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Each Number in Its Place: Teaching Place Value to First Graders

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Introduction

Place value: the value assigned to each place in a numeral, depending on the base of the number system. This is not exactly a definition that is accessible and useful to a primary teacher! Of course we need to teach this essential concept to our math students as the foundation of our notation system and maybe more importantly as a foundation of their future mathematics study. These young students are cognitively ready to learn, understand and use purposefully the idea of place value. Through this unit primary teachers themselves will gain a more complete understanding of place value as a real life skill and subsequently use the strategies and lessons for their students to have the same result. The key ideas of this unit are adapted from the Singapore mathematics program, a consistently successful approach to teaching elementary school students.

As a first grade teacher in a self-contained classroom in New Haven, my class is 26 six and seven year olds with an occasional eight year old. Our neighborhood/ magnet setting is a rewarding environment, with students coming to school each day from a variety of home circumstances and with differences in academic levels. As a result of these variables, the children have differing levels of background knowledge and life experiences. The classroom is a mixture of varied ethnicities, economic strata and social and emotional strengths and weaknesses.

During the last two years our school district has been piloting new elementary mathematics curricula. My school in particular has worked with two different mathematics programs, both in their early trial years. The first year was Math Expressions, published by Houghton Mifflin and developed by The Children's Math Worlds Research Project. The second, which is this current year, is ThinkMath, published by Harcourt and developed by Education Development Center, Inc through the National Science Foundation. These two piloted programs each present different math foundation strategies. It is difficult to know how the students will perform in future grades having had these two separate approaches during their first and second grades. We are continuing with ThinkMath for the coming year which will provide some consistency. By incorporating this unit into our mathematics curriculum, I hope to effectively teach the students the concept of place value, certainly an essential skill no matter what publisher we finally identify for the district!

Rationale

The responsibility of each elementary school mathematics teacher is to use effectively the required math curricula, ultimately guiding their students through a sequential path of concepts and practice. The choice of "concept paths" is decided and formatted by the authors and publishers of each curriculum. We, as professionals, are given some resources to guide us. These take the form of teacher manuals, professional development, team meetings and planning sessions. These resources are essential to the success of the teaching year for each of us and the learning year for the students. The particular teaching strategies and mathematics concepts within the scope and sequence of a curriculum need to be digested and fully understood by the teachers using it.

As a result of the No Child Left Behind law, states are looking to increase performance in mathematics to then improve scores on the state testing that must occur each year from third to eighth grade. Because of the pressure to be successful in this goal, school districts are turning to new texts, increasing technology and finding ways to prepare the students to achieve higher scores. Currently, many schools and teachers are experiencing what our district is - trying out several options. The variable that is being overlooked is the teachers themselves. Good teachers must be at the core of a solid mathematics education.

Teaching mathematics effectively depends on a solid understanding of the material. Teachers must be able to do the mathematics they are teaching, but that is not sufficient knowledge for teaching. Effective teaching requires an understanding of the underlying meaning and justifications for the ideas and procedures to be taught, and the ability to make connections among topics. Pedagogical content knowledge, the knowledge of how to teach a concept or subject, must be aligned with subject matter knowledge to create a productive learning experience for the student. Together these allow for understanding student's responses and determining the goal of the lesson.

An article by Hill, Ball and Schilling on pedagogical content knowledge suggests that although teachers' knowledge of content and students is not necessarily easy to measure, it appears from initial study that the news is good. Teachers in this study demonstrated a familiarity of students' mathematical thinking, for example, predicting common student errors. Understanding how students think is distinct from pure content or pedagogical knowledge and shows teacher skills and insight needed for successful student learning and growth. Because teachers see student work, hear student statements and see students solving problems, they are constantly considering what students are doing and how they are thinking. Designing instruction and motivating students to learn are the potential positive results of this unique knowledge. [1]

My intent and goal for writing this curriculum unit is twofold. Initially the unit will focus on increasing conceptual knowledge of place value and order of magnitude. This first objective will allow a more successful outcome for the second objective, which is to then have the first graders develop through mathematics activities the concept of place value. This unit covers 2 digit numbers and ends with the first 3 digit number in our number system, 100. The key goal for these students is to learn that a two-digit number is made of a certain number of tens and a certain number of ones. The key estimation principle to learn is that the tens are bigger than the ones, and most specifically, given two numbers, the one with the larger tens digit is larger. The core idea of this concept is what a digit at a certain place stands for. This will lead to the next level of instruction which is adding and subtracting of two digit numbers.

This unit will use the months before the 100 th day of the school year to prepare for the celebration of that

day. It has become a standard practice in primary classrooms across the country to celebrate the 100 th day of school. Certainly this is monumental not only as a number but as a benchmark for teachers and students as the general school spans 180 days. This of course this can become a fun, exciting, anticipated day of activity, practice, and reinforcement.

To be successful with this, subject knowledge of place value for the teacher will help to ensure a deeper understanding for the students. Place value appears to be a simple enough concept and is certainly found in our first grade textbooks or workbooks with guides for teacher language. So it must be that we are prepared and ready to teach. However, it is not enough to know how, one must know why. As a teacher of very young children, my task is to give them a strong and complete foundation to use in their mathematics classes and throughout their learning. It is in fact too important to overlook and breeze through quickly and unprepared.

Singapore Math

In a widely accepted comparison of global math skills, the Trends in International Mathematics and Science Study, Singapore continually finishes first among participants. Singapore is a prosperous, multicultural, multilingual nation of 4 ½ million people whose fourth- and eighth-grade achieve this standing. [2] Singapore's world-class mathematics system has quality components aligned to produce students who learn mathematics to mastery. These components include Singapore's highly logical national mathematics framework, mathematically rich problem-based textbooks, challenging mathematics assessments, and highly qualified mathematics teachers whose pedagogy centers on teaching to mastery.[3]

Beginning math students often have difficulty grasping abstract mathematical concepts. Students in first grade, and even beyond, need visual representations of mathematical ideas. Numbers are ideas. A strategy of clear illustrations to present the idea and reinforce the skill is a critical part of this curriculum. Because the six to seven year old has not achieved abstract thought and has not had enough relative experience, the event or experience needs to be visual, auditory, and/or tactile. This seems clearer and more logical possibly when discussing literacy to assist with connecting words to pictures, such as, "The cat is in the tree." The student would be reading the sentence while looking at an illustration showing the same. We must recognize that it even applies equally to mathematics.

The Singapore illustrations feature a concrete to pictorial to abstract approach. This approach allows students to begin to learn math in a meaningful way, understanding the mathematical concepts before learning the "rules" or formulas. To give an example, the Singapore first-grade text begins with a blank rectangle and the number and word for "zero." Below that is a rectangle with a single robot in it, and the number and word for "one." Then a rectangle with two dolls, and the number and word for "two," and so on. The page is very pictorial, but it refers to something very concrete. Next to the pictures are dots. Beginning with the number six (represented by six pineapples), the dots are arranged in two rows, so that six was presented as one row of five dots and a second row with onedot. [4]

Teaching to mastery is another key conponent of this approach. Topics are covered in detail and taught to mastery. Immediately after new concepts are taught students are engaged with a variety of mathematically rich problems to ensure their deep understanding. Singapore Math is designed to produce mathematical thinkers by walking students through component parts of a problem before asking them to solve the whole

problem. Making sure that students really understand allows for more success for them when new concepts are introduced.

"The Singapore texts don't make a huge deal about the concepts, but they present them in the correct and economical form," said Roger Howe, a professor of mathematics at Yale University. "It provides the basis for a very orderly and systematic conceptual understanding of arithmetic andmathematics." [5]

This curriculum will cover the introduction of place value after the mathematics readiness skills have been mastered, including learning number names to 10 (and their serial order) and using these numbers to count quantities, developing familiarity and facility with numbers, practicing with numbers, saying numbers, writing and reading them followed by simple addition and subtraction, or as the language of this unit will be, composing and decomposing numbers to 10.

Keeping the scope narrow will be important to mastering this skill. Students will need to draw on previous learning to be prepared to comprehend place value. Proficiency at composing and decomposing a ten will supply significant support for learning addition and subtraction within 20.

I would characterize this as more of a conceptual unit than a procedural curriculum. The classroom activities will include math facts to 20 or addition with composing and subtraction with decomposing within 20, working with manipulatives, and ultimately using this conceptual knowledge when working with two-digit numbers beyond 20. Because of the close connection between this unit and the Singapore math approach, the strategies in this unit will mirror theirs. The goal here is to promote a solid learning of this topic in a logical and sequential order.

To begin, digits at different places have different meanings or stand for different values. When students learn regular addition and subtraction, place value becomes more meaningful for them for they have to line up the digits with the same place value. After that, when learning addition with composing and subtraction with decomposing, students learn the idea of composing and decomposing to a unit of higher value. The composition and decomposition of a unit are important aspects of the concept of place value. It is important for the students to know that place value is a mathematical concept not a label for the columns for lining up digits.

One strategy to teach the students is to have them think about addition when they are doing subtraction, the inverse operation. This will help to facilitate their conceptual learning. It brings awareness to the fluid movement between these two operations. Inverse operations are one of the main principles that underlie the relationships among the operations of mathematics.

Composing and decomposing are a different way to learn addition and subtraction. These two terms allow the students to know that numbers can go together and come apart. Addition with carrying is called addition with composing - subtraction with regrouping is called subtraction with decomposing. Composition and decomposition of a unit are important aspects of the concept of place value. The goal is to emphasize that 1 ten is composed of 10 ones and can be decomposed into 10 ones. [6]

Students should have a clear idea about the rate for composing a higher value unit so that they can better understand why a higher value unit is decomposed into 10 lower value units.

Order of Magnitude

A clear understanding of the concept of order of magnitude will increase the effectiveness of teaching this unit in the classroom. This level of information is too complex for the early elementary grades but having this background concept knowledge follows the philosophy of truly understanding before teaching. Because students are at different levels, being prepared to further explain this concept is empowering the child to understand more fully.

Orders of magnitude are generally used to make very approximate comparisons. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of aroundhttp://en.wikipedia.org/wiki/100_%28number%29 100. Two numbers of the same order of magnitude have roughly the same scale: the larger value is less than ten times the smaller value.

For example, let's say the average height of a human being is about 1.7 meters (about 5'7"). For the sake of simplicity, let's round off 1.7 meters to the nearest power of 10, which is 10^{0} m (or 1 m). This does not mean the average height of a person is a mere 1 meter, but rather the average height is closer to 1 meter (or 10^{0} meters) than it is to 10 meters (or 10^{1} meters). Similarly, rounding the height of an ant, which is about 8×10^{-4} meters, to the nearest power of ten results in 10^{-3} meters. Another way of saying this is that the order of magnitude of the height of an ant is 10^{-3} meters. Now, if we compare the height of a human being (10^{0} meters) with the height of an ant (10^{-3} meters), we come up with the ratio human height/ant height = $10^{0} / 10^{-3} = 10^{0-(-3)} = 10^{3} = 1000$. A human being is roughly 1000 times (or 10^{3} times) taller than an ant. In other words, a human being is 3 orders of magnitude (3 powers of 10) taller than an ant.

Our common decimal system is a highly sophisticated method for writing whole numbers efficiently. It uses only 10 symbols (the digits: 0,1,2,3,4,5,6,7,8,9), arranged in carefully structured groups, to express any whole number. Further, it does so with impressive economy. To express the total human population of the world would require only a ten-digit number, needing just a few seconds to write.

This introduction and explanation from Dr. Howe's essay on place value gives an example of the order of magnitude concept. As he clarifies in this essay, the first thing to know about decimal notation is that the base ten expression of a number implicitly breaks the number into a sum of numbers of a very special type.

$7,452 = 7,000 + 400 + 50 + 2$

This is called expanded form. Although this is not a first grade skill, it is important to build the conceptual knowledge of the order of magnitude. Limiting subject matter knowledge to a narrow scope restricts a teacher's capacity to promote conceptual learning to their students. Knowing the importance of this foundation skill will improve the ability of the students to understand why instead of just how.

The summands in the expanded form of a number the place value components of the number. Each place value component is a digit (possibly 0) times a power of 10:

$7,000 = 7 \times 1000 = 7 \times (10 \times 10 \times 10) = 7 \times 10^{3}$

$400 = 4 \times 100 = 4 \times (10 \times 10) = 4 \times 10^{2}$

$50 = 5 \times 10 = 5 \times 10^{1}$

$2 = 2 \times 1 = 2 \times 10^{0}$

A single place number is a digit times a denomination.

The order of magnitude also may be described as the number of zero digits used to write the denomination, or one less than the total number of digits used. By the order of magnitude of a (non-zero) place value number, we mean the order of magnitude of its denomination. Finally, the order of magnitude of a base ten number is defined to be the order of magnitude of its largest (non-zero) place value component. Thus, 7,452 has order of magnitude 3, the same as its largest decimal component, 7000; and the 400, the 50 and the 2 have orders of magnitude 2, 1 and 0, respectively. [7]

Within this unit of working with two-digit numbers, the important principle to remember is that a two digit number is so many 10s and so many 1s, e.g., 47 is four 10s and seven 1s. Then the adding of the 10s and the 1s should become natural with guidance and discussion. Again logical sequence is important. Before two-digit addition is presented, it is important to establish the "make a ten" principle. Using the model of Singapore math, addition facts are not just a list to memorize, but an opportunity to begin understanding the structure of the number system. The emphasis is placed on the process of forming ten ones into a 10 when the sum of the digits is greater than 10, and of decomposing a 10 back into ones in the corresponding subtraction problems.

And as stated earlier, subtraction should be linked with addition until the students think of it as undoing addition. Keeping in mind that fact families (or in this curriculum, number bonds) are not limited to one-digit numbers. This process of linking subtraction problems to addition problems helps the students to make sense of subtraction.

Number Bonds

Number bonds are similar to fact families although not presented as an equation. The students must be taught how to identify parts in a whole. Examples with easily identifiable features, like size or color, should be used to allow student to easily split the whole into two parts Countable items, picture cutouts and picture cards can be used to present colorful and varied examples for the students to practice identifying parts and whole.

The idea in presenting number bonds is to use pictures to express that one big number is composed of two smaller numbers and that what you need to make the big number starting from one of the small numbers is the other small number. The equations that express what the number bond is saying would be a fact family. Again, this is a skill that is introduced pictorially before numbers are used. An example for beginning students might be a picture of 5 apples, 3 are small and 2 are large. The students would then be encouraged to tell a story about the picture. There are 5 apples; 2 are big; 3 are small.

Numbers to 20

Counting and Comparing

The students will learn how to count, read and write within 20. Two-digit numbers within 20 are introduced as being 1 ten and additional ones and 20 as being 2 tens. In order to count objects such as picture cutouts, students first make 10, then they may count on from 10. Number cards are used to help student recognize the numeral representing the number within 20. At the same time, the related number-word can be written to help students associate the number word with the symbolic representation.

Once the students are familiar with the numbers within 20 they will learn to count backwards within 20 as well. This will help them recall the number sequence within 20 which in turn will help with the comparison of numbers within 20.

Students are first introduced to comparison of the number of objects like animals and counters, between two or more sets before the abstract comparison of numbers. They will use the terms greater, greatest, smaller, and smallest during the comparisons.

Addition and Subtraction

Students will learn to add by making 10 first. Picture cutouts are used to demonstrate this method. Number bonds are then used to show the symbolic form of addition which helps the student move to adding without visual aids.

With subtraction, students are first taught to group objects in ones and tens before performing the act of taking away the desired number of objects during subtraction. They count the remaining objects to find the answer to the subtraction. Number bonds are used again to display the symbolic form of subtraction and show that the taking away can take place to the ten or the ones.

Numbers to 40

Counting

As in learning to count to 10 and 20, students will first learn to read and write the number symbols and number words for numbers up to 40. Picture cutouts again are used to display the quantity while the number symbols and number words are taught alongside the quantity.

The students are progressively taught how to count to 40. First they are introduced to counting up to 30, then to 40 by making ten first. Using picture cutouts and number cards, students will learn to recognize the logic ad pattern of counting beyond 20 is the same as that of counting beyond 30.

Students will learn the number sequence and the concept of more than and less than through activities involving finding a number 1, 2, or 3 more or less than a given number. After understanding these concepts, students will be able to compare numbers and arrange a given set of numbers in increasing or decreasing

order with more ease.

Addition and Subtraction

Because the students have learned how to interpret a two-digit number in terms of tens and ones, this better prepare them for addition and subtraction with and without renaming.

Students are first presented with cases of addition and subtraction without renaming. Addition and subtraction without renaming can be demonstrated using a combination of picture cutouts and number bonds. Using picture cutouts, students are shown that such addition and subtraction take place within the objects not grouped within a group of ten and there is no change in the objects grouped within groups of ten. Corresponding number bonds are used to show what happens at the abstract level. The addition and subtraction are shown to affect the number in the ones place while the number in the tens place is unaffected. The converse is shown to occur when the addition and subtraction of a ten takes place.

Students will also learn to apply the count on and count back method of addition and subtraction to adding or subtracting 1, 2, or 3 from a given number for numbers up to 40.

When showing addition involving renaming using rectangle cutouts, students need to learn how to make 10 to derive an answer. With subtraction, students will learn that they need to subtract from ten to derive the answer. Demonstrations of addition and subtraction with renaming involve the use of rectangle cutouts before moving on to demonstrating them at the abstract level using number bonds.

The Number 100

Tens and Ones

Students will now learn to count, read, and write whole numbers to 100. They will first be introduced to the tens, i.e. 50, 60, the number symbols and the number words. Then using countable items and number cards, students will be led to see that they can count in tens and ones and to write the corresponding number symbols. They should also be able to do the reverse, which is to tell how many tens and ones a given two-digit number is.

Estimation

Students will learn to estimate the quantity of an item within a container by comparing the proportion of the unknown quantity with a known quantity within another container of the same type. This method of estimation is useful for quantities that differ by about ten or in multiples of ten.

For students to arrive at reasonable estimates, they need to experience various scenarios requiring estimation, such as number of an item that can be held in containers of various sizes. This is to develop skills such as judgments of the size of the item and the container and making reasonable estimates.

Classroom Activities

Lesson One

Objective: The students will count within 20 by making (building up from) 10 and read and write numerals 11 to 20 and their corresponding number words.

Materials: Magnetic cutout pictures to attach to the chalkboard, students practice books

Procedure: Display a set of picture cutouts, such as apples or flowers. Make sure the number of cutouts is less than ten. Have the student count the number of cutouts. Add picture cutouts one by one until the total is ten. Each time a picture is added, ask student to call out the total. Add another cutout right under the first cutout, showing one row of 10 and one more. Have students say "eleven". Write the number word "eleven" and the numeral "11" on the chalkboard. Have students copy both the number word and the number into their exercise books. Indicate to the students that eleven is written as "11" to show "a ten and one" as the row of ten items and an additional one item. The example would look like:

eleven

11

Add another cutout to the second row, neatly below the first row's second cutout. Have the students say "twelve" and write the number word and the numeral. Remind them that for twelve there is one row of ten and two additional items.

Continue to add picture cutouts one by one until the total is 20. Each time write the number, both as a number word and as a numeral. Have students say the number word aloud and also write both the number word and the numeral in their exercise books. When it is 20, explain that the symbol means "two tens and nothing" as seen by the two rows of ten items.

Display a set of picture cutouts with a total between 10 and 20. Have the students count to the total, one by one, while pronouncing the number word correctly. Repeat this with different numbers of cutouts.

Write the number that is between 10 and 20 on the board. Have the students draw the same number of dots in their exercise books. Have them draw a group of ten, and then the extra ones in a separate group. Repeat this with different numbers.

Display a set of picture cutouts distributed randomly on the board. Have one student come to the board to select 10 cutouts and rearrange them into a neat row. Have another student take cutouts, one by one, from the remaining cutouts. Ask the student to use the cutouts to form another row while counting the new total by

starting from 10, one by one. When they are done, have them say the number name. Write the numeral and ask, what is the 1 telling us? What is the (number in the ones place) telling us? Ask whether they can hear the parts of the number in the name. That is, can they hear the "ten" in "fourteen"? (It sounds a little different doesn't it? But if you think about it, you can hear "ten". And can you hear the four more?

Repeat this process using different numbers of picture cutouts. At the end, highlight this method of separating out 10 objects first and then count onward from 10.

Lesson Two

Objective: The students will add two one digit numbers, using the "make a ten" strategy.

Materials: magnetic picture cutout cards for the chalkboard

Procedure: Display a set of 10 picture cutouts. Ask the students to count them. Add 2 picture cutouts under the first set. Ask student to count the number of added items. Referring to the two numbers, write the question on the board, 10 + 2 = ?. Ask the students to find out the answer by counting the total number of items. Then have them write the completed addition sentence in their exercise books.

10 + 2 = ?

10 + 2 = 12

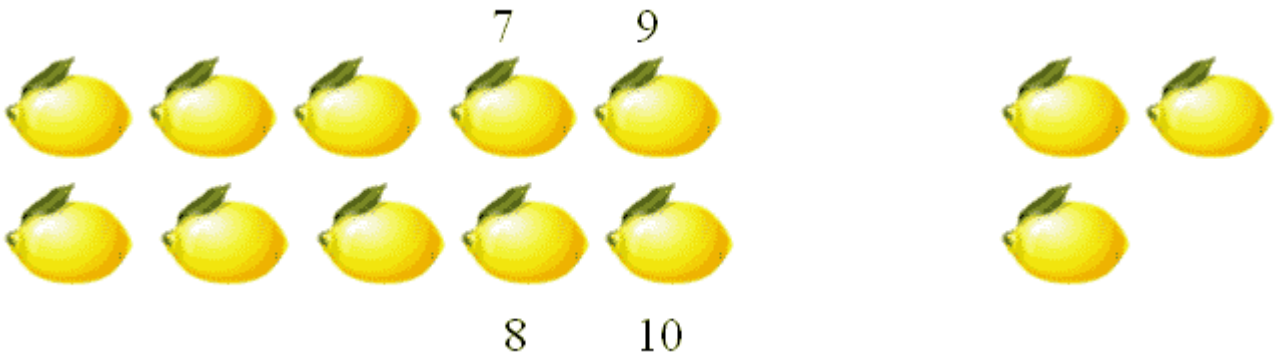
Display two clusters of picture cutouts on the board. Ask the students to count the number of items in each cluster. (6, 7)

Referring to the two numbers write the question on the board, 6 + 7 = ?. Have the students find the total by counting the items. (13) Introduce another way of counting to get the total number of items. Ask students to count only the second cluster, starting with one more that the number of items in the first cluster. (7, 8, 9, 10, 11, 12, 13) Have the students write the completed addition sentence in their exercise book.

6 + 7 = ?

6 + 7 = 13

Repeat the process of counting starting with the next number, this time moving the items one by one to the other cluster as they are counted, until the total becomes 10.



Ask students to count the number of items left in the second cluster. (3) Then explain that the total can now be determined by adding 10 to the amount remaining in the second cluster. Have the students say: 6 + 7 = 13 is the same as 10 + 3.

Lesson Three

Objective: The students will count within 40 by making tens first. They will read and write number symbols 21 to 40 and the corresponding number words.

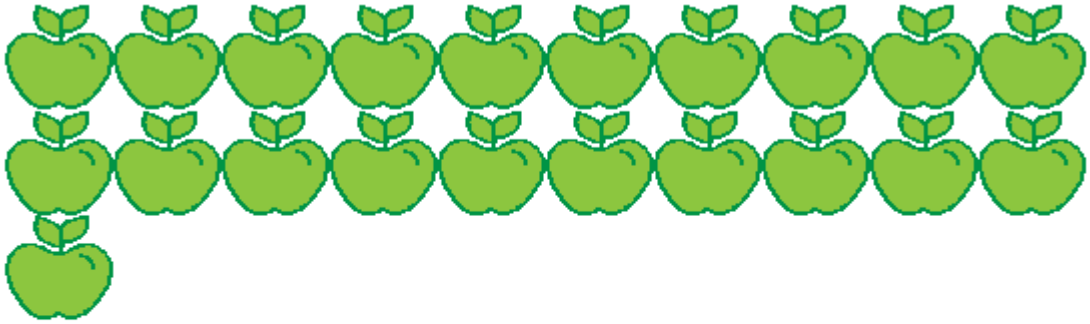
Materials: magnetic picture cutout cards for the chalkboard, number cards 20, 30, 40 and 1-9

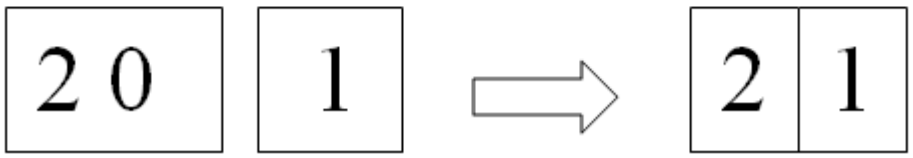
Procedure: Display 16 picture cutouts. Have students count them. Add more cutouts, one by one, until the total is 20. This would become two rows of ten cutout pictures. Each time have the students call out the current total. Remind students that numbers 11 through 19 each has a special name. Write the numbers on the board in a column. Call a student to the board and have him or her write the number word next to each.

20

twenty

Add one more cutout. Tell students that the total now is twenty-one. Have students say twenty-one aloud. Show the students that this is 2 tens and 1 more. Now show the number cards for twenty-one by overlapping the number card for twenty with the number card for one. Write the numeral 21 on the board and write the corresponding number word.





21 twenty-one

Have the students copy the numeral 21 in their exercise books and write "twenty-one" next to it. Explain that we put a dash between "twenty" and "one" to show the connection between the two words. This is the way we write numbers that are above twenty.

Add one more cutout to the display. Tell the students that the total now is twenty-two. Have the students say twenty-two aloud. Write the numeral 22 on the board and show the corresponding number word card. Have the students copy the numeral 22 into their exercise books and write twenty-two next to it.

Continue this process of adding cutouts one by one until the total is 29, writing the numbers and number words to form a clear sequence to that the students can see the pattern. Now write the numbers 21 through 29 in a column. Have the students call out the number words and write them next to each corresponding number. Have students copy them in their exercise books in the same way.

This same procedure will be followed when introducing the numbers from 31 to 40.

Lesson Four

Objective: The students will add a one-digit number to a two-digit number without renaming.

Materials: number cards or chalk board

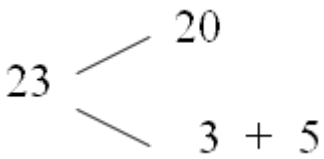
Procedure: Display 23 picture cards in a cluster. Have the students count them. Display five cutouts in another cluster. Have students count them also. Write the addition sentence on the board.

23 + 5 = ?

Rearrange the first set of cutouts into two rows of ten each and place the remaining three cutouts in a third row. Add the second set of cutouts to the third row. Have student count the items in the first two rows and verify for themselves that each row contains ten items. Then have them count the number of each cutout in the third row find that row's total.

Represent the situation using a number bond. Guide the students to say 23: is 2 tens and 3 ones. Then guide

the students to say: 23 plus 5 is 2 tens and 3 ones plus 5 ones.

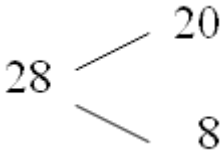


Write the addition sentence for the ones. Have the students provide the answer (8) without counting. Using the answer, modify the number bond.

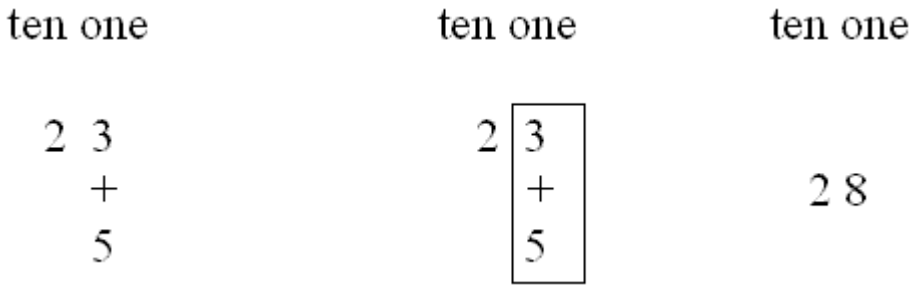
3 + 5 = ?

Write the corresponding addition sentence and complete the number bond. Guide the students to say: 23 plus 5 is the same as 20 plus 8 ones.

20 + 8 = 28



Explain the addition process using this illustrated example



Repeat the process for other pairs of two-digit and one-digit numbers. Make sure that the total stays less than 40 and the sum of ones is less than 10.

Lesson Five

Objective: The students will count by tens, count within 100 by making tens, and read and write the number symbols and the number words. They will be able to tell how many tens and ones make up a given two-digit number.

Materials: 10 bundles of 10 straws each, base ten blocks, counters (beads, beans, buttons)

Procedure: Show the students the bundles of straws. Have 10 students count how many straws are in each bundle. Point out that the number of straws in each bundle is the same; that is they are equal groups. Therefore, by counting the bundles, we can find out how many individual straws there are. Ask the students if they can think about how this could be done. Then point out (even if they have) that we can count the straws in terms of tens because each bundle has ten straws. Write the list of numerals as follows:

1 ten = 10 ten

2 tens = 20 twenty

3 tens = 30 thirty

4 tens = 40 forty

5 tens = 50 fifty

6 tens = 60 sixty

7 tens = 70 seventy

8 tens = 80 eighty

9 tens = 90 ninety

10 tens = 100 one hundred

Show the students base ten rods (10 units) and again have them count how many in each rod. Show the students three rods and ask how many altogether. Encourage the students to count in tens. Repeat this exercise using a different number of rods (up to 10).

Endnotes

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1. Heather C.Hill, et al. "Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Students."

2. Mitchell Lansberg. "In L.A., Singapore Math has Added Value"

3. Elizabeth Witt, Principal Editor .*What the United States Can Learn from Singapore's World-Class Mathematics System (and what Singapore can learn from the United States): An Exploratory Study,*

4. *Singapore Primary Mathematics*, 1A.

5. Mitchell Lansberg. "In L.A., Singapore Math has Added Value

6. Liping Ma. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.*, 9-10.

7. Roger Howe, *Taking Place Value Seriously: Arithmetic, Estimation and Algebra.*

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Baldridge, Scott and Thomas Parker. *Elementary Mathematics for Teachers*. Septon Ash Publishers

Carpenter, Thomas P. et al. *Children's Mathematics: Cognitively Guided Instruction.* Portsmouth, N.H.: Hinemann, 1999.

This book portrays the development of children's understanding of basic number concepts. The authors offer a detailed explanation and numerous examples of the problem-solving and computational processes that virtually all children use as their numerical thinking develops. They also describe how classrooms can be organized to foster that development.

Garelick, Barry. "An A-Maze-ing Approach to Math *Education Next* Spring 2005 Vol 4 No 2 http://www.hoover.org/publications/ednext/3220616.html

The author, while tutoring students in math, discovers some basic issues in the current mathematics curricula, from his perspective.

Garelick ,Barry. "Miracle Math" *Education Next* Fall 2006, Vol 6, No. 4, http://www.hoover.org/publications/ednext/3853357.html

The author discusses the Singapore Math approach as used in a suburban area of the United States, Montgomery County, Maryland.

Hill, Heather C., Deborah Loewenberg Ball and Stephan G. Schilling. "Unpacking Pedagogical Content Knowledge: Conceptualizing and Measuring Teachers' Topic-Specific Knowledge of Students," *Journal for Research in Mathematics Education,* Vol. 39, No.4, July 2008.

This article describes an effort to conceptualize and develop measures of teachers' combined knowledge of content and students.

Howe, Roger, *Taking Place Value Seriously: Arithmetic, Estimation and Algebra.* http://www.maa.org/pmet/resources/PlaceValue_RV1.pdf

Ma, Liping. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States.* Mahwah, New Jersey: Lawrence Erlbaum Publishers,1999.

This book describes the nature and development of the "profound understanding of fundamental mathematics" that elementary teachers need to become accomplished mathematics teachers, and suggests why such teaching knowledge is much more common in China than the United States, despite the fact that Chinese teachers have less formal education than their U.S. counterparts.

Singapore Primary Mathematics. Singapore: Times Media Private Limited, 2003.

Colorful and simply designed textbooks and workbooks designed for classroom instruction.

ThinkMath!. Education Development Center, Harcourt, 2008.

Lansberg, Mitchell. "In L.A., Singapore Math has Added Value," *Los Angeles Times*, March 9, 2008, http://articles.latimes.com/2008/mar/09/local/me-math9

Weinstein, Lawerence and John A. Adams, *Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin.* Princeton

University Press. 2008.

Witt, Elizabeth, Principal Editor .*What the United States Can Learn from Singapore's World-Class Mathematics System (and what Singapore can learn from the United States): An Exploratory Study,* Alan Ginsburg, Steven Leinwand, Terry Anstrom, Elizabeth Pollock http://www.air.org/news/documents/Singapore

Materials List

Base Ten Blocks - This is a set with unit cubes, rods (10 units), flats (10 rods), and a cube (10 flats). It is helpful to use ones that can be displayed from the front of the class, either from an overhead projector or ones that can be stuck onto the board. Also you can draw them, but the students should be familiar with the concrete objects.

Countable items such as counter, buttons, cubes, pencils

Straws or craft sticks that can be bundled into tens with tape or rubber bands

Picture cards and picture cutouts - This unit uses colorful identical picture cutouts to demonstrate groups of ten. Suggestions are be construction paper cutouts of objects (fruits, shapes) or pictures from magazines or newspapers.

Number cards - Large cards to display on the board in the numbers 10, 20, 30, 40, 50, 60, 70, 80, 90, and 1 through 9. The ones digit cards should fit over the 0 digit on the tens cards.