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Applying System of Equations to Real-World Scenarios: A Practical Curriculum

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Introduction

Word problems are a problem. Students of all levels continually struggle with word problems; however, there is a solution to this "problem". The objective for this curriculum unit is to show students the multiple ways that system of equations can be used to solve real-world problems. In order for students to be engaged and interested in learning, they need to see the real-world practicality behind the math concept being learned. Without a lot of practice, System of Equations can be difficult for students to compute correctly and efficiently. When difficulty is combined with a lack of interest or a question of relevance, the student can become frustrated and disengaged. Therefore, in order to capture the attention and interest of my students, I will first discuss what a system of equations is and then explain how to solve one using three different methods. Utilizing the substitution method, addition/subtraction elimination and multiplication/elimination methods, we will review how to solve a system of equations.

Once the students understand those concepts, I will discuss and review I = Prt and d = rt. Using the simple interest formula, I = Prt, the students will be exposed to how their money can grow in a year. This will undoubtedly be of interest to any student. The students will also be able to see the advantage and real-world practicality of calculating the distance an object can travel, given the rate of speed and time using the d = rt formula. Then I will combine the distance and interest formulas together with a system of equations and demonstrate how to solve real-world scenarios and exciting problems. I will also provide practical examples of problems involving wind and water currents, chemistry problems involving mixtures and solutions, more interest problems involving the time value of money, and other practical examples that involve systems of equations. I will spend two days on each of the three methods. Next, I will spend another three or four days introducing the interest and distance formulas, incorporating them into system of equations. It is critical that the students get a feel for solving the problems before I introduce the more complicated word problems. The distance formulas and simple interest formulas are not extremely complicated, and my students typically pick up on them quickly. I have also introduced them earlier in the year and use them as warm-up problems several times per week.

The first system of equations I will introduce is one that will require the substitution method to solve. For example;

$y = 4x$ $3x + 4y = 36$

A word problem that can be solved by the substitution method is;

"Teddy has 8 more songs on his iPod than Tyler. Together they have 112 songs. How many songs does each person have?"

Another example of a system of equations solvable by substitution is;

$x + 3y = 9$ $2x - 5y = 27$

The next class of systems of equations that I will present are solvable by the addition/subtraction method. An example would be;

$2x + 4y = 33$ $2x + 6y = 54$

In this system, the coefficient of x is the same in both equations. Therefore, if we subtract the first equation from the second equation we will be able to isolate the y variable. A word problem that would require the use of the addition/subtraction method would be;

"Teddy cuts grass in the summer for $ 20 a lawn. His neighbor, Gavin is very competitive and decides to cut grass as well but only charges $ 18 a lawn. Their total combined revenue one summer was $ 348. Teddy made $ 132 more than Gavin. How many lawns did they each cut?"

The last type of system of equations that I will speak of is one that requires the multiplication elimination method. An example of this type of system is;

$3x + 6y = -6$, $5x - 2y = 14$.

An example of a word problem that we could use the multiplication elimination method to solve is:

"The air-mail rate for letters to Europe is 45 cents per half-ounce and to Africa as 65 cents per ounce. If Shirley paid $ 18.55 to send 35 half-ounce letters to Europe and Africa, how many did she send to Africa?"

This is just a brief overview of the types of problems that I will cover with my class in this unit.

Background Information

I teach 8th grade Algebra in a suburban middle school in Charlotte, North Carolina. In this particular class, I have relatively few behavior problems. Talking at inappropriate times is my students' biggest fault. Carmel Middle School is on a block schedule and my Algebra class meets every day for approximately 85 minutes. I have a pacing guide, created by the county I teach in, that I follow. There is not an abundance of time allocated for systems of equations, so I tend to move rather quickly. I typically have about 30 students in an Algebra class and I have approximately 20 to 25 students in my lower level classes.

The population at my school is as follows: 61.4 % Caucasian, 21.5% African American, 11 % Hispanic and 4 %

Asian. Twenty six percent of our students are enrolled in the Gifted and Talented program, while 12.6% of our students have some degree of disability. We have 30% of our population on free or reduced lunch and the other 70% pay or bring bag lunches.

The percent of students at or above grade level in math is 80%, and the students at or above grade level in reading is even higher- 91.5%. These percentages put us above the Charlotte Mecklenburg District average in both categories. For the last few years, Carmel Middle School has consistently had 100% of Algebra and Geometry students performing at or above grade level.

The District itself is growing larger and larger every year; it currently enrolls 122,000 students and employs 10,050 teachers. Over the last couple of years, the Charlotte-Mecklenburg School System has increased the number of students by approximately 6,000 students per year. The district population is 43% African American, 37% Caucasian, 12% Hispanic, and 4% Asian.

Strategies

First, I will start with introducing the three ways to solve a system of equations. The first way is called the substitution method. I spend two days on substitution with my class. Typically, the problems are set up such that there is only one variable on each side of the equation and at least one variable has a leading coefficient equal to one. The other equation has two variables, as well, each on the same side of the equation. An example is as follows:

$x = 3y$, $2x + 4y = 20$.

Equations in the above form are the simplest and easiest way to start a substitution problem. The students need to understand that when there is more than one unknown in an equation, it cannot be solved as is. Since $x = 3y$, we can simply "substitute" the 'x' variable in the second equation. This would produce the equation $2(3y) + 4y = 20$. I ask for a volunteer to tell me if we now have more than one unknown variable. The rest of the equation should be solved easily. The students know how to solve multi-step equations by combining like terms with relative ease at this point. I make sure also to plug in my answer for 'y' and solve for 'x,' as well. The students need to see that it does not matter which equation they plug the value of 'y' into. They will get the same value of 'x' regardless.

I would also demonstrate problems where the variables are on the same side for both equations, but again one of them has a leading coefficient equal to one. For example:

$x - 7y = 21$ $x + 2y = 14$.

I would demonstrate how easy it is to place the first equation, $x - 7y = 21$, in the form $x = 7y + 21$, and then substitute the value of x into the second equation and solve. The students could have also set the second equation, $x + 2y = 14$, equal to 'x' by subtracting '2y' from both sides to get $x = -2y + 14$. It is important for the students to see they have options and that there is more than one way to solve these problems. The important thing to remind them is that any way they solve it, they will always get the same answer. In order to satisfy the visual learners, I will demonstrate two of each type of problem on the board. I then will give the students a few minutes to solve a problem independently. We will review the answer and I always make sure that no one has any unanswered questions. Please see problems #1-4 in the Appendix for more examples of systems solvable by substitution.

I tend to do a lot of group work with my class. This is beneficial in many ways. It reinforces the concepts for the students who are explaining the problem to other students, allows the students an alternative way of approaching the equations, and, finally, helps me to answer more questions than I have time to get to alone with a large class. I break the class into groups of two and have them work cooperatively towards solving a set of problems involving substitution. I do not focus on word problems on the first day; word problems can overwhelm my students and I ultimately want them to build confidence and not confusion. However, I will assign a couple of word problems for homework that night. Before class ends, I demonstrate how to use the graphing calculator to find the solution. This is an incredibly important learning tool, giving the students a graphic representation of how their answer is derived by the intersection of the two linear equations. It is incredibly important for them to understand not only how to solve the problem, but also what exactly it means when they do. My school has a TI-84plus calculator that I connect to an LCD projector, which allows me to show the entire class. I go through a couple of examples with the students. I explain how to enter the equations in slope-intercept form when pressing the 'y =' button. Then I show them how to graph the two equations by pressing the 'graph' button. The intersection of the two lines is clearly visible and the students are able to get a great graphic representation of their answer. I then allow the students to check their work with the graphing calculator, while at the same time placing great emphasis on the requirement that they show all of their work.

The next day we go over the problems from the homework. I have the students volunteer to come to the board and show their answer and all of the work it took to achieve that answer. We do this for each answer. Student demonstrations are a great learning tool. Students are able to see different ways to approach a problem, assuming the student demonstrations are correct. If these demonstrations are not correct, it gives the class and me an opportunity to discover why not. Students love pointing out each other's mistakes and in return will help to reinforce problem solving strategies. The student who makes the mistake sees what he did wrong and is given ways to correct it. However, certain students do not like being singled out in front of their peers. In the event that one of these students is wrong, I make sure to be cognizant of that and shower him with praise, assuring them that his error was just a minor one. The students will then spend some more time in collaborative groups going over word problems, while I pull aside the students that struggled with the problems on the homework, working with them one-on-one. At this point the students should be fairly proficient at solving the problems. Here is a sample solution of a word problem that can be treated by the substitution method:

"Margaret loves to buy shoes. Margaret has 18 more pairs of shoes than Jane. Together they have 184 pairs of shoes. How many pairs of shoes does each girl have?" This will not give whole numbers for the answers.

It is absolutely imperative that the students establish what the unknown is. I insist that the students write down what they are looking for and establish the unknown. In this case the questions gives away what we are looking for, "How many pairs of shoes does each girl have?". Therefore,

> Let 'm' = the number of pairs of shoes that Margaret has Let 'j' = the number of pairs of shoes that Jane has

The students must write this down before they begin any work. This helps the students begin the process of translating the word problem into an equation. The translation process is typically very difficult for students. Practice and writing these "let" statements to define their variables are the only way to get better at them.

The next step is to write an equation. "Margaret has 18 more pairs of shoes than Jane." This can be written as m = j +18. I then have the students take a look at the third sentence. Together implies addition. Therefore we can write the next equation as m + j = 184. Now we have two equations and we can easily solve them using the substitution method.

After I go over a few examples and the students work together in groups, I assign homework involving word problems. Please see the Appendix, # 5-9, for examples. Many of the students like to use the guess and check method for these problems. They do not want to spend the time writing down the problem and solving it. I repeatedly remind them that the only way to get better at these problems is to practice, ask questions, and listen. I do not assign many word problems for homework because I want them to slow down and take their time doing them. If they have too many, they will typically rush to get them all done and not produce the quality of work they need to be successful.

The next method for solving a system of equations is the addition/subtraction elimination method. I will introduce this concept on the third day of this unit. Before I introduce this new method to the class I go over the homework problems from the night before the same way I did previously, by calling volunteers to the board. Once we are done reviewing the homework I introduce the addition/subtraction elimination method by modeling multiple examples on the board and spend some time going over any questions they may have. The addition/subtraction method is typically used on problems similar to the following:

2x + y = 12 x - y = 24.

The key feature of such systems is that the coefficient of one of the variables in the first equation is the same number as the coefficient of the same variable in the second equation. I instruct my students to line up the equations vertically and analyze the equations to determine which variable would be easier to eliminate. In this case the 'y' would be the easiest to eliminate by vertically adding them together. I would also present a problem such that the students would have to subtract to eliminate one of the variables. For example:

3x + 7y = 10, 3x - 4y = 8.

The leading coefficients of the 'x' variable are equal and therefore can be subtracted to eliminate the 'x' variable. Writing the equations in vertical format, one over the other , makes this easy to do. Additional examples can be found under problems #10- 14 in the Appendix. The main idea with this method is that you must be able to eliminate one of the variables so that you are left with only one unknown. Whether they need to add or subtract is all they need to figure out to be successful in solving this class of systems.

At this point, I also make sure to point out that the students are learning different ways to solve these problems and thus have options. This empowers the students, which excites them, providing them with a sense of ownership.

Another common problem arises when the variables are not in the same order for each equation. For example:

3y + 7x = 10, 3x - 4y = 8.

As this system is written, the 'x' in one equation is not above the 'x' in the second equation. Likewise, the 'y' is not above the 'y'. When a student tries to subtract vertically, they will be combining un-like terms and the result will probably by nonsense. I show them how to move the terms around, using the commutative property, so that x variables align vertically and the y's align vertically. Once I am done explaining and modeling and answering any questions I break the students into collaborative groups and work to solve a set of problems. I circle the room to assist any students that are struggling with the new concept, making sure to remind the students that the graphing calculators are not a substitution for work. They must show their work and the calculators are simply a way of checking to make sure they have done the work correctly. We go over the answers at the end of the block and I answer any questions they may have. The homework assignment for that night will have a limited number of word problems in order to again build the confidence of the students.

The next day we go over the homework as described above. I pinpoint the students having problems with the new concept and take them aside. I work with them as the rest of the class works in groups to solve word problems (please refer to the Appendix, #15 - 18.) Figuring out how to translate these problems into equations is very tricky for students. It takes practice and patience. I remind my students that they will need to have two variables, x and y. They need to read the problem and decide what is going to be x and what is going to be y. Once they have this established, they need to write an equation to satisfy one of the sentences in the problem. For example:

> "The length of a rectangular garden is three times the width. What are the dimensions of the garden, if the perimeter is 32m?"

Do we know what the width or length is? No, we do not. Therefore it is safe to assume let x stand for the width in meters and let y stand for the length in meters. (The students must write this down). Then we can say that since the length is three times the width, the equation, $y = 3x$, describes the relationship between x and y. To finish the word problem, we must analyze the sentence, "What are the dimensions of the garden, if the perimeter is 32m?" Again, these students should know the meaning of perimeter. I require my students to draw a rectangle so that they can see for themselves that when you add the up the lengths of the sides, it should look like, $x + x + y + y = 32$. This simplifies to $2x + 2y = 32$. Again, I present the students with options. They can use substitution or addition/subtraction elimination to solve. I assign some problems to the students to work on collaboratively in groups and I circle the class to address any concerns or questions. Once the end of class is near, we go over the answers and I assign homework consisting largely of word problems using the addition/subtraction elimination method.

Lastly, the multiplication/elimination method is another way to solve a system of equations. This lesson is done on the day five of this curriculum unit. It must be done after the substitution and addition/subtraction elimination methods have been taught. Since the multiplication/ elimination method is a combination of the previous two methods, the students must also be able to effectively analyze a problem and decide which method would be the best approach to solving that particular problem. If they are able to do that then they are ready for multiplication elimination method. In order to find out if my students are ready, I give them a list of system of equations and ask them to identify, without solving, which method you would use to solve each problem. I also ask for volunteers to create their own problem and come to the board and write down a system of equations. Then the class has to decide which method to use and why. Students enjoy coming the board and writing down problems. This is typically a big hit with them and a great way to keep them engaged and interested. It also helps me to see who is grasping the concepts and who is not. This activity helps the students think about what is involved in a substitution or addition/subtraction elimination problem in order to put one together. This is a challenge to most of the students and fosters a higher order learning.

A typical problem that involves the multiplication elimination method follows:

> $x + y = 9$ $3x - 2y = 10$

In this case the student has a couple of different options. They could use what they already know about the substitution method and solve the first equation for x. I also point out that they could solve the problem by first multiplying the equation x + y = 9 by either 3 or 2. I make sure to ask them quest it does not matter ions such as, "does it matter which one I pick?" I show them that both procedures give the same answer. The key step is to eliminate one of the variables so that you have only one unknown. Therefore, if they multiply x + y = 9 by 3 they would simply subtract the answers vertically and eliminate 'x', or if they multiply x + y = 9 by 2 they would add vertically and eliminate 'y'. Either of these procedures will give you the same answer for 'x' and 'y'. It is important to allow the students to feel as if they are in control and have the ability to choose which method they feel the most comfortable with. I make sure to model many examples of problems. Each time I ask for volunteers to tell me if there was an alternate route that I could have taken to solve the problem.

Another type of problem the students might see is in the form:

> 2x + 9y = 7, 3x + 7y = 4

In this example none of the leading coefficients are the same and none of the leading coefficients in the first row can by multiplied by a number to equal a leading coefficient in the second row or vice-versa. In this case I explain that it will take multiplying both equations to get a leading coefficient that is the same. I point out that the 'x' in both equations are factors of 6. I can therefore multiply the first equation by 3, 3(2x + 9y = 7) to get 6x + 27y = 21. I show them that I can also multiply the second equation by 2, 2(3x + 7y = 4) to get 6x + 21y = 8. Now we have the leading coefficient of the 'x' variables in both equations equal to each other. We can subtract them vertically and solve for the unknown variables. After I have answered questions and modeled a couple of examples I then proceed to place them in collaborative groups to solve a set of problems. I circle the room and assist any students that need help. At the end of class we go over the answers and I answer any more questions that they may have. Just as I did in the previous days, I make sure to stress that graphing calculators are not a substitution for showing work. I assign homework consisting mostly of simple two-equation, two-variable problems requiring them to multiply by a constant and then add or subtract to eliminate a variable. The next day we repeat the process, going over the homework, answering questions, and attempting more practice problems in collaborative groups, while I answer any questions they may have. Examples of these types of problems can be found in the appendix, #29 - 32.

On the second day of teaching the multiplication elimination method I repeat the same process as I mentioned before. We complete the warm- up, review the homework, answer any questions, and then I model examples of how to solve a system of equations in word problem format using the multiplication elimination method. An example I would use with the students to show a word problem that would involve the multiplication method is;

> "Jack and Luke were selling lemonade. The difference in the number of cups Luke and Jack sold was 5 cups. Jack was selling his for $ 3 a cup and Luke was selling his for $ 2 a cup and together they made $ 15 in revenue. How many cups did Jack and Luke sell each?"

Immediately the students should know that they must define the unknown variables before they attempt to solve the problem. In this case the question at the end again tells us what we are looking for, "How many cups did Jack and Luke sell each?". Therefore, I have the students write;

> Let x = the number of cups of lemonade that Jack sold Let y = the number of cups of lemonade that Luke sold

By analyzing the word problem they should be able to pick up on key words like "difference" and "together". The first equation could be written as the difference of the number of cups Jack and Luke sold, or x - y = 5. The word "together" implies addition so we could write the last equation as $ 3 for every cup Jack sold and $ 2 for every cup Luke sold and together they sold $ 15 total. This would translate into 3x + 2y = 15. We now have two variables, two unknowns and we can solve this equation using the multiplication elimination method. We have some choices to make in solving this equation. I would stress to the students that it does not matter if we multiply the equation x - y = 5 by 2 or 3. Either way we will get the same solution. I model two more examples of word problems and then answer any questions the students may have. I would also have them attempt to solve a problem on their own then pair-share with their neighbor and compare answers. Then I would go over the answer and see how they did.

If time permits it would be beneficial for the students to spend another day on these word problems. Typically students pick up the process of solving the problems mathematically. The difficult part is actually taking the information from a word problem and creating two equations. Chances are it would be helpful for the students to practice another day on them. Unfortunately, in many school districts taking another day is not an option.

I play a game with the students that they really enjoy. The students break off into groups of two; I then randomly place 10 word problems around the room. Each is written on the inside of a folded piece of paper, so that you cannot see the word problem without lifting the piece of paper. On the outside of the paper is an answer to one of the problems. The students must lift a piece of paper, solve the problem on the inside, and then search the room to find the answer on the outside of one of the other pieces of paper that matches the answer they calculated. Once they have found that answer, they lift the paper again and solve the word problem on the inside of the paper and repeat the process. At the end they should have 10 answers listed on their paper. The first group to find them and be seated in their seat wins. This game involves some patience on the teachers' part because the students do some running around and get a little loud due to the excitement of the race to the finish.

Another great review game involves using the TI-Navigator system. I place the students into groups of two and the students log on. I send the students a question through the calculator they are logged onto and then whichever group to respond correctly gets a point. We keep tally of the points on the board and continue as long as time permits. The students really enjoy working in the groups of two and competing against other. In order to keep the students from rushing, I do not time them or give points only to the top finishers.

Hopefully, at the end of these six to seven days the students are adept at deciding the correct method to solve a system of equations by simply looking at it. They must also be able to create two linear equations from a word problem. If they can do this, then it is safe to move on. This next section is where the fun really begins. Using the formulas I = Prt and d = rt, I model a few word problems and show how to use these formulas. I do not do many problems because they should be fairly familiar with these formulas already. I select word problems that focus on real world scenarios; I start with word problems that involve money. Students are always fascinated by how their money can grow and I find that using problems involving interest is a great way to feed that fascination. I will start by modeling problems similar to:

> "Teddy invested $ 5,000, part at 11% annual interest and the rest at 13% annual interest. If he receives $ 610 interest at the end of one year, how much did he invest at each rate?"

The most difficult part is deciding what your x and y variables should stand for. The question at the end gives you a big hint. "How much did he invest at each rate?" So, we write, let x = the amount invested at 13% (in dollars), let y = the amount invested at 11% (in dollars).

The sum of x and y must equal $ 5,000 and therefore we can write as x + y = 5000. The formula, I= prt, is used in the second equation. In this situation, t = 1. From our definition of the variables, we see that the interest on the 13% account is .13x, and the interest on the 11% accounts is .11y. The combined interest is $ 610, which is the sum of the interest on the two accounts. We can express this by the equation .13x + .11y = 610. To finish we just need to pick which method to use to solve the equation. I also ask the students if they have any money in savings. I call for students to volunteer how much money they have and I take one of them and create a scenario. For example, if one of my students said that he or she had $ 500 in a savings account, we could create a scenario where they needed to have $ 52 in interest by the end of the year. They invested part of the $ 500 at 12% and the rest at 8%. How much of each would they have to invest their money? In order to get their attention, I stress, that the interest is free money. They do not have to do a single thing in order to get this money. This always piques their interest. You can see more examples of this type of problem in the Appendix, # 17-20. After modeling a couple of examples of this type of problem with the students I allow them to spend some time working in groups on their own. I pass out an activity sheet containing word problems that are similar to the ones we just went over. As the students are working in groups to solve the problems, I remind them that once they have solved their answer using paper and pencil they should graph their answer using their graphing calculator to make sure that their answer is correct. I also make sure to reinforce finding what it is that you are looking for and writing that down before they get started writing their equations. As class is ending, I assign the rest of the sheet for homework.

The next day after going over the warm-up and checking the homework, I introduce our next objective, using a system of equations to solve problems involving mixtures. All of my students will have to take Chemistry at some point in their high school careers and this is a valuable tool to help them solve problems in Chemistry. As in previous lessons, I model a couple of examples for my students. I start with the following problem:

> "How much cream that is 20% butterfat should be mixed with milk that is 5% butterfat to get 10 gallons of cream that is 14% butterfat?"

I explain to the class that it is important to establish what it is that we are looking for and write it down. In this case we are looking for gallons of cream. I open the class up to discussion and see what they come up with. As the answers come in I write them all on the board. Once I have everyone's answer written on the board, I have the class vote on which one is correct. Hopefully they decide on gallons of cream. This class discussion and voting enables me to see how many students are grasping the concept and if I need to back up and clarify any misconceptions. I again ask for volunteers to create the "let" statements. The class should come up with the following:

- let x = the number of gallons of cream at 20% butterfat
- let y = the number of gallons of milk at 5% butterfat

Once they have established what the variables mean it is safe to move on. At this point I have them establish the first equation. 10 is the total number of gallons given in the equation, so we can write x + y = 10. Then I explain that to find the amount of the cream that is butterfat we simply multiply the decimal form of the percent by the number of gallons of the cream. As a result, the equation would look like .20x + .14y = .14(10). This should not be too confusing to the students because they have all covered finding the percent of a number in the 6th and 7th grades. At some point, you should take note of the fact that the form of these equations is exactly the same as the interest rate problems. Solving the rest of the equation is simply a matter of preference at this point. Typically, the students have developed a sense of what method is best to use in certain situations. I mention that all these problems, interest rate and mixture with fixed total, can be solved by the method of false position, almost eliminating the need for setting up the system. I would again vote and have a discussion on which method would be best to solve the rest of this equation. The big idea would be that there is more than one way to solve this problem. The only thing that matters is if you get the answer correct or not. After doing a few more examples, I break the students into groups of two again. I assign them more mixture problems from an activity sheet and circle the room monitoring student progress. I again stress to the class that they should check their answers using their graphing calculators. At the end of the class we go over the answers. Before class ends, I lead a class discussion on the importance and relevance of system of equations to get some feedback as to whether the students are beginning to realize its importance or practicality.

The final lesson in this curriculum unit has the potential to be the most difficult. Combining wind and water current problems with system of equations can be a little tricky if not explained correctly. However, the ideas and concepts behind it are fun and can be solved easily as long as the vocabulary is understood. I start by defining the following words:

Air speed: The speed of the airplane through still air (including units) **Wind speed**: The speed of the wind relative to the ground (including units) **Tail wind**: A wind blowing in the same direction as the one in which the plane is heading **Head wind**: A wind blowing in the direction opposite to the one in which the airplane is heading. **Ground Speed**: The speed of the airplane relative to the ground with a tail wind:

> With a tail wind: ground speed = air speed + wind speed With a head wind: ground speed = air speed - wind speed

When I am done going over these definitions, I hold a class discussion on these words. I ask for volunteers to talk about why the ground speed formula makes sense. Sailboats are a great way to demonstrate the power of the wind, and I also like to mention sticking your hand out the window when driving. If you open your hand so it is perpendicular to the road, you can feel the push of the wind on your hand as the car moves. If you try to move it forward it is difficult, but if you move it backwards in the direction of the wind, it is much easier. This relation makes reasonable the formula, ground speed = air speed + wind speed. The idea behind this discussion is to make sure that the students understand how these formulas are derived, which makes the students that much more powerful. It also makes it easier to remember the formula. Students always have a difficult time memorizing something that doesn't make sense. At this point we are ready to begin and I model a couple of examples with the class. For example:

"With a tail wind, a light plane can fly 720km in 2 hours. Going against the wind, the plane can fly the same distance in 3 hours. What are the wind speed and the air speed of the plane?"

As we already know, we must establish the unknowns. Therefore,

> Let x = the air speed in km/hr Let y = the wind speed in km/hr

The students should be familiar with the formula 'distance = rate x time'. The equations must be set up in this format. I ask the class what the distance is in this equation. They should respond by saying " 720 km ". I then ask them what the time is. They should respond 2 hours with the tail wind and 3 hours with a head wind. I ask them what part of 'distance = rate x time' do we have left to find? "Rate!" should be the overwhelming answer. At this point I ask the class to write in their notes what they think is going to be the two equations we write. I give them a few moments and then I have them pair-share their answers with someone that is within one desk of theirs. After a few minutes of discussion, I go over the answer they are looking for. The air speed and wind speed are x and y. With the tail wind the time was 2 hours and this can be written as 'rate times time' or (x+y) times 2 which we write as 2(x + y). With the head wind we can write 'rate x time' as (x-y) x 3 which is the same as 3(x - y). We can then set both of these equal to the distance because 'distance = rate x time', 720 = 2(x + y) and 720 = 3(x - y). I then ask the students to calculate the answer on their own and when they are finished pair-share it with their neighbor and compare answers. Once I have given them enough time to do all of that, I ask for a volunteer to come to the board and demonstrate how to calculate the problem.

It is also important to point out that there are multiple ways to solve this problem. I could distribute the 2 and the 3 resulting in two equations that look like 720 = 2x + 2y and 720 = 3x - 3y. Then I could multiply them both so that their coefficients are the same and then add or subtract. Another way to solve would have been to divide by 2 on the first equation and divide by 3 on the second equation. This would result in two equations that look like 360 = x + y and 240 = x - y. I explain that the next step is a matter of choice. I could add the equations and eliminate the 'y' variable, or I could use substitution and eliminate either of the variables. The students have options and they can decide which way they feel more comfortable using. Once I am done modeling a few more examples for the class, I answer all questions they may have. We break into collaborative groups and work on solving some problems from the activity sheet that I hand out. I remind my students to graph their answers on their graphing calculators and I circle the room to help students. More examples of these types of problems can be found in the Appendix, # 25-28.

Appendix

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1. 3y - 2x = 11

y = 9 - 2x

2. y - 3x = 5

y + x = 3

3. x + y = 6

x - y = 2

4. x = 3y

2x + 6y = 12

5. Susan has seven more fish than Tammy. They have 43 fish altogether. How many fish does each have?

6. Bob is three years older than his brother. The sum of their ages is 33. How old is Bob?

7. Two angles are supplementary. The measure of one angle is 30 degrees more than the measure of the other. What is the measure of the larger angle?

8. The length of the rectangular garden is three times the width. If the perimeter is 32m, what are the dimensions of the garden?

9. x + 2y = 0 -x + y = -3 (2, -1)

10. x + 4y = -24 x - 4y = 24

11. 3x + y = 9 -3x + y = 3 (1,6)

12. 2x + y = 4 x + y = 3 (1, 2)

13. Two small pitchers and one large pitcher can hold 8 cups of water. One large pitcher minus one small pitcher constitutes 2 cups of water. How many cups of water can each pitcher hold?

14. The sum of two numbers is 15. The difference of the same two numbers is one. What are the two numbers? (7,8)

15. Twice a number, minus another number is equal to -10. The sum of these two numbers is 1,130. What are the two numbers? (750, 380)

16. Ted just produced a CD. He sells his new CD for $ 5.00. Brett just released a new CD as well. He sells his for $ 6.00 each. How many CD's would they each have to sell each if the difference in sales was $ 30.00 and the total sales were $ 90.00?

17. A total of $ 12,000 is invested in two funds paying 9% and 11%. If the yearly interest is $ 1,180, find out how much money they invested in each fund.

18. Teddy invested $ 5,000, part at 11% annual interest and the rest at 13% interest. If he receives $ 610 interest at the end of on year, how much did he invest at each rate?

19. Stuart invested $ 1,000 in two different funds. One paid 10% interest and the other paid 9% interest. At the end of the first year he made $ 94. How much did Stuart invest at each rate? (400, 600)

20. Margaret invested $ 510 in two stocks. The first stock had a return of 13% and the second had a return of 7%. The resulting interest after one year was $ 59.70. What was the amount of money she invested in each stock?

21. How many ounces of a 6% iodine solution needs to be added to 12 ounces of a 10% iodine solution to create a 7% iodine solution?

22. How many gallons of a 7% acid solution should be mixed with how many gallons of a 15% acid solution to equal 20 gallons of a 12% acid solution?

23. Dr. Hekyl plans to combine a 12% acid solution with a 30% acid solution to make 72 liters of a 20% solution. How many liters of each should be used?

24. How many liters of a solution that is 18% chlorine must be mixed with a solution that is 30% chlorine in order to get 50 liters of a solution that is 27% chlorine?

25. Flying against a head wind, a plane could fly 3000 km in 6 hours. The plane would require only 5 hours for

the return trip with no change in wind. Find the wind speed and the air speed of the plane.

26. A boat travels 4 km in 20 min with the current. The return trip takes 24 min. find the speed of the current and the speed of the boat in still water.

27. Walking down a long moving escalator, Phil covered the 75 m distance in 25 sec. Walking back up against the motion of the escalator, the distance was covered in 75 sec. What was the speed of the escalator?

28. Steve flew his experimental plane 56.25 km with the wind in 45 min. The return trip took 75 min with no change in the wind. What was the wind speed?

29. 3x + 5y = 11 6x + 4y = 16 (2,1)

30. 3x + 6 y = -6 5x - 2y = 14 (2, -2)

31. 3x + 4y = -25 2x - 3y = 6 (-3, -4)

32. 7x - 5y = 76 4x + y = 55 (13, 3)

33. A landscaping company placed two orders with a nursery. The first order was for 13 bushes and 4 trees, and totaled $ 487. The second order was for 6 bushes and 2 trees, and totaled $ 232. The bill does not list the per-item price. What is the cost of one bush and of one tree?

34. The air-mail rate for letters to Europe is 45 cents per half-ounce and to Africa as 65 cents per ounce. If Shirley paid $ 18.55 to send 35 half-ounce letters abroad, how many did she send to Africa

35. Lucy and Desi are driving across the Mojave Desert when they run out of gas. Desi starts walking east to find a gas station at the same time Lucy walks west to find a phone. After 2 hours they are 4.2 miles apart. Desi walks .4miles per hour faster than Lucy. Find their rates of speed. (d=rt)

Bibliography

Sterling, Mary Jane. (2001). *Algebra for Dummies.* New York: Hungry Minds.

Wingard-Nelson, Rebecca. (2004). *Problem Solving and Word Problems.* Berkeley Heights, NJ: Enslow Publishers.

Larson, Roland; Kanold, Timothy; Stiff, Lee. (1995). *Algebra 2: An Integrated Approach.* Lexington, MA. D.C. Heath and Company.

Bloom's Taxonomy. Learning Skills Program. 4 April 2004 http://www.coun.uvic.ca/learn/ program/hndouts/bloom.html

Lampert, Magdalene. *Teaching Problems and the Problems of Teaching.* New Haven: Yale University Press, 2001.

Ma, Liping. *Knowing and Teaching Elementary Mathematics.* Mahwah: Lawrence Erlbaum Associates, Publishers, 1999.

Piaget, Jean. *To Understand Is to Invent: The Future of Education.* New York: Grossman Publishers, 1973

Slavin, Robert E. *Educational Psychology: Theory and Practice*. Boston: Allyan and Bacon, 2000.

Larson, Kanold, Stiff. *Algebra 2: An integrated approach*. Toronto: D.C. Heath and Company, 1995.

Implementing District Standards

Charlotte Mecklenburg Schools Mathematics Standards

Algebra

Competency Goal 4: The learner will use relations and functions to solve a problem

4.03 Use systems of linear equations or inequalities in two variables to model and solve problems. Solve using tables, graphs, and algebraic properties; justify results.

8th grade Math

Competency Goal 5: The learner will understand and use linear relations and functions

5.01a Identify relations and functions as linear or nonlinear. 5.01d Interpret and compare properties of linear functions from tables, graphs or equations. 5.04 Solve equations using the inverse relationships of addition and subtraction, multiplication and division 5.03 Solve problems using linear equations

Competency Goal 1: The learner will understand and compute with real numbers.

1.02 Develop flexibility in solving problems by selecting strategies and using mental computation, estimation, calculators or computers, paper and pencil.

Briefly annotate those school district academic standards that your unit would implement in a significant way.
