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Take Your Best Guess: Exploring 1, 10 and 100

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Introduction

You enter my classroom as a second grade student and are asked, "What's the difference between 1, 10 and 100? How much bigger is 100 than 10? Is it a big difference?" You try to answer, but you're not quite sure. Then I ask, "OK, well let's look at the number 11. What digits are in the number 11 and what do they stand for?" Again, you don't understand. "What's a digit?" you think to yourself. This is how most of my students react when we start to learn about place value.

I teach second grade in an unusual school. We are located within a school district that serves mostly underprivileged minority students. However, we are located amongst a fairly white, upper-middle class neighborhood. We have come to operate like a "magnet" school in that we take students from all over the City of Richmond when we have open spaces. There are often open spaces because a majority of the students in the surrounding neighborhood attend local private schools. These spaces are usually filled by students from schools in the district that are seen as "failing" because they are not making adequate yearly progress. The implication of this for my classroom is that I have a large variety of students from different backgrounds, with sometimes vastly different support systems and early childhood experiences. While one student may have a very supportive parent who is able to provide a lot of extra help at home, another may have a parent who works two jobs and isn't able to be as involved. Some of my students are very well off, and others qualify for free lunch. Many of my students did not start at our school in Kindergarten, and may have just joined our school because a slot opened up.

Just as there is a huge range in the personal backgrounds of my students, they are also often at very different levels in their academic developments. I may have a student who has a very deep understanding of place value, has memorized the math facts to 20, can do two-digit addition, and is ready for multiplication and division when he or she walks into my room in September. On the other hand, a different student may not have any fluency with math facts to 10 and does not have any strategies for one digit addition and subtraction. However, year after year, I have noticed that the majority of my students would not be able to answer the questions presented above. It is the goal of this unit to create a scope and sequence that increases awareness of the meaning of decimal notation and enable my students to use this more accurate number sense to help them estimate lengths and quantities in the real world.

Overview

This unit is intended to teach second grade students that there is a significant difference in the value of a digit when it is written in different places in decimal notation. Specifically, it seeks to show them that tens are a lot bigger than ones, and hundreds are a lot bigger than tens. It also seeks to further enforce the difference in the "places" using digits other than 1. For example, we'll explore the difference between 5, 50 and 500. These extensions will help solidify the idea that place value is a significant aspect of our numerical notation system. These concepts will be practiced in a small unit on measurement and estimation in the 3rd quarter of the school year.

However, the path I would use to guide the students to this understanding of place value and order of magnitude starts at the beginning of the year and winds through lessons on digits, composing and decomposing numbers to 10, number bonds to 20, expanded form, multiples of 10 and order of magnitude. In order to be able to successfully engage in the estimation activities described in this unit, I will begin to teach them to have a new perspective of our number system starting at the beginning of the year. I believe very strongly that we need to strengthen our students' sense of number and help them gain fluency in breaking apart and combining numbers. They need to readily be able to see a multi-digit number in many different ways and break it into its place value components before we can ask them to compare large quantities and understand the usefulness of rounding larger numbers. A guide for following this path is included in this unit as an appendix.

Nevertheless, I understand that not all math curricula allow as much flexibility as the one I teach with. If you are not able to implement the full three quarters worth of lessons focusing on a structured path toward greater number sense, I believe that you could take the estimation and measurement activities and insert them into your unit however you see them connecting to your current curriculum. While students' skill in estimating is largely based on the strength of their foundation with the concept of ten and order of magnitude, this does not mean that there is a completely linear progression from number sense to estimation. "Mathematics instruction...should promote numeracy by making students more sensitive to order of magnitude, and to the estimation capabilities of place value notation." [1] Experiences with measurements and quantities that differ by a power of ten will strengthen each student's understanding of place value, order of magnitude and relative size.

Rationale

The study of place value and order of magnitude is essential to constructing a solid understanding of our decimal notation system. A substantial grasp of decimal notation is the foundation for rounding, comparing numbers and computing. In Virginia, the Standards of Learning and benchmark tests put a strong emphasis on students being able to successfully round a value, compare two values, and perform the 4 basic operations with multiple numbers. I feel that a key factor for helping our students achieve success in these areas is to approach building a more thorough sense of number in early childhood education. We must help young children start to comprehend the complexity of the way we communicate mathematically *before* we can expect them to manipulate and compute within that system. A key factor in their success in simple computations is an appreciation of exactly what each digit in a number stands for and that where you put a digit can make a huge impact on the value of the number.

In second grade, students start to explore the way we order digits to represent different values. However, it is not often presented to them from this perspective. Instead, seemingly out of the blue, we ask them to answer questions like "Which digit is in the tens place?" or "What is the value of 4 in 402?" As adults, we see these concepts as transparent. However, from the viewpoint of a child who has been counting groups of objects from 1-10 since he or she could talk, but has not gone much beyond ten, the fact that our numbers are communicated in specific combinations using decimal notation is NOT readily apparent. Most of the students I have worked with do not understand the concept of "digit" when we begin mathematics in September. They were not introduced to "base-10" as a concept before they got to my class. When they think of 10, they see it as a complete picture representing ten objects. Most often, they do not see 10 as representing 1 ten and 0 ones. I think we are doing a disservice to our students when we don't emphasize that 10 is a two-digit number well before 2nd grade. Our students learn that there are 26 letters by learning a nifty song. Then they learn that when you combine those letters they begin to represent specific sounds that hold meaning (words). Why do we not do the same thing in early mathematics teaching? Why is there not a digit song that all children are encouraged to learn at a very young age? (See Appendix D.)

Once a student understands the concept of "digit," he or she can begin to look at the pieces of a multi-digit number. However, they first need to be able to explain the relationships between single-digit numbers. Most of my students do not enter second grade with this ability. They cannot readily explain the relationship between a set of numbers. If I asked them, "How are 7, 3 and 4 related? Can you show me with a picture?" they wouldn't be able to successfully answer. This is an indication that they don't have sufficient experience composing and decomposing numbers. If they can't pull apart and recompose a number, then they can't begin to manipulate its pieces. Before I can ask them to manipulate a two or three-digit number, they need to be able to compose and decompose smaller numbers with accuracy and fluency. (Activities for building these skills are included in Appendix B.)

The way in which we see base-10 numbers determines our success in doing computations with them. "The fact that base ten numbers are sums has a pervasive influence on the methods for computing with them…the procedures for carrying out the four arithmetic operations with base ten numbers are largely determined by the fact that base ten numbers are sums of their place value components." [2] We must begin to see base 10 numbers as sums of their components. When we begin to do this, we can manipulate them with greater efficiency and accuracy.

After a second grader is able to see the relationships between pieces of numbers, the activities integrating order of magnitude can be implemented. As we start to talk about multi-digit numbers in terms of their place value components, the activities in this unit become relevant. Subsequently, we can begin to use "very round numbers" to estimate quantities and lengths.

If we don't give students the opportunity to find the proof for the rules of math on their own, then how can we expect them to truly understand mathematical processes at the elementary level? Sure, we could tell them the age-old adage "Because I said so," but we would be leaving them with a very superficial concept of number and our decimal notation system. Children need to repeatedly see the difference in magnitude for each place value component so that they have an internal understanding of the significance of position of the decimal notation system. It is only when children build a solid concept of digits, place value and order of magnitude that they will truly be successful in computation, estimation and the further manipulation of numbers.

Background

Terminology

Digit: A digit is a number (0,1,2,3,4,5,6,7,8,9) that can be used alone or in combination with other digits to communicate a value.

Place Value Notation/ Decimal Notation/ Base-10 Notation: Our way of writing numbers is a complex system based on groups of ten. The order in which we arrange digits determines the value the digit represents. We have many "places" and each place has it's own value, hence "place value." A digit placed directly to the left of the decimal point stands for ones, the next place to the left denotes groups of ten, the next place to the left from that denotes groups of hundreds, and so on. We can represent an infinite number of possible values using just ten digits because of our notation system. We don't have to write 200 + 30 + 9, we can just write 239 and the previous values are assumed. This allows us to write large numbers very compactly.

Place Value Components/ Very Round Numbers: This refers to the value of each digit in a multi-digit number when it is considered on its own. It is a digit times a power of 10. For example, the number 3,461 has four place value components. They include 3,000, 400, 60 and 1. [3] A *very round number* is a number with only one non-zero digit, e.g., 3000, 400, 60 and 1. Our base-ten place value system expresses every whole number as a sum of very round numbers. For example, the number 3,461 is the sum 3000 + 400 + 60 + 1. When very round numbers are combined like this to make some number, they are called the *place value components*, or *very round components*, of that number.

Order of Magnitude: This term refers to the number of zeros used to write a very round number. [4] For an arbitrary whole number, the order of magnitude is one less than the number of digits.

Composing/Decomposing Numbers

Chinese equivalents for the terms "compose" and "decompose" are prevalent in Chinese mathematics learning. [5] However, these are relatively unfamiliar terms to early childhood math teachers in the United States. [6] If we begin by focusing on what these words mean, we will help ourselves and our students understand how the terms relate to mathematics. *Compose* is a verb that means "to make or form by combining things, parts, or elements." [7] In mathematics, when we compose a number, we are making a number by combining two or more smaller values. Essentially, we are looking for ways to group the possible parts of a number. *Decompose* is a verb that means "to separate or resolve into constituent parts or elements." [8] In mathematics, when we decompose a number, we are pulling that number apart into smaller values. There may be a variety of ways to compose or decompose a number, depending on how many digits it contains. We can compose 3 in a few ways. (1 + 1 + 1) or (2 + 1) or (1 + 2) or (0 + 3) are all ways of making 3. We can decompose 9 in a numerous ways. (There are 30!) Some examples are: 9 can be broken into (2 and 7), (5 and 4), (1 and 8) or (2 + 3 + 4). This concept is integral to early childhood mathematics education, and I believe we should introduce not only these terms, but also these concepts, with young children as they begin to build concepts of number. We can use "make" or "combine" as synonyms for compose, and we can use "unmake" or "break apart" as other ways of explaining "decompose," but we should use these terms interchangeably.

Expanded Form

Expanded form expresses a number as the sum of its place value components. Expanded form helps us visualize a number by looking at its place value components separately. Each digit is considered on its own, as a separate value. When we consider each digit separately, its order of magnitude becomes more apparent. Then we see that none of the digits can represent the same order of magnitude. [9] Examples of expanded form:

67 = 60 + 7	4,532 = 4,000 + 500 + 30 + 2
503 = 500 + 3	11 = 10 + 1
8030 = 8,000 + 30	83 = 80 + 3

If we take the time to explicitly teach children to break apart numbers in this way, they will be able to utilize this strategy when doing operations. The study of numbers in expanded form should be introduced after students have broken numbers into tens and ones first. They need to be guided to see each digit as a certain number of tens and ones (or thousands and hundreds) first. Then you can begin to use expanded form.

Order of Magnitude

Once you understand expanded notation of multi-digit numbers, you can begin to explore the concept of order of magnitude. When you break a given number into expanded form, you can see each digit represents a different power of ten. We can refer to the pieces of a number shown in expanded form (these multiples of 10) as "place value components." None of the place value components can represent the same place value. I can write 1,111 in expanded form like this, 1,000 + 100 + 10 + 1, and we will see that it has 4 place value components. Then we can further analyze these components.

1,000 =	1 x 10 x 10 x 10 =	1 x 103
100 =	1 x 10 x 10 =	1 x 102
10 =	1 x 10 =	1 x 101
1 =	1 x 1 =	1 x 100

This chart helps us begin to name each order of magnitude. 1,000 is order of magnitude 3 because it is 1 x 10^3. Any 4-digit number is order of magnitude 3, which includes numbers from 1,000-9,999. 100 is order of magnitude 2 because it is 1 x 10^2. Any 3-digit number from 100-999 is order of magnitude 2. 10 is order of magnitude 1 because it's 1 x 10^1. Any 2-digit number, 10-99, is order of magnitude 1. 1 is order of magnitude 0 because it is 1x 10^0. Any other 1-digit number is also order of magnitude 0.

The chart also helps us to see that with each successive digit to the left, the value of the digit is ten times bigger. 10 is ten times bigger than 1. 100 is ten times bigger than 10. 1,000 is ten times bigger than 100. If you want a deeper understanding of the implications of order of magnitude in estimation, I would point you to pages 27-30 in R. Howe's *Taking Place Value Seriously.* [10]

Understanding order of magnitude helps us more accurately conceive the difference between very large numbers that can often be abstract. It is hard for us to know just how significant the difference is between 1 million and 1 billion. However, if we begin to analyze the decimal notation system in term of order of magnitude, we should at least know that 1 billion is a thousand times bigger than 1 million: ten groups of a million is ten million; ten groups of ten million is one hundred million; and ten groups of one hundred million make a billion. In all, this is 1,000 groups of a million. Then you begin to realize that that is a huge difference, and it is this realization that can help us more accurately estimate.

Second graders do not need to have an advanced understanding of order of magnitude. We needn't try to explain the role of exponents in noting order of magnitude. However, we do want to build a foundation by emphasizing that 10 is ten times bigger than one and 100 is ten times bigger than 10. This foundation will help students grasp the significance between each of the places in our decimal notation system. It will also help them be able to estimate more readily by giving them experience with the difference in size between values that are different order of magnitude.

Strategies

Estimation and Measurement with Number Bonds to 100

The base-10 manipulatives we use in teaching place value have a very useful attribute; they are measured in centimeters cubed. 1 ones block is 1cm long and a tens rod is 10 cm long. We could use the ones block and a tens rod to begin to use our knowledge of order of magnitude to help us estimate lengths. We will find things that are about the same length as a ones cube or a tens rod, making a chart in our math journals. We will talk about how the items that are about the same length as a tens rod are 10 times longer than the things we found that are about the length of 1 ones cube. [11]

We could further integrate measurement into our exploration of multiples of 10 to use the base 10 blocks to create our own centimeter rulers. Each student will be given a tens rod and a strip of paper about 2 inches thick and 50 cm long. They will take the tens rod and line it up on the line already drawn on the paper. They will mark the end of each rod and record 10cm. They will continue to move down the line marking lengths of 10cm until they have measured 50cm. They will go back and use the rods to mark each cm length. This will give each child an experience measuring. (See Lesson 1 for detailed instructions.)

Additionally, these rulers will also serve as a number line to model addition of larger 2-digit numbers. I might ask the students to line up, end-to-end, 2 tens and 3 ones on the centimeter ruler. They will see that these 2 tens and 3 ones have a length of 23cm. If they were to move the blocks around, they would still get a length of 23cm. My hope is that the students will understand that no matter which order we add the pieces, 10 + 10 + 3 = 23. This activity provides an automatic connection between the pieces we are combining and the whole number they represent once combined. The students can SEE that 10 + 10 + 3 = 23. In comparison, when we just model a 2-digit number with the same base-10 blocks on a "Tens & Ones" chart the two-digit representation of "23" is missing. The visual model is there, but the number that corresponds to that visual model is not immediately connected. However, when we model a number on the centimeter ruler/number line there is an immediate visual connection between the model and the number it represents. I can use this centimeter ruler/number line in a vast number of activities to solidify each student's understanding of how to compose and decompose larger 2-digit numbers.

After we use our centimeter rulers as number lines for a few lessons, we will be ready to add on to them so that they measure 0-100cm. This will be the first lesson in the discussion that 100 is the same as 10 tens. Therefore, 100 is ten times bigger than 10. All of our experiences using this ruler as a number line will help us be more familiar with length in centimeters. The exposure to lengths from 0-100 centimeters should allow them to estimate lengths in relation to this experience. I could begin to ask questions such as "Is this piece of string closer to 10 cm or 100 cm? Is this line segment closer to 20 cm or 50 cm.? Is a cat 10-20cm long or is it 20-50cm long? Is your leg about 40cm or about 100cm long? Is 40cm to the knee, or to the hip?"

It is here that I would begin to point out that the leading digit of a number represents most of the value of the number. I would ask "Why don't I ask, 'Is this about 25?' or 'Can you guess exactly how many marbles are in this jar?'" The point of asking this question is to point out that until now we have been working with very round numbers (multiples of 10). We do this because usually we are only really concerned with the leading digit when we estimate. I would then try to explain this through modeling with the base-10 blocks. I would ask them to show 78 in tens and ones on a "Tens & Ones" chart. I would then ask, "Where are there the most blocks? Are there more blocks in the tens place, or are there more blocks in the ones place?" The answer to this question may not be readily apparent to the class. I may need to remind them that 7 tens = 70 individual items. "If we think about the number 78 in expanded form as 70 + 8, which is more, 70 or 8? Right, 70 is more. You can see that most of 78 is in the tens place. This means that the 7 tens is more important to us than the 8 ones." I would repeat this conversation with other 2-digit numbers. I might choose many 2-digit numbers where the tens' digit is smaller than the ones' digit. This sets up a direct conflict with the idea that the larger digit in a number must represent the larger value. Many of my former students have often automatically looked for the biggest digit when asked to identify the digit that represents the larger value. However, in the example 78, the eight is the bigger digit, but 8 is not bigger than 70. In the number 78, the digit that shows the bigger value is the 7.

I would then extend this conversation to support the use of very round numbers in estimating quantities. "This is why we estimate with very round numbers. Most of the important information about a number is in the first digit, so we can just take the first place value component of a number. If we do that, we are guaranteed to work with a very round number. And, as you know, very round numbers are easy to work with because they end with zeros." Then we could try this with a few two-digit numbers.

I would continue by explaining that when we estimate we want to estimate numbers under 100, we use multiples of 10. However, instead of just picking a random very round number out of the air, it is more meaningful for us to say that a quantity is between __(x)___ and __(y)__. When estimating, we want to express that we couldn't possibly be sure of an exact answer. It is easier to show the uncertainty of our estimate if we say our guess is between two numbers. When we do this we are setting boundaries for our estimates. This allows us some flexibility with our estimate. We can be more positive that a quantity is between two very round numbers than we can be that it is about one specific quantity. Think of 35 + 47. Since 30 35 40, and 40 47 50, then 30 + 40 = 70 35 + 47 40 + 50 = 90.

After discussing this crucial aspect of estimation, we would discuss various collections of objects. We would work to create collections of different objects that vary by order of magnitude. I would involve the students in the creation of these collections in order to give them the chance to manipulate collections of objects that have 10 items or 100 items or 1,000 items. They could work together in small groups to make collections of beans that differ in size from order of magnitude 1 to order of magnitude 3. You could split the class into three groups and ask them to make a collage. However, group one makes a collage with 10 beans, group 2 makes collages with 50 beans, and group 3 makes collages with 100 beans. They could begin to think about tackling problems such as estimating how many people are in the cafeteria by estimating it between two very round numbers. The chances for extension are vast.

We could also further explore the relationship between centimeters and meters as an extension to the activities listed below. Following the study of 100 cm = 1 meter, you could discuss and explore the differences between 1 meter, 10 meters and 100 meters.

Activities

Lesson 1

Objectives

- To explore the difference between 1 and 10 through linear measurement using base-10 manipulatives
- To measure items around the classroom in centimeters using base-10 manipulatives
- To report lengths as estimates between two precise measurements when using non-standard measuring tools
- To record information gathered in order to informally assess understanding

Materials

- math journals
- ten base-10 cubes and one base-10 rod per student
- up to 50 base-10 cubes for teacher to display
- 1 centimeter ruler for every 2 students

Procedure

The lesson begins with a whole group discussion about a tens rod. (We would have already discussed the manipulatives and used them in building two digit numbers.) Give each child a base-10 tens rod and ask, "How many cubes are in a rod?" Show the whole group 10 ones and ask, "Which manipulatives are easier to handle, the rods or the cubes? If you had 30 or 40 cubes, would you rather have them in separate cubes or grouped together in tens?" Let the class discuss, but make sure to reinforce that one tens rod is ten ones neatly packed into a form that is easy to handle. (The Cuisenaire rod activity described in Appendix B under Composing and Decomposing 10 would be a good activity to do prior to this lesson.)

Now pair the students up and give each pair 1 base-10 cube and 1 rod. Ask them to measure both in centimeters using the centimeter ruler with their partner (you may need to guide them with this, depending on their prior experience). Have them trace each one in their math journal and record the length next to each outline. After they try to measure on their own, discuss that the cube is 1cm and the rod is 10-cm/1 decimeter.) Then ask, "If one rod is ten centimeters, how long is 2 rods? How long is 3 rods? How long is 4 rods? How long is 5 rods?" They should be able to figure out that you can count by tens, so 2 rods is 20 cm, 3 rods are 30 cm, etc.

Lesson 2

Objectives:

- To explore powers of ten through linear measurement using base-10 manipulatives
- To construct a 50 cm ruler that can be used to measure items around the classroom

Materials

- 1 base-10 tens rod for each child
- base-10 cubes
- 5 base-10 tens rods taped together
- strips of cardstock precut measuring 1 inch wide and 50 cm long with a straight line pre-drawn down the middle

Procedure

Show the class the extra long rod that is 5 base-10 rods taped together. Ask, "How many rods did I use to make this extra long rod? If I used 5 rods, how many centimeters long is this?" Then prove that the rod is actually 50 centimeters by measuring it with a meter stick.

Next, explain that everyone is going to make his/her own 50 cm ruler to use for the next few weeks. Demonstrate how to make the ruler by lining up one rod at one end of the strip along top of the pre-drawn line and marking the other end of the rod with a pencil line. Write "10 cm" under the pencil mark. Then move the rod down to the pencil line you just made and measure out another 10 cm length. Do this until you have measured and marked 50 centimeters in 10-centimeter increments. You can then go back with the tens rod and mark each individual centimeter along the ruler using the lines built into the rod, and then label all 50 centimeters. Then give each student a strip, a tens rod and a pencil and ask them to make it themselves. Find a safe place to store the ruler until the next lesson (they can be folded and stored in Ziploc bags), or you can have them temporarily tape them to the top of their desks.

Lesson 3

Objectives

- To compare 10 centimeters with 100 centimeters
- To introduce the terms "decimeter" and "meter"
- To visually see the difference between 10 and 100
- To associate 10 tens with 1 hundred
- To see 100 as ten times bigger than 10
- To introduce the prefix *deci-*
- To reinforce fractions as a pieces of a whole through a concrete exploration
- To see fractions as directly applied in everyday life

Materials

- math journals
- 2 previously made 50 cm rulers per pair of students
- a few meter sticks for display

- one extra long rod created by 10 tens rods taped together
- 10 tens rods per pair of students

Procedure

Call the class to the rug to make a circle. Have each student bring his or her 50 cm ruler with him or her. Pair students up with a person sitting next to them. Give each pair 10 tens rods and ask them to line them up end to end. Ask "How many centimeters long is one rod? (10cm). How many rods do you have all together? (ten) Can you figure out a way to quickly count how many centimeters you have in 10 rods?" They should be able to count by tens to find that there are 100 centimeters total. Explain that 100 centimeters is equal to 1 meter. Show a meter stick and line it up underneath one groups train of tens rods. They should be exactly the same length. Pass out a few meter sticks and have the groups take turns aligning their 10 tens rods with the meter stick. Summarize again that 100cm is equal to 1 meter. Then say, "How many rods made up a meter? (ten) So, each rod is 1 out of 10 pieces. That means each rod is 1/10 of a meter. There is a special name for 10 centimeters. We call it a decimeter. Deci- is a prefix that means one tenth (1/10). So, a decimeter is 1/10 of a meter. Thus, each tens rod is a decimeter. A ones cube is a centimeter, a tens rod is a decimeter (or 10 cm), and 10 tens rods is a meter (which is the same as 100 cm or 10 decimeters)."

Then ask the students to line up both of their 50-centimeter rulers end to end. These two rulers should be the same length as the 10 tens rods because 50 cm + 50 cm = 100 cm. If they wanted to measure things using a meter stick, they could just combine their 50 centimeter rulers to make one 100 centimeter ruler = 1 meter.

Lesson 4

Objectives

- To relate previous experiences measuring in centimeters with opportunities for estimating lengths of objects
- To create a visual display of objects about 1 cm long, 10 cm long or 100 cm long in order to make it easier to compare relative lengths
- To give students a chance to estimate lengths of items and distances in the school

Materials

- three lengths of thick yarn, 1 cm, 10 cm and 100cm
- slips of paper to write on
- plastic bags
- digital camera(s)
- push pins
- 1 ones cube, 1 tens rod, and 2 50cm rulers taped together for each pair of students

Procedure

Explain that the class will make a display by finding objects and distances in the school that are about 1 centimeter, 10 centimeters/ 1 decimeter, and 100 centimeters/ 1 meter. Pair up the students and give each pair 1 ones cube and 1 tens rod. Have them tape each of their 50-centimeter rulers together to make a meter stick (or they could each create a line of 10 tens rods taped together to use as a meter stick). They are going to use these manipulatives to find items around the room that are about this length. Demonstrate how to use the different measuring tools to measure objects around the room. Then show them that they should leave the tool next to the object in order to take a picture of the object for the bulletin board. If they find portable items that are 1 cm or 10 cm, they may opt to collect the item in it's own plastic bag instead of taking a photograph of it. They can also measure distances between two things/places. They may find that it is 1 meter between the rug and the bookcase. They can lay a meter stick in that space and take a photo of the distance between two points.

After the class has finished, print the photographs. Ask each pair of students to make a caption for each photo using small strips of paper. The caption should say "The __________ is about ____ centimeters." or "It is about _____ cm from ____________ to __________." Pin the lengths of string on a bulletin board, leaving room to add the pictures above each length. Add a label under each yarn declaring how many centimeters long it is (you may also want to add "decimeter" and "meter.") Pin the photos above the length of string corresponds the measured length of the object/distance. You can continue to add to this display on later dates.

Lesson 5

Objectives

- To make polygons that have perimeters of 10 cm, 20 cm, 30 cm, 40 cm, 50 cm, etc. up to 100cm
- To integrate linear measurement with a study of perimeter and plane shapes
- To provide further exposure to lengths that increase by powers of ten in order to improve student ability to estimate lengths

- Materials

- Wikki Stix (or pipe cleaners) cut or combined into lengths of 10cm, 20cm, 30cm, 40cm, 50cm, 60cm, 70cm, 80cm, 90cm, and 100cm - enough for each student to have 2 different lengths
- strips of paper to make labels
- ten 10 cm Wikki Stix for demonstration

Procedure

In a whole group setting, display the different lengths of Wikki Stix (WS). Point out that they are cut into lengths that are multiples of ten. Show the 10 cm WS and then the 20 cm WS. Line up two 10 cm WS alongside of the 20 cm WS. Explain, "The 20 cm WS is two times bigger than the 10 cm WS. It is the same length as two 10 cm WS, so we say that it is *two times bigger.*" Show the 30 cm WS next. "This WS is the same as three of the 10 cm WS, so we say that it is *three times bigger*. 30 is three times bigger than 10." Continue this until you get to the 100 cm WS. "This WS is the same length as ten 10 cm WS, so we know that this WS is *ten times bigger* than the 10 cm one. SO, 100 must be 10 times bigger than 10!"

Now explain that you are going to use all of these different lengths to make different polygons. Demonstrate how to bend a WS to make a triangular shape, a rectangular shape or a trapezoidal shape. Explain that each student will get two different lengths of Wikki Stix. They should trace the length of each WS in their math journal. Then, they should use their 50 cm ruler to measure the length and record it under the line segment they just drew for each WS. This should look something like this (not to scale):

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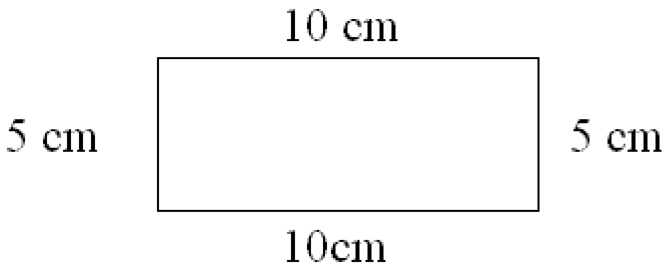
10 cm 30 cm

After they have recorded this information, they can bend the WS to make whatever polygon they would like. Once they have made their two shapes, they will draw models of the shapes in their math journal. They will then use their 50 cm rulers to measure the length of each side of their shapes. They should record the length of each side on the drawings like so (not to scale):



Next, they need to add up all the measurements of each side to find the perimeter of their shape. 10 cm + 5 cm + 10 cm + 5 cm = 30 cm. The perimeter should be the same as the length the Wikki Stix were before they were bent into a polygon.

Appendix A: Singapore Math as a Model for this Unit

There has been a lot of research that the Singapore and Chinese models of structuring math in the early grades gives students a stronger foundation. (See Ginsburg, Geary and Ma.) Therefore, I think it is very important to examine the scope and sequence of the math curriculum in Singapore.

The Singapore curriculum splits the first grade year into two volumes of study 1A and 1B. While there are other topics of study, these units have a strong focus on number sense. Additionally, the preface of the Primary Mathematics textbooks states the following,

> "The main feature of this package is the use of the Concrete à Pictorial à Abstract approach. The students are provided with the necessary learning experiences beginning with the concrete and pictorial stages, followed by the abstract stage to enable them to learn mathematics meaningfully. This package encourages active thinking processes, communication of mathematical ideas and problem solving." [12]

1A starts with counting numbers 0-10 and exploring number bonds. The students are guided through number stories and visual representations of how you can combine numbers to make other numbers. They prompt the students to find pairs of numbers that make values up to 10. They also integrate early algebraic thinking by asking students to find missing addends, instead of just focusing on calculating sums. Then they move into subtraction, defining the operation, using word problems and pictures to give practice, and linking addition with subtraction. Once the students are proficient with number bonds to 10, they move on to counting and comparing numbers to 20. The numbers from 11-20 are introduced pictorially by showing a group of ten plus a number of separate ones. The students are then guided through exercises that emphasize that 13 is 10 and 3 or 15 is 10 and 5, etc. Then they are prompted to add using the strategy of making a 10 first and then counting on. When subtracting, they are prompted to split the larger number into a 10 and its other piece first. Subtract the smaller amount from the 10 and then add on the ones that were ignored initially. Also, they are asked to compare problems like the following 8 - 3 = 5 and 18 - 3 = 15.

1B starts with comparing numbers to 10. It quickly moves into studying numbers to 40. This is where they continue to use expanded form to represent each two-digit number. For example, 23 is 20+3 and 28 is 20+8. Then they begin to compare these larger numbers as well. I think it's important to note that with each number there is a picture that emphasizes groups of ten with some ones left over and that these pictures/diagrams take a variety of forms. Multiplication is introduced as repeated addition and division is introduced as sharing a number of items by splitting them into equal groups. By the end of 1B the students are working with numbers to 100. They start by counting by tens, 50, 60, 70, 80, 90 and 100. Then they add and subtract multiples of 10 to a number. They also do addition and subtraction using the expanded forms of numbers, teaching students to look for tens to add and subtract with first.

While this is their 1 st grade curriculum, I think it frames a great beginning for my 2 nd graders. Our Houghton Mifflin curriculum is sequenced in a very different manner, and it does not place such an emphasis on the decimal structure of our number system. I have designed this unit to bring elements of Singapore's curriculum into my classroom.

Appendix B: Extended Scope and Sequence

Number Bonds to 10

My students come to second grade with a large deficit in their understanding of the relationships between numbers contained in what we usually refer to as "fact families." I think it is important to concretely show students the relationship between the numbers in a fact family. "How are 7, 3 and 4 related? Can you show me with a picture?" When we start to represent the relationship between the members of a "fact family," we are dealing with pictorial representations called "number bonds." It is the manipulation of these number bonds that will help our students see and remember the connection between the numbers in a fact family.

What is ten?

Very young children in the United States are not taught to understand that our number system is based on a decimal notation. They are not usually taught to appreciate what "base-10" means until about 2 nd grade. I feel that this is entirely too late to introduce such an integral and fundamental explanation of our number system. [13]

I start my math curriculum by focusing on what a digit is because this is the foundation for later knowledge of place value and order of magnitude. I explain that there are ten digits (0,1,2,3,4,5,6,7,8,9) that we combine in a variety of ways to represent any number we can imagine. I further explain that the way in which we arrange these digits holds meaning. I reintroduce the digits from our decimal system with a picture of the value they represent.

		:	:.	::	::.	:::	:::.	::::	::::.
0	1	2	3	4	5	6	7	8	9

I then show the students a number of counters pictured on the overhead. They have to choose the correct symbol to represent that number, and once we agree, we write the digit on the board. I do this with many quantities of counters from 0-9.

Once the class seems familiar with the digits, I show them ten counters. I ask them how they would write this number using the digits. I often ask, "Is there a digit for this value? Can we write this number using *just one* of the digits?" When they answer, "No!" I take this opportunity to explain that in our decimal notation system we love to work with groups of ten. "Working with groups of ten is easy. 10 is a very round number, which makes it easy to deal with. Adding something to zero is simple because any number plus zero leaves you with the number you started with ($n + 0 = n$). Zero doesn't ask us to change anything. When we get a full group of ten, it deserves a higher ranking, so we bundle that ten together and call it by a new name. Instead of having ten single items, we now have 1 group of ten." I then explain that this is how we "compose a ten", and we show that we have one ten by putting the digit for one in the tens place on a tens and ones chart. "This is how we communicate that we have one group of ten." Then I ask, "But do we have anything else to count? Are there any counters left over after we make a ten?" When they answer correctly, I assert that since there aren't any counters left we need to account for that in our number, so we write a 0 in the ones place to show that we have made one group of ten and have no extra pieces left to count. I can also ask, "If we didn't write the zero, how would we know that we are talking about 1 ten and not just 1 one?"

I would then have them break into pairs to quickly practice this with amounts from 0-10. Partner A puts a number of counters into the ones place on a "Tens & Ones" chart. Partner B counts the counters and decides if they represent a one-digit number or a group of ten. If the counters equal 10 then Partner B must pick up all of the counters and put them in a cup. The cup is then moved to the tens place on the chart. If he or she is correct, they switch roles and continue to play. This may seem like a very low level activity for second graders, but I feel like it is extremely worth investing the small amount of time it would take to have this discussion and practice the process.

Composing and Decomposing 10

The foundation has now been laid for further exploration of how to compose a ten. The lessons taught in the first quarter are focused on giving the students the opportunity to compose and decompose 10 in a variety of ways. The goal is that the students be able to readily retrieve all the possible ways of combining single-digit numbers to equal ten. I want them to see 2 and automatically know that if you added 8, you would be able to compose a ten. Conversely, I want them to think that if they started with ten and needed just 6, they would have to break apart that very round number into 6 pieces and 4 pieces. There are a number of activities that you can do to reinforce the composition and decomposition of a ten.

One such activity involves displaying 10 beans in a variety of combinations on popsicle sticks. Each child gets 5 popsicle sticks, and they are asked to glue a1 bean on the left side and nine small beans on the right side of the stick, leaving a space in the middle. For example, the popsicle sticks might look like this, where the 0's are beans.

Model of Popsicle Stick	Number Bonds Represented
0 000000000	10 = 1 + 9 or 9 +1
00 00000000	10 = 2 + 8 or 8 + 2
000 0000000	10 = 3 + 7 or 7 + 3
0000 000000	10 = 4 + 6 or 6 + 4

00000 00000 10 = 5 + 5

This is a visual representation of a number bond. They can physically see that 1+9=10. Furthermore, if they flip the popsicle stick around, they see the commutative property of addition in noticing that it's also 9+1=10. They will do this with 2 and 8, 3 and 7, 4 and 6, and 5 and 5. The sticks can be saved in bags for further explorations.

Another way of reinforcing the composition of 10 is to use Cuisenaire rods. They are measured in cm and are in lengths from 1 unit to 12 units. Therefore, you could have each student measure each different colored rod with a cm ruler and have him/her mark on the rod how long it is. The students could then be prompted to find all of the combinations of rods that are equal length to the 10 rod. This might look like this:

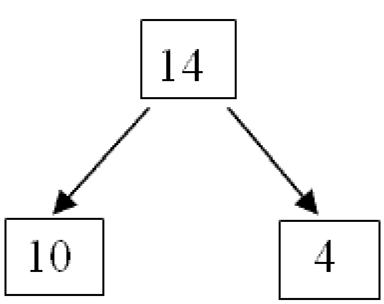
10cm rod	
2cm	8cm
4cm	6cm
7cm	3cm

There are many more activities that one can use to increase student confidence in composing and decomposing ten. I want to stress how important this skill is as a building block for later understanding.

Number Bonds to 20

Once the students can readily compose and decompose a 10 (noting that I will use these terms alongside of "make" and "unmake"), they will be ready to move to addition and subtraction facts to 20. It does not make sense to teach students to just rely on the "count on" and "count back" methods in adding and subtracting to 20. This does not instill in them an appreciation for the efficient system that is decimal notation. Instead, it implies that our mathematics is without a logical system for communicating numbers, when in reality our system is very sophisticated.

One possible way to avoid deficits created through just counting on and counting back, is to emphasize that the numbers from 10-20 are composed of one ten and a number of ones. For example, 17 should automatically be seen as one ten and seven ones. We can help young children internalize this view (for immediate retrieval) by consistently pairing the combined digits with the expanded form of the decimal notation and a visual representation. This may look like the following:



If we are consistent in showing this triplet in the same ways/forms then we are helping our students see the "teen" numbers in a more useful and meaningful way. The development of this perspective is pertinent because it gives students a uniform view of the logic of our decimal notation system. You may want to draw their attention to the illogical way we name the numbers from 11-19 in English. We call 1 ten and 1 one eleven. It would make more sense if this was called "tenty-one" because it would follow the pattern set later with "twenty-one" or "fifty-one," for example. It would also be helpful to try to point out how to hear the "ten" in "fourteen." "Can you hear the "ten" in 14? It sounds a little funny, but it's there at the end. So, "four-teen" means we have four more than ten, and if you listen carefully, you can hear the "four" and the "ten." This would have to make accommodations for the number names of 11, 12, 13 or 15. For example, in "fifteen" neither the "five" nor the "ten" are obvious in the word "fifteen."

To reiterate, if we only began to talk about "regrouping" when we deal with numbers larger than 20, we do not give any recognition to the structure of the decimal notation system. And thus, we end up with children who are confused and unable to successfully perform higher-level addition and subtraction computations. In an attempt to counteract this, we will spend a large amount of time doing "teen" computations with the decimal notation system in mind. We will continue to compose and decompose a 10 whenever necessary. The process may look something like this,

14 + 6 = (10 + 4) + 6

We can now see that we can work with 4 + 6 first.

So, 14 + 6 becomes 10 + (4 + 6) and 4 + 6 = 10

14 + 6 = 10 + (10)

The new 10 plus the original 10 from 14 is equal to 2 tens or 20.

14 + 6 = 20

When we start to represent familiar numbers as a sum of its parts, then it becomes easier to look at all the possible relationships. We could look at 17 in a variety of ways. It could be 10+7 or 11+6 or 12+5, etc. The ability to see the teen numbers in this way will give students the opportunity to approach problems from many points of reference. They could solve 17-9 by thinking first that 17 = 10 + 7. They could then proceed to do 10 - 9 = 1, and then 1 + 7 = 8. Or a child might see that 17 = 9 + 8. They then would easily recognize that they could just subtract that 9 and be left with the 8, so they would successfully solve 17 - 9 as equaling 8.

Exploring Multiples of 10 and Order of Magnitude

Towards the end of the 2 nd quarter focused on doing these sorts of activities with numbers to 20, we will begin to look at multiples of 10 up to 100. Now that the students can compose and decompose a 10 readily and are strong with facts to 20, we can then begin to explore expanded notation and order of magnitude. I think a logical activity to start with is to concretely explore the relationship between a single digit number and a multiple of 10 with the same leading digit.

We will start with 20, since we at this point we are focused on all computations dealing with numbers up to 20. I will talk about 20 in reference to the way we discussed 10. We will compose and decompose 20, using base 10 blocks as a way to see the actual bundling and breaking apart of the 2 groups of 10.

It is now that I will also introduce the idea of multiples of 10. We will talk about 20 as being ten times bigger than 2. We will see a quantity of 2 alongside a quantity of 20. I will ask, "Which group looks bigger? Do you know how much bigger this group is? It's ten times bigger than the group of 2!" We will then proceed to split the 20 pieces into 10 groups of 2. (This is an opportunity to also reinforce counting by 2s.) In this way, they will see that if you have ten groups of 2 items, you have 20 things. We will then talk about 3 and 30 in the same fashion, making the point that 30 is 10 times bigger than three. We will continue this with 40 and 50. I have chosen to stop at 50 at this point for three reasons. First, looking at five multiples of ten should be enough to establish a pattern. Second, I don't want to explore the concept of 100 equaling ten 10s until I have laid a stronger foundation. Third, we are going to make centimeter rulers that measure up to 50 centimeters, and this activity will be a perfect compliment to the idea that 10 is 1 ten, 20 is 2 tens, 30 is 3 tens, 40 is 4 tens, and 50 is 5 tens.

Appendix C: Implementing District Standards

The Virginia Mathematics Standards of Learning for second grade that support this unit are as follows:

1. 2.12 The student will estimate and then use a ruler to make linear measurements to the nearest centimeter and inch, including measuring the distance around a polygon in order to determine perimeter. [14]
2. The unit offers activities specifically designed to teach this standard. You could easily extend upon the activities included in this unit to teach 2.13-2.17.

The NCTM standards that support this unit are as follows:

In prekindergarten through grade 2 all students should -

Understand measurable attributes of objects and the units, systems, and processes of measurement and should -

- Recognize the attributes of length, volume, weight, area, and time;
- Compare and order objects according to these attributes;

Apply appropriate techniques, tools, and formulas to determine measurements and should

- Use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meter stick
- Develop common referents for measures to make comparisons and estimates [15]

This unit focuses on length, but could easily be expanded to include area and volume. One of the activities included involves the students lining up objects to compare objects based on length. It also involves an activity designed to use base-10 rods to measure things longer than 10 centimeters.

Resources

Annotated Teacher Bibliography

Burns, Marilyn, *About Teaching Elementary Mathematics.* Sausalito: Math Solutions Publications, 2007. A wonderful resource for methodology and pedagogy. A wealth of lesson ideas!

Geary, D.C., *Children's Mathematical Development: Research and Practical Applications.* Washington, D.C.: American Psychological Association, 1994. A research-based psychological perspective of early childhood mathematics teaching and learning.

Ginsburg, Alan, Steven Leinward, Terry Anstrom, and Elizabeth Pollock. *What the United States Can Learn from Singapore's World Class Mathematics System*. Washington, D.C.: American Institutes for Research, 2005. A research based comparison of the mathematics pedagogy in Singapore and the United States. It looks at grades 1, 3 5 and 6 in both countries and compares specific mathematics curricula from the U.S. with that of Singapore.

Howe, Roger, *Taking Place Value Seriously*, Preparing Mathematicians to Educate Teachers, http://www.maa.org/pmet/resources.html. An extremely helpful guide for a deeper understanding of elementary level operations and their implications in higher level mathematics.

Ma, Liping, *Knowing and Teaching Elementary Mathematics.* Mahwah, N.J.: Lawrence Erlbaum Associates, 1999. Provides a wonderful analysis of mathematics instruction and conceptualization in the United States as compared to China. This comparison is thorough in it's analysis of how teachers craft their explanations. A great place to start!

Singapore primary math texts, U.S. Edition Curriculum Planning and Development Division, Ministry of Education, Singapore: Federal Publications. These texts are textbooks and workbooks for students based on the texts used in Singapore. They align nicely with the framework of this unit and are a great resource for visual aids.

Wood, Terry, "Second-Grade Classroom: Psychological Perspective" and "Creating an Environment for Learning Mathematics: Social Interaction Perspective." *Journal for Research in Mathematics Education, Monograph Number 6*, Reston, VA: National Council of Teachers of Mathematics, 1993: 7-20. This monograph is full of psychological research on mathematics pedagogy and largely focuses on classroom discourse.

Annotated Children's Bibliography

Clements, Andrew. *A Million Dots.* New York: Simon & Schuster, 2006. This book is for older grade levels because it deals with much larger numbers and tries to convey the magnitude of a million through pictures. It is fun to read to second graders; I have found they love being presented with a more concrete representation of "million." They also seem excited about the largeness of the numbers.

Jenkins, Steve, *Actual Size.* Boston: Houghton Mifflin Company, 2004. This is a wonderful book about the size of animals from around the world. Sizes are given in inches and feet, so it would be a good book to add as an extension of linear measurement with customary units. It is also a great motivator because children often find animals interesting. You could read the book and then ask them to estimate the lengths of other animals not mentioned in the text.

Jenkins, Steve. *Hottest, Coldest, Highest, Deepest.* Boston: Houghton Mifflin, 2004. This book compares different geographical features around the world in terms of size and other measurements. It would be a great extension on linear measurement and relative size. It tries to make large measurements more understandable by comparing them to the average height of a person. This would be a great way to tie geography into the topics covered in the unit.

Goldstone, Bruce, *Great Estimations.* New York: Scholastic Inc., 2006. This is a great resource for photos of collections of objects. It has pictures of groups of 10, 100 and 1,000. The first nine pages easily relate to the concepts covered in this unit. The photos may give students a broader reference point to answer questions like, "Is it about 10 items or about 100 items? Does this look like 100 or 1,000?"

Notes

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1. R. Howe, *Taking Place Value Seriously,* 3.

2. R. Howe, *Taking Place Value Seriously,* 5.

3. R. Howe, *Taking Place Value Seriously,* 3.

4. R. Howe, *Taking Place Value Seriously,* 3.

5. L. Ma, *Knowing and Teaching Elementary Mathematics,* 1-27.

6. D.C. Geary, *Children's Mathematical Development,* 67.

7. http://www.dictionary.com

8. http://www.dictionary.com

9. R. Howe, *Taking Place Value Seriously,* 4.

10. R. Howe, *Taking Place Value Seriously,* 27-30.

11. M. Burns, *About Teaching Mathematics,* 75.

12. Singapore Primary Math Texts, Curriculum Planning & Development Division, Ministry of Education, Singapore, vol. 1A*,* 5.

13. D.C. Geary, *Children's Mathematical Development,* 44-46.

14. Commonwealth of Virginia, Board of Education, *Standards of Learning*, http://www.doe.virginia.gov/VDOE/Superintendent/Sols/math2.pdf

15. National Council of Teachers of Mathematics. (2000) *Principlesand standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.