied by six and division by six will result in X = 15.
- Now we are ready to answer the question, remember we were asked to find the length and the width of the rectangle so the answer is *not* 15, the width is 15 and the length is 35. We can check to see if 2 width and 2 lengths equal 100 or 2(15) + 2(35) = 100 or 30 +70 = 100.

Equal Problems

The New Bill Phone Company charges $ 30.00 sign-up fee and 12 cent per minute. The Best Buy Phone Company charges $ 10.00 sign-up fee and 15 cents per minute. How many minutes can I talk to make sure my bills will be *the same?*

- What is the question? (When will both phone plans be *the same* price)
- How do you represent New Bill Phone Company in an equation? (.12X + 30)
- How do you represent Best Buy Phone Company in an equation? (.15X + 10)
- Can you rewrite the question in symbols? (.12X + 30 = .15X + 10) or .03X = 20 or X = 666.67.
- Answer the question. I can talk 667 minutes and it won't matter which company I choose because my bill will be the same - $ 110.04.
- Nice Job - pat yourself on the back!!!!

Janelle has $ 20.00 and is saving $ 6..00 per week. April has $ 150.00 and is spending $ 4.00 per week. When will they both have *the same* amount of money?

- What is the question? (When will Janell have *the same* amount of money as April.)
- How do you represent Janelle's money in an equation? (6X + 20)
- How do you represent April's money in an equation? (-4X + 150)
- Can you rewrite the question in symbols? (6X + 20 = -4X + 150) or 10X = 130 or X = 13.
- Answer the question. In 13 weeks Janell and April will have the same amount of money in their accounts.
- Nice Job - pat yourself on the back.!!!!

A box of fruit has four more apples than oranges. Together there are 52 pieces of fruit. How many of each fruit are there?

- What language is telling you what to do? (Four more than)
- Can you draw a picture, sketch, line to show this problem situation? ____ + ____ = 52
- Tell me which one of those do you know nothing about. (I know something about the apples, but I don't know anything about the oranges - let call the oranges X)
- Therefore apple plus oranges can be represented by (X + 4) + (X) = 52, by combining like terms our equation becomes 2X + 4 = 52. Since we are only interested in the value of X, we need to get all the other numbers away from X, so starting with the number that is furthest away we need to eliminate the positive four by adding a negative four to both sides of the equation, or 2X = 48 Finally we need to eliminate the two and since the two is multiplying the X we can "undo" multiplication of two by dividing by two by or X = 24.
- Lets make sure we have answered the question - How many of each fruit is the question - the answer is *not 24,* rather there are 28 apples and 24 oranges or 28 + 24 = 52
- All our statements are true. Nice Job - pat yourself on the back!!!!

Two Type Problems

Cashews cost $ 5.50 per pound and peanuts cost $ 2.00 per pound. I want a 7 pound bag with a mixture of both nuts but I only want to spend $ 28.00. How many pound of each kind of nuts can I buy?

- Lets look at the numbers we are given and what they represent.

5.5 represents the price per pound of the cashews (pounds)

2 represents the price per pound of the peanuts (pounds)

7 represents the number of pounds I need to buy (pounds)

28 represents the money I want to spend

- What two things do these numbers represent? (money/pounds)
- Lets make two separate equations that make true statements.

The the weight of the cashews and the weight of the peanuts combined should

equal seven pounds (C + P = 7) and the cashews cost $ 5.50 per pound, while

the cost of the peanuts is $ 2.00 per pound and I want a combination of both

with a final cost of $ 28.00. I can combine this information into a equation

stating (5.5C + 2P = 28).

- Lets make sure our equations match our sentences.

C + P = 7, this is true, and 5.5C + 2P = 28, this is also true.

- Since we now have two equations, we can make a system and solve.

C + P = 7

5.5C + 2P = 28

By multiplying the first equation through by -2 we can eliminate one of the variables.

-2C + -2P = -14

5.5C + 2P = 28

This will result in 3.5C = 14 which becomes C = 4 and since C represent the pounds of cashews we know that the number of pounds of peanuts I can buy must be 3.

- Lets check to see if we are correct.

4 + 3 = 7 True

4(5.5) + 3(2) = 28 True

- Please note that this is not the only way to solve this problem and student will be encourage to use whatever method they are most comfortable with.
- Example problems unsolved

Two Equation Method

The two equation method is a more advanced algebraic solution to the picture problem. I have included these examples for the benefit of the reader, however my students are not ready for this approach when they enter my classroom. This unit leads my students in that direction.

X + Y = 126

2Y = X

- Use substation to make one equation.

2Y + Y = 126

- Combine like terms.

3Y = 126

- Divide both side of the equation by three to solve for Y.

Y = 42

- Plug the Y value into the equation.

2(42) + (42) = 126

- Check for accuracy

In the football game, Rockey gained three times as many yards as Bullwinkle. Rockey also gained 10 more yards than Boris. The three players gained a total of 410 yards. How many yards did Boris gain?

In science class, Hector decided to measure the mass of his math, science and Spanish textbooks. He discovered that the mass of his Spanish text is five grams more that four times the mass so his math book, while the mass of his science text is 110 grams less than his Spanish book. Combined, the mass of the texts is 1,178 grams. Find the mass of each book.

West High School's population is 250 students less than twice the population of East High School. Combined they serve 2,858 students. How many students attend West High School?

The perimeter of a triangle is 76 centimeters. The second side is twice as long as the first side. The third side is four centimeters shorter than the second side. How long is each side?

The length of a rectangle is three centimeters more than twice the width. The perimeter is 60 centimeters. How long and how wide is the rectangle?

The length of a rectangle is three times the width. The perimeter is 16 feet. What is the width of the rectangle?

The notebook costs $ 0.15 more than the pen. The total cost of the pen and the notebook is $ 2.25. How much does the pen cost?

An apple tree produces three times as much fruit as a pear tree. During one season, two apple trees and three pear trees produced a total of 126 pieces of fruit. How many apples does an apple tree produce in one season?

In Harry's backyard, his peach tree produces two pieces of fruit for every three pieces that grow on his apricot tree. If the two trees have a total of 265 pieces of fruit on them, how many pieces of fruit does each tree have?

It is now 7:51 pm. The movie that you have waited three weeks to see starts at 8:15 pm. Standing in line for the movie, you count 146 people ahead of you. Nine people can buy their tickets in 70 seconds. Will you be able to buy your ticket before the movie starts?

The length of a rectangle is 4 less than twice the width. If the perimeter is 244, find the length, width and area.

Valentine's Day is quickly approaching and Pablo is arranging flowers for his girlfriend. As a rule, he always uses twice as many roses as carnations and three more tulips than roses. If the arrangement he's designing will take 98 flowers, how many of each flower will he need?

The number of wild horses at the Lazy Z Dude Ranch could be found by counting them, but when Hank visits the ranch, he suggests that 23 fewer than five times the number of horses at the ranch is the same as 58 more than twice the number of horses on the property. If hank is right, how many horses does the Lazy Z have?

Juan and his friends are going to an amusement park and discover that they have two ticket options. One option is to buy an admission ticket for $ 5.00 and then pay .25 for each ride. The other option sis to buy an admission ticket for $ 2.00 and then pay .75 a ride. What do you think Juan should do?

Two car rental agencies are competing for business. Deluxe Driving charges $ 35 a day and .15 a mile while Rent-It-Cheap charges $ 20 a day and .35 a mile. Which agency should you choose?

Highland has a population of 12,200. Its population has been increasing at a rate of 300 people per year. Lowville has a population of 21,000 which is declining by 250 people per year. Assuming that the rates do not change, how many years will the population be equal?

It is the end of the semester and the clubs art school are recording their profits. The Science Club started out with $ 20 and has increased its balance by an average of $ 10 per week. The Computer Club saved $ 5 per week, but started out with $ 50 at the beginning of the semester. The Math Club was just formed this year and managed to raise $ 15 per week. Which club is the wealthiest at the end of the 12th week?

Future Problems

Students will progress to the following type of problems throughout the year. The skills that they have mastered will make this progression easy for them. I hope that they will be able to see the similarities to the examples we have gone over.

Three times a number increased by three is equal to 30. Find the number.

If two times a number minus six is equal to 20, find the number.

A person has eight coins consisting of quarters and dimes. If the total amount of this change is $ 1.25, how many of each kind of coin are there?

In a bank, there are three times as many quarters as there a half dollar and six more dimes than half dollars. If the total amount of the money in the bank is $ 4.65, find the number of each type of coin in the bank.

Bill is 8 years older than his brother. In 3 years, Bill will be twice as old as his brother. Find the present age.

A craftsman has two alloys of silver. The first one is 70% pure silver and the second one is 50% silver. How many ounces of each must be mixed to have 12 ounces of alloy which is 65% silver?

Annotated Bibliography

Sallee Kysh Kasimatie Hoey, Tom Judy Elaine Brian. College Preparatory Mathematics.

Second Edition. Cal: CPM Educational Program,2002.

Sterling,MaryJane. Algebra for Dummies.First Edition.NY:Hungry Minds Inc;2001.

Cutler,Ann. Instant Math.First Edition.NY:Doubleday & Company,Inc,1962.

Saunders,Hal. When Are We Ever Gonna Have To Use This?Updated third Palo Alto,Cal: Dale Seymour Publications,1988.

Bluman,Allen. Word Problems DeMYSTIFIED.First Edition.NY:McGraw Hill,2005.

Stapel, Elizabeth. "Mixture' Word Problems." Purplemath. Available from

http://www.purplemath.com/ modules/mixture.htm. Accessed 07 May 2007

Staple, Elizabeth. "Work' Word Problems." Purplemath. Available from

http://www.purplemath.com/ modules/workprob.htm. Accessed 07 May 2007.