Curriculum Units by Fellows of the National Initiative
2008 Volume IV: Bridges: The Art and Science for Creating Community Connections

The Design and Analysis of Structures

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Introduction

Science is designed to discover information about the universe in which we live and to determine how this information can be organized into meaningful patterns. The ultimate purpose of science is to determine the arrangement that exists between the various facts. The most basic and first science is physics. At its core, physics is about understanding what lies behind everyday phenomena like rainbows, red sunsets, blue skies and bridges as well as the more revolutionary concepts of quantum theory, relativity and cosmology. One of the key ideas in physics is that, behind the complexity of the world around us, there is an underlying simplicity and unity in nature. The simplicity and unity in nature is expressed through comprehensive fundamental concepts, such as Newton's Laws of motion and the conservation of energy. Such concepts, when put to work using mathematics, provide explanations for every-day events. The application of the concepts of physics provides the platform for most of the other sciences and engineering.

The purpose of my curriculum unit is to use bridges as a structure to facilitate my students' understanding of the interconnectivity of all "things". How subatomic particles, protons, neutrons and electrons are joined in structures to make the elements. Elements are bonded using electrical forces to form molecules and compounds. All materials are made of these molecules and compounds and are engineered to form structures such as bridges, homes, trees and even humans. The students will learn to analyze the forces in the most basic static design and then extend that same principle to more complex structures until they can analyze the forces in a simple bridge design. My curriculum unit will include an inquiry laboratory element to design and construct a bridge structure and test the mechanical efficiency of the bridge structure.

Classroom Environment

I teach at an urban high school with a population that is about 46% African American, 39% White, 11% Hispanic and 4% other. I teach introductory algebra-based physics to tenth, eleventh and twelfth grade students on a semester block program. I have introductory physics students for ninety minutes every day for one semester. The tenth grade students are part of the International Baccalaureate (IB) magnet program at my school. I teach between forty and sixty tenth grade IB students who are self-selected for this program, they take introductory chemistry in the same year, therefore some students have completed chemistry, but not all. The upper classmen have all completed chemistry and are divided between honors and regular level, which they choose. The curriculum unit is developed for the introductory physics course. I also teach

Advanced Placement (AP) algebra and calculus based physics courses and will use most of the curriculum unit materials in the algebra based AP physics class, expanding the portion of the unit on rotational statics.

Objectives

The most important part of analyzing any structure begins with the application of Newton's first law. Newton's first law states: An object at rest or moving at constant velocity continues its current state unless acted upon by a net force. The guiding questions that students need to determine are: What is a force? Are there different types of forces? And how do we determine the force in a structure?

Any force has three characteristics; they are *magnitude*, a *point andline of action* and a *direction* along its line of action. The magnitude is the amount of force say, 100 Newtons or pounds of force. The point of application is where on an object the force is applied, such as at the corner or in the center of mass of the object. The line of action is in the direction the force is acting such at 30 degrees above the horizontal plane through the point. There are two types of forces, external forces and internal forces. The effect of an external force on an object is independent of its point of application on the line of action. The effect of an internal force on an object is very much dependent on the point of application. Forces will be categorized as either contact or long range forces. Contact forces are all forces acting on the object that result from the contact between the object and its surroundings at the object's boundaries. These forces include forces of static and kinetic friction, tension and compressive forces and the normal force. Long range forces result from the object's interaction with a force field, such as magnetic, electric or gravitational fields the most common in introductory physics is the force due to gravity or the weight of the object.

For students to apply Newton's laws to solve force problems they first must be able to represent the forces that act on the object. This representation is called a free-body-diagram (FBD). Analysis of free body diagrams are one of the most difficult skills for most introductory physics students to grasp. This curriculum unit was developed to help students become proficient at drawing and analyzing static force problems and apply these skills to design, analyze and build a bridge. The purpose of developing force diagram is that the diagram simplifies the problem for analysis. Consider, for example, a weight of 100 Newtons hanging from a wire. We begin the analysis by drawing the free body diagram of just the weight, free of the connection to the wire.

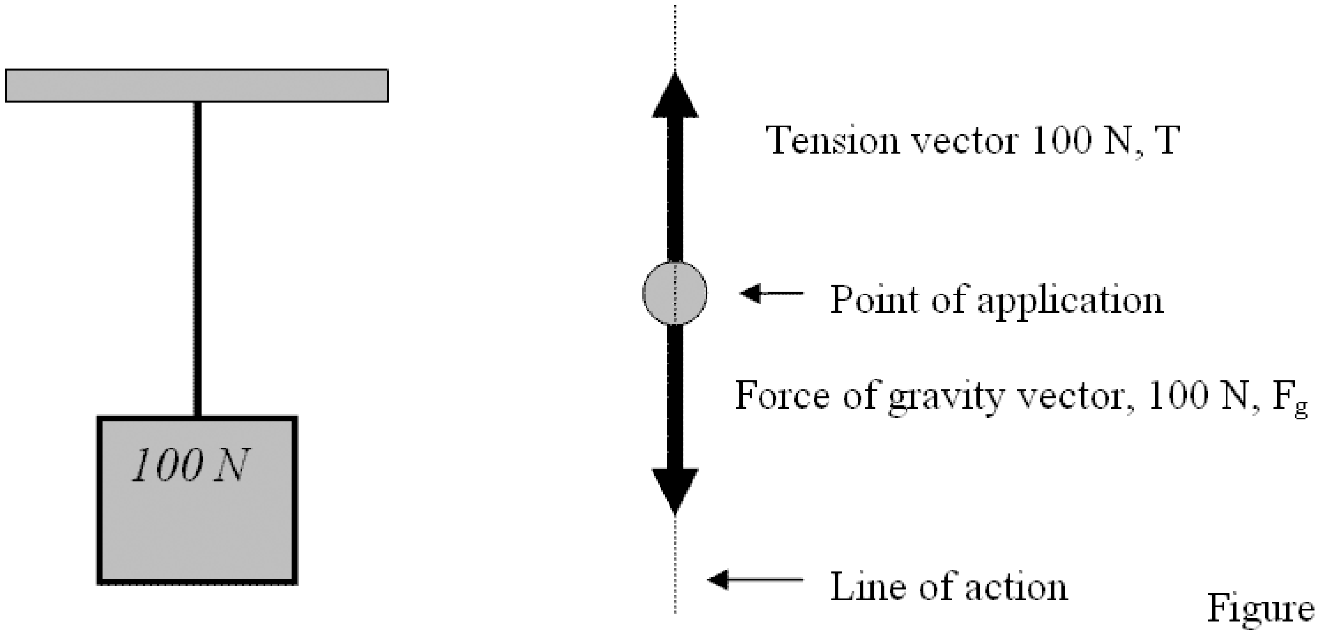


Figure 1

The arrow indicates the direction of the force and the 100 Newtons is the magnitude of the force. The point of application is at the center of mass of the object or the centroid. Labels are used to represent the forces; I will use F_g for the force due to gravity, T for a force in tension, C for a force in compression, F_f for a frictional force, F_N for the normal force and a F for a general push or pull force. The weight is in static equilibrium which means that the force due to gravity is just balanced by the tension in the wire. Therefore the tension in the wire is equal to 100 N.

In this simple example determining the tensile force in the wire was straight forward and almost common sense but, the physics principles that was applied was Newton's laws of motions and specifically the condition for static equilibrium. The condition of static equilibrium can be expressed mathematically, as the sum of the forces in that act on the object is equal to zero. To do this we arbitrary assign upward forces as positive and downward forces as negative and the equation is simply:

$\sum F_y = 0$.

The subscript y represents the vertical direction or the y-axis on the coordinate system. This elementary application of Newton's first law, that the sum of the forces in the vertical direction equals zero is the basics of design of every structure, from the simplest to the most complex. It is one of three fundamental conditions of static equilibrium that apply to all objects in a two dimensional system. The other two equations are;

$\sum F_x = 0$

$\sum T_o = 0$

The first equation is the sum of the forces in the horizontal direction or x-axis are equal to zero and is a logical extension of the sum of the forces in the y-axis. The second equation is the sum of the moments or torque around any point of rotation, o, are equal to zero. The torque is the rotation force on an object caused by forces where their lines of action do not cross a common point. For my introductory physics all lines of action will cross a common point of application therefore the torque will be zero. Torque or moment will be addressed when the students analyze the forces in a section of the truss bridge they design.

Referring back to the free-body diagram of the weight held by a wire, some students will observe that there is a 100 Newtons pulling down and 100 Newtons pulling up and conclude that the tensile force in the wire is 200 Newtons; this is not true. If we make an imaginary cut through the wire at any point and draw a free-body diagram of only that portion of the wire that lies above or below the cut, the force inside the wire is always 100 Newtons. [1]

Once the students have an understanding of the simple system of one wire holding a weight free-body diagram, we will explore what would change if two vertical wires support the weight, then three and so forth. The next step in teaching analysis of static structures using free-body diagrams would be to introduce a horizontal force in addition to vertical forces. For example, instead of supporting the weight with one vertical wire I will have the student analyze the same weight supported by two wires pulling at the same angle as shown the Figure 2a below.

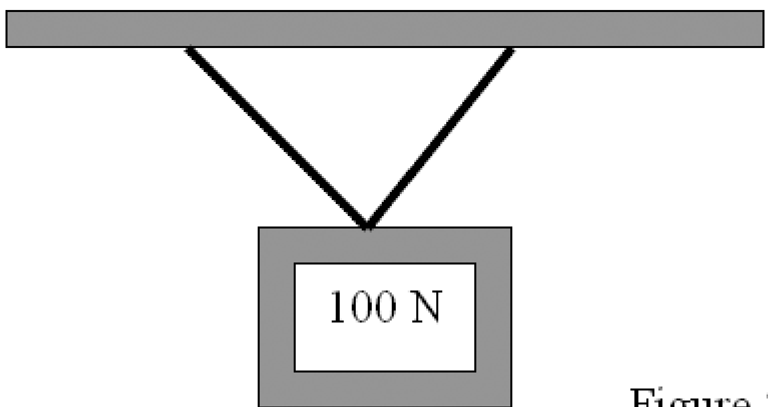


Figure 2a

The completed free-body diagram of this situation is show in Figure 2b. The student must draw an arrow to represent each of the wires at the point they attach to the 100 Newton weight. They need to include an *x-y* coordinate axis and the 100 Newton gravitation force.

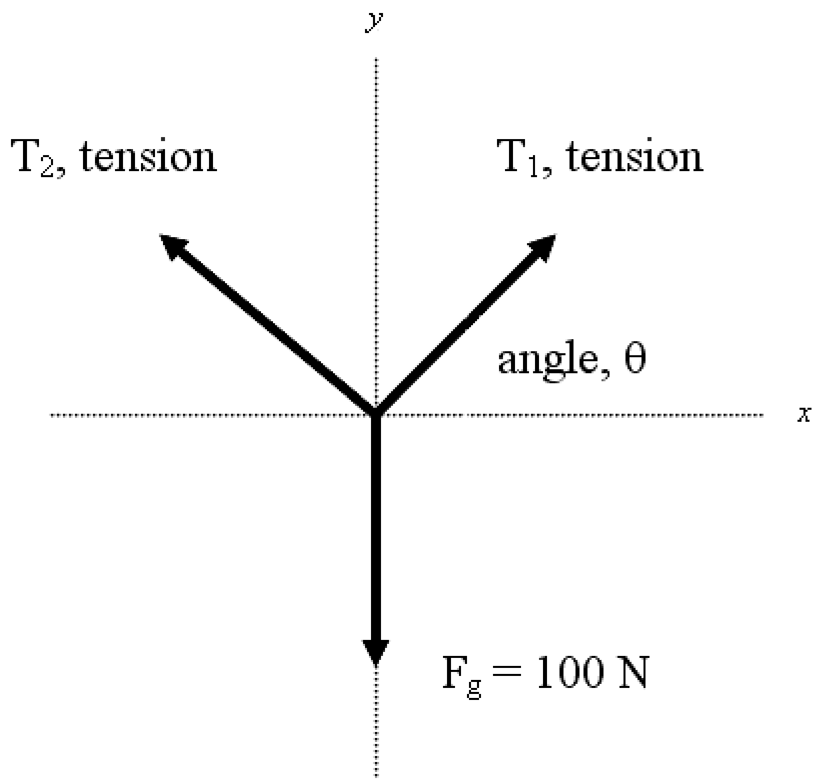


Figure 2b

The previous example dealt with collinear force, forces in only one direction, which allow us to easily add and subtract the vectors. The forces in the above example are not collinear even though they are concurrent, which means that their lines of action all pass through a common point. To find the sum of concurrent forces that are not collinear, we must use vector addition.

Vector Addition

There are five vector quantities that are discussed in introductory physics. They are displacement, velocity, acceleration, force and momentum. A vector is a physical quantity that has both magnitude and direction, such as force. There are two methods of vector addition, graphical and mathematical.

Graphical addition of vectors

Graphical addition of vectors requires students to draw each vector to a set scale in a head to tail method using a ruler and protractor. The resultant (sum) is the vector drawn from the starting point to the head of the last vector; using the scale the student determines the magnitude and direction for the force using a ruler and protractor. Figure 3 below demonstrates how the three vectors in the free-body diagram above are added using a graphical method.

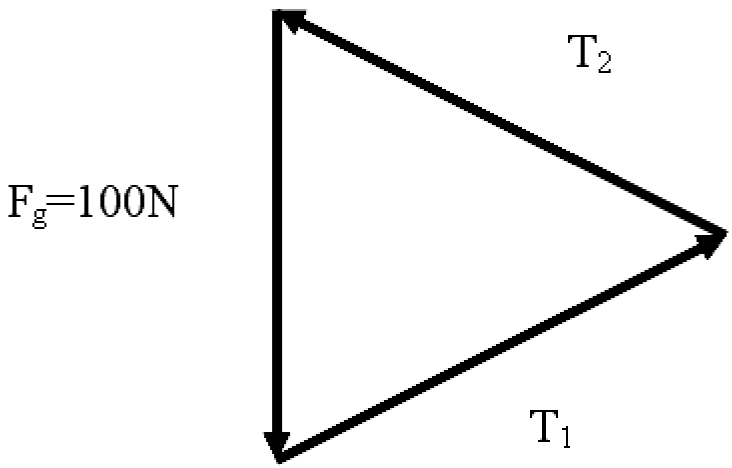


Figure 3

Notice how the vector addition results in a closed triangle, if a structure is static where the sum of all the forces are equal to zero then the vector addition will always have a sum of zero. We can use this fact to graphically solve for unknown forces. When graphically adding vectors the order in which you draw the scaled arrow does not matter, you will always have the same resultant independent of order. Figure 4 below shows the graphical addition of a five force system solving for one unknown force added in two different orders.

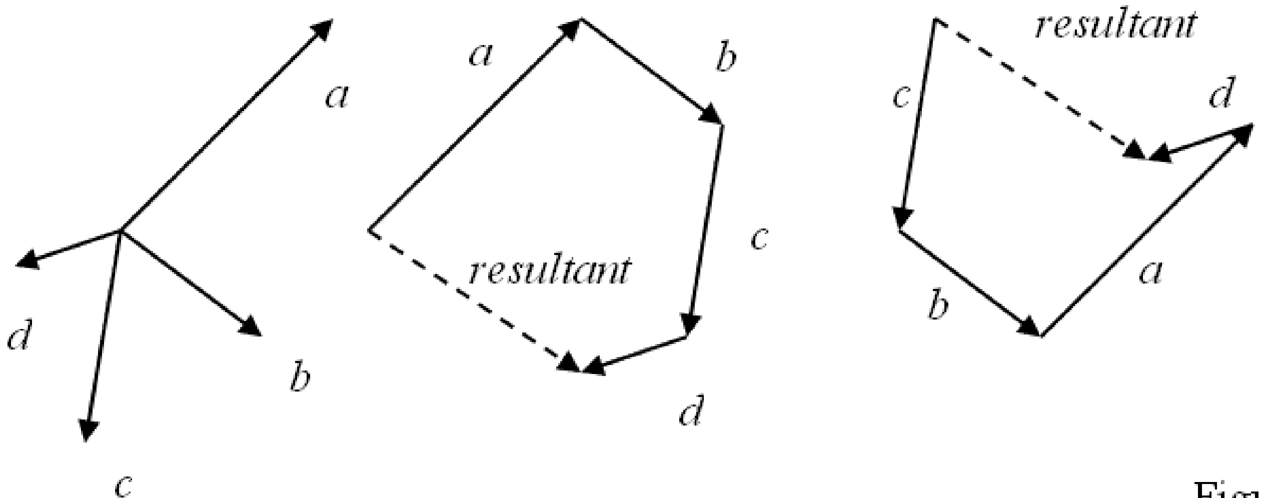


Figure 4

Both orders of adding the vectors concluded that the fifth vector resultant force was in a downward right direction and of the same length which represents the magnitude of the force. Both dashed line in figure 4 are identical.

Mathematical addition of vectors

To add vectors mathematically, the student must first resolve any force not in a vertical or horizontal direction into their horizontal, x, and vertical, y, components using trigonometry. For example in Figure 5 the tension in the wire is composed of a horizontal and vertical component, the horizontal component is equal to $T_x = T\cos\Theta$ and the vertical component is equal to $T_y = T\sin\Theta$. To solve for the unknown horizontal pull of the second wire, $T_{2,}$ the horizontal components are then summed to zero resulting in the following mathematical formula which can be solved for the unknown horizontal quantities.

$\sum F_x = 0$, $0 = T_2 - T\cos 60^\circ$, or $T_y = T\cos 60^\circ = 58$ N

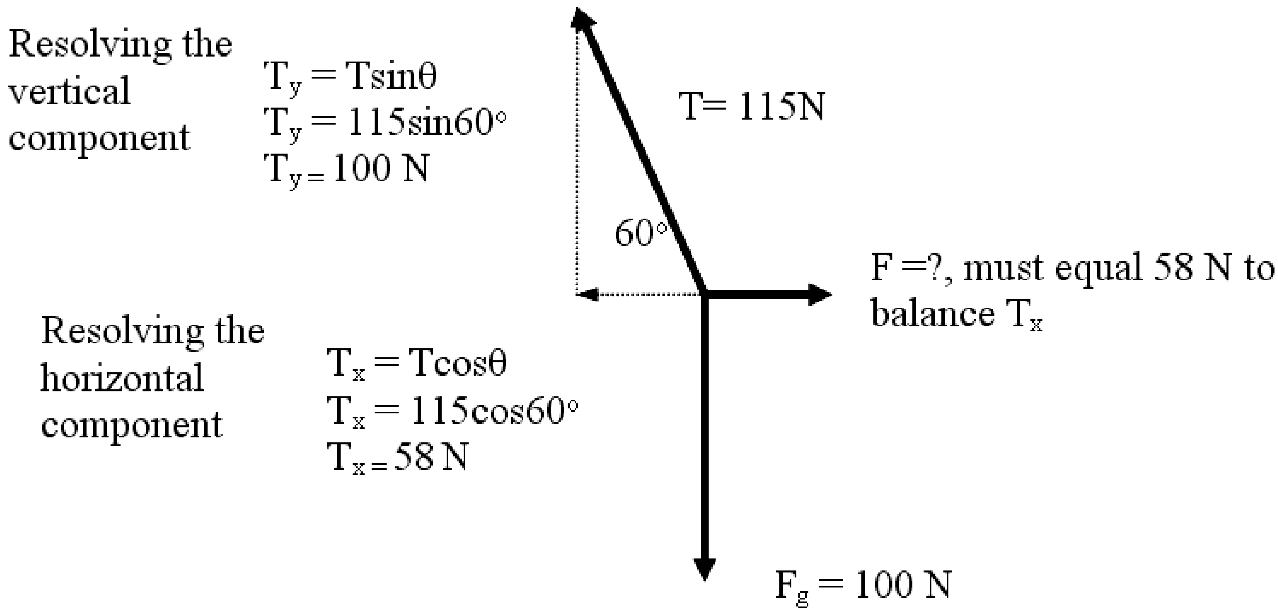


Figure 5

The vertical components are also summed to zero resulting in a mathematical formula which can be solved for any unknown vertical, y, quantities. If there is more than one unknown quantity in each mathematical formula a series of systematic equations is required to solve for the two unknown quantities.

As you can see, these problems can become very complex and students will struggle with the mathematics initially, but the more they practice with resolving vectors into their horizontal and vertical component and applying Newton's first law, the more proficient they will become at solving static force problems.

Strategies and Classroom Activities

Drawing Free-Body Diagrams

The objectives demonstrate the process involved in the analysis of any object in static equilibrium. After demonstrating to the students how to draw a free-body diagrams I will have the students work in pairs at two desks moved together to draw free-body diagrams from a worksheet. Each group is given handouts with eighteen diagrams of objects in different physical situations from which they must draw and label the free-body diagrams. They will first draw the free body diagrams on large (two feet by four feet) white boards. The large white boards allow me to circulate in the room and help students on an individual basis, the students are not permitted to move to the next free-body diagrams until I have checked their board. Students in this situation can be self-paced and with the partner have discourse about their diagrams. The students will copy the correct free-body diagram onto their worksheet to have a permanent record of the picture and the free-body diagram just like in figures 1 and 2 above. To summarize the activity a discussion will follow to find the similarities and differences in the different situations. I want the students to recognize that the free-body diagram for a weight hanging from a wire is the same as a book at rest on a table except the book is in compression and the wire is in tension. There are many of these similarities in drawing free-body diagrams.

The next step will be to have students develop free-body diagrams not from drawings or pictures of the physical situation, but from written descriptions of physical situations. For example the student will start with the following description of a physical situation: *A student pushes on a desk with a force of 50 Newtons at an angle of 30 $^{\circ}$ and the desk does not budge. Draw and label the free-body diagram.* The student must use the textual clues to develop the free-body diagram. For example the word budge means the desk is in static equilibrium and the sum of the forces must equal zero. Then they must conclude that when "a student pushes on the desk" there is resistive force acting on the desk that is not clearly defined. That resistive force is static friction. When using word problems as opposed to diagrams, the solutions can vary from student to student. For example one student will draw the student pushing to the right and in a downward direction (the 30 $^{\circ}$ angle above the horizontal) and another might draw the force from the left and pushing upward (the 30 $^{\circ}$ angle below the axis). The lack of detailed information in the above example can lead to different interpretations of the problem. There will be class discussion on how word problems of this type can be interpreted differently and how the free-body diagrams would differ but the process of determining a solution would be the same.

Vector addition strategies

Graphical analysis

Students will practice the graphical analysis of vectors addition using rulers and protractors. Initially, I will model graphical analysis as direct instruction in a whole group format. As a group we will step through a problem, setting the scale, drawing each vector to scale in a head to tail fashion and determining the solution by drawing and measuring the vector from the tail of the first vector to the head of the last vector. The student will then practice a problem independently, and again as a homework assignment. Students generally do not have difficulties with the process of graphical analysis of vectors. Their problem arises with making the scale either too large or too small. A few students need extra help with using the protractor and understanding what is meant by a direction in the form of, 50 $^{\circ}$ East of South. Students will get frustrated with the graphical addition because the time commitment to make a scaled drawing. That leads to teaching the mathematical method of vector addition.

Mathematical addition of vectors

Prior to the unit on Newton's Laws and forces the students have been taught how to resolve vectors into their horizontal and vertical components using right triangle trigonometry. Therefore the only new skill to develop in this unit is how to sum the vertical and horizontal components. The free-body diagram is vital in this process. The student must examine the free-body diagrams to add and subtract all the components of the forces in the horizontal direction and then set that sum equal to zero. They will then repeat the same process in the vertical direction. If the free-body diagram is not correct then the formula developed from this diagram will be incorrect. The equations of static equilibrium developed will then be solved using algebraic techniques for any unknown forces. The ability to analyze a free-body diagram in to a mathematical relationship is the most difficult skill in the introductory physics classes; therefore the students must practice the skill repeatedly.

Bridge Building

The purpose of the bridge building project is to help students practice the analysis of a real structure in static equilibrium. The students will design and build a bridge and apply their knowledge of how to analyze forces to determine the forces in one portion of their bridge structure. The dimensions of the elevated bridge will be 35 centimeter long, 15 centimeters above the ground level and 5 cm wide. Each student must draw a scaled drawing of their proposed bridge structure which reinforces the same type of skills as graphical analysis of vectors. I will introduce the inquiry bridge building project by showing a power-point presentation of various bridge designs. The power point will show a truss system, an arch support and a suspended bridge design. In each system we will discuss how the forces are transferred from the bridge deck to the ground support. Students will have to limit their model design to a trussed bridge constructed from dry pasta and glue to limit the materials that must be made available. Dry pasta will be used because it is inexpensive, available in various sizes, such as spaghetti, linguine, fettuccini, and lasagna for the road bed and requires no tools to cut. In addition to the scaled drawing the students will have to show the free-body diagram for a section of their bridge and determine the force in each member for a given load of 100N. Upon completion, the bridges will be tested for their mechanical efficiency. The mechanical efficiency of a structure is the ratio of the mass the structure can support compared to the mass of the structure.

To determine the forces in each member of a truss panel, instruction on how to sum the torque or moment around a point must be accomplished. The students will do a numerical solution to the bridge project and a graphical solution. To begin a numerical solution the students must draw a free-body diagram of one panel of their truss bridge design as shown in figure 6, below.

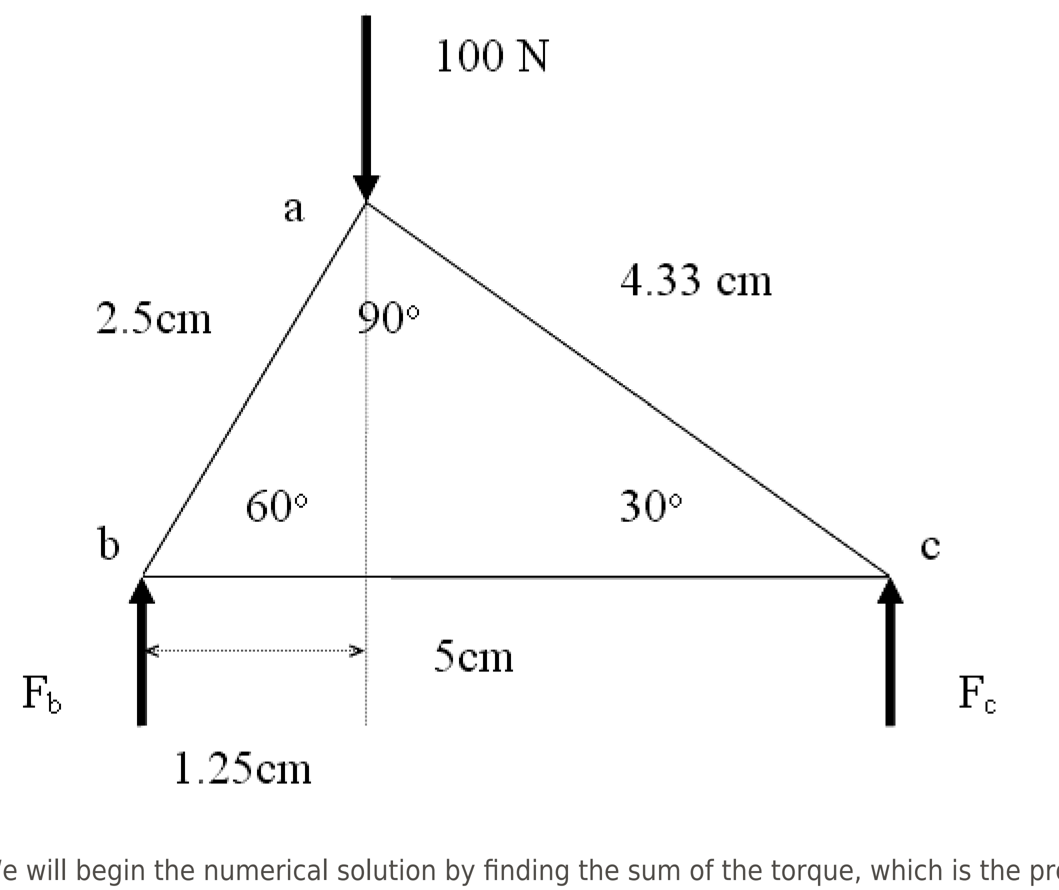


Figure 6

We will begin the numerical solution by finding the sum of the torque, which is the product of the force and the moment arm, around point b and setting it equal to zero. The result will be the following:

$\sum T_b = 100N(1.25cm) - F_c(5.0cm) = 0$

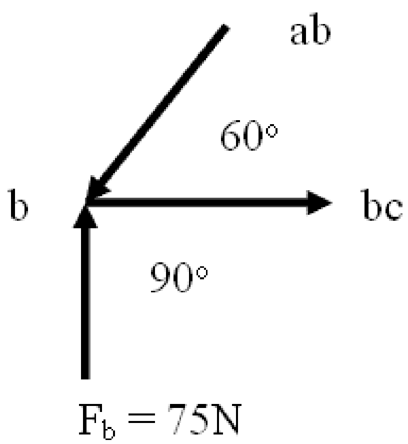
Solving for $F_c = 25$ N

Next the students will sum the forces in the vertical direction and set the sum equal to zero to find F_b the result will be the following.

$\sum Fy = 100N - 25\ N - F_b = 0$

Therefore $F_b = 75$ N

It is customary to begin finding the forces in each member of a truss at the left reaction point F_b in figure 6. We begin by drawing the free-body diagram of joint b as shown below in figure 7 below. Also shown in the figure are the sum of the vertical forces to solve for the force in the *ab* member and the sum of the horizontal forces to solve for the forces in the *bc* member of the truss. The solutions are $F_{ab} = 86.6$ N and $F_{bc} = 43.3$ N.

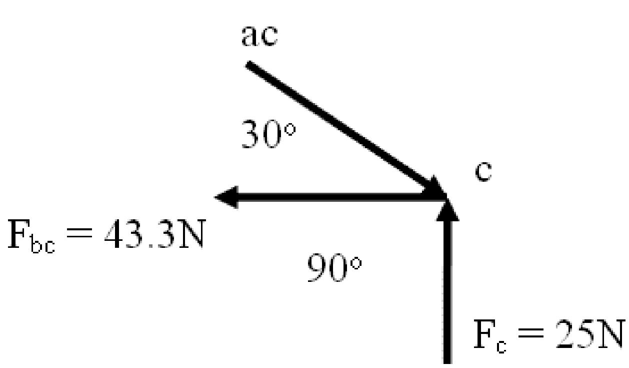


$$\sum F_y = 75N - F_{ab}\sin 60° = 0$$
$$F_{ab} = 75N/\sin 60° = 86.6\ N$$

$$\sum F_x = 86.6N\cos 60° - F_{bc} = 0$$
$$F_{bc} = 43.3\ N$$

Figure 7

An identical process would be followed for the c point on the truss panel to solve for the force in the ac member or we can solve for ac member at the point *a*. The free-body diagrams and the force equation for the c point are show in figure 8 below.



$$\sum F_y = 25N - F_{ac}\sin 30° = 0$$
$$F_{ac} = 25N/\sin 30° = 50\ N$$

Figure 8

The final summary diagram that shows all the forces on and within the truss panel is shown in figure 9 below. A compressive force is indicated by a *(c)* next to the magnitude and a tensile force with a (*t*). [2]

In a final discussion of the student's bridge analysis, I will ask the students the following questions about the forces in the members of one part of their bridges based on their free-body diagram analysis. Which members are in tension and which are in compression? Which members have the most force in them and the least amount of force and why? How can you change the material you use to optimize the design of your bridge?

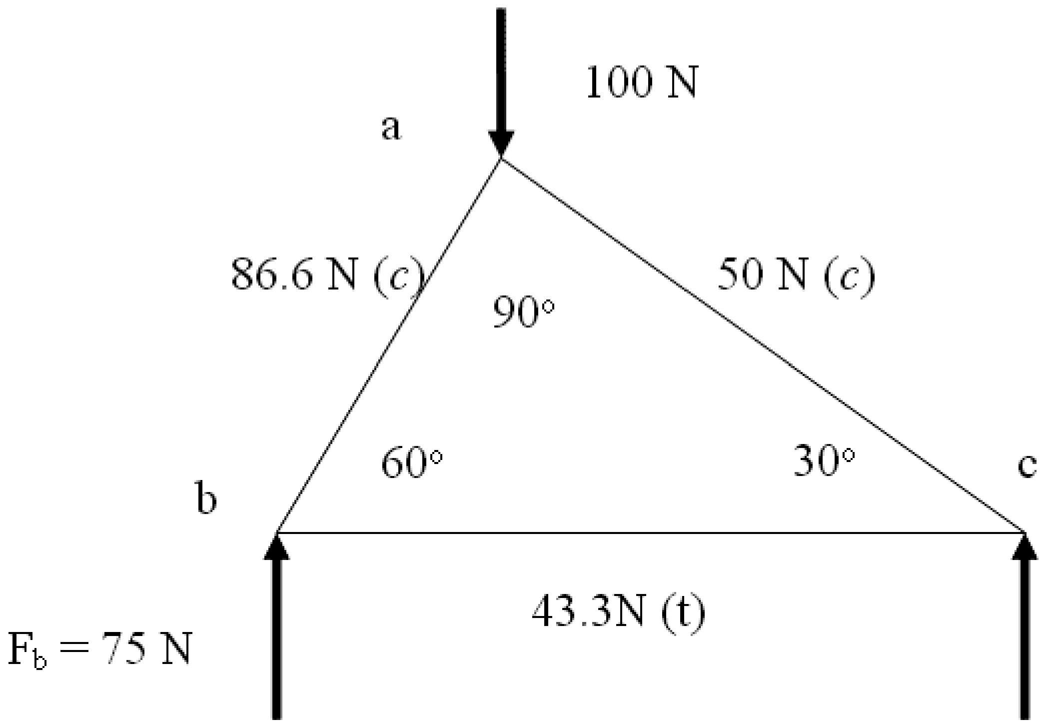


Figure 9

When the models are complete, they will be tested for the mechanical efficiency by loading the bridge on the road bed until failure occurs. Then, in a final discussion that includes a power point of pictures of truss structures other than bridges, such as buildings boat, and airplanes. The class will have a whole group discussion about the connection between the bridge structure and analysis of other structures in the world.

Bridge Building Inquiry Lab

Materials

Large sheets of paper (11 x 17)

Rulers and protractors

Dry pasta, spaghetti, linguine, fettuccini, and lasagna

Hot glue and glue guns

Cardboard or foam board to build on

Waxed paper or plastic tape to protect pattern from glue

Pins and/or tape to hold pasta to pattern

Objective

The students will design and build an elevated trussed bridge, analyze the forces in one panel and determine the mechanical efficiency of the bridge. Prior to designing the bridge students have completed the unit on static force analysis and have researched bridge designs.

Directions

Students will draw a full scaled trussed bridge design on the paper. One side view, top view and bottom view. This will be their building pattern.

The bridge must be 35 centimeters long, 15 centimeters above the ground and 5 centimeters wide.

Students will analyze the forces in one panel of the truss and show the free-body diagram and analysis.

Students will construct their bridge from dry pasta and hot glue using their drawing as the pattern.

Testing

Completed bridges will be weighed. A test block 5 centimeters by 5 centimeter with a chain attached will be placed on the bridge to provide a load. A bucket will be attached to the chain and sand will be added to the bucket until the bridge fails to support the load. The bucket of sand and chain will then be weighed. The mechanical efficiency of the bridge is the ratio of the mass of sand the bridge held divided by the mass of the bridge. For example, if the bridge held 2.8 kilograms of sand and had a mass of 42 grams the mechanical efficiency would be 2800 grams/24grams = 116.7 or the bridge can support around 116 times its own weight.

Bibliography

Zalewski Waclaw, , and Edward Allen. *Shaping Structures Statics*. New York, NY: John Wiley & Son, 1998.

Teacher and student annotated bibliography

Since I teach honors level Physics to high school students the resources for students and teachers would be the same. Many of the references with pictures of bridges will be available to the students to use for ideas on their bridge designs.

Brown,David J. *Bridges: Three Thousand Years of Defying Nature.* Buffalo: Firefly Books, 2005.

An overview of the origins of bridges, The influence of materials in the engineering of bridges. Many historic and new forms of bridges

Cortright,Robers S. *Bridging:Discovering the Beauty of Bridges.* Tigard, Oregon: Bridge Ink, 1998.

Many illustration of bridges with their environmental settings. Shows the many different types of bridges.

Cowan,Henry J. *The World's Greatest Buildings: Masterpieces of Architecture and Engineering.* San Francisco, CA: Fog City Press, 2000.

This book has a focused discussion of building types and synthesis of form and function.

Dupre,Judith. *Bridges.* New York: Black Dog & Leventhal , 1997.

Describes the many basic principles of bridge making with many illustrations.

Eberhart,Mark E. *Why Things Break.* New York: Harmony Books, 2003.

Understanding the world by the way it comes apart, written by a chemist about the role of material in construction of structures.

Engel,Heino. *Structure Systems.* Stuttgart, Germany: Deutsche Verlags-Snstalt GmbH, 1997.

Discussion and illustrated diagrams and models of the may structural system types and subsets of criteria. Will help provide clarity for the design and development of form, support and elements in creating structural systems.

Gordon,J.E. *Structures: or Why Things Don't Fall Down.* London: Plenum Press, 1978.

This book explains structures with simple facts of reality combined with significant observations that together make the understanding of living structures memorable.

Gottemoeller,Frederick. *Bridgescape:the Art of Designing Bridges.* NewYork: John Wiley & Sons, 2004.

The design detail for a bridge as view from an engineer's perspective. It illustrates an attempt to understand and explain the qualitites of bridge design including many practical environmental factors.

Graf,Berhard. *Bridges that Changed the World.* Munich: Prestel, 2002.

Illustrations of bridges thest cover a broad array of bridge types with a discussion about the bridge's unique role within the community.

Hewitt, P. (1993). *Conceptual physics*. New York, NY: Harper Collins.

Excellent conceptual physics text book

Leonhardt,Fritz. *Bridges: Aesthetics and Design.* Stuttgart, Germany: Verlags-Anstalt, 1982.

This author explores many design issues including issues about the physical and mechanical properties of materials.

Mainstone,Rowland J. *Developments In Structural Form.* Cambridge: The MIT Press, 1983.

The book describes the study of the relationships between structural forms, actions, materials and constructions.

Orton,Andrew. *The Way We Build Now: Form, Scale and Technique.* Berkshire, England: Van Nostrand Reinhold, 1988.

Schodek,Daniel L. *Structures.* Upper Saddle River, NJ: Prentice-Hall, 1998.

This book covers basic statics of structural systems including bridges. Includes good three dimensional pictures of structures

Tzonis,Alexander. *Santiago Calatrava: The Complete Works.* New York: Rizzzoli, 2007.

Designs by Calatrave especially covers the conceptual images and dynamic studies leading to project design. Many beautiful bridges and buildings by Calatrava.

Tzonis,Alexander. *Santiago Calatrava: The Poetics of Movement.* New York: Universe Publishing, 1999.

This book contains many beautiful pictures of Calatrava's designs. It captures the extraordinary achievement of how the dynamics of art, movement, engineering and building are interconnected and realized.

Whitney,Charles S. *Bridges of the World: Their Design and Construction.* New York: Dover, 2003.

Describes the influence of materials on bridge design and form.

Zalewski, Waclaw, and Edward Allen. *Shaping Structures: Statics*. New York, NY: John Wiley & Son, 1998.

A great book on statics and the graphical analysis of structures. It covers the basic principles of forces, their direction, lines of action and resolutions. The book includes an interactive disk to solve problems.

Zitzewitz, P., Elliott, T, & Haase, D. (2005). *Physics principles and problems*. New York, NY: McGraw Hill.

Physics textbook assign to my class

Video sources

Ray and Charles Eames, *The Power of Ten*

Appendix 1 State Objectives for North Carolina Physics

The following are the North Carolina Physics objectives that this curriculum unit addresses.

4.01 Determine that an object will continue in its state of motion unless acted upon by a net outside force (Newton's First Law of Motion, The Law of Inertia).

4.02 Assess, measure and calculate the conditions required to maintain a body in a state of static equilibrium. Describe forces as interactions between two objects, including contact and forces at a distance. Recognize that force is a vector quantity. Define normal force. Represent the forces acting on an object using a force diagram. Analyze force diagrams to calculate the net force on an object. Determine that the net force acting on an object in static equilibrium is zero.

Design and conduct investigations of objects in static equilibrium. *(Torque and rotational equilibrium are enrichment topics.)*

4.05 Assess the independence of the vector components of forces. Resolve forces into components. Apply Newton's Laws of Motion to the perpendicular components of force in the following examples: objects pulled or pushed along a horizontal surface by a force at an angle to the surface; objects sliding down an inclined plane; three concurrent forces acting on an object in static equilibrium.

4.06 Investigate, measure, and analyze the nature and magnitude of frictional forces.

Distinguish between static friction and kinetic friction. Solve quantitative problems with frictional forces. (*Coefficient of friction is an enrichment topic*) Identify interaction pairs of forces for contact forces and forces at a distance. Analyze Newton's Third Law.

3.01 Define *vector* and *scalar*, incorporating magnitude and direction. Clarify that a positive value for a force indicates motion in one direction while a negative value indicates motion in the opposite direction.

3.03 Resolve vectors into vertical and horizontal components. Recognize that vector components are independent of each other.

1.01 Identify questions and problems that can be answered through scientific investigations.

1.02 Design and conduct scientific investigations to answer questions about the physical world.

1.03 Formulate and revise scientific explanations and models(dpi)

Appendix 2 Teacher and student annotated references

Since the curriculum unit is developed for high school physics student, teacher and student references are identical.

Endnotes

1. Zalewski, Waclaw, and Edward Allen. *Shaping Structures Statics* (New York, NY: John Wiley & Son, 1998), 3.

2. Ibid.,82.