Curriculum Units by Fellows of the National Initiative
2007 Volume VI: Keeping the Meaning in Mathematics: The Craft of Word Problems

Assisting the Transition from English to Mathematical Language

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by Paula Shaffer-Roche

Overview

This unit maintains a focus on various strategies and approaches to implementing word problems. For example, the lessons utilize mental math and tables to help aid to the mathematical thinking involved in problem solving. Furthermore, the lessons emphasize starting with simple mathematical language, then completing more complex problems. The goal is to build a strong problem solving foundation. The lessons are built around the idea that math is similar to learning a foreign language. Hence, students have to be prepared to handle mathematics with the idea in mind that with more exposure the language becomes much easier to translate. The unit seeks to illustrate methods that help address the difficulties that mathematical language presents to students. Classroom activities will analyze meaning in word problems in order to arrive at the correct answer. My goal for this unit is to have students be able to attack word problems without fear and frustration. I hope the strategies explored here inspire educators to analyze word problems with their students more easily and effectively.

Primarily, the lessons are geared towards a ninth grade audience enrolled in an Algebra I course at the high school level. However, the unit may be of interest to teachers at other levels. The items covered may be adapted to any level math course because word problems are relevant to all layers of mathematics. Furthermore, the approaches illustrated in the unit can be used in various curricula. Problem solving is an imperative skill in numerous subjects.

Rationale

Teaching is a relatively new territory for me. Currently I have only three years of teaching experience under my belt, but I try to take every opportunity available in order to enhance my instruction. Presently, I am a math teacher at Taylor Allderdice High School in the Pittsburgh Public School District. The 2006-2007 school year was my first year there. I taught Algebra I, Algebra II, and AP Statistics. Taylor Allderdice has the reputation for being one of the best Pittsburgh Public Schools. Therefore, I was excited to be given the opportunity to teach there. The students that I see throughout the day range greatly in ability level. My AP classes are taken by students that have been declared gifted at a very young age. The students are highly intelligent and extremely motivated. On the other hand, my Algebra I and Algebra II students are classified as mainstream. In general, mainstream classes possess students with a wide range of abilities. As educators we are constantly faced with this dilemma. However, experience teaches us wonderful lessons in order to face the challenges of teaching a range of students in a single class.

The Algebra I class I had this year had a variety of ability levels. However, an underlying characteristic among the majority of my students was a low reading level. Most students were operating on average a fifth - sixth grade reading level. The district had adopted a new curriculum this year, in which there was a heavy emphasis on reading. Students were tested periodically throughout the semester. The exams contained many word problems related to concepts that were taught in the classroom. I struggled throughout the year teaching word problems to the population of students that I was given this year. Hence, I knew I needed to seek out assistance to improve my skills in this area. Therefore, I accepted the challenge of taking part in the national seminar in order to investigate strategies for improving my students' problem solving abilities.

The word math provokes a lot of fear. Many individuals possess so many negative views on the discipline. When I first meet people and tell them I am a secondary math teacher, they look at me as if I were from another planet. Some try to sympathize about dealing with the attitudes and behaviors of adolescents. However, the majority groan at the very mention of math. Then, remarks that they absolutely hated the train word problems and they never understood any word problems the teacher would present usually follow. The linkage between math and word problems is indispensable to learning mathematics, but the motivation and deep understanding often gets lost in translation by many students.

One of the main reasons that people loathe mathematics is that they fail to recognize the relevance of the material to real world applications. Fortunately word problems create an avenue for teachers to present concepts to students simultaneously thru real world scenarios. However, as previously mentioned, people have a tendency to shy away from word problems. In fact, word problems are not well liked by teachers or students. Many of my colleagues admit that they do a poor job on teaching word problems. Unfortunately, some even will only spend a very limited amount of time on real world applications and feel the need to move on to more routine and structured problems. In an average high school math classroom, the majority of the time is spent on presenting a concept to students and then practicing with some examples from the textbook. As math educators we should not overlook the fact that mathematics is not primarily number crunching. Mathematics is an incredible field. The discipline offers learners a drastic new way of thinking and a universal language. Math requires one to discipline the mind to actively engage into what may seem to many a foreign language. I think exposing students more to real world applications by problem solving will help to build more positive outlooks in the field of mathematics.

In regards to the negativity surrounding mathematics, I do struggle with teaching my students strategies and concepts through word problems. I want to maintain a strong focus on word problems in the classroom because I know that becoming a strong problem solver is invaluable to a well rounded education. Furthermore, word problems play a prominent role on standardized tests. I want my students to feel completely confident in attacking and peeling apart layers in any problem that may be presented to them. Too many times I have seen a student glance briefly at an open ended problem on a standardized test and then leave the entire question blank. When students are asked why they did not even attempt the problem, most say they had no idea how to get started on the problem. My goal is to have students have the desire and motivation to tear apart the problem and hopefully find a successful path that leads to the correct answer. I do believe that changing students' negative attitudes about word problems will not be an easy task, but I am more than ready to take on this necessary challenge. Therefore, I felt compelled to take advantage of the wonderful opportunity to take a seminar at Yale University whose sole focus was on the investigation of word problems.

The seminar has opened my eyes to the world of possibilities in the realm of word problems. The unit I have designed will incorporate many strategies and approaches learned from the class. Primarily, the unit is designed for a ninth grade Algebra I course, but the lessons can be adapted to fit into any math classroom for any level. Furthermore, educators teaching outside mathematics may want to consider some elements in order to improve problem solving abilities among their students. In general, the seminar reinforced the idea of peeling apart the layers of problems in order to find the best approach to implement in the classroom. Problems, ranging from a wide variety of concepts and levels, were analyzed and discussed in detailed during class. Investigating numerous word problems helped me to formulate a clear focus for my unit.

Mathematics is similar to teaching a foreign language. The author, G. Polya, of How to Solve it proclaims that especially setting up equations resembles translating from one language to another (174). This mathematical language presents problems for both educators and students. Students have an extremely difficult time translating ordinary English into mathematical symbols and teachers have a challenging time presenting various concepts existing within the discipline. In order to translate from English to Spanish, we can not translate word for word; we have to completely understand what the sentence is trying to communicate to others. Also, one has to recognize the uniqueness existing within the Spanish language, in which there may be certain phrases existing in one language that may not be found in the other and vice versa. In an ideal world, languages could be translated word for word. However, we know this not to be true in this world. Some phrases do not exist in other languages. In order to make translation smoother, the whole meaning should be established and the sentence may need to be rearranged. Similarly creating mathematical equations to represent real life scenarios involve translation. Sometimes the translation is simple and direct, but sometimes it is highly complex. The latter situation puts emphasis on retrieving the entire meaning of a verbal statement.

In order for a student to achieve an overall understanding of a problem, all the terminology present in the statement should be recognizable to the learner. Hence, one of my goals for the unit is to present terms to students and have them develop a conceptual understanding of the mathematical vocabulary before they start developing equations to represent an event. Primarily, I plan to present my students with word problems that are easy to read. Then, as their skill of reading math becomes stronger, the language of the problems can become more complex. In the article, *Effects of Teaching Sixth Grade*, the authors describe an experiment completed with sixth grade students on the processing of word problems by the young mind. The study illustrated that simplifying the vocabulary and shortening sentence length are factors that help make a word problem more readable and in turn help the learners develop a generalized meaning of a scenario (Cohen, Stover, 1981).

After establishing an overall meaning of a word problem, then a plan of attack has to be formulated by the student. In order to aid in this process, the teacher can help to encourage students to develop habits that they can use while attacking problems. One method would be to encourage the use of diagrams. Most students are visual learners and the use of a drawing can help with finding a solution. Also, teachers can help students learn to eliminate extraneous information. Furthermore, teachers need to assist students in rearranging information in the problem that may be interfering with the student's ability to generate an overall meaning.

In order to ease the teaching of word problems for educators, an analysis of problems is needed to provide teachers knowledge of the content presented and to help guide the teachers on making informed decisions for his/her classroom based on the needs of their individual students. The problems are attached in the appendix. I believe knowing the differences between the problems and the different approaches to solve the problem will help to enhance instruction.

The problems are organized along two dimensions, one variable and two variable word problems. Dimensions define the general concept involved in the problem. There are existing subgroups within these dimensions. The one variable word problems contain one-step, two-step, multi-step, and mixture problems. The two variable word problems contain linear systems with different values or percentages for the two unknowns involved in the problem. For both dimensions I tried to include the dimension of complexity with regards to numbers. I included percentages and decimals frequently. Following this paragraph is an exploration of the problem territory. The beginning of the problem set can be viewed as glorified arithmetic. However, I feel these problems serve as a good transition into problems that may require more symbolism. Similar to foreign language, in mathematics we have to give our students a strong background so they can build and enhance their skills. The detailed explanations are not necessary to be relayed verbatim to students, but the exploration helps educators develop a strong foundation of the mathematics involved in each problem.

Problems A. 1. Through A. 4.

Primarily the problem territory begins with the simplest kinds of equations, one-step equations. Specifically, the problem set starts with one variable equations, in which one of the four standard mathematical operations is used to solve for the variable. A word problem that represents this equation is the following:

A 40 foot rope is cut into 2 pieces. One piece is 4 foot, what is the length of the other piece?

The focus of all lessons is translating mathematical language into an algebraic equation. The problem is solved by subtracting 4 from 40. In elementary school, teachers expose this type of problem to children and label it as a subtraction problem (of the missing addend type). However, we want students to make the transition into algebra. Therefore, we stress the importance of creating an equation to model the situation. A student can identify what the problem is asking for and develop the equation $x + 4 = 40$. Even at this early stage of creating an equation to model a scenario, students need to develop the habit of defining the meaning of the variable(s) being utilized in the mathematical representation. In this case, let x = number of feet of the other piece of rope. Then, to solve the algebraic equation, the student simply subtracts 4 from both sides of the equations.

The previously discussed problem needs to be paired with a problem involving subtraction in order to emphasize the fact that adding and subtracting are inverse operations of each other. Similarly, a one step equation formulated by multiplication, and one requiring division, are presented in the problem set. Again, the operations can be paired together to represent the existing inverse relationship. All word problems requiring the creation of a one step equation can be found in the appendix under A1. In addition to the concern of having the student write a correct equation to model a problem, attention needs to be directed to the student's ability to decipher which operation to use to represent a situation. In order to help translate the mathematical language more easily, common mathematical terms and their corresponding meanings are located around the room.

The completion of one step equations leads into the investigation of two step equations, in which two of the standard mathematical operations are utilized in the creation of the equation. A word problem that illustrates this type is:

How old am I if 400 reduced by 2 times my age is 244?

The key word "reduce by" signifies to students subtraction and they create the equation, $400 - 2a = 244$. The variable a stands for my age in years. When solving for the variable students have a difficult time seeing how to remove the number in the front because most do not see a positive sign in front of the number. In order to solve this type of equation, students have to understand how to reverse what the order of operations tells them to do. Hence, the first step is to subtract 400 from both sides of the equation. Then, divide both sides by a -2. Some students may forget the negative sign in front of the two and their answer would result in a negative number. So, this would be a teachable moment to reinforce rereading the problem in order to ensure that the answer made sense. Does it make sense to say someone is negative years old? Hence, students need to check their answer by substituting their answer to the original equation and see if a true equation results.

Instead of equations with a constant in front, sometimes the variable is the first term in the equation. Consider the word problem:

John has three sacks of apples and three more apples in his pocket. Each sack contains the same number of apples. Altogether he has 33 apples. How many apples are in each sack?

The student defines the variables as a = the number of apples in each sack or any other letter that he/she may choose. Then, the equation $3a + 3 = 33$ is created to model the problem. When the variable is in front the student has an easier time seeing that they have to start by subtracting three from both sides and then divided both sides of the equation by three.

In order to increase difficulty of these two-step equations, fractions may be incorporated in the scenarios. Examine the following problem:

Jill sold half of her comic books and then bought 16 more. She now has 36. With how many did she begin?

The mention of half signals that fractions can be used to represent the relationship of the variable rather than a whole number. Of course, decimals may be used instead of a fraction because most students feel more comfortable with them. Sometimes, a problem is given and students may not know immediately what decimal is equivalent to the fraction and starts to feel overwhelmed, especially if no calculators are available. Hence, students should know how to solve equations involving fractions. For this problem let c be defined as the number of comic books Jill has from the beginning and then the equation, $\frac{1}{2}c + 16 = 36$ models the word problem. The student subtracts 16 from both sides and then can use the concept of multiplying both sides of the equation by the reciprocal of the fraction. Reciprocal is a term that most students are familiar with, but review on multiplying a whole number by a fraction will be necessary. In addition to highlighting the concept of multiplying both sides by the same number, the use of fractions can ignite conversation on how a lot of scenarios are not represented by whole numbers.

Furthermore, in the examination of two step equations, a word problem can involve two unknowns with one variable. Consider the problem:

A 9 foot rope is cut into 2 pieces so that one piece is twice the other. How long is each piece?

Now the student can define the variable x = the length of one piece (in feet) and 2x = the length of the other piece (in feet). The problem is stating a relationship along one variable of interest. So, the equation $2x + x = 9$ is created. The skill of combining like terms comes into play and then $3x = 9$ revert back to a one step equation. The student will have to recognize there is an understood one in front of the lone x, in order to combine like terms successfully.

Another twist on problems involving a relationship and using the skill of combining like terms is to use more complicated numbers. The following problem uses percentages.

Two ropes are together 275 yards long. One rope is 50% longer than the other. How long are the ropes?

The two variables are defined as follows: x = the length of one rope (in yards) and x +.5x = 1.5x which is the length of the other rope (in yards). Then the equation x + 1.5x = 275 represents the situation. The students have to recall adding with decimals and then again the problem reverts back to a one-step equation. The problems that are meant to be created by a two-step equation can be found under A2.

After students see problems where one and two step equations can be created to model the problem, then scenarios can start to represent multi-step equations. In the previously discussed problem, there is a relationship along one variable of interest. Multi-step problems revolve around this idea and in turn some problems become quite complex. First, we can have students solve the problem:

A length of a board was 10 inches shorter than another length. Together the boards were 20 inches. How long were the boards?

The variables are defined as x = the length of one part of the board in inches and let x - 10 = the length of the other part of the board in inches. The equation x + x - 10 = 20 will be solved by combining like terms first; then the equation reduces to a two-step equation.

Problems which are best represented by a multi-step equation may require more complex algebraic manipulations. For example, examine the problem:

A pile of 18 coins consists of quarters and dimes. If the total value of the coins is $3.00, find the number of quarters and dimes the child has.

The variable involve is d = how many dimes and 18 - d = how many quarters. Then the student produces the equation, .10d + .25(18 - d) = 3.00. The numbers are slightly more complex in this problem and also the students has to have an understanding of the distributive property. The student would simplify the problem to .10d + 4.50 - .25d = -.15d + 4.50. Of course, the student could choose to let q represent quarters instead of dimes and go from there.

The problems can become more complex as the number of relationships increase. For example:

A 12 foot rope is cut into 3 pieces so that the second piece is 1 foot longer than the first and the third piece is 1 foot longer than the second. How long is each piece?

This problem can be formulated using one variable and two existing relationships. If we let x = the length of one piece of the rope in feet, then we can express the length of the second piece as x + 1, and the length of the third piece as (x +1) + 1, which we can quickly simplify to x + 2. Then the condition of the problem can be written as x + (x +1) + (x + 2) = 12. The problem requires the student to combine like terms with the variables and combine like terms with the constants. When all the simplifying is completed, the problem is seen as a simple two step equation in disguise. The multi-step equations are located in the appendix under A3.

In addition to increasingly complex relationships, some problems become more complex conceptually. Mixture problems are a difficult concept for many students to understand. Using tables are usually a good method to help students visually understand the concepts involved in these types of problems. A multi-step equation is still going to be an option for solving. For example:

How much cream that is 20% butterfat should be mixed with milk that is 5% butterfat to get 10 gallons of cream that is 14% butterfat?,

is a question involving mixtures. A possible solution to the problem would be: letting x = the number of gallons of cream and 10 - x = the number of gallons milk that is 5% butterfat. The number of gallons of butterfat in the combined solution would be .14(10). Then the following equation can be written:

$.20x + .05(10 - x) = .14(10)$

and this is a linear equation with the unknown on one side.

Linear equations can also have a variable on both sides and mixture problems are a way to introduce this concept to students. The algebra involved in these types is usually a lot more difficult for students because they demand a high degree of skill of algebraic manipulating. Consider:

How much of a 10% vinegar solution should be added to 2 cups of a 30% vinegar solution to make a 20% solution?

Again, the variable will be defined as such, let v = the number of cups of a 10% solution that one should add. Then v + 2 = the number of cups of the combined solution. The amount of vinegar in the 10% solution is .10v. The amount of vinegar in the 30% solution is .3(2). The amount of vinegar in the combined solution will be .2(v +2). Since this vinegar solution is the sum of the two original mixtures, we can create the equation

$.10x + .3(2) = .2(x + 2).$

The linear equation requires distributing the (.2) and then combining like terms in order to get the one variable on one side of the equation. All mixture problems are found under A4.

Problems from B 1.

Now the problems are taking a progression from one variable equation to two variable equations. These problems can be solved a variety of ways. If we look at these problems as two variables then, we can take the approach of looking at them as setting up a linear system and solving by elimination, substitution, or graphing. The following problem can be formulated using two different variables.

You are selling tickets for a musical at your local community college. Student tickets cost $5 and general admission tickets cost $8. If you sell 500 tickets and collect $3475, how many student tickets and how many general admission tickets are sold?

Let s = the number of student tickets sold and g = the number of general admission tickets sold. The total number of tickets sold is s + g = 500. The total amount of money collected is 5s + 8g = 3475. The students have a variety of choices to solve the system of equations. In this problem as in others, students must read carefully to be sure they are answering the question that is asked of them. Sometimes finding only one variable is required.

Using two variables gives the student a variety of methods to solve the problem, but for some problems, substitution may be the best way to approach the situation. For example,

You just purchased a cellular phone and are trying to decide the best cellular phone company to which to give your business. When you contacted the Talks - A - Lot company, they were offering a monthly plan of $40 for 500 minutes and $0.25 for each minute over the 500 minutes. Chat - Away is charging $35 for 500 minutes and $ 0.30 for each additional minute over 500. How many minutes would you have to talk over the 500 minutes for the cost to be the same with both companies? What would be the equal cost?

Primarily, to approach this problem, let m = the number of minutes talked over the 500 minute limit and y1 = the total cost of the bill for Talks - A - Lot and y2 = the total cost of the bill for Chat - Away. Then considering the initial cost for Talk - A - Lot is $40 per month and $0.25 for each additional minute over 500, the equation created would be $y1 = 40 + 0.25m$. Hence, the formula for the Chat - Away plan is derived similarly, $y2 = 35 + .30m$. Knowing that we want how many minutes over 500 would the companies be the same cost, we could set the two equations equal to each other then we would have a linear equation with an unknown on both sides. Two variable problems are located under B1.

Objectives

In this unit, students will investigate a variety of word problems to help them achieve several objectives. Students will be able to:

- Create one, two, and multiple step equations in order to represent a real life situation.
- Solve linear systems by using elimination, substitution, or graphing methods.
- Analyze a specific set of word problems and communicate differences between the various questions.
- Create a word wall containing mathematical terminology in order to make translations easy in the classroom setting and in turn making the words part of the students' permanent vocabulary.
- Create and solve two equations set equal to each other.
- Create and solve mixture problems by drawing tables.

Strategies

Word problems should not be taught in isolation. Hence, I plan on teaching them throughout the year. Word problems force students and teachers to take a break from routine problems. Polya asserts the definition for a routine problem is one that a student solves by plugging numbers into a formula and starts crunching away or one that requires substitution in a step by step manner. Both approaches lack originality and mental activity. However, routine problems are essential and help to build a strong mathematical background. It is important to remember that routine problems need to be supplemented with problems that require higher level skills. Polya makes an interesting analogy to only teaching routine problems in the classroom. He remarks that demonstrating only the mechanical operations is using less brain power than when using a cookbook. He believes that kitchen recipes leave more room for creativity and judgment of the cook than mathematical recipes (Polya, 1988, page 172).

Specifically, the unit will be taught during the first semester because this is when the students are beginning to learn how to formulate expressions and equations. I am anticipating the unit to take about two to three weeks to complete. Then, the second dimension of two variables will be in the first part of second semester. However, the unit may be shortened or lengthen to fit the needs of students' individual needs. For example, adjusting the number word problems dealing with particular concepts would be a common adaptation.

Investigating and analyzing various word problems in seminar helped to develop many useful strategies to implement in the classroom. Primarily, an underlying strategy throughout the entire unit is the idea of beginning with simple word problems and gradually building a foundation that will ideally help students solve complex scenarios and utilize higher level thinking processes. In addition to building their word problem solving abilities, I hope students' confidence will rise as a product of their knowledge. Confidence will help students attack problems on standardized tests, which are purposely not routine problems. The breakdown strategy will hopefully cater to students who possess lower reading levels. Simple word problems may involve only one translation. In order to take into account many abilities, mathematical terminology will heavily be stressed in the beginning of the unit. An emphasis on vocabulary will help to build a strong foundation needed to breakdown word problems. A word wall will be created by the students to aid vocabulary development. The wall will encompass words associated with all four operations and will be located on the side wall for the first semester in order for the students to reference while formulating equations and solving problems.

The purpose of the word wall is to help students identify the operation suggested in word problems. However, an individual word in a problem may suggest one operation, but the overall meaning calls for another.

Another strategy implemented during the unit will be referred to as *false positioning*. The method is similar to a mental math approach. The unit will still maintain a focus on setting up and solving equations, but I wanted to offer another technique in order to see if students may grasp the approach better. I feel that false positioning may help students to internalize mathematical concepts and in turn develop a true meaning of what the problems are asking them to do. For example, consider the common ticket problem.

You are selling tickets for a concert at your local community college. Student tickets cost $5 and general admission tickets cost $10. If you sell 100 tickets and collect $575, how many student tickets and how many general admission tickets are sold?

The false positioning method would have an individual consider if you sold all of one kind of ticket and then deduct to find the true amount. In the above example, you may want to think if only student tickets are sold the revenue would have been $500, but the problem is stating that $575 is made. So we know that to account for the difference of $75 another kind of ticket was sold, general admission. The difference between the two types of tickets is $5, so for each general admission ticket that is sold instead of a student ticket; you gain $5 in revenue. When divide $75 by $5 you get 15 general admission tickets were sold, and in turn 85 students tickets were sold. The false positioning method turns a two variable problem into a one variable problem.

Lastly, the strategy of drawing tables will be utilized in the unit. Drawing tables is especially helpful when solving a mixture problem. These types of problems normally frustrate students tremendously. My hope is that a table will organize the important information in such a manner that will aid in attacking the problem. Consider the question,

How much of a 10% vinegar solution should be added to 2 cups of a 30% vinegar solution to make a 20%

vinegar solution?

A table, as shown below would be illustrated in class in order to help students organize the information given in the problem:

Cups of solution% vinegarTotal Vinegar solution10% solutionX.10.10X30% solution2.302(.3)MixtureX + 2.20.2(X + 2)

(table 07.06.10.01 available in print form)

Now the student can create the equation .10x + 2(.3) = .2(x + 2) and use the concepts of Algebra I to solve for the one unknown quantity of how many cups of 10% vinegar solution is needed for the desired 20% vinegar solution.

Classroom Activities

The following outline details the activities designed to help students achieve the objectives of this curriculum unit. The following is only a suggested timeline. The unit can be adapted to fit the needs of all students.

Day 1

Students will be given two words from the mathematical language sheet (see appendix). Also, each student will be given two blank sheets of paper. On each piece of paper the student has to make visual representations of the mathematical words given to them. Each drawing should include the following components: word(s), definition, example, and illustration. Students should be encouraged to draw a rough sketch of the drawings on notebook paper before starting on blank paper. Also, the teacher should show several examples of acceptable pieces of work in order to give students complete understanding of the expectations needed to be met. This activity should not take more than one day to complete. If some students are not finished with their work, then words can be taken home and completed for homework. All words should be posted on the bulletin board and throughout the year words may be posted if a term may be deemed as necessary to add on the word wall.

Day 2

Teacher will instruct how to translate one - step word problems involving addition and subtraction by illustrating several examples. The emphasis will be on translating words into mathematical symbols. Then, students will form groups of four and analyze ten problems. Then, each group will be asked to present any two problems and solve them on the board.

Day 3

Teacher will instruct how to translate one-step word problems involving multiplication and division by illustrating several examples. Again, the emphasis will be on translating words into mathematical symbols. Then, students will form groups of four and analyze ten problems. Then, each group will be asked to present any two problems and solve them on the board.

Day 4 - Day 5

Teacher will instruct how to translate two-step word problems involving combining like terms. For independent practice, each row will be given a white board. The teacher will show the class, on the overhead, a two-step word problem. The students will write the equation and show all work on the white board. When finished the student will hold up the board and pass back for the next student.

Day 6

Teacher will give assessment on creating word problems for one and two step equations. Teacher needs to stress that students need to write an equation and show the steps for solving for the variable.

Day 7 - Day 9

Teacher will lead discussion on translating a multi-step equation word problem. The first problems should include ones that involve combining like terms then solving for the variable. Then, students should solve problems involving more complicated relationships between the variables.

Day 10 - Day 14

Teacher will introduce mixture problems by illustrating the table method. Students will participate in guided practice and independent practice sessions in the classroom.

Day 15

Teacher will discuss false positioning and go thru several problems using the technique. All students will be given one unique problem and then several students will be asked to present their findings on their problem scenario.

Day 16 - Day 17

Teacher will take the idea of false positioning and create a linkage to setting up a system of equations. Teacher will lead discussion on translating a system of equation word problem. Guided and independent practice will be occurring within the classroom.

Day 18

Four students will be asked to come to the board at the same time. The students at the board will be asked to solve the following problem: John has 100 coins in his piggy bank some dimes, others quarters. His total capital is $ 13, $ 16, $ 19, or $ 22. (Each student at the board is given a different capital amount) Find the number of dimes and quarters that John has in his bank. All still seated students are required to choose one of the four versions and work on it at the same time. The students may choose to use one, two, or without algebra at all in order to solve the problem. All possible approaches that students may have chosen will be discussed in class. (Toom, 2007)

Annotated Teacher Bibliography/Resources

Cohen, S., & Stover, Georgia (1981). Effects of Teaching Sixth Grade Students to modify format variables of math word problems. *Reading Research Quarterly,* Retrieved July 4, 2007, from http://www.jstor.org/view/00340553 /ap020064/02a00010/0.

The authors give detailed data on experiments performed on sixth graders. The research was conducted in order to investigate how young minds perceive word problems.

Lampert, M (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

The author gives a very comprehensive account of her teaching experiences in regards to teaching problem solving skills.

Landsberger, J (2007, March 16). Studyguides and Strategies. Retrieved July 4, 2007, Web site: http://www.studgs.net/mathproblems.htm

This website gives a detailed outline of common mathematical vocabulary used in equations.

Polya, G. How to solve it a new aspect of mathematical method. Princeton University Press, 1988.

The author gives a detailed account of thinking processes required in the field of mathematics. The book offers approaches and strategies to a wide variety of common concepts found in the discipline.

Toom, A (2007). How I teach word problems. Retrieved July 9, 2007, Web site: http://www.homeschoolmath.net/ teaching/teach-solve-word-problems.php

The author describes his methods on teaching word problems.

Annotated Student Bibliography/Resources

Bluman, Allan. Math Word Problems - Demystified. New York: McGraw Hill, 2005.

This resource book offers students a lot of practice problems that can be found in Algebra I textbook. All problems have a detailed explanation of the solution.

Larson, Ron, and Boswell, Laurie. Algebra I. Evanston, IL: McDougal Littell, 2007.

This textbook provides students with detailed explanations and numerous practice problems.

Appendix A: Mathematical Language Sheet

ADDITION

1 Increased by

2 More than

3 Combined, together

4 Total of

5 Sum

6 Added to

7 Plus

SUBTRACTION

1 Less than

2 Fewer than

3 Reduced by

4 Decreased by

5 Difference of/between

6 Minus, less

MULTIPLICATION

1 Of

2 Times

3 Multiplied by

4 Product of

DIVISION

1 Quotient of

2 Out of

3 Per, a

4 Ratio of

5 Percent (divide by 100)

Appendix B: Sample Word Problems

A. One Variable (1 unknown)

1. One-Step Equations

a. A 40- ft rope is cut into 2 pieces. One piece is 4 ft, what is the length of the other piece?

b. The length of a rope is reduced by 10 feet. Now the rope is 100 feet in length. What is the length of the original rope?

c. The length of a rope is 100 feet and is cut into 5 equal pieces. What is the length of one piece?

d. The average length of 5 pieces of rope is 20 feet. What is the total length of all 5 pieces?

e. In the 2004 Olympics, Shawn Crawford won the 200 meter dash. His winning time was 19.79 seconds. Find his average speed to the nearest tenth of a meter per second.

f. In Everglades National park in Florida, there are 200 species of birds that migrate. This accounts for 4/7 of all the species of birds sighted in the park. Find the number of species of birds that have been sighted in Everglades National Park.

2. Two-Step Equations

a. A 9- ft rope is cut into 2 pieces so that one piece is twice the other. How long is each piece?

b. Two ropes are together 275 yards long. One rope is 50% longer than the other. How long are the ropes?

c. How old am I if 400 reduced by 2 times my age is 244?

d. How old am I if 40 increased by 3 times my age is 100?

e. Jill sold half of her comic books and then bought 16 more. She now has 36. With how many did she begin?

f. Jill sold Â¾ of her comic books and then gave her friend 2 of them. She now has 100. With how many did she begin?

g. John has three sacks of apples and three more in his pocket. Each sack contains the same number of apples. Altogether he has 33 apples. How many apples are in each sack?

h. Kim has a job where she makes $8 per hour plus tips. Yesterday, Kim made $53 dollars, $13 of which was from tips. How many hours did she work?

i. Tyler paid $124 to get his car repaired. The total cost for the repairs was the sum of the amount paid for parts and the amount paid for labor. Tyler was charged $76 for parts and $32 per hour for labor. Find the amount of time it took to repair his car.

j. A skateboarding park charges $7 per session to skate and $4 per session to rent safety equipment. Jared rents safety equipment every time he skates. During one year, he spends $99 for skating charges and equipment rentals. How many sessions did Jared attend?

k. the capacity of a landfill is 4,756,505 tons. The landfill currently holds 2,896,112 tons. A cell is added to the landfill everyday, and each cell averages 1600 tons. After how many days will the landfill reach capacity? Round your answer to the nearest day.

3. Multi-Step Equations

a. A 10- ft long piece of wood is cut into 2 pieces so that 1 piece is 2 feet longer than the other. What is the length of the two pieces?

b. A length of a board was 10 inches shorter than another length. Together the boards were 20 inches. How long were the boards?

c. A 12-ft rope is cut into 3 pieces so that the second piece is 1 foot longer than the first and the third piece is 1 foot longer than the second. How long is each piece?

d. An 80 inch board is to be cut into 3 pieces so that 1 piece is twice another and the third piece is 10 inches more than the second. Find the length of each piece.

e. A pile of 18 coins consists of quarters and dimes. If the total amount of the coins is $3.00, find the number of quarters and dimes the child has.

f. A small child has 6 more quarters than nickels. If the total amount of the coins is $3.00, find the number of nickels and quarters the child has.

g. A person has twice as many dimes as she has pennies and three more nickels than pennies. If the total amount of the coins is $1.97, find the numbers of each type of coin the person has.

h. A person has quarters and dimes that total $2.80. The number of dimes is 7 more than the number of quarters. How many of each coin does the person have?

i. The length of a rectangle is 3.5 inches more than its width. The perimeter of the rectangle is 31 inches. Find the length and the width of the rectangle.

j. A ticket agency sells tickets to a professional basketball game. The agency charges $32.50 for each ticket, a convenience charge of $3.30 for each ticket, and a processing fee of $5.90 for the entire order. The total charge for an order is $220.70. How many tickets were purchased?

k. A roofing contractor gives estimates for shingling a roof in cost per square, where a square is a 10 foot by 10 foot section of roof. The contractor estimates $27.50 per square for materials, $17 per square for labor, $30 per square for overhead and profit, and a total of $750 for miscellaneous expenses. If the contractor gives an estimate of $2314.50, about how many squares does the roof have?

4. Mixture Problems

a. How much cream that is 20% butterfat should be mixed with milk that is 5% butterfat to get 10 gallons of cream that is 14% butterfat?

b. How much of a solution that is 18% fertilizer must be mixed with a solution that is 30% fertilizer to get 50 gallons of a solution that is 27% fertilizer?

c. A candy store owner wants to make 30 one-pound boxes of candy costing $5.00 a box. If she wishes to use 12 pounds of candy costing $8.00 per pound, what should be the cost of the other type of candy she should use?

d. How much of a 10% vinegar solution should be added to 2 cups of a 30% vinegar solution to make a 20% solution?

B.Simultaneous Equations (2 unknowns)

1. Different Values/Percentage

a. You are selling tickets for a musical at your local community college. Student tickets cost $5 and general admission tickets cost $8. If you sell 500 tickets and collect $3475, how many student tickets and how many general admission tickets are sold?

b. You are selling tickets to a concert. Student tickets cost $1.25 and adults cost $2.25. If you sell 560 tickets and collect $916, how many student tickets were sold?

c. Ed invested $2000, one part at 3% interest and one part at 1.5% interest. If he got $41.25 simple interest in one year, how much did he invest at each percentage?

d. Paul invested $3000, part in a 1-year CD paying 1.20% and the rest in municipal bonds that pay 3% a year. The annual return from both accounts was $72. How much was invested in bonds?

e. Margie is responsible for buying a week's supply of food and medication for the dogs and cats at a local shelter. The food and medication for the dogs cost twice as much as those supplies for the cats. She needs to feed 164 cats and 24 dogs. Her budget is $4240. How much can she spend on dogs?

f. A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totaled $487. The second order was for 6 bushes and 2 trees, and totaled $232. The bill does not list the per-item price. What is the cost of one bush and of one tree?

g. A passenger jet took 3 hours to fly 1800 miles in the direction of the jet stream. The return trip against the jet stream took 4 hours. What were the jet's speed in still air and the jet stream's speed?

h. You just purchased a cellular phone and are trying to decide the best cellular phone company in which to give your business. When you contacted the Talks-A-Lot company, they were offering a monthly plan of $40 for 500 minutes and $0.25 for each minute over the 500 minutes. Chat - Away is charging $35 for 500 minutes and $0.30 for each additional minute over 500. How many minutes would you have to talk over the 500 minutes for the cost to be the same with both companies? What would be the equal cost?

i. Dan and Sydney are getting high-speed Internet access at the same time. Dan's provider charges $60 for installation and $42.95 per month. Sydney's provider has free installation and charges $57.95 per month. After how many months will Dan and Sydney have paid the same amount for high-speed Internet service?

k. For $360, a rock-climbing gym offers a yearly membership where members can climb as many days as they want and pay $4 per day for equipment rental. Nonmembers pay $10 per day to use the gym and $6 per day for equipment rental. Find the number of visits after which the total cost for a member and the total cost for a nonmember are the same.

Appendix C

Pennsylvania Academic Standards

2.1.11 A. Use operations (e.g., opposite, reciprocal, absolute value, raising to a power, finding roots, finding logarithms).

2.1.8 G. Use the inverse relationships between addition, subtraction, multiplication, division, exponentiation and root extraction to determine unknown quantities in equations.

2.2.11 A. Develop and use computation concepts, operations and procedures with real numbers in problem-solving situations.

2.5.11 A. Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems.

2.5.11 C. Present mathematical procedures and results clearly, systematically, succinctly and correctly.

2.5.11 D. Conclude a solution process with a summary of results and evaluate the degree to which the results obtained represent an acceptable response to the initial problem and why the reasoning is valid.

2.8.11 D. Formulate expressions, equations, inequalities, systems of equations, systems of inequalities and matrices to model routine and non-routine problem situations.

2.8.11 H. Select and use an appropriate strategy to solve systems of equations and inequalities using graphing calculators, symbol manipulators, spreadsheets and other software.
