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Quadratic Equations in Word Problems Students Can Relate To

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Objectives

Does your math textbook provide enough word problems for students to feel confident about the subject matter? Can students relate to the problems in the text, or are they mostly artificial and contrived? In this unit, I have compiled a collection of word problems about quadratic equations. I hope they will have more appeal to today's teenagers than standard textbook collections. Also, they are organized in a way that is different from any math textbook I have seen. The premise is that by categorizing a large number of word problems and arranging them in increasing order of difficulty while only changing one aspect of the problem at a time, students will gain a better understanding of the subject matter. As students progress through the categories of word problems, their quadratic-solving skills should improve and they should gain a better understanding of how each small change affects the solution and/or the choice of solution method. These principles were suggested to me during my Yale Teacher Institute National Seminar on word problems, led by Dr. Roger Howe.

I teach at a comprehensive vocational-technical high school where students spend up to one-half of each day in their chosen career area and the remainder of their day in academic classes. The school is a "choice" public school and our students are held to the same academic standards as all public school students in the state. Our math classes are generally grouped heterogeneously and we find a wide range of abilities. In recent years I have taught primarily tenth grade students in either Level 2 or Level 3 of our integrated math program. Students choose our school for a variety of reasons. Some are focused on what they want to do when they finish high school and use the vo-tech school to get a head start; some have been moderately successful students and are looking for a route to success other than a four-year college, and some are avoiding their "feeder" school. All students ask the question, "Why do I need to learn this?"

Teaching at a vocational school offers opportunities in mathematics to find relevant problem situations. I have assembled word problems related to as many career areas as I could. However, the problems are intended to be relevant for high school students in general. For the past 10 years (of the 13 years that I've been teaching math) I have made it a personal mission to improve students' understanding of the idea that doubling both dimensions of a figure QUADRUPLES (not doubles) its area. I have used models, had them draw pictures, do the calculations, etc. to find the relationship between scale factors and area and volume. I do think I have made progress; that is, I believe most of my students understand why doubling two dimensions, in fact, quadruples the area of a figure. However, they don't "own" that concept; their automatic answer, especially on a multiple-choice-type test, would still be that the area doubles if the dimensions are doubled. To further my mission, I chose to focus this unit on quadratic word problems as yet another approach to help students internalize the scale factor relationship between changes in dimensions and changes in perimeter, area and volume.

This unit begins after students have studied the skills needed to solve quadratic equations. They should be able to find x-intercepts by factoring, using the Quadratic Formula, or examining a graph or table on a graphing calculator. They should also be familiar with finding the coordinates of the vertex of a quadratic function. While quadratic functions apply to many problem territories, including projectile motion, geometry, economics, rates, and number patterns, I chose to begin this unit with projectile motion. I selected problems that relate to sports whenever possible because most teenagers can relate to sports, either as a participant or an observer, and because the parabolic path of objects in flight as a function of time is visually represented by the graph of the quadratic function.

Once students complete the projectile motion problem suite, I switch them to the geometry problem suite where they will gain much-needed practice in setting up area and volume equations based on information given in word problems. By breaking the problems into different categories, I hope that my students will gain confidence in approaching word problems, interpreting the information that's there, and write and solve equations to answer the questions posed.

Finally, when they have mastered the art of writing area and volume equations, and they are adept at solving them, I can continue on my personal mission by having students study the effects of dilations (increasing or decreasing dimensions by some multiple) on perimeter, area, and volume. According to Magdalene Lampert, in her book Teaching Problems and the Problems of Teaching, students will see the big ideas if they are given the opportunity to analyze them in multiple situations. Then, if they can abstract a mathematical idea from those situations they should be able to apply it to new situations (Lampert (2001), p.255). By the end of this unit, students will have worked with quadratic functions in multiple situations, and should, one can hope, be successful when asked to apply their knowledge in the future.

Analysis

There is background knowledge required for students to work on the problem suites in this unit. Assuming they recognize the general form of a quadratic function as $ax^2 + bx + c$, students must, at the lowest level, be able to solve equations by using tables and/or graphs on a graphing calculator. At a higher level, students should be able to solve quadratic functions by algebraic methods including square roots, factoring, completing the square or using the Quadratic Formula. Students should also be able to find the vertex (coordinates of the maximum or minimum point) by using a graphing calculator or algebraically from any form of the quadratic function.

We spent considerable time in our seminar categorizing problems in a problem suite according to similarities and differences. The first order of business is to define a problem territory. My problem territory is Quadratic Functions, which I am breaking down into two subgroups, namely Projectile Motion and Geometry. The second order of business is to designate the dimensions that I use for grouping and categorizing the problem suites that I assembled. In this section, I will describe the dimensions in detail using examples. Many more word problems can be found in Appendix B, broken down according to the dimensions I describe.

Problem Suite A: Projectile Motion

The projectile motion problems in my problem suite come from the equation (which is derived from the laws of physics)

$h(t) = h_0 + v_0 t + \frac{1}{2}at^2$

where h(t) describes the vertical height of an object with respect to time, *t* (seconds), and

h_0= initial height

v_0= initial upward velocity

a = acceleration due to gravity (*a* = -32 ft/s or -9.8 m/s).

Dimension 1A: Write the equation

The problem suite begins with students practicing writing projectile motion equations. I would expect students to extract the initial height and initial upward velocity from the information given in the word problem and substitute these values for h_0 and v_0, respectively, in the equation given above. They also need to select the appropriate value for *a*, depending on the units (feet or meters) used in the problem.

Dimension 2A: Evaluate the equation

The simplest question to ask students is to find the height of an object at a given time. In this case, the student simply substitutes the time (in seconds) in place of *t* in the equation. By doing the arithmetic (square the number of seconds and multiply by the *a* value times ½, then add the product of *bt*, and then add the value of *c*), the student is evaluating the equation to find the height.

Dimension 3A: h_0 = 0; find the time it takes an object to return to the ground

If a projectile is launched from the ground, the initial height is zero, or, in terms of the quadratic function $ax^2 + bx + c$, $c = 0$. For example, consider a soccer ball goal kick that a defender kicks from the 6-yard line at an initial upward velocity of 52 ft/s. Since the velocity is given in ft/s, the acceleration in this problem will be -32 ft/s, leading to the equation, $h(t) = -16t^2 + 52t$. An equation in this form will always be factorable by factoring out the variable, t, giving h(t) = t(-16t + 52). This equation can be factored further by factoring out a common factor of -4, giving h(t) = -4t(4t - 13). A quadratic equation in this form can be solved for x-intercepts ("zeroes") or coordinates of the vertex, as described below. The Quadratic Formula will yield the same result, but the factored format leads to solutions quickly, as demonstrated in this section and the next.

To find the time it takes for the ball to return to the ground, first students must set the function equal to zero because the height of the ball on the ground is zero. Next, they need to find the x-intercepts, also known as the roots or the zeroes of the equation. Mathematically, when they find the roots of an equation where $h_0 = 0$, they will find two of them. One of the roots in this case will always be zero because the object is on the ground at the start. So, it's the other root that answers the question of when the object returns to the ground.

Continuing with the example started above, solving the equation -4t(4t - 13) = 0 can be done by setting each of the two factors equal to zero. So, -4t = 0 when t = 0 and 4t - 13 = 0 when t = 13/4. Therefore, the soccer ball will return to the ground after 13/4 = 3.25 seconds in the air. Again, the Quadratic Formula will work to find the "zeroes." And, it's always a good idea to confirm the answers by checking them against a table or graph on the graphing calculator.

Dimension 4A: Find the time it takes an object to reach its maximum height

Here, students must recognize that this question is asking for the x-value (time) that would give the maximum y-value. In other words, they are looking for the x-coordinate of the vertex. Since we can rewrite quadratic functions in vertex form by "completing the square," we know that every quadratic function is a parabola with a vertical line of symmetry that passes through the vertex. Because of that symmetry, two points on the parabola having the same y-value (as in the "zeros") must be reflections of each other across the line of symmetry. Therefore, the line of symmetry must be halfway between them. There are several ways for students to find the coordinates of the vertex point, but I will continue with the soccer example that is already in factored form. We found that the x-intercepts are 0 and 3.25 seconds. To find the line of symmetry, we find the average of 0 and 3.25, which is 1.625. Since the vertex is the only point on the parabola with the maximum y-value, it must be on the line of symmetry. So for this example, the time it takes the soccer ball to reach its maximum height will be 1.625 seconds.

Dimension 5A: Find the maximum height reached by an object

Once you know the time it takes an object to reach its maximum height, what you really know is the x-coordinate of the vertex. So, to find the maximum height, simply evaluate the quadratic function for that *x*-value. For some reason, my students often forget that they know how to "plug" a number (x-value) into an equation to find its corresponding y-value. Perhaps, now that I included Dimension 2A (evaluating) in this problem suite, my students will be more successful at remembering to use the x-value of the line of symmetry to find the corresponding (maximum) y-value of a function.

To complete the soccer example, the maximum height of the soccer ball can be found by evaluating h(1.625) = -4(1.625)(4-1.625 - 13) = 42.25. Therefore, the maximum height reached by the soccer ball is 42.25 feet. Again, we should verify our answers for the two coordinates of the vertex by finding them on the graphing calculator.

Often, one problem will ask students to find all of the things I separated into different dimensions: the time it takes an object to return to the ground, the time it takes to reach a maximum height, and what that maximum height is. I am choosing to keep the questions separated so that students must consider what they need to find, rather than just going through a process of finding "everything."

Dimension 6A: h_0 ^{1} 0; find the max, find the time to reach max or ground

When the initial height of the object is not zero, the quadratic function in the form $ax^2 + bx + c$ will contain all three terms with $c = h_0$. Students will be asked to answer the same three questions previously discussed. The difference will probably be in the solution method. Most likely, the quadratic function cannot be factored easily and students will use the Quadratic Formula to find the x-intercepts. When $h_0 > 0$, one of the x-intercepts will be negative. I would hold a discussion to be sure students understand why a negative time for the ball to be on the ground does not apply to these situations. Looking at a graph of the function on the calculator and seeing that the y-intercept is equal to h_0 (i.e. the graph shows the ball starting above the ground represented by the x-axis on the graph) should help them see that the graph to the left of the y-axis is excluded in this situation and the positive x-intercept represents when the ball hits the ground. This time shows up clearly on the graph, as well.

I will use another soccer example to demonstrate two other algebraic methods for finding the coordinates of the vertex. Suppose a player bumps the ball with her head. If she is standing so that her head is 5 feet above the ground when she bumps it and the ball goes straight up with an initial velocity of 12 ft/s, then the equation would be h(t) = $-16t^2 + 12t + 5$. The first method for finding the coordinates of the vertex is "completing the square." The steps in the process would be:

$$-16t^2+12t+5=-16(t^2-\frac{12}{16}t-\frac{5}{16})=-16(t^2-\frac{3}{8}t+\left(\frac{3}{8}\right)^2-\left(\frac{3}{8}\right)^2-\frac{5}{16})=-16(t-\frac{3}{8})^2+16\left(\frac{29}{64}\right)$$

So, the original equation in the form $ax^2 + bx + c$ has been transformed into the vertex form $(x + h)^2 + k$ where (-h , k) represents the coordinates of the vertex. By transforming the original equation, we can see that the vertex point (in a more simplified form) is $\left(\frac{3}{8},\frac{29}{4}\right)$. Returning to the example, the soccer ball reaches its maximum height of 29/4 = 7.25 feet in 3/8= 0.375 seconds.

The second method for finding the coordinates of the vertex uses the Quadratic Formula. Once again, using the fact that the vertex of the parabola lies on the line of symmetry, we can find the line of symmetry from the first part of the Quadratic Formula, namely, x = (-b/2a)x. For the same soccer example, the line of symmetry occurs at x=-12 / -32 = 3/8 = 0.375 seconds. Then evaluating the equation h(0.375) = $-16(0.375)^2$ + 12(0.375) + 5 = 7.25 feet agrees (fortunately) with the result we got above.

Dimension 7A: Find the time(s) to reach specified height, h(t)¹ 0

If students are solving these equations using tables and graphs on a calculator, this dimension is a non-issue. They are just looking for the x-value(s) that corresponds to a different number in the y-column of the table, or a specific y-value on the graph.

This dimension does add complexity to solving quadratic functions algebraically because the quadratic expression is set equal to a number other than zero, as in $ax^2 +bx+c = h$. However, all algebraic solution methods that we teach are based on finding the x-value(s) that make y = 0. So, students must manipulate the equation to make something equal to zero. The manipulation involves subtracting the specified height, *h*, from both sides of the equation. Only the *c*-value is changed on the left-hand side, and the resulting equation $ax^2 +bx+c' = 0$ ($c' = c - h$) is still quadratic, but now the quadratic expression is set to zero. In this form we can solve it by factoring or using the Quadratic Formula to find the roots.

Example: Suppose a baseball is thrown straight up with an initial velocity of 19 m/s from a height of 2 m above the ground. When is the ball 15 m above the ground?

The equation to solve is $-4.9t^2 + 19t + 2 = 15$. To begin, subtract 15 from both sides of the equation giving $-4.9t^2 + 19t - 13 = 0$. Next, I would apply the Quadratic Formula giving x = 0.89 seconds and x = 3.0 seconds. There should be two times that a ball is at the same height-once on the way up, and once on the way down. Of course, we should confirm these times by checking a graph, table, or substituting the results into the original equation.

Dimension 8A: Find the initial upward velocity

For more practice with algebraic manipulations, as well as solidifying the projectile motion ideas, problems in this dimension give information about a certain point on the graph (time, height) and ask for the initial upward velocity. For example: If a softball player hit the ball from a height of 1.2 m above the ground and it hit the ground after 2.75 sec, what was the initial upward velocity of the ball when it was hit? This problem does not provide a lot of information outright, but we know the force of gravity, and we know that the height of the ball when it hits the ground in 2.75 sec is zero. With this added knowledge, we can write the equation $0 = ½(-9.8)(2.75)^2 + v_0(2.75) + 1.2$ and solve algebraically for $v_0 = 13$ m/s.

Another way to ask for v_0 would be to give the time and height of the maximum and ask for the initial upward velocity. For the same softball situation, the problem would be: If a softball player hit the ball and it reached its maximum height of 9.8 m in 1.33 sec, what was the initial upward velocity of the ball when it was hit? Substituting the vertex (k,h) into the quadratic $y = a(x - k)^2 + h$, we get $y = -4.9(x - 1.33)^2 + 9.8$, which can be written in expanded form as $y = -4.9x^2 + 13x + 1.1$. From this we see that $v_0 = 13$ m/s which agrees with our answer above!

Dimension 9A: Find the initial height

Similar to Dimension 8A, we can give students enough information to solve for the initial height of an object. They would need to take the information given, add some implied information (i.e. gravity, using the correct units) and substitute into some form of the projectile motion quadratic equation. Solving for h_0 then requires applying algebraic skills.

Dimension 10A: Interpret the result/compare result to information given

So far, all of the problems in the suite have asked students to find the value of one of the variables in the word problem. In this group, students must figure out what variable they are looking for and then use the result to answer a question. For example:

A woodland jumping mouse hops along a parabolic path given by $y = -0.2x^2 + 1.3x$ where x is the mouse's horizontal position and y is the corresponding height, both in feet. Can the mouse jump over a fence that is 2 ft high? The answer is yes. But to find the answer, students must find the maximum height the mouse can jump. Although this problem brings in horizontal distance as the x-variable, rather than time, the question still requires finding the y-value (height) of the vertex point by any method they choose. The maximum height the mouse jumps occurs at a horizontal distance of 3.25 ft and is 2.11 feet. Since the maximum height is greater than the fence height, yes, the mouse can jump over it.

Dimension 11A: Including the x and y components of velocity

If I have a very advanced group of students, or ones that solve all problems in the problem suite described so far, I would challenge them with problems that require using trigonometry to determine both the vertical and horizontal components of the initial velocity. These problems are typical of what they will see in Physics. In our curriculum they have already studied trigonometric relationships, so these problems are within their grasp. I think the greater challenge will come from the multiple steps required to answer these questions. I am including some of these problems in the Appendix, but will not include any examples here.

Problem Suite B: Geometry

Within the Geometry problem suite, students will encounter many of the same dimensions that I discussed within the Projectile Motion problem suite. They will encounter problems where $c = 0$ and $c ^1 0$. They will find problems where they must manipulate the equation to equal zero (as described in Dimension 7A above) before applying one of the algebraic solution methods. They will be asked to find the dimensions that yield the maximum area or volume and/or what the maximum area or volume is. In some problems they will need to interpret their answer in order to answer the question. Since students already worked with these dimensions as they related to projectile motion, I am assuming they are fairly adept at solving them, and I will not repeat them here. Instead, the dimensions I will describe are concerned with how to set up the quadratic equations that need to be solved.

For each of the Geometry problems, I would strongly recommend drawing a picture to visualize the problem and labeling the dimensions given. At first students may need help labeling the dimensions in terms of only one unknown, so that they have only one variable in the equation. They will also need to know, or have available to them, basic area, surface area and volume formulas for different shapes and figures.

Dimension 1B: Find the maximum area, given the perimeter

Beginning with rectangular areas, there is a category of problems that provide a perimeter and ask students to find the maximum area that can be enclosed. For example, if you have a 500-foot roll of fencing and a large field, and you want to construct a rectangular playground, what is the largest possible area, and what are its dimensions?

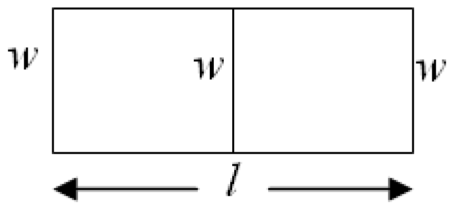
I would first insist that my students draw a rectangle to represent the playground area. Next, they need to label the dimensions. The names "l" and "w" work, but that means there are two variables to solve for. The 500 ft is the perimeter and can be used to relate the length and width of the playground. In other words, $2l + 2w = 500$. Solving for l (it could be w instead) and simplifying, $l = 250 - w$. Now, using the area formula for a rectangle, we can write $A = lw = (250 - w)w$, which is a quadratic function of w. Since we are looking for the maximum, we can leave it in this factored form to find the roots, $w = 0$ and $w = 250$. The maximum will occur halfway between the roots, on the line of symmetry at $w = 125$. So, the width of the playground area should be 125 ft, and, substituting, the length should be 250-125 = 125 ft, and its maximum area would be $125^2 = 15{,}625 \text{ ft}^2$.

After doing several problems of this type, I would hope that some students recognize that the maximum area for a given perimeter occurs when the rectangle is a square. I would review that observation during a short class discussion. It is an observation that many of my students remember from previous math classes, but it never hurts to reinforce things when they reach the same conclusion from another direction.

There are further subcategories for finding the maximum area, given the perimeter. Students may be asked to find the maximum area of a rectangular area when one side uses a physical boundary and the perimeter refers to only three sides of the rectangle. Altering the playground problem above, if one side of the playground is bordered by a school building, what would be the maximum area, and what are its dimensions? In this case, $500 = l + 2w$ (or $2l + w$), so $l = 500 - 2w$. The quadratic function for area would be $A = (500 - 2w)w$. The zeroes would be $w = 0$ and $w = 250$, and the maximum would occur at $w = 125$. The dimensions do change, however. While the width of the maximum area is still 125 ft, the length would be $l = 500 - 2(125)$

=250 ft and the maximum area for the playground would be (250)(125) = 31,250 ft ^{2} (twice as large as the previous example!).

Another subcategory occurs when the perimeter must enclose 2 or more areas that need to be maximized. Continuing with the playground example, if the 500 ft of fencing must enclose two separate playgrounds for different age groups and both must enclose the same area, the picture would look like this:



Then P = 2*l* + 3*w* = 500 and *l* = 250 ñ (3/2)*w*. Area = (250 ñ (3/2)*w*)*w*. The zeroes are *w* = 0 and *w*= 500/3, so the maximum area will occur when *w* = 250/3. The maximum area for both playgrounds together would be approximately 10,417 ft ^{2} with dimensions of 125 ft by 250/3 ft. The area for each playground would be approximately 5,208 ft ^{2} with dimensions of 62.5 ft by 250/3 ft.

The final subcategory is to vary the shape of the area enclosed by a given perimeter. In other words, students may need to use the area formula for shapes other than rectangles, depending on the information given in the word problem.

Dimension 2B: Find the dimensions, given the area and perimeter

In my search through textbooks and Internet sites, I found many word problems that state the perimeter and required area for a region, and students are asked to find the dimensions that satisfy both. An example of this type would be: A student environmental group wants to build a rectangular ecology garden. The area of the garden should be 800 ft ^{2} to accommodate all the species of plants the group wants to grow. A construction company has donated 120 ft of fencing to enclose the garden. What should the dimensions of the garden be? In this case, P = 2*l* + 2*w* = 120, or *w* = 60 - *l*. Then A = *l*(60 - *l*) = 800. To solve, I would distribute the *l,* subtract 800 and rearrange the order to get $-l^2 + 60l - 800 = 0$. There are two solutions, *l* = 20 and *l* = 40. In this example, both solutions work (the garden doesn't know which is length and which is width), and both solutions yield the same dimensions. The garden should be 20 ft by 40 ft.

Dimension 3B: Borders

Another category of area problems that results in quadratic functions involves borders. In each problem, the border will be a uniform width, *x*, surrounding the inner region. This dimension can be broken down into four subdivisions, two of which have a very subtle difference. Sometimes, the word problem presents the specific dimensions (as in length and width of a rectangle) of the inner area (we can calculate the area from the dimensions) and the area of the entire region after the border area has been added. Other times, we are given the specific dimensions of the outer area, and the area of the inner region. For rectangular examples of these two types, we either add 2*x* (*x* in each direction) to each of the inner dimensions, or subtract 2*x* from each of the outer dimensions (again, *x* in each direction).

Example: An elementary teacher wants to paint a 4-square court in the center of a 20 ft by 30 ft fenced area. If the teacher wants a walkway of uniform width around the court that leaves a court area of 336 ft ^{2} , how wide is the walkway? The outer (original) area is 20 x 30 = 600 ft^2 and the inner area is 336 ft^2. Since the walkway must be the same width on all four sides of the rectangle, the inner width can be represented by 20 - 2x, and the inner length can be represented by 30 - 2x. Then the inner area will be (20 - 2x)(30 - 2x) = 336. Expanding, subtracting 336, and simplifying gives us $4x^2 - 100x + 264 = 0$. The two solutions are x = 3 and x = 22. Since the walkway cannot be wider than the width, x = 22 is impossible, and the walkway must be 3 ft wide. (Check: 14x24 = 336 ft^2)

The third subdivision is very similar to the first two, except that the area of the border is given. Problems of this type require adding the border area to the inner area or subtracting the border area from the outer area when writing the representative area equation. The fourth subdivision would be for shapes that are not rectangular. The formulas would differ, but they are solved in the same manner.

Dimension 4B: Volume

The most common variety of volume problems that result in quadratic functions are those that begin with a rectangular piece of cardboard/metal. If a square is cut from each of the four corners and the sides folded up, it forms a box/tray without a lid. In some of the problems, students are given the side length of the squares cut out, while in other problems they are given the dimensions of the original material and must find the size of the square cutout.

Example: A square piece of cardboard was used to construct a tray by cutting 2-inch squares out of each corner and turning up the flaps. Find the size of the original cardboard if the resulting tray has a volume of 128 in^3. Since the original cardboard is a square of length *x*, the length of each side of the base of the tray after cutting out the squares can be expressed as *x* - 4 (2 in from each end). Then the volume formula for a "box" gives V = *lwh* = $2(x - 4)^2 = 128$. After expanding, distributing, subtracting 128 and simplifying, we get $2x^2 - 16x - 96 = 0$. The two solutions are x = 12 and x = -4. Since a length cannot be a negative number, the original length of each side of the cardboard was 12 inches. (Check: 2x8x8 = 128 in^3)

Dimension 5B: Pythagorean Theorem

I must admit that the nearly all of quadratic problems that I found that required the Pythagorean Theorem are contrived problems. However, I include them in this unit because they are good reinforcement for quadratic functions, algebraic manipulations and Pythagorean Theorem.

Example: A nature conservancy group decides to construct a raised wooden walkway through a wetland area. To enclose the most interesting part of the wetlands, the walkway will have the shape of a right triangle with one leg 700 yd longer than the other and the hypotenuse 100 yd longer than the longer leg. Find the total length of the walkway. This problem is asking students to find the perimeter of the triangle. We start by expressing the lengths of each side in terms of the length of the shortest leg, *x*. Then the longer leg has length *x* +700, and the hypotenuse has length *x* + 800. Applying the Pythagorean Theorem, we get $x^2 + (x + 700)^2 = (x + 800)^2$. After expanding, rearranging, simplifying, etc., we have the equation $x^2 - 200x - 150{,}000 = 0$ to solve. The solutions are x = 500 and x = -300. Again, since length cannot be a negative number, the length of the legs are 500 yd and 1200 yd, and the length of the hypotenuse is 1300 yd. The length of the walkway is then 500 + 1200 + 1300 = 3000 yd. (Check: $500^2 + 1200^2 = 1300^2$)

Dimension 6B: Surface Area

I only found a few problems involving surface area, but they were different enough to include in this unit for a change of pace. The problems can be found in the Appendix but can be omitted because of time constraints, if necessary.

Dimension 8B: Dilations

Dilations form their own problem suite. I use area problems, described in the dimensions above, as a basis. I ask students to double or triple the area, make a prediction about the new dimensions of the figure. Then they calculate the new dimensions, and finally, compare their prediction to their calculated dimensions. Or, I ask students to double (for example) the dimensions of a figure, predict the new area, calculate the new area and compare the two. As students compare their predictions to their calculations, I expect them to reason why their predictions were correct or incorrect. Their reasoning will be a source of classroom discussion to help students internalize the effects of scale factors on area, perimeter and volume, my personal mission!

Example: A plumbing contractor realized he needed more storage space for his supplies. If he wants to double the space that he has now, a 10 ft by 12 ft shed, by adding the same amount to both the length and width, what are the new dimensions of the shed? I would expect students to predict the new space to be 20 ft x 24 ft (even though they are ignoring the condition of adding the same amount to length and width). To calculate the new dimensions, let *x* be the number of feet added to each dimension. Then, (10 + x)(12 + x) = 2(10-12) = 240. After expanding and manipulating, the equation to solve is $x^2 + 22x - 120 = 0$, yielding x » 4.5 and x » -26.5. Since length cannot be negative, the amount to add to each dimension is 4.5 ft. Thus, the new storage area would be 14.5 ft by 16.5 ft giving an area of 239.25 ft^2, essentially double the original 120 ft^2, as desired. If each of the dimensions were doubled (as in the prediction above), the new area would be 480 ft^2; four (2^2) times the original area! Furthermore, the average ratio of new to old dimensions (14.5/10 & 16.5/12) is 1.41»√2, an observation that I will be sure to point out if my students don't see it themselves. Also, a follow-up discussion on similarity with respect to multiplying versus adding to alter dimensions might be appropriate.

Strategies

I always begin class with a Warm-Up activity. Sometimes it is general review to keep concepts fresh, and sometimes I use the activity to lead into a new lesson. I write the Warm-Up activity on the chalkboard. When students enter the classroom they are supposed to copy the questions, along with the date, into the proper section of their notebooks while I take attendance or deal with other issues. They should do their best to answer the questions themselves, but are allowed to consult with classmates in their groups, or nearby. I always review the Warm-Up questions, and I expect students to record the correct answers and reasoning in their notebooks.

While I vary seating arrangements from traditional rows to semicircular rows to pairs to groups, I typically have students seated in groups of 3-4 in the classroom. I arrange the groups so that at least one person can usually help the others. For problem-solving lessons like these, I would assign roles for the group members. One person would read the word problem aloud, another would restate the information given that they will need to use in a formula. The third person would restate the question that they are trying to answer. If there is a fourth member of the group, I would assign him/her the role of Time Manager to keep everyone on task, moving forward, and at the same place at the same time. For groups of 3, one member has to do "double-duty." I would also rotate the roles, either problem to problem, or partway through the class period.

Classroom Activities

Lesson 1: Projectile Motion

To lead into the Projectile Motion lesson, I would have students practice evaluating expressions for given values of the variables. In particular, I want students to recall that the product of any number of factors is zero if any one of the factors is zero. This is a key concept behind factoring quadratic functions that my students sometimes lose sight of. A possible Warm-Up activity might be: Evaluate 18ab(c + d)(e - f) when

a = 1, b = 0, c = 4, d = -8, e = 100, f = 73
a = 4, b = -2, c = 0, d = 1, e = 7, f = 7
a = -3, b = 7, c = -6, d = 6, e = 2, f = 5
a = 2, b = 1, c = 2, d = 0, e = 3, f = 1

The discussion afterward would highlight the different ways that the same expression resulted in a product of zero.

Before beginning the word problems, I would define the variables and describe the physics (height would increase linearly forever, except that gravity becomes a greater force over time because of t^2 to pull the object back down to earth) behind the projectile motion formula $h(t) = h_0 + v_0t + \frac{1}{2}at^2$. Next, I would demonstrate how to write the equation given the information in a problem. Students would then begin to work on the sports-related word problems in their assigned groups. Appendix B provides an assortment of problems, but I might give a more extensive list to students so that they can have some choice in which problems they do within each category. You can tweak the problems to fit the sports that most interest your own students; however, be cautious with your choice of parameters and units to ensure that they're realistic.

As groups reach Dimension 7A (solve for a specific height), be sure to check that they manipulate the equations so they equal zero (as described earlier) before applying any algebraic solution method. I would also be prepared for a class discussion to emphasize the need to set the equation equal to zero if many groups don't recognize it themselves.

Within 2 or 3 90-minute block periods, I would expect all students to complete, and be held accountable for, word problems from Dimension 1A through 9A. Because of the range of ability levels within most classrooms, I know not every group will work at the same pace, but there are additional problems available for those that are prepared to move on. Ideally, I would love for my serious athletes to apply the principles relating the horizontal and vertical components of velocity to their own sports to see how they might improve their game, but I think it will depend on time, interest and ability.

Lesson 2: Geometry

I expect this geometry lesson to last about 2 days on a 90-minute block schedule. I will review basic perimeter, area, surface area and volume formulas for a variety of 2- and 3-dimensional shapes in my Warm-

Up activity for the quadratic geometry problem suite. Some of the questions are trivial, but some require multiple steps.

Part I. Find the area and perimeter of a) square with side length 15 cm, b) rectangle with length = 40 in, width = 24 in, c) isosceles right triangle with hypotenuse = 3 m, d) equilateral triangle with side length = 8 in, e) circle with radius = 6 cm. *Part II*. Find the volume and surface area of f) cylinder with radius = 2 in and height = 10 in, g) box with length = 70 mm, width = 60 mm, height = 130 mm, h) box with square bottom with area = 81 ft^2, height = 20 ft. *Part III*. Given the perimeter of a rectangle = 18 cm and length = 4cm, find the width. Given the perimeter of a rectangle = 50 cm and width = *x*, find the length (in terms of *x*).

From previous experience, I expect my students to have trouble writing the equations for the geometry word problems, especially using the perimeter to write dimensions in terms of just one variable. Therefore, before assigning the word problem set, I will do one or two examples with the full class. Again, students will work in their groups so they will have support as they practice writing and solving quadratic equations. Hopefully, students will make some observations as they work through the geometry problems. I will let their observations and difficulties lead to full-class discussions.

Once all groups have completed the first five categories (the "faster" groups will get to surface area), I will have students find a partner (or triple) that is in the same career area. The assignment for the pairs is to write and solve a minimum of three word problems related to their career area. (Non-vocational students can create problems about anything of interest to them.) One problem should focus on perimeter, one on area, and the third on volume. To help them, I will talk about the baseboard molding of the classroom measuring the same as its perimeter (this would work for a student's bedroom, also). The tiles on the floor cover the area of the floor, and the air in the room, or cabinet space are measures of volume. By the way, I will save these student-generated problems as a source of future problems! I don't expect the students to create three quadratic problems, and that's OK; they need to recognize the difference between quadratic and linear equations.

Lesson 3: Dilations

One more day for geometry, but this one focuses on dilations. As a Warm-Up, and reinforcement, I would take a problem or two from the previous geometry problems and change the numbers. Continuing with the pairs from the same career area, I will hand out a set of problems related to an assortment of careers, and have students select 3-4 problems of their choice. Since I only wrote one or two problems per career area, they will have to do some unrelated ones, also. However, by doing multiple problems they should start to see the relationship between changes in dimensions (scale factor) and changes in area.

I have some general instructions and tips for this problem suite. First, pay attention to units! Second, compare (by ratio) the original dimensions to the new ones; record the ratio (aka, scale factor). Third, compare (by ratio) the original and new area; record the ratio. Fourth, compare the ratio of areas to the scale factor. In each problem, students are asked to predict new dimensions or area and compare predictions to calculated answers. So, fifth, reason why predictions are right or wrong. After doing several problems, I hope students will be making correct predictions because they've learned that area increases/decreases by the square of the scale factor. That is, when the area is doubled, the dimensions only increase by a factor of √2» 1.4, but when the dimensions are doubled, the area increases by a factor of $2^2 = 4$!

The follow-up part of this lesson is for the pairs to write and solve another (quadratic this time) problem related to their career area and create a poster illustrating the problem. Next, I will have the partners split up and find new partners from a different career area. The new partners will each be an expert (good for selfesteem) and explain their problems to each other. Finally, everyone will solve his/her partner's problem. If time allows, I will also have pairs present the problems posed on the posters to the rest of the class. Again, I will keep the student-generated problems for future use since they know more about their career areas than I do. I can also use them to add to the problem set so future classes will have more choices.

Resources

These two books served as general background reading for teaching mathematics. They refer to elementary topics, but the ideas apply to any level.

Lampert, M. (2001). *Teaching Problems and the Problems of Teaching*. New Haven, CT:

Yale University Press.

Ma, Liping. (1999). *Knowing and Teaching Elementary Mathematics*. Mahwah, NJ:

Lawrence Erlbaum Associates, Inc.

I used the following list of textbooks to find quadratic word problems related to sports and geometry; however, any math or physics text would serve the same purpose.

American River College, & University of New Orleans. (2004). *Intermediate Algebra* (9th ed.).

Boston: Pearson Addison-Wesley.

Burger, E. B., Chard, D. J., Hall, E. J., Kennedy, P. A., Leinwand, S. J., Renfro, F. L., et

al. (2007). *Algebra 2*. Orlando: Holt, Reinhart and Winston.

Dossey, J. A., & Embse, C. B. V. (1996). *Secondary Math, An Integrated Approach*.

Menlo Park, CA: Addison-Wesley.

Hirsch, C. R., Fey, J. T., Hart, E. W., Schoen, H. L., & Watkins, A. E. (2008). *Core-Plus*

Mathematics. (2nd ed.). New York: Glencoe/McGraw Hill.

Holliday, et al. (Eds.). (2005). *Algebra 2*. New York: Glencoe/McGraw-Hill.

Larson, R., Boswell, L, Kanold, T. & Stiff, L. (2004). *Algebra 2*. Evanston, IL:

McDougal Littell.

Lial, M. L., Hornsby, J., & Schneider, D. (2005). Precalculus. (3rd ed.). Boston: Pearson

Addison-Wesley.

Wilson, J. D., & Buffa, A. J. (1997). *Physics* (3rd ed.). Upper Saddle River, NJ:

Prentice Hall.

The following list provides additional sources of word problems, including puzzles.

Carroll, L. (1958). *Symbolic Logic* and *Game of Logic*. New York: Dover Publications,

Inc.

Gardner, M. (Ed.). (1959). *Mathematical Puzzles of Sam Loyd*. Mineola, NY: Dover

Publications, Inc.

Kordemsky, B. (1972). *The Moscow Puzzles*. New York: Dover Publications, Inc.

Members of NCTM can access calendar problems from *Mathematics Teacher* magazine and search for ones appropriate for any topic via the website: http://www.nctm.org

An Internet search on "quadratic equations and word problems," "quadratic equations and applications," "quadratic equations and sports," etc. all provide a multitude of sample problems. One such site, Purple Math, always comes up and has 3 pages of examples.

Stapel, Elizabeth. "Quadratic Word Problems: Projectile Motion." Retrieved July 12,

2007 from http://www.purplemath.com/modules/quadprob.htm

Materials for Classroom Use

Graphing Calculators, if possible, are recommended

Word Problems - I provide a collection of word problems, grouped according to the dimensions described in the Analysis section, in Appendix B. I had to limit the collection because of space. Teachers, feel free to select any variation of them or add to them to suit the needs and interests of your own students.

Poster Paper and Markers - In Lesson 3, I assign students to make posters illustrating a problem.

Appendix A - Implementing District Standards

Our district standards align with state standards, so the following is a list of State of Delaware Mathematics Standards that are addressed by this unit.

Content Standard 2 - Algebraic Reasoning:

Students in grade 10 will be able to use linear, quadratic and cubic functions to describe length, area and volume relationships and also estimate solutions to…quadratic functions using tables and graphs.

Students in grade 11 will be able to use algebraic techniques to identify the vertex and intercepts for quadratic functions and also apply the quadratic formula to solve problems.

Content Standard 3 - Geometric Reasoning

Students in Grade 8 will be able to demonstrate the effects of scaling on volume and surface area of rectangular prisms. (NOTE: I find this to be an area of weakness, despite it being an 8 th grade standard, so the 3 rd lesson in this unit is trying to reinforce it from another approach.)

Students in Grade 10 will be able to find missing dimensions of a shape given the area, volume, or surface area.

Process Standard 5 - Problem Solving

All students in Grades K-12 will be able to build new mathematical knowledge, solve problems that arise in mathematics and in other contexts, apply and adapt a variety of appropriate strategies to solve problems, and monitor and reflect on the process of mathematical problem solving. (NOTE: I believe more exposure to word problems should improve problem-solving skills.)

Process Standard 8 - Connections

All students in Grades K-12 will be able to recognize and use connections among mathematical ideas, understand how mathematical ideas interconnect and build on one another to produce a coherent whole, and recognize and apply mathematics in contexts outside of mathematics. (NOTE: This standard summarizes the goal of this unit. Quadratic functions relate to many contexts, and, in this unit, students are given the opportunity to practice the mathematics of quadratic functions in multiple contexts.)

Appendix B - Collection of Word Problems

Problem Suite A: Projectile Motion

Dimension 1A: Write the equation

> Avery throws a football straight up in the air with an upward velocity of 27 m/s from a height of 1.5 m. Write the equation describing the height of the football as a function of time.
> A soccer player sets up a free kick by putting the ball on the ground near the referee. If she kicks it with an initial upward velocity of 68 ft/s, what equation describes the height of the ball as a function of time?
> If a golf ball is hit with an initial upward velocity of 20 m/s, write the equation describing the height of the golf ball *t* seconds after it is hit.

Dimension 2A: Evaluate the equation

> A basketball player launched a shot from beyond midcourt just 3 seconds before the final buzzer. If the

ball was launched from a height of 8 feet with an initial upward velocity of 41 ft/s, the equation describing height off the ground as a function of time would be $h(t) = -16t^2 + 41t + 8$. How high would the ball be 2.5 seconds after the shot was launched?

A boat in distress launches a flare straight up with a velocity of 190 ft/s. If the path of the flare is modeled by $h(t) = -16t^2 + 190t + 20$, how high is the flare 10 seconds after it was launched?

The height *h* in feet of a person on a waterslide can be modeled by the function $h(t) = -0.025t^2 - 0.5t + 50$, where *t* is the time in seconds. At the bottom of the slide, the person lands in a swimming pool. How high is the person after 1 second on the slide?

Dimension 3A: $h_0 = 0$; find the time it takes an object to return to the ground

A soccer goalie kicks the ball from the ground at an initial upward velocity of 40 ft/s. How long will it take the ball to hit the ground?

A golf ball is hit from ground level with an initial upward velocity of 62 ft/s. After how many seconds will the ball hit the ground?

Dimension 4A: $h_0 = 0$; find the time it takes an object to reach its maximum height

Suppose a baseball is thrown straight up from a height of 4.5 ft with an initial upward velocity of 60 ft/s. At what time will the maximum height be attained?

A football player attempts a field goal. The quarterback holds the ball on the ground as the kicker kicks with an upward velocity of 50 ft/s. How long does it take the ball to reach its maximum height? It its horizontal velocity is 18 ft/s, how far has it gone?

Dimension 5A: $h_0 = 0$; find the maximum height reached by an object

Suppose a baseball is shot straight up from a height of 4.5 ft with an initial velocity of 60 ft/s. What is the maximum height reached by the ball?

A golf ball leaves the tee with an initial upward velocity of 18 m/s. What is the ball's maximum height? If its horizontal velocity is 6.5 m/s, how far has it gone?

Dimension 6A: $h_0 \,^{1}\, 0$; find the max, find the time to reach max or ground

Brandon threw a baseball with an upward velocity of 50 ft/s from a height of 6 ft. How long will it take the ball to reach its maximum height? What is the maximum height the ball reaches?

A player bumps a volleyball when it is 4 ft above the ground with an initial vertical velocity of 20 ft/s (equation would be $h = -16t^2 + 20t + 4$). What is the maximum height of the ball? If the volleyball were hit under the same conditions, but with an initial velocity of 32 ft/s, how much higher would the ball go?

In a volleyball game, a player on one team spikes the ball over the net when the ball is 10 ft above the court. The spike drives the ball downward with an initial velocity of -55 ft/s. Players on the opposing team must hit the ball before it touches the court. How much time do the opposing players have to hit the spiked ball?

Jason lobbed (hit) a tennis ball upward with a velocity of 48 ft/s from a height of 4 ft above the ground. How long does his opponent have to get to the ball before it hits the ground?

Dimension 7A: Find the time(s) to reach specified height, $h(t) \,^{1}\, 0$

A baseball player hits a high pop-up with an initial upward velocity of 98 ft/s, 4.5 ft above the ground. How long does a player on the opposing team have to catch the ball if he catches it 5.6 ft above the ground?

A basketball player passes the ball to a teammate who catches it 11 ft above the court, just above the rim of the basket, and slam-dunks it through the hoop (an "alley-oop" play). The first player releases the ball 5 ft above the court with an initial upward velocity of 21 ft/s. How long is the ball in the air before being caught, assuming it is caught as it rises?

A baton twirler tosses a baton into the air. The baton leaves the twirler's hand 6 ft above the ground and has an initial upward velocity of 45 ft/s. The twirler catches the baton when it falls back to a height if 5 ft. For how long is the baton in the air?

Dimension 8A: Find the initial upward velocity

A tennis ball hits a winner from 0.5 m above the ground that hits the sideline 1.8 sec later. What was the initial upward velocity of the ball?

A golfer hits his second shot from the ground. It reaches a maximum height of 100 ft in 2.5 sec. What was its initial upward velocity?

A football punt reaches a maximum height of 68 ft in 2 sec. What was the initial upward velocity of the football?

Dimension 9A: Find the initial height

A baseball line drive was hit with an initial upward velocity of 3 m/s. It was caught by the 3 rd baseman 0.1sec later at a height of 1.1m. What was the initial height of the ball when it was hit?

A diving volleyball player bumped the ball with an initial upward velocity of 18 ft/s. Another player was able to set the ball 1 sec later at a height of 5 ft. What was the height of the volleyball when it was bumped?

Dimension 10A: Interpret the result/compare result to information given

A baseball is popped up into foul territory with an upward velocity of 42 ft/s from a height of 3.5 ft above the ground. If the left fielder is 100 ft away and runs at an average speed of 18 ft/s, will he be able to reach the ball before it hits the ground?

A player throws the ball home from a height of 5.5 ft with an initial upward velocity of 28 ft/s. The ball is caught at home plate at a height of 5 ft. Three seconds before the ball is thrown, a runner on third base starts toward home plate, 90 ft away, at a speed of 25 ft/s. Does the runner reach home plate before the ball does?

Dimension 11A: Including the x and y components of velocity

A golf ball leaves the tee with an initial velocity of 30m/s at an angle of 37° to the horizontal. At what time(s) will the golf ball be at 10m above the ground? What is the maximum height reached by the ball? What is its range (horizontal distance traveled by the ball)?

A quarterback passes a football with a velocity of 50ft/s at an angle of 40° to the horizontal toward an intended receiver 30 yd downfield. The pass is released 5ft above the ground. Assume that the receiver is stationary and that he will catch the ball if it comes to him. Will the pass be completed?

Problem Suite B: Geometry

Dimension 1B: Find the maximum area, given the perimeter

You have a 500-foot roll of fencing and a large field. You want to construct a rectangular playground area. What are the dimensions of the largest such yard, and what is the largest area?
Steve has 120 ft of fence to make a rectangular kennel for his dogs. What dimensions produce a kennel with the greatest area?
Joe has 30 ft of fence to make a rectangular kennel for his dogs, but plans to use his garage as one side. What dimensions produce the greatest area?
A roll of aluminum with a width of 32cm is to be bent into rain gutters by folding up two sides at 90°angles. A rain gutter's greatest capacity, or volume, is determined by the gutter's greatest cross-sectional area. Find the length of aluminum that should be folded up on each side to maximize the cross-sectional area.
Suppose a stream borders our land, and we want to make a right-triangular garden with the stream as the hypotenuse. If we have only 80 feet of fencing, what is the maximum area of our garden?
To create a temporary grazing area, a farmer is using 1800 ft of electric fence to enclose a rectangular field and then to subdivide the field into two equal plots. What is the largest area of the field the farmer can enclose?

Dimension 2B: Find the dimensions, given the area and perimeter

An ecology center wants to set up an experimental garden using 300m of fencing to enclose a rectangular area of 5000 m^2. Find the dimensions of the garden.
A student environmental group wants to build a rectangular ecology garden. The area of the garden should be 800 square feet to accommodate all the species of plants the group wants to grow. A construction company has donated 120 feet of iron fencing to enclose he garden. What should the dimensions of the garden be? If additional plants are donated that require 110 ft^2 of space, will the 120 ft of fencing be enough for the enlarged garden?
A kennel owner has 164 ft of fencing with which to enclose a rectangular region. He wants to subdivide this region into 3 smaller rectangles of equal length. If the total area must be 575 sq ft, find the dimensions of the entire enclosed region.

Dimension 3B: Borders

Tonya wants to buy a mat for a photograph that measures 14 in. by 20 in. She wants to have an even border around the picture when it is mounted on the mat. If the area of the mat she chooses (before it is cut) is 352 in^2, how wide will the border be?
A landscape architect has included a rectangular flowerbed measuring 9ft by 5ft in her plans for a new building. She wants to use two colors of flowers in the bed, one in the center and the other for a border of the same width on all four sides. If she has enough plants to cover 24 ft^2 for the border, how wide can the border be?
A family has a round swimming pool in their back yard with a diameter of 48 ft, and they want to build a circular deck around it. If the space available for the pool and deck is 2300 ft^2, and they want the deck to be a uniform width, how wide can the deck be?
A ring of grass with an area of 314 yd^2 surrounds a circular flowerbed, which has a radius of 10 yd. Find the width of the ring of grass.

Dimension 4B: Volume

A square piece of cardboard has 3 in squares cut from its corners and then has the flaps folded up to form an open-top box. What original length would yield a box with volume 432 in^3 ?
You are designing the ventilation hood for a restaurant's stove. The hood is to be made by cutting squares from the corners of a piece of sheet metal, then folding the corners and welding them together. The piece of sheet metal is 5 ft wide. The length of the finished hood should be 9 ft, and its volume must be 22 ft^3 . The height of the hood should not exceed 1 ft. What will be the height of the completed ventilation hood?

Dimension 5B: Pythagorean Theorem

A nature conservancy group decides to construct a raised wooden walkway through a wetland area. To enclose the most interesting part of the wetlands, the walkway will have the shape of a right triangle with one leg 700 yd longer than the other and the hypotenuse 100 yd longer than the longer leg. Find the total length of the walkway.
The perimeter of a TV screen is 88 in. Find the least possible value of the length of the diagonal. What are the dimensions of the TV screen?
A kite is flying on 50 ft of string. Its vertical distance from the ground is 10 ft more than its horizontal distance from the person flying it. Assuming that the string is being held at ground level, find its horizontal distance from the person and its vertical distance from the ground.

Dimension 6B: Surface Area

The surface area of a box with open top has a square base and a height of 4 in. If the surface area of the box is 161 in^2 , find the dimensions of the base.
A manufacturing firm wants to package its product in a cylindrical container 3 ft. high with surface area 8p ft^3 . What should the radius of the circular top and bottom of the container be?

Dimension 7B: Dilations

For each problem,

- a. predict the answer,
- b. calculate the answer,
- c. compare your calculation to your prediction, and
- d. reason why your prediction was right or wrong.

OFFICE/WORK SPACE: A company bought office space measuring 14 m by 20 m. They want to create cubicles or work areas in the center, surrounded by a hallway that is the same width all the way around. In the first design, the area of the cubicles is equal to the area of the hallways. What is the width of the hallways? If the width of the hallways is cut in half to provide more work area, what is the corresponding area remaining for the cubicles?
WORK SPACE: The manager of an auto body shop wants to expand his business and enlarge the work area of his garage. If the original garage area is 30 ft by 80 ft. and he plans to double the work area, what are the new dimensions of the enlarged work area if it is enlarged by the same amount in each direction?
WORK SPACE: The manager of an auto body shop wants to expand his business and enlarge the work

area of his garage. If the original garage area is 50 ft by 60 ft. and he plans to double both the length and width, what is the increase in work area?

DRAFTING: A house plan shows a center entranceway with rooms off of it on three sides (left, right and back). The homeowner wants to cut the area of the entranceway in half by moving the 3 walls in by the same amount to give each of the surrounding rooms more space. If the original entranceway was 18 ft by 18 ft, how far should each wall be moved?

LANDSCAPING: A student environmental group wants to build a rectangular ecology garden. How many feet of fencing does the group need if the maximum area they expect to plant is 500 ft^2 ?

If the group decides to double the maximum area, what is the increased length of fence needed?

If the group is given twice as much fencing as they need, how much additional area could they plant?

CARPENTRY: A builder found 80 ft of "vintage" crown molding to use for a custom home. What is the area of the largest room he can design to display all of the molding? If he chooses to split the molding evenly between two rooms, what is the maximum area of each room?

CARPENTRY: Suppose the builder chooses to use 80 ft of "vintage" crown molding in a 12 ft by 15 ft room with a tray ceiling (the ceiling has a rectangular recessed area surrounded by a uniform border on all sides like a picture frame). What are the dimensions of the "tray" if the molding is used for the perimeter of the room AND the perimeter of the tray?

Students in the Early Childhood class were assigned the task of designing a new fenced playground. They had a total of 120 ft of fencing to work with. What are the dimensions of the largest possible play area? If they were given twice as much fencing, what are the new dimensions and area for the playground?

MASONRY: A homeowner wants to double the area of his 15 ft by 25 ft brick patio by adding a different-color-brick border on 3 sides (one of the 25 ft sides is against the house). If the border has a uniform width, how wide should the border be? What are the dimensions of the enlarged patio?

AUTO: The specifications for a Ford F150 truck show it's a 6-cylinder, 4.2 L engine. Each cylinder has a bore (diameter) of 9.68 cm and a stroke (assume it's the height) of 9.5 cm. If the design engineer decided to cut the diameter of each cylinder in half, but maintain the same displacement (volume per cylinder), how much change would there be in the height of each cylinder?

CULINARY: A cake batter fills two 9-inch (diameter) round cake pans to a level of 1.5 in. What radius would be needed for all of the batter to fit in one round pan filled to the same level?

ELECTRICAL: For every six increases in gauge numbers, wire diameter is cut in half. No. 18 AWG has a diameter of 0.403 in and No. 24 AWG has a diameter of 0.0201 in. What is the change in cross-sectional area from No. 18 to No. 24?

HVAC: Although it usually over-sizes them, one rule of thumb used by some contractors to calculate the size for a cooling unit is 1 ton of air conditioning for each 600 ft^2 in the house.

According to this rule of thumb, what size unit (in tons) would be needed to cool a 1-story house that measures 40 ft by 35 ft?

If the original house is doubled in both dimensions to 80 ft by 70 ft, what size cooling unit would be needed?

If the family can afford a cooling unit twice the original size, and if the original house must be enlarged by the same amount in each direction, what are the new dimensions of the house?

umbing Suppliers lists the following specifications:

peSize | Outer Diameter

" | 0.840" o.d.

" | 1.050" o.d.

| 1.315" o.d.

1/4" | 1.660" o.d.
/2" | 1.900" o.d.
| 2.375" o.d.
/2" | 2.875" o.d.
| 3.500" o.d.
| 4.500" o.d.
What is the volume of PVC used to make a 1½" pipe that is 8 ft long?
What is the volume of PVC needed to make a 3" pipe that is 8 ft long?
A building site plan originally called for ½-inch pipe to be used. However, the plans needed to be changed so that the pipe could carry twice the amount of flow from the site. What is the change in pipe diameter required to allow for twice the flow volume?