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The Amazing Zero; Starring in: "Place Value, Estimation and Order of Magnitude"

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by Vivienne Bartman

Overview

Ask a child what 'zero' is and they will either say "a number" or "nothing". But there is a lot more to zero than being "a number" and "nothing." This unit, " *The Amazing Zero Starring in Place Value, Estimation and Order of Magnitude* " will show the children how important the number zero is in mathematics. It will especially focus on the use of zero in our base ten place value system for writing numbers. It will also delve into how numbers with lots of zeros, round numbers, help us with estimation by concentrating our attention on the relative sizes of numbers, rather than being mesmerized by their digits. The unit will begin with the importance of zero in place value and then finish with the importance of zero in order of magnitude in real life situations such as adding and subtracting. The main objective for this unit is to build a strong foundation of the concept of Order of Magnitude. Within order of magnitude falls the concepts of place value, place holder, with zero being the main character in the scenario.

In the Pittsburgh Public Schools, we believe that all students are capable of making sense of mathematics. Classroom environments are respectful of all ideas; students are invited to take risks as problems are posed and students wrestle with solutions. The focus is on students figuring things out, testing ideas and making conjectures, developing reasons and offering solutions. Students share and discuss in groups, in pairs and individually. Reasoning and hard work is celebrated, as students defend their methods and justify solutions. In Pittsburgh Public schools students build understanding and develop skills from their own experience. Mathematics is more meaningful when it is rooted in real life contexts and situations. Children begin school with more mathematical knowledge and intuition than previously believed. Our program builds on this, and gradually moves children from a concrete to an abstract and symbolic understanding. Mastery of mathematics concepts and skills comes with repeated exposure and practice, not after one lesson. This enables students to make new connections and build on the mathematical content they already know while gradually learning more difficult and challenging content. All students in the Pittsburgh Public Schools have the capability to be empowered mathematically, or become mathematically literate. In providing opportunities for improvement of quality of life, it widens choices for personal and professional life pathways. Our goal is to provide a high quality curriculum that empowers students to cope with the mathematical demands they will face in the future. [1]

My school is located in one of the highest crime and poverty areas in Pittsburgh. There is a lot of instability in the neighborhood and most of the families are non-traditional families. Many of the children come to school for stability, hot meals, and social interactions. My school is a kindergarten through eighth grade center where the average student body is 250 students with 99.2% African American. Discipline within the building is very strict and requires that each child wears a uniform. Class size ranges from fifteen to twenty-five children. The current elementary math curriculum is the Everyday Math Series. The Everyday Math curriculum is based on hands-on-activities. The lessons are taught through the spiral method in which the children learn a little bit about a topic each year. The children are learning a lot of important ideas but I have found as a middle school math teacher the children come to sixth grade with big holes in their prior knowledge of basic math skills. Their concepts get mixed up and they do not have a solid foundation in math facts. It is my job as a third grade teacher to give the children the solid foundation needed for a successful education. The children need to be given a safe and nurturing environment in which they are able to focus on their education. I need to give them the foundation that will be ever-lasting.

My third grade classroom is based on positive self-image and mutual respect for anyone that steps through the door. Rules, procedures, consequences and rewards are all part of my classroom management. Many of the children come into school with negativisms and issues that effect not only their education but all of their peers' education as well. This unit needs to be started the very first day of school so as to begin the classroom atmosphere immediately. Built into the unit is a positive self-image and support of classmates. To get the children coming to school and learning they have to be happy and feel protected from outside influences. I need to have the dual responsibility of planning situations and activities to facilitate their math development while being flexible enough to capitalize on opportunities as they arise incidentally during the day to capture the 'teachable moments'. One of my aims is to help my children become independent learners. This can only happen if I allow the children to make decisions and take responsibility for their learning. By involving my children in daily decisions about evaluating their work and encouraging them to set and review personal goals, I can help my class to become avid learners. Self-evaluation encourages them to think about what they are doing and what they need to do.

Students in the primary grades are capable of learning the skills of estimation, and they need instruction to do so. We will all agree that there is a great need for activities that will lead students into an awareness and understanding of estimation. It seems very apparent that children can only be successful estimators of computational solutions if they first have a sense of number to form the basis for problem solving and logical reasoning skills. Estimation plays such a pervasive role in out-of-school settings that children must have a wide variety of experiences with the skill. Over 80% of out-of-school problem solving situations involves estimation. I believe that a topic with such wide uses in later life should not be studied as spiral activities. If a child does not have a firm foundation with estimation at a very young age I feel that they will be playing catch up throughout their education. Spiraling will not have the immediate effect that is needed to build the desired foundation.

The Final Report of the National Mathematics Advisory Panel 2008 also state: "A focused, coherent progression of mathematics learning, with an emphasis on proficiency with key topics, should become the norm in elementary and middle school mathematics curricula. Any approach that continually revisits topics year after year without closure is to be avoided. By the end of Grade 6, children should have a robust sense of number. This sense of number must include an understanding of place value and the ability to compose and decompose whole numbers. A strong sense of number also includes the ability to estimate the results of computations and thereby to estimate order of magnitude, for example: How many people fit into a stadium or how many gallons of water are needed to fill a pool?"

It is my opinion that a complete understanding of estimation needs to be given full attention not just an "oh by the way" kind of lesson. Many of the curriculums I have seen uses estimation as a secondary way to solve the problem. For example: an open-ended problem is taught by having the children show their work, explain their work, and check their work using estimation. With estimation being so important in everyday life it should be taught to estimate the answer first then solve the problem using the usual steps. If the answer is similar to the estimation then the problem should be right. Finding the ball park figure first helps the child understand reasonableness of their answer. This strong conviction brings me to the conclusion that every lesson given here needs to be continued until it is mastered.

Objectives/Rationale

The hustle and bustle of our high tech world is leaving very little room for our children to estimate. Children are constantly being entertained and stimulated by both active and interactive shows and games. They come to school expecting the same type of programming. When it is not received, the children become bored, have unfinished assignments, and suffer academically. If this attitude starts in the lower grades it will be much worse in the upper grades. It is our job as teachers to find activities that will stimulate their academic needs and get them to become active learners.

Children of all academic levels need to be given time to be wrong, time to think, and time to struggle, with the thought that it is ok to be wrong the first time. If they are to experience mathematics in depth, they must have enough time to become engaged in real mathematical problems. The ideal situation for a teacher is to be able to step back and be come the facilitator. The teacher should learn by listening to what the children have to say. Reading, journal writing and class discussion should be a part of every class.

There is evidence from research that elementary school children do not understand our numeration system, especially place value.(Kamii and Joseph, 1988;Ross,1986, 1989; Smith 1973) According to the research, only 33% of third graders knew that the numeral 16 means 1 ten and 6 ones. But with carefully planned activities of counting and grouping by hundreds, tens and ones, a third grader can be expected to understand and work with three digit and larger numbers. [2]

It should be obvious that it is often easier to calculate using estimates rather than using exact figures. It is less obvious that then, even with calculators and computers taking the work out of computation, estimating may make things a lot easier without an important loss in the quality of the answers. In fact, answers derived using shrewd estimates may be more reasonable and more realistic than those that attempt to be exact. [3]

In order for a child to be a successful mathematician we as teachers must build a strong foundation in not only the basic skills of adding, subtracting, multiplication and division but also a strong foundation in the use of zero as a place holder in place value, the zero in rounding, and the zero in order of magnitude, thus, using estimation as a tool. Elementary teachers need to give the child all tools needed to be successful. Many children think that a math tool is a calculator, ruler, base 10 blocks, or compass. The children don't realize that tools can be algorithms, formulas or just a firm knowledge of the subject.

The lessons will be created so as to invite **all** students into mathematics- boys and girls, members of diverse cultural, ethnic, and language groups and students with different strengths and interests. The unit will help

the children with their growth in the enjoyment and appreciation of mathematics, develop flexibility and confidence in approaching problems, fluency in using mathematical skills and tools to solve problems and proficiency in evaluating their mathematical thinking. Because third grade is typically a self contained classroom reading and writing skills will also be an emphasis.

This unit will use several stories and songs to reinforce the concepts. Children need an opportunity to reflect upon and explain or justify their ideas and solutions, both orally and in writing. Children's literature provides a context through which mathematical concepts, patterns, problem solving and real-world contexts may be explored. The children's books, focusing on math, makes mathematics readily accessible to all children. Therefore, I will be using a variety of math related books throughout the unit. The class will have written or verbal reflections or responses. In order to present the solution and write mathematically they need to compile their information in an orderly way.

It is important that the children find their own ways of organizing and recording their work. They need to learn how to explain their thinking with both drawings and written words, and how to organize their results so someone else can understand them. For this reason it is suggested that a teacher does not always provide a student recording sheet, for example; if a tally sheet is provided a child will automatically figure out the problem using tallies rather than finding their own strategy. [4]

The idea of nothingness and emptiness has inspired and puzzled mathematicians, physicists, and even philosophers. What does empty space mean? If the space is empty, does it have any physical meaning or purpose? The use of zero in estimation can be a hard concept to understand. A young child needs constant repetition and a complete understanding of what it means to estimate. The process is more important than an exact answer. Using real-life situations that require estimations will give the children a chance to become comfortable with estimating. An estimate is created by using prior knowledge. Everyone gets better at estimation with practice. From the schoolroom to the supermarket and lots of places in between we must perfect estimation in order to survive.

Estimation starts and ends with zero.

The zero has many responsibilities in math. First and foremost zero will always be the digit before one when counting. It may not be stated when counting but everyone knows that before one of something there was nothing, or zero. The zero has the ability to turn a two, into twenty and then two hundred. Each time a zero is added to make a number bigger it multiplies that number by 10.

Place value enables a person to distinguish between the face value of a digit and its value because of its particular position in a numeral. In teaching place value vocabulary words such as *ones, tens, hundreds, thousands, ten-thousands, hundred-thousands* etc are required. However, the key point that students need in estimating, especially with front end estimation, is that each place is ten times bigger than the next smaller place.

The order of magnitude is a concept that is used in class but is not made explicit as such in the Elementary and Middle grades. The idea of order of magnitude is based on powers of ten. A power of ten is ten multiplied by itself a certain number of times. A power of ten is written as a 1 followed by some zeros, and the number of zeros tells the power. Thus, the powers of 10 are 1 = 100, 10 = 101, 100 = 102, 1000 = 103, and so forth. The order of magnitude of a number is based on the largest power of ten less than the number. For example: 1000 7, 452 10,000, so the order of magnitude of 7,452 is 3. To simplify this example: the order of magnitude is 3 because there are 3 digits that follow the 7 in 7,452. To make it clear: 100 256 1000, so the order of

magnitude of 256 is 2. Simplified: the order of magnitude is 2 because there are 2 digits following the leading 2 in 256.

Using the exponent a ten can be raised to whatever power is required to represent a size. The exponent can be a positive to represent a number larger than one or a negative number to represent a number less than one. Now you might be thinking "Well what does this have to do with the zero?" That is easy! The exponent represents the number of zeros that must be after the 1 in the ten therefore 10^5 is 1 with 5 zeros or 100,000. This concept leads into scientific notation. For example: My savings account has $1.20 x 10^5 (one dollar and twenty cents times ten to the fifth power). Ten to the fifth power would be a one and five zeros or 100,000. multiplied by $1.20 and I have $120,000 in my savings. (I wish!). If the order of magnitude of my savings account were six, I would have over a million dollars. The order of magnitude can have an infinite amount of zeros with each one representing a multiple of 10. Wow! That zero has a lot of power.

Reys (1986) describes five strategies for estimation: front-end, rounding, clustering, compatible numbers and special numbers. This unit will utilize the front end and leading digit strategies but first we will begin with the zero and place value. The leading digit rounding lesson will coincide with the development of the order of magnitude concept. Base ten blocks will be used to discover how easily one can add and subtract using the expanded form. Base 10 blocks consist of unit cubes (1/2" x 1/2" x 1/2"), ten rods (1/2" x 1/2" x 5"), hundred flats (5" x 5" x 1/2") and thousand cubes (5" x 5" x 5"). These blocks can be used to represent the expanded form to give a visual representation of a number.

The **Rule for front-end estimation is:** For any number in the calculations: between 1 and 10, round off to the nearest whole number; between 10 and 100, round off to the nearest multiple of 10; for example: choose from 10, 20,30..... 100 , between 100 and 1000, round off to the nearest multiple of 100, 200, 300.....etc. An example of this would be as follows: To add 347 + 129 we first add the hundreds (300 + 100) then the tens (40 + 20) then the ones (7 + 9) to obtain 476. This method is using the expanded form of each number and adding the hundreds, tens, and ones separately. Vertically this problem would look like this:

347

+ 129

300 + 100 = 400

40 + 20 = 60

7 + 9 = 16

476

Mathematicians at Rutgers state that the *precise* definition of Leading digit estimation is "A reasonable approximation, then, of a multi-digit sum or difference can always be made by considering only the leftmost places and ignoring the others. This strategy is referred to as front end estimation and is the main estimation strategy that many adults use. In third and fourth grades, it should accompany the traditional rounding strategies." [5]

The leading digit captures most of the number, the next digit captures most of what is left, and after that, the digits are less important, as far as size is concerned. Within the lessons we will discuss the fact that when you add very large numbers the relative size of the number really does not change much when you look at each

digit. For example: 1,354,596 + 2,485,439 relatively could be rounded to the nearest million and still be in a ball park figure of estimation. Here we would add 1,000,000 + 2,000,000 » 3,000,000. The importance of zero is very prominent here and should be dwelled upon.

Background Information

Zero is often equated with "nothing," but that is not a good analogy. Zero can be the absence of a quality, but it can also be a starting point. On the number line zero is exactly between the negative and positive numbers. The zero is the starting point or origin on a graph with an x-axis and a y-axis. The word origin sounds very important. The origin is the beginning. Well, so is the zero. It was so important mathematicians actually feared it and refused to use it. It came in and out of history and it is really hard to pin-point the exact moment the zero was created.

Counting is a universal human activity., However, most people do not begin their counting with the zero. Counting first began when people would use their fingers to keep track of "how many", even a small child can hold up fingers and say "I am this many". What would happen when you turn 11? No more fingers, perhaps you could include the toes but we don't have an unlimited number amount of toes either. That is why many groups of people invented their own "Tally System", which is still in common use today. In this system, you make a small vertical line for one unit, then another next to it for the second, and so on until you reach the fifth- which crosses the other four. You then start a over on the next set of five. This turns out to be a great system for keeping track of a small number of objects over time when you may need to add another at any time. Of course, as any of you that have used the tally system in grade school will know, it quickly becomes unwieldy to repeatedly count several thousand tallies. Many centuries passed before anyone even considered including zero as a number.

To appreciate the significance of zero in counting, look at the decimal and Roman number system. In the decimal system, all numbers are composed of ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. After counting to nine, the digits are repeated in different sequences so that any number can be written with just ten digits. Also, the position of the number indicates the value of the number. For example, in 407, 4 stands for four hundreds, 0 stands for no tens, and 7 stands for seven. The Roman numeral system has the I and V that are used in different amounts and orders to represent 1-8 but at nine you don't start over you add a new letter X. And how do the Romans represent the zero?

Zero is a unique number, belonging to neither the positive nor negative number set. It was the very last number symbol to be developed. Many ancient cultures, such as the Sumerians, Indians, Chinese, Egyptians, Romans, and Greeks, developed different symbols and rules for counting. But the concept of zero did not appear in number systems for a long time; and even then, the Roman number system had no symbol for zero. Sometime between the sixth and third centuries B.C.E., zero made its appearance in the Sumerian number system, which was sexagesimal (base 60), as a slanted double wedge.

Among the ancient Babylonian civilizations of 600 b.c. and earlier, a space was sometimes used to indicate an "empty place," however often as not, spaces were not used. Therefore, using Akkadian symbols and the Babylonia base of sixty, the numbers 1, 60, 3600, and 216,000 were all written with the same symbol. Despite its potential for confusion, the method appears to have been acceptable for several centuries. Around 300 b.c.

the Babylonians invented a special sign to serve as a placeholder where a numeral was missing. However, this symbol was apparently used only for intermediate empty positions, so the Babylonians never achieved an absolute positional number system.

The first zero symbol was believed to have originated in the fourth century b.c. by an unknown Indian mathematician. Wanting to record a more permanent answer from his beaded counting board, he used a simple dot, called a sunya, to indicate columns in which there were no beads. The beads were representations of numbers and the dots were zero the place holder. In a.d. 150 the astronomer Ptolemy began using the omicron (the first letter of the Greek word meaning "nothing") in the manner of our zero as a place holder. There is no evidence, however, that he regarded this symbol as a number that could be used in computations. The omicron is the 15th letter of the Greek alphabet. In the Greek system of numeration, it has a value of 70. It is rarely used in mathematics because it is indistinguishable from the letter "o" and easily confused with the zero .

The genius of our numeration system lies in the combined characteristics of place value and a placeholder numeral, that is, a symbol for the number zero. Although zero is first introduced as a number, children can be asked to do numeration activities that have them specifically think of zero as a placeholder such as asking a child "I have 8 hundreds and 3 tens. What's my number?" This step will begin to help a child to answer the question: Why is a zero needed in the number?

Strategies

Because zero is so important, we as teachers need to get the children to think about the zero all the time and reinforce that without the zero we could not count, tell time, measure, or use money. Much of the daily routine will incorporate the zero which needs to begin with the first day of school and continue throughout the school year. If a child learns it once they will forget but if we as teachers build that neural pathway with constant reinforcement and representation they will have a solid foundation.

This unit needs to be done beginning with day one of school. In order for the children to understand the power of zero the classroom must be constantly reinforced with the topic. A third grade child will need to become familiar with the fact that zero does not mean "nothing", that it has great value in everyday situations. Depending on the prior knowledge of the class the entire unit of lessons could last from a week to several weeks and then reinforcement throughout the year. It could be to your advantage as a teacher to do it every Friday as part of the classroom weekly procedure. The unit comprises seven separate lessons, with some of the lessons having several activities.

The unit will begin with an introduction to the zero in a unique manner so that the children will be intrigued and excited to learn. Some of the other lessons will include a reintroduction to the order of magnitude using money, an odometer to represent the order of magnitude, watching short video clips about the value of zero, creating zero poems, reading literature about estimating and zero, learning about place value with a game called "Flip it", creating order of magnitude with base ten blocks, using order of magnitude through leading digit estimating and front end addition. Having zero as the place holder will be discussed. One of zero's many useful functions is as a 'place-holder'. For example, in the number 600, the zero immediately to the right of 6 informs us that the 'tens' column is empty; the zero to the right of that tells us that the 'units' column is also

empty. The only column that has any values in it is the 'hundreds' column. But if we did not somehow indicate that the two right-hand columns were empty we would write '600' as '6', which is not the number that we mean. We will finish the unit with a class-created book about Zero.

The math literature books are very important to the development of the unit and for many of the lessons they are the starting point. The first book in use is *The History of Counting*. This is a well-researched, intriguing account of how counting has evolved. Schmandt-Besserat recounts how the Paiela, who cultivate orchards in the highlands of Papua New Guinea, "count by pointing to parts of their body. . . . The number 1 is called 'left little finger,' 11 is 'left neck,' 16 is 'right ear,' etc. The Veddas of Sri Lanka count without numbers, collecting pebbles to indicate how many. She also painstakingly charts the long evolution of counting through abstract counting, the use of ten digits, and the advantages of Arabic numerals over the older systems. Her rigorous, scientific approach to the subject ensures that readers will never take the counting system for granted again. The full-color illustrations make history beautifully visual, while a glossary sets forth concrete definitions for readers to peruse. The next book is *Zero. Is it Something? Is it Nothing?* This book shows clearly that zero can be both "something" and "nothing," depending on its position in a number. Specific examples show some ways in which zero figures in arithmetic computations and its place in all kinds of measurements. Occasionally there are not enough examples of each operation for readers to reason out the rule behind the example. Directions are given for constructing a paper odometer. Historical and anecdotal facts add further to the book's interest. Cheerful watercolors of children of different ethnic groups and animal characters illustrate the text and make the material more appealing to young readers. The final book we will be discussing is *Estimation.* The reader of this book will learn to make better estimates. He/she may learn to be on time. He/she may learn to stay within his/her allowance. Estimation strategies will be discussed showing the children how easy it is to make a good estimate rather than guessing at the amount or size of an object.

The mathematical goal of this unit is an understanding of leading digit estimation. You might think, why pick that method? Well, that's simple. In my state the third graders are to take the state standardize assessment test (PSSA) for the very first time. In my research I found out that the third grader can not use the calculator at all. If I can give them a strategy to be able to eliminate several of the multiple choice answers I have served them well, and done my job. The few extra points that this gives a child could be the difference between an advanced student and proficient.

Classroom Activities

Year round activities

During the morning message time I have my children write in their journals. As part of this activity we will include counting the days of school with straws. Each day of school one straw will be placed into a cup. To begin the development of Order of Magnitude on day 10 we will bundle the 10 straws together and place them into the tens cup. The class should continue to bundle the sets of 10 until we reach 100 and naturally bundle all 100 together.

To assist the children in understanding that zero is really important they will need to associate what they do in class with that of the importance of zero. In a non-mathematical way my class will understand that whenever they have done something extraordinary they can be proud of themselves. They will be able to show the

entire class that they are important. We will have a Super Zero the Hero mark on the floor which will be a huge zero stuck on the floor. Anyone that achieves a victory in academics during the school year deserves a victory dance to represent a hero's job well done. Just like the football player who has just run for a touchdown it deserves a celebration. This activity however must be done so that the teacher decides who deserves a victory dance. The connection will be that with out the zero math would be very hard and without success learning would be very hard.

Beginning every math class with the chant Teacher: "Are we nothing?" Students: "No! We're Zero." Teacher: "Are we something?" Students: "Yes! We're Zero." Teacher: "Are we small?" Students: "No! We're Zero?" Teacher: "Are we important?" Students: "Yes! We're Zero." This is just a fun interaction that will reinforce the zero and get the children excited about math .

The final year long activity will be to have a weekly visit to the computer lab to practice what we have learned. Please refer to the recommended interactive websites in the appendix to choose the games and activities available for all teachers to use.

Lesson 1

Narrative: "Today's lesson is your very first math lesson of third grade. How many times have you had a third grade math lesson before now?" The answer should be "zero". " I thought so!" (hinting to the lesson) "We are going to play a game called 'Guess my Number' I am thinking of a number no bigger than 10. I would like everyone to pick a number and hold that many fingers high up in the air." When the children have done so announce how many children have the right answer.(zero) The children then need to take a few minutes to look around the room to see which numbers haves been represented (if three children have zero and I said three they look to see which number has three.) If no one guesses zero, you can say: The right number of people have guessed the right number. Without telling the children who were correct have them change their number to the number they now think it is. Count how many children have it correct again have them use mental math to make a better guess on the correct number. Eventually all the students should have zero fingers in the air. Zero can either be represented with no hands in the air or one fist in the air. The ideal scenario would be to have the child who never answers questions be the one that gets the number right first, because he will probably not hold any fingers up. Have a discussion of their prior knowledge of zero include the following leading questions. "Is zero important?" "Was there always a zero?" "Why do you think we don't count using zero?" "When do we use zero?" Following the discussion watch " Zero my Hero video from Schoolhouse Rock. Following the video ask if the children have anymore thoughts they would like to add about zero. Read the book *History of Counting* in part or in whole depending on prior knowledge.

Hands on activity: Have one child go to the board and write the number 1, and hand him/her a penny " I gave you one penny how do you feel about that? "What can you do with one penny?" Ask another child to write a zero to the right of the one. "What number do we have now? 10, right! I will give you 10 pennies. Would you like that many pennies? What can you do with that many pennies?. Continue with 100. Do the same process. and yes Give the child a bag of 100 pennies or 2 rolls of pennies. I would stop at this point unless you are in a district that pays you what you are worth ! To avoid jealousy among the other students plastic or paper pennies could be used but may not have the effect that I am looking for.

Problem of the day: With your partner describe what will happen next. You may draw a picture, make a chart, or show it in numbers, but be sure to describe it in words. Give them time to work together leaving enough time for a discussion about 1000.

Lesson 2

Review the penny activity from the previous lesson. Explain the hallway activity: Using dots and the dot paper arrays we will represent up to 1 million dots. This activity will increase by multiplying the previous amount by 10 or adding a zero to the previous number. As a group start with the first several numbers that will be represented and post them on the wall in the classroom. (1, 10, 100). As the discussion continues remind the students that each time another zero is added it increases the order of magnitude. With each number display the array of dots. With 1000 ask if anyone could guess what arrays we might use to represent 1000. Remember you could use 10 of the 100 arrays to equal 1000 and that is what point we want to find. ? If you paste these 10 arrays on a paper and copy it, it will then be easy to represent 10,000 and so on.

Hands-on-activity: Once everyone is clear about how to create the arrays write down the next several numbers that will follow in the pattern. Have each group pick a number to create and supply enough dot arrays for them to produce their number. This activity might seem like an involved tedious problem. If you don't make it all the way to 1 million that is the beauty of it. The lesson is designed for the children to discover how large a million really is. For ease of production the book *Building on Numbers you know* has display sheets of 5,000 dots and 6,300 dots to utilize any way you would like. You can find these pages on page 197 and 198 of this book above text. The display can then be placed in the hallway as a class project. As a closing to the lesson read the book One Million Dots.

Lesson 3

To begin the lesson read the student book *Zero. Is It Something? Is It Nothing?* The book is an introduction to the use of the zero. There are activities, poems and riddles all about zero. It shows rounding, zero as the place holder and counting. It is a fun book to get involved with.

Hands-on-activity: Following the reading have each child make an odometer. The directions begin on page 15. Be sure to include the zero. When the odometers are finished give the students several numbers to create using the odometer. They need to write the number on a piece of paper, write how many thousands, hundreds, tens and ones. Continue until the class have mastered the activity. The next step is to use a deck of number cards and flip over 4 numbers have the children make the largest and smallest using only each digit on the cards once. They need to write the numbers on the paper and use the odometer if they need to. The final activity for the day would be to have them in pairs and play the "*greater than less than*" *g*ame. Each child will create number using a different number in each place value (stops them from using 9999 and 0000 all the time) Once they have created their four digit number they do not let the other person see the number. They will take turns (without looking at the opponents number)deciding whether their number is greater than or less than their opponent. If they are correct they get one point. The game then goes so the other person has chance to guess. The children need to discover for themselves that if the leading digit (the thousand) is really the only digit they need to look at if they have different first digits. If the thousand digits are the same then they need to look at the hundred digit. Strategy: A number closer to 9999 will win if they say greater than and a number closer to 0000 will usually be less than the opponents.(Let them discover this themselves). They must keep score and write down the strategies they may have come up with. They should play this game until they understand the systematic strategy: put the largest digit in the thousands place, the next largest in the hundreds place, the third largest in the tens place, and the smallest in the ones place.

Lesson 4

Where the zero falls! For the opening of class watch the video clip "Zero the Hero" discuss what the video was

about then begin the activity.

Hands-on-activity: Make number cards with oak tag or construction paper give each child a zero and keep the rest of the number a side. Also have a card with a decimal point on it. The teacher should pick a number card and stand to the left of the front of the room. Hold the card up and say: "I am a 2 (or what number you picked) can anyone make me bigger?" Pick someone to come up and stand where they think they should stand and say what number it has become. Also ask them, how many times bigger than the previous number is it? Keep adding zeros until they no longer can say the number. Tell them the name of this larger number. Say they could keep on doing this forever, and that each time they add a zero on the right, the number gets ten times bigger. The children need to understand that you don't need to many zeros to have a really big number.

Activity 2: Have 2 children hold a two non-zero two digit number. Ask if someone could show a way to make the two digit number bigger using a zero, and then another way (12 could become 102 or 120). Have them explain what each number means, and the difference between them, and which one is larger. Then ask if there was a way to make the 12 stay the same but still use the zero (012). Probably this will need discussion, and in particular, what it means, or if it means anything, will have to be talked about. They are not used to having a zero on the left. Have the children then put the three numbers in order from smallest to largest. This activity needs to be very thorough and complete. It is a huge concept to understand. The key objective to this lesson is to make sure that the children really understand that the zero as the place holder can make the number a lot bigger.

Activity 3: Extension activity for advanced students: Bring out the decimal point and stand next to the zero in 012. Ask the children to think about what has just happened. Talk about it with their partner. Have a brief discussion then they should write, draw and explain what happened with the decimal point and the zero. Example of an Answer: When the zero is on the left, and to the left of the decimal point, it does not change the value of any other digit. Similarly, if you add a zero on the right to the right of the decimal point, it does not change the value of any other digit. However, when you insert a zero between non-zero digits, some digits are moved to different places, and so they get different values.

Lesson 5

Remind the students of the lessons with the pennies and the dots. Discuss with them what they remember. The ending to the discussion should be a connection with order of Magnitude. Bring out a set of base 10 blocks. Ask if a student could show one unit. After doing so ask to show 10 using unit blocks followed by representing it using a rod. Continue in the same manner with rods, flats and the 1000 cube. Demonstrate how the base ten blocks can be used to represent numbers for example 4,381 would have 4 cubes, 3 flats, 8 rods and 1 unit. Practice with several different numbers until all students have mastered the concept. Discuss the idea that the number is a sum, of 4 thousands, 3 hundreds, 8 tens and 1 one.

Activity 1: Give each group a set of base ten blocks and the directions to show: an addition problem using the base ten blocks and two single digit numbers, an addition problem with two double digit numbers, two triple digit numbers and one with carrying. Have each group represent their data in pictures and numbers in expanded form (4000 + 300 + 80 + 1)

Lesson 6

Preparation: Before class be sure to have large number cards (oak tag or construction paper). One set of the cards should have the digits 1-9 and a zero on the back. Another set of cards should have the digits 1-8 and

one digit higher on the back (1 should have 2 on the back, 2 should have 3 on the back etc.) The second set is for the leading digit of any number created. The first set is for the following digits in any number. You will also need to make a plus, minus and equal sign.

Activity 1: Write the problem 3,265 + 2,613 on the board or any such problem. Hand one child the 3 card with the 4 on the back and give other student the corresponding card with a zero on the back have a plus sign on a card for a child to hold. Repeat the same process with the second number. Ask the rest of the students if 3,265 is closer to 3000 or 4000 they should say 3000 have the children flip the digits 265 over to reveal the zeros. Next ask the children to round 2613to the closest thousand (leading digit) Is it closer to 2000 or 3000? It is 3000 ask the children if they know what we should do next. They should answer "Flip the 2 over to 3 and flip the 613 to zeros" Now ask the children if they need a calculator to add the two numbers. Their response should be "no" because it is easy to add 2 + 3. See if any one can put the right answer after the = sign. Of course they must use more people. Repeat this process until the children are confident they understand rounding to the leading digit. Give them several problems to work on independently. As always they need to write out the problems in pictures, numbers or words.

Activity 2: Even the day after begin with a review but ask how different would it be if we subtracted. Discuss their ideas and demonstrate any suggestions. Have the class practice what they have learned. Keep several examples for the book and hang some on the wall. Read the book *Estimation* to the class and have a discussion.

Lesson 7

The Poetic Zero: read several poems written about zero then create short poems. Include doing the Number Stomp. Make a class book about "*The Amazing Zero Starring*

in: "Place Value, Estimation and Order of Magnitude" Can include pictures, poems, stories, math problems and the History of zero . A cover should be created and laminated and the pages could be bound with a binding machine.

Teacher Reading List

Cathcart, George and Pothier, Yvonne. 1997. *Learning Mathematics in Elementary and Middle Schools*. second ed. New Jersey: Prentice Hall. Original edition, 2000. This book is about learning mathematics - about children learning mathematics. It is also about teachers creating a learning environment that supports and encourages children to build understandings, make connections, reason, and solve problems.

Ifrah, George. 1985. *From One to Zero, A Universal History of Numbers*. New York: Viking Penguin Inc.This uniquely wide-ranging and original book is described by the author as "an archaeology of numbers". The book is made vivid and tangible by hundreds of illustrations.

Kaplan, Robert. 1999. *The Nothing That Is, A Natural History of Zero*. New York: Oxford. This story of zero reveals the truths about the naure not only of math but of human thought itself. The beauty of math is that even though we invent it, we seem tobe dicovering something that already exist.

Katzman, Pamela and McClees, Jackie. *A Reasonably Close Encounter*. USA: Dale Seymour.1983. This is a workbook with activities for estimation. This book would be ideal for upper level students or as an enrichment to the above lessons.

Russel, Susan and Tierney, Cornelia. *Building on the Numbers You Know*. White Plains, New York: Dale Seymour.1998. This Investigations curriculum embodies a new approach based on lyears of research about how children learn math. Eacfh grade level consists of a set of separate units. This is a grade 5 unit.

Russel, Susan and Russel, Cornelia. *Measurement Benchmarks*. White Plains, New York: Dale Seymour. 1998. his Investigations curriculum embodies a new approach based on lyears of research about how children learn math. Eacfh grade level consists of a set of separate units. This is also a grade 5 unit.

Seife, Charles. *Zero, The Biography of a Dangerous Idea*. New York: Penguin Group. 2000. This book follows the number frokm its birth as an Eastern philosopical concept to its struggle for acceptance in Europe and its apotheosis as the mystery of the black hole.

Threewit Fran. *Estimation Destination*. San Leandro, CA: Watten/Poe. 1992. This curriculum unit takes the elementary student on a estimating trip using a passport. The unit comes complete with lesson plans, worksheets and directions.

Weinstein, Lawerence and Adams, John H. *Guesstimation*. Princeton N.J. Princeton University Press. 2008. This book is filled with estimating problems similar to the Fermi problems. The difficulty level starts out easy and continues to increase with each problem.

Wood, Chip. *Yardsticks, Children in the Classroom Ages 4-14*. MA: Northeast Foundation For Children.1997. A resource for teachers and parents of children in the classroom ages 4-14. The book contains concise descriptions of the universal characteristics of children at different ages.. A practical resource on child development.

Student Reading list

Brimmer, Larry Dane. *Monkey Math*. CT: Children's Press. 2007. A simple counting book using basic estimation.

Clements, Andrew and Reed, Mike. *A Million Dots*. New York: Simon and Schuster Children's Publisher. 2002. A picture book show the child what a millon dots would look like.

Franco, Betsy. *Time to Estimate*. Mankato,Minnesota: Capstone Press. 2002.

Franco, Betsy. *Counting Our Way to the 100th Day*. New York; Margaret McElderry. 2004. Poems and rhymes about the 100 th day of school

Hoban, Tana. *Let's Count.* New York: William Morrow and Company. Inc. 1999. A counting book in which the origin of numbers is discussed.

Johnson, Stephen T. *City by Numbers.* New York: Penguin Books. 1998. A picture book showing how numbers are represented in everyday situations

Linn, Charles F. *Estimation*. New York: Thomas Y. Crowell Company. 1970. This book shares different strategies for estimating. It also shows how important it is to know how to estimate.

Murphy, Stuart J. *Betcha*. New York: HarperCollins Publisher. 1997. A story about winning a contest by estimating.

Murphy, Stuart J. *100 Days of Cool*. Harper Trophy. 2004. A story about the 100 days of school and children in the story being cool for all 100 days.

Schmandt-Besserat,Denise. *History of Counting.* New York: Harper Publisher 1999. A child's historical book about counting and numbers.

Schwartz, David. *On Beyond a Million*. Double Day. 1999. A story about numbers higher than a million and what they look like. Scientific Notation is shown as well as the powers of ten. The googol is discussed (100 zeros)

Zaslavsky, Claudia. *Zero. Is It Something? Is It Nothing?* New York: Franklin Watts. 1989. Riddles, poems, and activities about numbers.

Internet Resources

McPhee, Isaac M. http://math.suite101.com/article.cfm/the_number_zero *The Number Zero The History of Formulating Nothing*

Ladhani, Rafiq. "Zero." [Available Online] Available from http://www.bookrags.com/research/zero-mmat-04/.[cited July 8, 2008]

O'Connor J J and Robertson E F http://www-history.mcs.st-andrews.ac.uk/HistTopics/Zero.html *A history of Zero*

Stories, songs, and poems to be used in Estimation

http://www.rif.org/assets/Documents/readingplanet/ReadAloud_Stories/zero_the_hero.html : "Zero the Hero" animated story done in cartoon by Jane Lester

http://www.kinderhouse.net/zero_the_hero.htm : Another Zero the Hero story

http://www.songsforteaching.com/jennyfixmanedutunes/zerothehero.htm : song and lyrics for another Zero the Hero song.

http://www.drjean.org/html/monthly_act/act_2006/01_Jan/pg03d.html : The number stomp By Dr Jean

Interactive Estimation Games and activities on line

http://www.shodor.org/interactivate/activities/Estimator/?version=1.6.0-oem&browser=MSIE&vendor=Sun_Microsystems_Inc. This is a great activity to show estimation strategies with length, area and population density. It teaches the child to make educated estimations rather than guessing.

http://education.jlab.org/placevalue/ The goal of the Place Value Game is to create the largest possible number from the digits the computer gives you. Unfortunately, the computer will give you each digit one at a time and you won't know what the next number will be. You are not allowed to rearrange any of the digits you have already placed, so think carefully before you lock a number in place! Good luck!

http://arcytech.org/java/b10blocks/instructions.html This site gives the young child the first introduction to the use of the base 10

blocks.

http://nlvm.usu.edu/en/nav/frames_asid_154_g_1_t_1.html This site uses base ten blocks up to 1000 and develops addition problems with the blocks.

http://www.funbrain.com/guess2/index.html The FunBrain Magician will give you a math word problem to solve. If your answer is too high or too low, FunBrain will let you know. Try to solve the word problems in the fewest possible tries.

http://www.aaamath.com/plc31e-placevalue-w2n.html encourages the child to change numbers written in words to the number it represents. Keeps track of correct answers. This would be a good assessment for ending a unit on place value.

http://www.aaamath.com/B/g5_31_x1.htm This site is similar to the previous site but has a lot of hidden zeros in the writing of the numbers.

http://www.aplusmath.com/Flashcards/rounding.html This site makes the child actually follow written directions. Each flash card is asking them to round to a different place value. After 10 flashcards the child is given a percent of correct answers

http://www.oswego.org/ocsd-web/games/Estimate/estimate.html This is a number line estimating activity ideal for a whole class activity using slates/chalk for all children to be involved. Different levels of estimating can be used for all levels of skill.

http://pbskids.org/curiousgeorge/games/how_tall/how_tall.html An important part of school readiness is getting kids used to measuring and estimating using standard and non-standard units of measure. For example, kids learn how to use their footsteps to measure the length of a room. This interactive game asks kids to estimate the height of an object using a variety of non-standard units of measure—tires, donuts, and coins.

http://pbskids.org/curiousgeorge/games/count_your_chickens/count_your_chickens.html

The goal of this game is to help kids hone counting and estimating skills. Along the way, they are also introduced to the comparative language of "more" and "less" as they help George catch some unruly (but adorable) chicks. This game has three different levels, but the difficulty of tasks within each level is determined by your child's current ability.

Appendix

Third Grade Standards for Estimation

Category 2.2.3: Computatin and Estimation

2.2.3.A Apply addition and subtraction in everyday situations using concrete objects.

2.2.3.E Use estimation skills to arrive at conclusions.

2.2.3.F Determine the reasonableness of calculated answers.

Endnotes

1. This paragraph was taken from a PowerPoint written by Dr. Robin Ittigson Math Supervisor of the Pittsburgh Public Schools for a curriculum writing seminar

2. (Learning mathematics pg112)

3. (Zelman, Bell)

4. (TERC, Tierney)

5. http://dimacs.rutgers.edu/nj_math_coalition/framework/ch10/ch10_03-04.html