

Curriculum Units by Fellows of the National Initiative 2008 Volume IV: Bridges: The Art and Science for Creating Community Connections

# The Art, Science, and Mathematics of Bridges: An Integrated Unit for Middle School

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# **Objectives and Rationale**

Teaching math can be like that Far Side cartoon where a dog is looking quizzically at a talking man; from the caption bubble we learn that what the dog hears is "Blah, Blah, Blah". Ever wonder just how many students are tuning out during math instruction or not understanding a word you say? Traditional methods of teaching mathematics as a disciplined study obviously fail to motivate a significant portion of students today and if you are at this website searching for lessons you have likely figured this out. I dedicate this unit to all the students that have been asking for a follow up to their previous bridge building experiences in elementary school and to everyone in the Yale Teachers Initiative on Bridges for their excellent suggestions regarding this unit.

This unit for 8 <sup>th</sup> graders, (good for grades 5-8), will address through a history of great thinkers the development of understandings about forces and give students a variety of opportunities, including hands on activities, to demonstrate and experience the dynamics and equilibrium of forces in bridge structures. It creates opportunities to develop understanding of balancing equations through looking at and working with cantilever bridges. Specifically, the lesson will use cantilever bridges to work with the equation force times distance from a fulcrum on one side must be equal to the force times the distance from the fulcrum on the other side, to be balanced. It will use this formula in its ratio form as well to solve for one unknown through the use of cross multiplication. Additional activities address proportion and measurement through students' scaled bridge and truss drawings, and graphing through plotting the stress vs. strain of various materials, (fishing line of various weights, string, wires). Students will begin to work with the following science concepts: force, compression, tension, stress, strain, elasticity, and plasticity. Use of these terms and concepts will be developed and practiced throughout the unit. Finally, this unit on bridges develops students' understanding of how math and engineering is used in the real world through examples, hands-on experience, and interacting with architects and engineers. For an example of standards covered, see Appendix A for specific NM standards covered in this unit.

The goal of this unit is to integrate mathematic standards into a unit on bridges but it could also be considered a science unit that integrates math in a unit on bridges. The art has been left in the unit's title for the importance of aesthetics in design.

#### Foundations of Engineering-history of the discovery and defining of basic forces

Here, right here, in the eye, here forms, here colors, right here the character of every part and every thing of the universe, are concentrated to a single point. How marvelous that point is! . . . In this small space, the universe can be completely reproduced and rearranged in its entire vastness! -Leonardo da Vinci 1

Leonardo da Vinci, (1452-1519), was a genius engineer, scientist, and artist that was initially educated by frequently being sent out doors by his uncle to observe and sketch the natural world around him. In his notebooks he notes that engineers began thinking about what works and why in his lifetime. The scientists you will read about below began seeing the world in a new way and in turn lead to our ability to build ever more complex structures. Documenting the entire history of structures and what influences affected the evolution of thought that needed to take place to get where we are with our understanding and knowledge about structures today would take a book. The following is a brief overview of the history of the study of structures.

Until the 1600s knowledge of building was largely based on intuition, common sense, and experience passed on through craftsmen. Certainly much of this is true today especially in building your average home, and certainly foremen and project managers without engineering degrees become knowledgeable about loads and their necessary components through experience. Engineering on some level however is necessary for many structures and is based on an understanding of forces like compression, tension, stress, strain, and a whole bunch of terms that we may not understand in their engineering context. An understanding of engineering begins with understanding when and how some great thinkers started considering and putting names to these forces, and how they became responsible for developing and organizing our patterns of thinking about materials. Things we take for granted, like why we don't fall through or into the Earth, are actually scientific questions. How we can get a bridge to stand up, to span a mile or more, are engineering problems solved by the science we have for the behavior of materials. In the case of the first question, scientists have constructed models of Earth based on how we see it behave in events like earthquakes for instance. In the case of the second question, scientist have defined terms to explain forces and have studied their interactions, and have developed materials to behave for a set of conditions or parameters. The study of forces and how a material will behave is known as material science, or more specifically, the study of elasticity. Structural engineers and material

scientists are the ones doing such studies.

The Beginning

The term engineer comes from the Latin word, *ingeniator*, meaning one with *ingenium*; the ingenious one.

Some sources credit Galileo's (1564-1642) unfortunate run-in with the church and his subsequent house arrest life sentence, (he supported Copernicus' theory that the Earth actually circumscribed the sun, and not vice versa), with his turn to studying what could today be called material science, however it appears that Galileo had already turned to other scientific interests and studies before this. (The first version is certainly possible though and certainly interesting to mention to students as far as giving them historical context for the climate in which Galileo lived.) What ever the case may be, there is evidence from the notebooks and letters left behind that towards the end of his life, while under house arrest, Galileo was allowed to study tension and correspond with his peers, in this case, a priest named Marin Mersenne (1588-1648) who was working in France and studying the strength of wires. . Later there will be another priest in France by the name of Edme Mariottee (1620-1684) who worked on the strength of metal rods both in tension and bending. What Galileo figured out during this time was: a rod pulled in tension has a strength proportional to its cross sectional area, or in other words, the larger the diameter of wire, the greater the wire's capacity. What this also tells us is that in the western world, we have evidence that material science was born around the early part of the 1600 and it started with looking at how wires behaved in tension.

Tension, compression, elasticity, plasticity, and strength

What is tension you may ask? Tension is when something stretches. Think about a long balance beam supported at each end. When you get to the center it deflects, (it is said to be 'in deflection'), it bows under the load of your weight. Picture what this balance beam looks like—it is an inverted arch and the material just under your foot is in compression because the material had to move closer together in some way -likely the bonds on the molecular level have moved closer together. Meanwhile, in the bottom part of this bow, or inverted arch, the wood had to stretch—this wood is in what we call tension. The molecules may be pulling to keep their shape. If when you get off the beam, the wood returns exactly to its original shape we can say that it is elastic. If it becomes permanently bowed in any measurement, it has reached its plastic state, and if it broke, it reached its breaking stress, also known as its ultimate strength.

Robert Hooke

"If I have seen further it is by standing on the shoulders of giants." Isaac Newton in a letter to Robert Hooke, 5 February, 1676.<sup>2</sup>

Chronologically, Robert Hooke is the next important figure in the development of material science, a.k.a the science of elasticity. He lived from 1635-1702 and is now considered one of the greatest scientists of his age. He was a true Renaissance man. Besides being an architect, coining the term cell, building Gregorian telescopes, and studying everything from physics, geology, biology, astronomy, and naval technology, he developed and invented an amazing array of devices and correct theories. Hooke realized that every kind of solid changes its shape by stretching and contracting itself when a force is applied to it. He defined tension by explaining that solids push back -that is, if a force is applied he explained that there is an equal and opposite push back. What is important to realize is that the deflection is not always apparent. Compare the deflection for example of sitting on a stone bench as compared to a sofa cushion. One should understand that they both deflect but that one may only be seen deflecting on a molecular level, a way that is not apparent. Molecules have chemical bonds. In solids they are generally strong and stiff. Buildings become shorter when we climb them to upper stories. Cement pillars bulge under a heavy load. The study of forces and deflections is known as elasticity. Elasticity is the property whereby one may load and then unload a structure without any lasting effect on that structure's shape. The object springs back to its shape exactly, (and exactly means more than what one can often perceive with the naked eye). When a substance, or structure if you will, does not completely recover and becomes distorted, this is referred to as plasticity, or the material's plastic state. The object has been permanently deformed.

What might be ridiculously obvious today is that Hooke "discovered" that the deflection of a material is in proportion to its load. This discovery was made about 1800 years after the oldest Roman bridge was built. Double the load and you will double the deflection. This was the state of affairs back in 1676. Hooke published his findings in a paper in 1679 and the principle stated above has been known ever since as "Hooke's law". A limitation of Hooke's law is that it does not consider size, geometrical shape, and the material of which it is composed as factors in the deflection of a structure. Compare for example a thin spring and a metal plate. Which deflects more? Consider shape and size. Now compare a spring of rubber vs. one of steel.

Of interesting note is that no one really made great progress on Hooke's law, at least of any note, for nearly 200 years. Rather sad, but true. Our scientists of yesteryear were much the same as people in all walks today, some egotistical and self-serving competitors. Hooke and Newton had many arguments and disagreements during their time. Newton, a great scientist in his own right, outlived Hooke, criticized him, and invoked a general disregard for this area of science.

#### For historical context

Shakespeare and Galileo were born in1564, the year Michelangelo died, and the year after Nicolaus Copernicus died. Copernicus had already stated that the Sun was the center of the universe and clarified that Earth revolves around the Sun. Hooke was born in 1635 and Newton 7 years later in 1642-the year Galileo died. Alas, Shakespeare died in 1616. By the time Newton and Hooke were born New York had been founded by the Dutch, Virginia by the English, Santa Fe New Mexico by the Spanish, Quebec by the French, and Harvard had been founded in 1636. Yale opens during their lifetimes in 1701. Worthy of note is that non-western societies had helio-centric theories prior to Copernicus but that Copernicus added the movement of the planets around the sun in elliptical orbits and published his findings in a scientific paper. Another interesting note is that calculus, a math that builds on algebra, trigonometry, and analytic geometry, has roots dating back to 1800 BC in Egypt. During the 17 <sup>th</sup> century Newton and Gottfried Wilhelm Leibniz brought calculus to a point that solidified it and they are credited with its invention. Several others are also credited with calculus' advancement, including a Japanese mathematician by the name of Seki Kowa. Calculus however appears to have evolved over time in much the same way as geometry for example. Calculus is the math used in engineering today though Algebra can be used to describe concepts.

#### Stress, Strain, and Progress in the Eighteenth and Nineteenth Century

Materials that one constructs structures from have a molecular structure bonded by its atoms and/or molecules. Depending on whether we are talking about an elemental material like copper for instance, cement which is an mixture, or steel which is an amalgam, a material has it's own set of physical properties and behaviors; all materials have a unique set of molecular properties. These factors affect how a material/structure will react to a load, that is, a force or weight. Compare something tall and narrow to something short and squat, each having the same amount of material. Which is more stable? Obvious, to a builder perhaps, but this concept was not something universally considered in materials and thus not mathematically accounted for until the nineteenth century. To summarize, the behavior of a material is not simply dependent on its composition, but on the geometric form it acquires as well.

The big names in moving material science forward in this period are Euler (1707-1783), Thomas Young (1773-1829) and Augustin Cauchy (1789-1857). Euler looked at cantilevers and their deflections, Young formulated the 'specific modulus' which calculated how much a column may compress depending on a load, and later in 1926 will have Young"s Modulus named after him- an important formula used today in engineering, and Cauchy created a formula to describe the breaking load of a material as well as defining two important concepts- stress and strain.

The term stress describes not only the breaking stress, but how hard molecules within a material are being pushed together or pulled apart as the result of a external load. Cauchy defined stress as the load divided by the area. It is a vector because the load has a directional force.

S = P/A

s=stress

P=applied force or load

A=cross sectional area of the material

Strain is another important concept and is defined as how far molecules or atoms are being pulled apart or pushed together, that is, how far bonds within atoms are being stretched or compressed. The formula for Strain = extension of the length/ original length. (This is essentially the same as the percent change formula: difference/ original x 100, just not expressed as a percentage, that is taught in the 8 th grade Pre Algebra curriculum). Note that these formula have become more complex today as a result of refined research on behaviors for particular conditions, (just try goggling Formulas for stress), however these simplified formulas are sufficient for this unit.

Augustin Cauchy (1789-1857) will define the breaking point at which a substance, say a wire for instance, will break. As mentioned above, elasticity is the property where a material under a load deflects and then upon removal of the load, recovers completely. Plasticity is where upon removal of the load, the material remains changed, or deformed din some way. The breaking point is such a load that fractures the material. Cauchy defined the breaking point as the force divided by the area of the fracture. Once the breaking point of a material was known this equation could be rearranged to figure out an unknown variable, such as the maximum load that can be carried for a particular size of material.

Young's Modulus plots strain on the x-axis (abscissa) against stress on the y-axis (ordinate). The resulting line communicates the stiffness of a material; the greater the slope of a line for a given material, the stiffer the material. The formula for Young' Modulas is E=Stress/ Strain. The resulting calculation remains a constant for any given material. It is worthwhile looking at some examples of stress strain diagrams. <sup>3</sup> Today engineers test for and understand the load that a substance can withstand as a result of these diagrams.

### Engineering Today

In the 19 th century the French did much work to expand upon these principals. Interestingly enough, as is so often the case with theory, it took time to put these scientific and mathematics discoveries into practice. People went on building as they had done for centuries and likely the builders were largely uninterested in listening to theoreticians. The same attitudes can pervade building today. Artisans who are intelligent and skilled understand how loads affect the materials they work with. A good engineer or architect listens to the craftsperson and vice versa. Ideally there is an open dialogue and mutual respect between builders, engineers, and architects. Not every structure needs to be designed by an architect or engineer today. House builders without formal educations often design their own for example but must comply with building codes. Building departments however can and do occasionally call for engineered drawings for certain elements in a building during a permit process. In today's world though we would not consider foregoing engineering in designing big bridges, big structures, and our roads.

By mid-nineteenth century engineering began working its way into architecture and today we have obviously come a long way. Materials are regularly tested; metals and cements for example are sampled, analyzed, and engineered for maximum strength. Research continues on polymers, ceramics, nano-technology, fiber options, semi-conductors, photovoltaic technology, and the list goes on. Engineering takes into account fracture mechanics and the concept of "creep". Calculus has become a regular tool of engineering, and computer-modeling programs are commonplace. More recently, we have people like Santiago Calatrava applying engineering to bring to life structures that none have thought to create before, pushing our ideas and our structures into new realms, all made possible due to current understandings of forces and principles of structural behavior. In Santiago Calatrava's case, we are fortunate to have an artist, architect, and engineer wrapped into a creative designer of bridges, buildings, and art.

Types of Bridges (Note that the following discussion of bridge types is arranged alphabetically as opposed to the previous chronological organization of the historical section on material science above.)

#### Arch

The arch is one of the oldest bridge types and was used in ancient civilizations like Mesopotamia, Samaria, Babylonia, and Egypt. Perhaps one of the oldest and most famous uses of an arch from this time period is the Ishtar Gate from 575 BC, a masterpiece and part of the ancient Wall of Babylon, today housed in Berlin. Romans went on to use arch construction, most famously perhaps for their aqueducts which largely

still stand today and China has a masterpiece arch bridge known as the Anji Bridge. Arches in bridges may be used in a variety of ways and are based largely on compressive forces but arched spans experience thrust and either abutments are necessary to support this outward force at the base, or a tie is required, or both. In a bridge this tie can be the roadbed itself. Span to rise ratios as well as material use will determine specific needs. Arched bridge designs are generally excellent in addressing spans between 30-800 feet depending on the design and material. There are a variety of ways that arches are incorporated into bridges as one will see with just a bit of investigation. A search for arch bridges will yield numerous examples and one will see arches supporting roadways from above and below the arch support. Round, segmental, pointed, and 3-hinged arches are all common in bridge design and are commonly constructed from steel, iron, concrete (pre-stressed, post-stressed, or reinforced), and stone. The keystone was an important discovery for early arch construction. It is the last stone placed into the top of a stone arch which structurally completes the arch structure.

#### Beam

Beams are the earliest and still most common form of bridges. A tree that has fallen across a river constitutes a beam bridge. Beams are ideal for spans up to about 70' and then usually prove themselves an inefficient use of material. Trusses then become a more efficient use of material and can span up to 150' to 200' effectively. Often beam construction integrates abutments or pier supports. Beams are structural members which typically have compression in the top fibers and tension in the bottom fibers.

#### Cantilevers

A cantilevered structural member is one which is like a beam continuous over a support, however it differs by projecting horizontally into space extending beyond its support. There are a variety of cantilever bridge designs. One often sees the use of cantilever design during the construction phase of suspension bridges. Most often, one sees

pre-stressed concrete cantilever bridges that are designed with cantilevers on either side, either connected directly to one another, or to a beam that bridges a gap between the two cantilevered sections. A highly engineered pin system joins this later design and must be designed allow for expansion and contraction at the joints. Cantilever bridges designed to carry heavy loads can be constructed from steel, iron, and concrete. The required structural design for this type of bridge is affected by the distance of the extension and the forces this bridge will be exposed to. One will often see use of arches in the lower section of the cantilever, supported on a pier. Cantilevers cause a reversal of internal compression and tension, depending upon where one looks. Sections between two piers act as a beam and have compression in the top part of the member, and tension below whereas in the cantilevered section, one finds tension in the upper part of the member, and compression below. Trusses are often adapted to this structural type.

### Cable Stayed

Cable stayed bridges are the most recent invention in the design of bridges, and perhaps due in part to this freshness strike one as incredibly artistic. Though they date back to the late 1700s, they are not common in history. The most recent incarnations of a cable stayed bridge is the Millau Viaduct in rural France, completed in 2004 and the Alamillo Bridge in Seville Spain, completed in 1992, both strikingly modern and inspired. The design of a cable stayed bridge incorporates a pylon from which cables are attached directly in support of the bridge deck. Cabled stayed bridges are ideal for spans up to a thousand feet and use much less cable than a suspension bridge and they use less material than a cantilevered bridge of similar span. Cable stayed bridges do not require the anchorages needed for suspension bridges and may be chosen when anchorages are a poor choice. One sees compression in the bridge deck supported by the tension in the support cables. Cable stayed bridges are generally categorized as either fan or harp in design, depending on the pattern of placing the cables.

#### Suspensions

Modern suspension bridges are unmistakably identified by their tall pylons, sweeping arched cables, and wire hangers. Modern suspension bridges incorporate trussed bridge decks, trussed piers, and span ever longer distances. Currently the Akashi Kaikyo Bridge in Japan holds the record for the distance between it's piers at a total length of 6,532 feet, (equal to 1.23 miles, or 1991 meters). Suspension bridges require anchors at both ends to support the forces in the suspended cables from which the road bed hangs. The main forces in a suspension bridge are tension in the cables and compression in the pillars and pylons. Suspension bridges are one of the earliest forms of bridges, having their roots in rope and vine foot bridges across rivers. These early form of suspension bridges still exist in less developed areas throughout the world are worthwhile examples of structural ingenuity.

#### Truss

Trusses are incorporated into so many bridges that it easy to forget that the truss alone with an attached bridge deck constitutes its own category. Truss bridges frequently take on the appearance of an x-ed in flattened arch and this structural component sits either above or below the deck. Early applications were constructed of cast iron and wood where as most trusses today are built of steel or wood. Trusses rely on the inherent stability of a triangle. When a load is transmitted across the bridge it is supported at differing times by tension or compression in the truss. In a variety of designs, the structure's skeleton is filled in with shapes of a K, or X, or simply a triangle with vertical supports where needed to carry a compression load. The truss bridge is best illustrated through diagramming:



figure 1: 4 truss designs.

Moving Bridges

There are a variety of types of moving bridges that incorporate truss, cantilever, and cable stayed elements in their design, and include draw bridges, swing bridges, vertical lift bridges, and old-fashioned transporter bridges. These are not covered in this unit but may be of interest to the reader.

# **Strategies and Suggestions**

Images and books on bridges should pervade the classroom environment as much as possible. Power Point Presentation should be prepared in advance for: the history of forces, including images of scientists and diagrams or graphs to illustrate concepts wherever possible and one on bridges that include examples of all basic bridge types, (see background information above). Examples of bridges should span time and come from countries across the world and should ideally include examples from countries of any recently immigrated ethnic groups represented in the classroom. The teacher may want to consider a chronological approach to presenting bridge images. Refer to the bibliography for books and the information above on the history of forces to identify appropriate images.

There is a hands-on component to this unit and some supplies will be needed. Ideally, the teacher will be able to furnish these supplies to the students. The teacher will obviously need to consider whether students will work on models independently or in pairs, consider classroom management, working space, storage of the bridges, and line up engineers and architects to visit the classroom during construction. It is highly recommended that the teacher make a bridge before assigning this project for an appreciation of the skill and time needed to accomplish such a construction. Furthermore, it is recommended that the teacher speak to an engineer for feedback on this finished teacher construction. This initial model would also be an ideal way to network with engineers and discuss ways they could interface with your class.

Incorporated but not discussed in the lesson plans is the use of Cornell Style Note taking and Interactive

Notebook approaches. Both are included as this unit is ideal as an opening unit for the year and an introduction to both methods is ideally addressed at this time.

In concluding activities students will be expected to construct a bridge and create a presentation on what they learned. This should cover both standards and student skills, (See parts 9 and 10 of the lesson plans below). It is recommended that teachers have students identify "student skills" goals as they proceed through this unit in addition to setting goals for learning standards. This unit can be easily assessed through a portfolio type assessment as well. The teacher will need to consider creating rubrics for hands on components.

This unit is laid out in 10 parts, developed for block scheduling, and will take approximately 8-9 weeks to complete if needed math skills and math standards as noted in Appendix A are covered fully where noted throughout lesson plans.

# **Lessons and Activities**

## Part One: Introduction (developed for an 85 minute block)

This unit begins with a power point presentation of images of bridges. During presentation of bridges begin to integrate basic bridge vocabulary. (See Appendix B for list of terms). Students will use their notebooks to begin sketching basic types of bridge structures and label components with vocabulary. The goals here are to introduce the topic of bridges while building vocabulary and knowledge of bridge types and cultivating interest and dialogue amongst students and teacher. Students will be furnished with books with images and bridge diagrams and expected to complete diagramming and labeling at least 2 bridge types. They will then be expected to reflect in their notebooks, (see interactive notebook styles), on one of the bridges they saw either in the power point presentation or books. Students will share their notebook drawings and reflections in groups of 2-3 to encourage peer dialogue while the teacher moves around the room and collects memorable quotations from dialogues to put onto an overhead to share with the class. Images of bridges will include global and local examples as well as examples from countries of students' origin for purposes of ownership.

# Part Two: Building vocabulary, considering why we need bridges, what kind of bridge when, and discussion of the value of aesthetics (developed for a 65 minute block)

Students will journal on "Do we need bridges in New Mexico?" (Or substitute your state here.) Students will address who needs them, what they are for, where they are found, why we need them, and when we started needing them. A vocabulary list of bridge terminology and the diagram of basic bridge types from the previous activity will be furnished for students to paste into their notebooks. A power point presentation of images of bridges that students have not already seen will be used to help students practice identifying bridges by type. While pointing to particular components of the bridge, students will develop vocabulary by looking at the information in their notebooks to guess, identify, and label the component on the diagrams of bridges. Efficient use of materials and appropriate choices of bridge types depending on spans will be discussed as well as how efficiency of materials and design should still consider aesthetics and artistic merit. Costs will be discussed. Community pride and possible economics affects of artistic and aesthetically pleasing designs will also be integrated. Include some images of Santiago Calatrava's bridges to illustrate this premise. Homework: Students should talk to their families and bring in bridge stories and memories or, do a web search of bridges and bring in an image of a bridge they would like to share at the beginning of the next class.

# Part Three: Hands on exploration of the truss. Introducing compression, tension, buttressing (developed for an 85 minute block)

Opening: Show students a video on the Tacoma Narrows Bridge collapse, also referenced as Galloping Gertie. <sup>4</sup> Follow up with a discussion on design flaws.

An inquiry activity: Students will construct two-dimensional triangles and squares from straws cut and simple straight pins, (straws can be cut to any size- teacher should encourage a variety of triangle types), sketch their constructions in their notebooks, and explore and record observations of the behaviors of each. Students will be instructed to stand the shapes up and push down on them to simulate what would happen when the shape is loaded with weight. Students may wish to explore additional shapes as well. Students will then be directed to construct a simple 3-dimensional truss bridge of either triangles or squares, again with straws and pins, to bridge a 24" gap between 2 desks. In student constructions, it is intended that students will create truss bridges with square shapes instead of triangles so that the differences in stability and strength can be compared. If this is not the case, ask some groups to do this. Students will load their completed bridges with books from either above or below, (using string if necessary if loading from below the bridge). Groups will orally report on and share their discoveries and discussions will ensue. The teacher will interject and introduce the following vocabulary: compression, tension, truss, and buttressing as well as list student observations and "discoveries" on the board. Students will be given time to write a reflection and conclusion on what they learned and how they learned. Before closing, the teacher will demonstrate bridging a table with a simple flat sheet of paper, and loading this sheet of paper, and then folding the same sheet of paper accordion style and loading it again. Students will be asked to consider which paper bridge was stronger, stiffer, and students should consider how they would label each of these paper bridges; beam or truss.

# Part Four: Tension and Compression, molecules, atoms, and elements (developed for an 85 minute block)

Begin by integrating a daily question, (see suggestions in Appendix C). Return to the Power Point Presentation or other visual images, (for example, books, or an actual bridge if there is an accessible local truss bridge), to look for trussed components in bridges. Students will be issued 12" sections of pipe insulation and will look at the compression vs. tension in the material. Although visible with the naked eye, hand held magnifiers may prove interesting. Students will be expected to make a connection between their observations of loads on trusses and compare this to what is going on internally in the material, but the teacher will need to ask students how the forces we see in a piece of pipe insulation (compression and tension), play out in a bridge's deck, (roadway). From here, segue to a preliminary lesson on atoms and molecules. Students will learn the difference between an atom and molecule, what an element is, and be introduced to the concept of molecular bonds through drawings, diagrams, and human demonstrations. Students will take notes in Cornell Style, (the summary should be done in class). For homework: students will be assigned the construction of a small truss bridge due in one week. Class will agree upon parameters of size, and discuss materials, and any limitations. Note: Atoms and molecules can also lead into a 2 days of math lessons on scientific notation.

# Part Five: Shape affects strength, Cantilevers, Levers and balancing equations, discovering that force x distance = force x distance (anticipated time is 3-4 blocks)

Begin by asking students to predict which shape will hold a greater load- a tall cylinder or a short one, each of the same surface area. What about a tall rectangular prism or a short one? And, then have them to them predict: a prism or a cylinder. Ask them why. Objective: students should come to understand that shape affects the ability to support loads. Ideally, the teacher has constructed simple models of each of these shapes so a simulation can be made. Follow with a discussion.

Students will create a human cantilever based on the Benjamin Baker style demonstration on why a cantilever is safe. An image for this lesson is easily retrievable from a book or the Internet. The teacher will need to consider what materials to have on hand. Students can take turns being part of this human cantilever. Students will then be directed to an image of the Firth of Forth Bridge and be asked to compare this bridge design to the human design cantilever. Students will then be given some type of material to use for a lever, (large popsicle stick or 2" x 12" piece of wood), Plastalina oil based clay, and some type of manipulatives to use as loads for either side of this lever system, (pennies, pattern blocks, math tiles, other). The clay will be used to construct a pier or fulcrum, (or substitute another appropriate material). Weights will be used to balance these levers as students move the position of the fulcrum away from the center. Students will be asked to look for relationships and patterns between masses on either side of the fulcrum and the length of the lever from either side of the fulcrum. They will need to measure the changing distances from the fulcrum. Recommend that students make regular changes, that is 2 centimeters at a time, or 1" at a time. Discoveries will be shared, and the formula fd=fd will be introduced in it's ratio form if students have not already discovered it. This lesson will then move into a discussion of how either side of equations must be equal in algebra in order to be "balanced" or equal, and students will begin to work on applications of this formula. It is anticipated that the focus on balancing equations will continue for some days and then segue into lessons on cross multiplication and the manipulation of the lever formula to solve for an unknown. Integration of images of cantilever bridges images at different points during these days will incorporate questions of balance and equilibrium of forces. Be sure students can make the connection that loads on bridges are the forces in a simple lever problem.

## Part Six: Checking in on truss structures (developed for one 80 minutes block)

Have students present their constructions and what they learned. Ask an engineer to give students positive and constructive feedback and time allowing, ask them to share how they design truss components.

# Part Seven: Proportion, scale, cross multiplication, and other applications (anticipated time is 5 blocks due to integration of math instruction, practice, and application)

Look at arched bridges and examine their span to height ratios through measuring images found in books with tracing paper and rulers. Then, explore proportions of spans vs. heights of other bridge types by using information found in books, or on the Internet. Class needs to agree if suspension and cable stay bridge height is a measure of clearance, height of pylons, or both. Before beginning, through discussion and peer sharing and teaching, students need to agree on ways that they will organize gathered information. Students should work in small groups of 3-4 and each collect one measurement for each bridge type and share their information with other group members.

Next, students will design a bridge, considering first it's purpose. Students will be expected to consider the ratios they discovered above, and abide by the span limits for each bridge type, (students have these numbers in their notebooks by this point). Students will create a scaled elevation drawing of their design. Scales must be cleared with instructor to prevent 1:1 ratios. These drawings will be used to construct a model in Part 9 below, and should be drawn such that they will be the actual size of the model. The teacher should therefore furnish large graph paper. Students will be given 2 dedicated class periods for the drawing of their bridges and will be expected to finish them in time for the sixth class, (in a block schedule scenario). Instructor will need to teach scale and cross multiplication to find unknown dimensions for their drawings. Peer tutoring

may be sufficient for the scales used in these drawings however. This drawing activity will segue to mathematics lessons and extensions on proportion and scale, (extensions will include finding the height of a tree for example by setting up a proportion of the tree's height to the length of its shadow and the length of another object, and measurement of this other object's shadow). Ideally, an architect or engineer will be in the classroom to assist students with feedback on their designs.

# Part Eight: Graphing the elasticity of fishing line and developing concepts of strain, stress, elasticity, and plasticity (developed for one 80 minute block and one 60 minute block)

Students will graph elasticity using 10 pound fishing line and weights. Needed supplies include fishing line, weights- 1-10 pounds, (or scales, sand, and plastic baggies), a system to attach weights to fishing line, large paper, (like the rolls of paper one finds in schools to cover bulletin boards). Have students take a 4-5 foot long piece. Students should work in pairs, or groups of 3. Have students set up a graph as seen below with even increments for 1-12 pounds along the axis marked weight. Paper should be set up in "portrait" format, otherwise, students may find they will need to add paper to the bottom of their graph as their fishing line stretches.

# Weight (force) (pounds)

Length of Fishing line (inches)

### figure 2: Elasticity of 10 pound fishing line

Students will then hang up their graphs and hang a section of fishing line on the vertical axis at the 0 pound mark, extend and straighten out their fishing line without stretching it and mark the length of the fishing line directly onto their graph. Students will then move their fishing line over to the 1 pound mark, attach 1 pound of weight, and measure the length of the fishing line again, (it should be longer this time). Students should then remove the weight and observe the resiliency of the fishing line. (They should mark the length of the fishing line fishing line after the weight was removed as well, using a different color, and determine if the fishing line material is resilient.) Students should repeat moving their fishing line over, attaching the appropriate amount of weight, (2#, 3#, etc), and marking the length of the fishing line until the line breaks.

Regroup as class. Discuss terms: stress, strain, elasticity, plasticity, breaking point, resiliency and have students define them in their notebooks. Ask students to analyze their graphs and label them accordingly.

Discuss application to bridge design; choice of materials and mathematical determination of size of material. Incorporate stress-strain diagrams, (see background knowledge above, and vocabulary in Appendix B). Students should write a conclusion and reflection to this activity in their notebooks using the Interactive style.

# Part Nine: A history of forces, assignment of a bridge model and authentic assessment for this unit, practical tips and practice for construction of models (anticipated time is approximately 4 blocks)

Students will construct a model of the bridge they drew in part 7. If they want to draw a new design they may. Students will spend one class period working with a variety of materials practicing and exploring bridge model construction methods. An architect in the classroom is ideal at this point. Students will decide upon appropriate materials for their bridge and purchase and bring materials to class, though as previously stated, the teacher providing all materials is a better scenario. As part of an authentic assessment, students will also work on a presentation in addition to their bridge models, ideally a Power Point Presentation that discusses what they have learned throughout this unit, although some other presentation that resonates with a students' learning style is acceptable as well. The teacher will provide a checklist of components that students should cover which will focus on standards, as well as "student skills", (for example, communication, participation, notebooks, interpersonal skills with peers, guests, ability to stay focused and on task for 45 minutes at a time, or any goals that students may have set for themselves at the outset).

Present a Power Point Presentation on the history of forces, as laid out in the background information above, to give students historical context and appreciation for development of thought. This can be done in one day, or at the start of each day, followed by time to construct models and work on presentations.

## Part Ten: Conclusion and presentations (developed for 2 blocks)

At least one engineer or architect will come in to give feedback on students' structures. Students will reflect on what they learned through each other's bridges. Students will present on what they learned in this unit, (refer to Part 9 above)

# **Appendix A: Implementing New Mexico Standards**

# Grade 8 Mathematics (Pre-Algebra II) and Grade 9 Mathematics, (Algebra I)

5-8 Benchmark A.2: Represent and analyze mathematical situations and structures using algebraic symbols.

5-8 Benchmark A.4: Analyze changes in various contexts.

5-8 Benchmark G.1: Analyze characteristics and properties of two-and three-dimensional geometric shapes and develop mathematics arguments about geometric relationships.

5-8 Benchmark G.2.1 Represent, formulate, and solve distance and geometry problems using representational systems.

5-8 Benchmark M.2.3 Use proportional relationships in similar shapes to find missing measurements.

9-12 BenchmarkA.1: Represent and analyze mathematical situations and structures using algebraic symbols.

9-12 Benchmark A.3: Use mathematical models to represent and understand quantitative relationships.

9-12.G.1.1 Understand that numerical values associated with measurement of physical quantities must be assigned units of measurement or dimension.

### Grade 8 Science Content, Benchmarks, and Performance Standards

5-8 Strand I: Scientific Thinking and Practice Benchmark 1: Use scientific methods to develop questions, design and conduct experiments using appropriate technologies, analyze and evaluate results, make predictions, and communicate findings.

Standard 1:Understand the processes of scientific investigations and use inquiry and scientific ways of observing, experimenting, predicting, and validating to think critically.

5-8 Strand II: Content of Science Standard 1: Understand the structure and properties of matter, the characteristics of energy, and the interactions between matter and energy.

5-8 Strand III: Science and Society

Standard I: Understand how scientific discoveries, inventions, practices, and knowledge influence, and are influenced by, individuals and societies.

# **Appendix B: Vocabulary for Bridges**

Abutments: the end support of a bridge structure. They are most easily identified as the structural support at the lower or back ends of an arched structure.

Anchor: this is where the ends of a suspension bridge's cables are connected and secured into either the land or man-made structure. A suspension bridge cannot function without anchors.

Buttress: to buttress means to support. Many arched bridge designs rely on buttresses structures on their outsides walls to support the resulting lines forces acting within them.

Bridge deck: this is the road way

Cables: this is the name for the wires that extend from one bridge anchor across the top of the suspension bridges' pylons, and back to the bridge anchor on the other side. In suspension bridges they are hung in an inverted arch formation. This is also the name for the cables that support a cable stayed bride.

Caissons: this refers to structures built below the water to keep water out during the construction of bridge piers.

Concrete: a mixture of cement, sand aggregate, gravel aggregate, water, and selected admixtures.

Concrete in bridges is either pre-stressed, post-stressed, or reinforced.

Dead load: is the load of the bridge upon itself self-weight of all permanent loads

Equilibrium: refers to a state of balance. In bridge design, engineers create a system whereby forces can be balanced.

Hangers: are the iron chain, rods, wires, or ropes that attach the cables to the bridge deck.

Keystone: the last stone placed into the top of a stone arch which structurally completes the arch structure.

Live load: is the load of the traffic upon the bridge the imposed temporary loads on a structure

Pylon: the section of bridge directly above the piers that support that bridge cables.

Piers: an upright supporting section of a bridge. They support the bridge from below. In cases of piers found surrounded by water, they rest on the bedrock at the bottom of the body of water.

Saddles - engineered components of a bridge which are clamped to the cables to support the hangers.

Span: refers to the distance a bridge member crosses or gaps. It is the distance between supports of a structure. The measuring of bridges record long bridges sometimes refer to the distance of the suspended section of bridge, and other times refers to the total length the bridge spans.

# **Appendix C: Daily Question Ideas and Extension Ideas**

### Daily Question Ideas:

How high can a stone building be built?

At what point will the weight of a building crush the lower stones?

How much cable is in the Golden Gate Bridge?

How much does steel cost and what would the steel cost to build the Washington Bridge in today's dollars?

What load was your chair designed for? What directional loads was it designed for? Why does your teacher tell you to stop tipping back on the chair and what does this have to do with forces?

### Extension Ideas:

- 1. Incorporate careers in bridge building through having students read about them. Have students introduce themselves and explain their lines of work via role-play.
- 2. Biomiocry, 'analogical recruiting', appropriation of forms from nature, and Calatrava.
- 3. Venn Diagrams to compare and contrast properties of materials
- 4. Explore interactive on line computer programs. Try PBS. Kids, search for other sites, incorporate computer softwares, and The Geometer's Sketchpad.

- 5. Incorporate Euler's explanation to the Koningsberg Bridge puzzle and have students create towns with bridges and their network solutions.
- 6. Building of Oakland Bay Bridge, (DVD), and other films on Bridges (Building Big Series)
- 7. Read The Great Bridge-Building Contest, by Bo Zaunders
- 8. Take field Trips to local bridges
- 9. Study bridge failures
- 10. Newton's 3 Laws of Motion in relationship to bridges
- 11. Moving bridges

# **Endnotes**

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Isle of Wight History: The Life of Robert Hooke. July 25, 2008.
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Graf, Bernhard, Bridges that Changed the World, Munich: Prestel, 1999

A resource of images with explanations on their significance in the

history of bridge building.

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A helpful resource book of activities and concepts involved in bridges,

for the teacher or student in grades 1-8

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A well illustrated book documenting the construction of a bridge.

Schodek, Daniel L., Structures, Upper Saddle River, NJ: Prentice-Hall, Inc., 1998

Contains essential basics on structures including tables on materials

strengths, graphs, and structural analysis.

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Images of Calatrava"s work, with Alexander Tzonis commentary on his premise on why Calatrava"s work can be defined in terms of "poetics of movement". Images are harder to read however than the larger images found in *Santiago Calatrava: The Complete Works.* 

Tzonis, Alexander, Santiago Calatrava: The Complete Works, New York: Rizzoli, 2007

An inspiring and excellent starting place in reading about bridges, includes many large images of Calatrava''s dramatic designs and structural inventions.

Whitney, Charles S., Bridges of the World, Their Design and Construction, Dover Publications, Mineola, New York, 2003

A chronologically arranged book detailing design, history, and construction of bridges includes many black and white images.

Zaunders, Bo, The Great Bridge-Building Contest, Harry N. Abrams, New York, 2004

A children's story about a bridge- building contest and its unlikely winner.

Historically accurate, it is useful in demonstrating the strength in bridge models.

# **Films**

Building Big; Bridges, David McCauley, WGBH Boston Video, 2004

Extreme Engineering Oakland Bay Bridge, Discovery Communications, Silver Springs, MD, 2003

Tacoma Bridge (collapse), August 1, 2008

http://video.google.com/videosearch?hl=en&q=Galloping%20Gertie&um=1&ie=UTF-8&sa=N&tab=iv#q=tacoma%20narrows%20bri dge%20collapse&hl=enTacoma Bridge

# Web Resources

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