Curriculum Units by Fellows of the National Initiative
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Crunching Numbers for Lunch

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Overview

This integrated unit teaches explicit estimation skills through examining environmental issues at our school site. The overall goal for the unit is for students to make sense of statistical reporting and very large numbers through evaluating use and waste habits of the school community. By using estimation techniques and manipulating quantities, students will be able to evaluate quantities over 1000. Beyond recognizing large numbers and understanding their value, these techniques will also provide them with the skills necessary to multiply and add large numbers and evaluate their own multiplication and addition processes to see if their solutions are reasonable. Besides merely recognizing and computing large numbers, students will be able to use estimation and relative place value to compute quantities of length, area and volume using various units of measurements.

In fourth grade a great deal of mathematical instruction is focused on the processes of long multiplication and division. Research shows, and testing increasingly requires, that best practices in mathematics instruction include teaching students to write about their mathematical thinking, explain their process, and evaluate their solution. Performing algorithms will no longer suffice. Evaluation of the reasonableness of solutions is critical in teacher evaluation and student self reflection of their processes to determine what is taking place; Is there a computational error? Are students using an algorithm correctly? More importantly, do students understand the basic premise behind the algorithm they are using? One of the goals, therefore, in this exercise, is to teach students estimation techniques (focusing on order of magnitude and relative place value) to evaluate their own processes and errors to determine the reasonableness of their solutions when applying new computation strategies to increasingly larger numbers.

These same principles apply to the teaching of measurement, an essential skill that is often relegated to the back of the book, the end of the pacing guide, and consequently left out of yearly cycles due to a lack of time. The teaching of measurement can be embedded in many other topics if we ask the right questions: What is the context of the problem? What kind of solutions are we looking for? How will we measure and report our results?

In order to reach these goals, I will have my students examine the problem of the amount of waste that is generated in our school cafeteria. Most of my students, young as they are, share their parent's vocal concern for the environment and affection for current "green" trends governing use and misuse of resources in the community. The school is actively recycling and is hoping to receive a grant to place solar panels on the roof to offset coal generated electricity use. These are big issues, and I would like to study them with my students in a context that is tangible and prevalent in our daily life - lunch.

The companion piece to the mathematical component of the unit would be to examine the environmental impact of these quantities, a socially important lesson which meets multiple social studies and science standards. Students would learn about the waste cycle from usage to landfill as well as the various alternatives that are available in our community, such as recycling. A natural follow up to the unit would be to examine alternatives to the current materials being used and analyze those statistics in order to make an informed evaluation of the costs and benefits of potential changes, such as rewashing plastic trays, which consumes water (a scarce resource in our community), and biodegradable disposables, which are more costly to purchase and require community buy-in to manage. Eventually, I would hope to apply the model to examining other waste vs. conservation situations. Other extensions would be for students to develop their own examinations of their environment, and hopefully their own campaigns for environmentally and socially responsible choices in the school community.

Rationale

> "According to the Environmental Protection Agency, the average American produces about 4.4 pounds (2 kg) of garbage a day, or a total of 29 pounds (13 kg) per week and 1,600 pounds (726 kg) a year. This only takes into consideration the average household member and does not count industrial waste or commercial trash...with the garbage produced in America alone, you could form a line of filled-up garbage trucks and reach the moon. Or cover the state of Texas two and a half times. Or bury more than 990,000 football fields under six-foot high (1.8 meter high) piles of waste." [a1a]

Whereas in literature we often enjoy or contemplate the experience of a subject with which we have no relationship, when discussing mathematical terms or quantities we have to have some background knowledge in order to make *logical* sense of the message. These days a great deal of media reporting focuses on "green" issues, i.e., environmental health, global warming, the food chain, and pollution. Such global issues naturally bring with them global numbers, which may be easy to understand, but may not be so easy to *comprehend*. Compounding this problem is the nature of mass media which strives to make the greatest impact regardless of the legitimacy of the statement. Take the above paragraph as an example. First of all, 29 x 52 = 1508, which would round to 1500 pounds per year instead of the more impressive 1600. Likewise, 13 x 52 = 676 kg per year, not 726. And about that layer of trash burying Texas...how deep would it be? Two and a half millimeters or two and a half feet? There's quite a difference in those two quantities, yet we are given no information with which we can make a reasonable assumption. Ironically, in the one instance where a recognizable number (one million) could have been appropriately used, the author chose to use the very ambiguous 990,000.

Children in particular may not have the life experiences that enable them to conceive of these great quantities. In addition, the life experiences they have had may lead them to misconceptions based on their concrete interpretation of observations; i.e., the sun and the moon *appear* to be approximately the same size. How do children make sense of very great numbers? How can we help them evaluate their ideas and correct misconceptions? Students need a firm understanding of the magnitude of numbers, place value and estimation in order to make sense of these kinds of statements and to be able to critically evaluate numeric statements to see if they are reasonable. The literacy strategies so heavily emphasized in primary grade pedagogy can also be used as instructional tools to help them develop their capabilities in reading and understanding mathematical statements. Another strategy I frequently use is to establish a line of questioning which will guide students toward a conclusion that I have already determined to be the main idea of the lesson. The conversations included here are merely examples of how I would structure that questioning.

One event that is consistent in the daily lives of students that can provide a context for our studies is lunch. The cafeteria provides multiple "units" that are consistently uniform and therefore easy to evaluate; For the purpose of this unit, we will focus on polystyrene trays. Students will use estimation to arrive at a variety of quantities which they could then extrapolate through different units of measurement. For example, students could examine the number of trays discarded by each person; classroom; grade; the entire school, or even all of the schools in the district. Then they could analyze how those quantities increase over time; days, weeks, months, and the school year. Students would also predict, analyze and estimate a variety of comparative measurements: How far would that stretch? What area would it cover? How large a container would it fill? How tall would it be? How much would it weigh? Lesson 4 is an example of what this instructional sequence could look like in the classroom.

When all of the data has been collected, computed, and checked for reasonableness, students will evaluate their data and create two reports; one non-text, such as a chart or graph, and one text statement. To reconnect to the starting point of the unit, students will create a logical comparison in order to develop an appropriate voice for the text statement. Depending upon the intended use of the data and the impact the student authors would like to make, the voice could be one of relief, shock, pride, etc. In order to establish a context and reference for this voice, we will be working on a parallel unit which will teach students about the processes involved in waste management, the impact on the environment, and responsible practices.

I have found the 4MAT System, developed by Bernice McCarthy at About Learning, to be an appropriate framework to use in developing units in which all components are related to a central idea. In simple terms, the 4MAT concept states that "...teachers as well as students need to understand the reasons for doing what they do." [a2a] I am also intrigued by Liping Ma's "four properties of understanding - basic ideas, connectedness, multiple representations, and longitudinal coherence - [as] a powerful framework for grasping the mathematical content necessary to understand and instruct the thinking of schoolchildren." [a3a] Therefore, I have attempted to structure this unit in a way that emphasizes the connections between the various topics.

Ideally this unit would be started early in the fourth grade year to review and reinforce place value concepts taught in third grade and establish strong understandings of whole number place value in order to better grasp the decimal and fraction concepts which will be introduced later in fourth grade. When I taught fifth and sixth grade, I was surprised at the difficulty many students had with computation, particularly long division and any operations with fractions and decimals. I hope that a "side effect" of this unit will give students the foundation they need in manipulating and estimating numbers so that later curriculum will be supported.

Mathematical Background

Assessing Prior Knowledge

Before starting this unit, I would assess to see if students could identify and name place value positions and whole numbers both orally and in written form up through 9999. The purpose of this is twofold; first, in order to evaluate numbers up to the millions, students need a firm grasp of the ordering of the periods. If they understand the "ones, tens, and hundreds" places of the hundreds period, they will be able to apply this knowledge toward understanding the place value names in the subsequent periods. Secondly, students also need to be able to write these numbers correctly in order to work with them accurately. A common error in third and fourth grade numeracy is for students to translate numbers literally instead of using place value notation. For example:

Teacher says: "Six thousand one hundred fifty two"

Student A writes: 6000100502

Conversely, another misconception with this number could present itself as such:

Teacher writes: 6,152

Student B says: "Six one five two"

Student A obviously understands the value of the individual place value components of the number, but needs assistance in writing it correctly. Student B is only understanding the number as a grouping of individual digits without place value. Much of the intent of this unit lies in an attempt to "undo" these misconceptions and redirect those personal understandings towards better fluency with the conventional notations and algorithms that students will encounter on tests and in later grades.

The practice of composing and decomposing numbers, or breaking the *standard form* of a number into its *expanded form* is one of the fundamental principles of the base ten system. An error such as one made by Student A provides an excellent opportunity to introduce the expanded form of a base ten number which will form the basis for our work on estimation. Rewriting Student A's response as a sum of four place value amounts shows the relationship between the standard form of a number and its place value components:

6152 = 6000 + 100 + 50 + 2

Each addend is a digit times a unit or a denomination of a given order of magnitude.

6000 = 6 x 1000 100 = 1 x 100

50 = 5 x 10 2 = 2 x 1

These numbers that have only one non-zero digit are the building blocks of the decimal system. We will call them *very round numbers*. They will form the basis of our approach to estimation. [a4a]

I am assuming that most beginning fourth graders have been introduced to addition, subtraction, and multiplication with multi-digit numbers. However, student mastery of the conventional algorithms will certainly vary widely, and it should be expected that these will need to be reviewed or perhaps discarded in favor of other methods that are more readily understood by the individual student. Although explicit teaching methods of these fall outside the scope of this unit, the concepts addressed here are designed to assist students in strengthening their number concepts in order to develop and assess their own techniques. Again, understanding of expanded form gives students greater flexibility and individual control over their approach to computation.

Comparing Large Numbers

One of the overall goals of this unit is that students grasp the meaning and value of large numbers in order to evaluate their purpose in the context of text and the author's intent in using that particular number. I also hope that students will begin to use mathematical arguments and expressions to strengthen their own research and writing. Exposure to a wide variety of exact and estimated large numbers in context will provide multiple opportunities to evaluate their meaning and purpose. For example, we might question why the author of the EPA statement mentioned earlier decided to use 990,000. What does this number represent? Is there another number close to 990,000 that would be easier to understand? What about one million? What's the difference between 990,000 and one million?

One of the principles of estimation is that the relative error between the actual number and the estimated number is less than ten percent. At the fourth grade level, I would not teach this explicitly, but would nevertheless examine the idea. For example, the difference 10,000 in the above example may seem like a big number by itself, but it is only 1/100 of a million and not very significant when compared to a million. This could be demonstrated visually so that students could see that it would indeed be reasonable to use one million as a replacement for 990,000 in that context.

Place Value and Order of Magnitude

A complete understanding of the fundamentals of place value and order of magnitude is essential to alleviate misconceptions in numeracy, errors in computation and notation, and understanding the measurements represented by numerical expressions. We will make manipulatives with which students may work with order of magnitude concepts visually and kinesthetically. (See Appendix 1). These manipulatives help students solidify their understanding of the above principle; that is, that all numbers are the sum of very round numbers and will improve students' fluency through composing and decomposing numbers. The terms "standard notation" and "expanded notation" will be applied to these exercises as they consistently appear on standardized tests and in many textbooks. These exercises also reveal the gradual increase in the number of digits numbers contain as the order of magnitude increases. Discussion of the role of zero will segue into preparation for estimation activities using very round numbers. To address the ten percent principle mentioned earlier, we will sometimes use pretty round numbers (i.e., numbers with a whole number other than zero in the first two places such as 25,000).

Importance of the Leading Digit and Very Round Numbers

When a number is expressed in its expanded form, i.e.,

7,543 = 7000 + 400 + 50 + 3

the addends or very round numbers in the sum are called *very round components* or *single place components* of the number. The largest one is called the *leading single place component*. [5] In this case it is 7000.

Examination of the value of the leading single place component in both arbitrary and very round numbers will help students understand that the leading single place component gives one a great deal of information about the value of the number; in fact, it is always more than half of the number and often quite a bit more than half. This information is crucial in estimating the approximate value of large numbers and the relative size of other numbers in comparison.

Application of Concepts to Computation and Self-Evaluation

At this point it would be appropriate to demonstrate the application of these strategies towards computation. Mastery of multiplication strategies is an essential component of the fourth grade curriculum, and is necessary to compute area and volume. Although mastery of addition is required in earlier grades, its use with large numbers is applied in fourth grade, and continued practice will strengthen those skills for application to division. Focusing on very round components of a number will help me demonstrate that when adding, we always add the very round components of the same order of magnitude. When multiplying, we multiply the very round components of one factor with the very round components of the other and add the products. This process lends itself well to estimating sums and products.

The Four Square model of problem solving requires students to evaluate their solutions for reasonableness, a strategy that is certainly helpful in solving any problem. However, students without explicit instruction will simply say "Yes, it's reasonable because I checked it." if they are not taught specific techniques. For the purpose of this unit, I will not address specific algorithms, as this varies from teacher to teacher. My intent is for students to develop a greater conceptual understanding of operations rather than adherence to a specific procedure. These activities demonstrate the use of estimation strategies as they relate to student self-assessment of their addition and multiplication computation skills.

Measurement

Using multiple units of measurement based on the metric system as well as non-standard units will provide continued practice with different units of measurement, as well as reinforce the idea that *any* object that can be replicated can be used as a unit of measure. Using conventional metric measurements will reinforce the base ten structure of our place value system. Computing length may require adding numbers with different numbers of digits, giving students the opportunity to practice using order of magnitude and place value knowledge to add correctly. Students computing area problems will demonstrate understanding of multiplication strategies and use order of magnitude with leading digits to evaluate whether solutions are reasonable. Volume problems may be solved by using repeated addition or multiplication. The relationship between area, length and volume will be explored to introduce students to conservation concepts and proportional space. For example, 1000 centimeter cubes in a line will stretch for 10 meters, but if arranged in a plane array, they will fit inside a square with sides of less than 1/3 meter. If they are stacked in a cube formation, they will fit in a cube with sides that are only 10 cm long.

Application of Knowledge to Experimentation and Reporting Techniques

The final portion of the unit demonstrates application of all of these strategies as applied to the "real-world" issue of waste management in the school cafeteria. Students will design their own investigations, experiment, compute, evaluate, and report their findings using both comparative statements and visual aids.

Strategies

The following sequence designates the scope and sequence of the entire unit from initial presentation to final project inclusive of the overall concept, mathematical instructional sequence, and integration with literacy and environmental sciences.

Lesson 1: Connect

Students will recognize that trash and waste accumulates into great quantities and this has an impact on the environment. Previous literacy instruction has developed their ability to infer meaning from fiction text, and using literature in math will help them learn how to do the same with mathematical concepts in fiction and non-fiction text. We will start by reading The Lorax by Dr. Seuss to establish the learner's connection to the environmental consequences of human practices, followed by class discussion.

Lesson 2: Attend

Students will clarify the reason for learning by connecting the main idea of The Lorax to a non-fiction text article describing a real-world problem. We will read "Altered Oceans: Plague of Plastic Chokes the Seas" (*Los Angeles Times,* August 2, 2006) in which author Kenneth R. Weiss makes the case that ocean pollution has a devastating effect on wildlife and human health. The leading question will be, "How did the author of the article make the point that there is a LOT of trash and that this is a problem?" Students will pull out all of the numbers and quantity related phrases (such as "about one billion pounds") in the article. We will examine the characteristics of these words and phrases and through discussion conclude that most are not exact numerical statements, and many of them refer to very large numbers. Students will evaluate examples of headlines containing numbers, and will label them as either "exact" or "estimated".

Lesson 3: Imagine

So far students have learned about the problem of large quantities of trash collecting in the world through examining fiction and non-fiction text. At this point they will engage in an experience that will emphasize their personal role in the problem and start them thinking about what skills they need to examine it further. I will remove all trash cans, bags, and recycling bins from the classroom before students arrive and tell the students they will have to keep all of their used materials on their desk for the day so we can easily sweep it off the tables when we get some trash bags. I would be sure to have students eat lunch in the classroom that day as well. By the end of the day, I predict that students will be having a hard time finding space to work and will be complaining like mad. Discussion will focus on identifying the problem and hypothesizing the impact of this problem when expanded to the worldwide community. We will then watch the video "Plastic Debris, Rivers to Sea" developed by the Algalita Marine Research Foundation to bring attention to the problem that a great deal of "land-based" trash ends up in the ocean, contaminating the environment. At this point I will start the parallel environmental science unit for students to learn about what happens to trash once it leaves a building, and have them build a "mini landfill" to simulate and evaluate municipal waste management practices. [a6a]

Lesson 4: Inform/Practice

This section forms the bulk of the mathematics content instruction and practice for skills mastery. After watching the video, which also contains phrases and statistics about large quantities, students will evaluate what kind of skills they need in order to make sense of these numbers. Activities 4a - 4c teach how to use relative place value and order of magnitude to assess the value of a number, and the importance of the leading digit in comparing numbers and evaluating size. Activities 4d - 4e put these skills to work in computation practice. Finally, Activity 4f addresses different measuring techniques, emphasizing the importance of units and factors of ten in the metric system.

Activity 4a: Comparing Large Numbers

I will start out by reminding students about all of the large numbers we've encountered in our readings and videos and start a discussion with the question "How did you determine if a number was exact or estimated in your reading?" We will start a chart titled "What We Can Do to Understand Numbers With Large Magnitudes". We will review the names of the place value positions up through 1000. Students will naturally be curious about the term "magnitude", so it will be entered in their vocabulary section, defined as "greatness of size or amount" [a7a]

Appendix 1 describes the manipulatives students will use for this series of instruction. I have found these manipulatives to be very helpful in demonstrating order of magnitude and relative place value through repeated construction and deconstruction of numbers. Lesson Plan A demonstrates the sequence of learning used to establish these concepts and develop skills using these tools.

Activity 4b: Place Value and Order of Magnitude

Throughout this activity I would continually make charts of the numbers and concepts the students are manipulating to have a running record of their thinking process and references for future use. The goal for this sequence of learning, demonstrated in *LessonPlan B Part 1*, is for students to recognize the differences in value obtained by placing digits into various place value positions, the comparative differences between the orders of magnitude, and the systemic nature of the base ten system.

Activity 4c: Importance of the Leading Digit and Very Round Numbers

Lesson Plan B Part 2 guides students toward making connections between the relative place value of digits in a number and the information we get about a number by simply looking at the leading digit or two digits. Although this lesson could be separated from part one, there is a natural segue from one concept to the next and I would attempt to keep the flow moving from part one to part two uninterrupted. After the main lesson, students should be given many opportunities to practice composing, decomposing and approximating numbers using the number of digits to determine order of magnitude and leading digits to get the most information they can. There are a variety of games that may be used for this purpose:

Building Great and Not-So-Great Numbers: Students have place value charts to the 1000's. Teacher states the goal for the round, i.e., create the greatest number, least number, a number between x and y, etc. Roll 10-sided die. Students place digit in place they think will get them towards goal, which cannot be erased once placed. After four rolls, students compare numbers and see who met goals. After several rounds, discuss strategies, i.e., "When you're trying to create the greatest number, what strategies do you use? (Put larger digits in the leading digit places) Smallest number? Why?" Later in the year students can play this game by placing numbers into equations to try to come up with the greatest or least sum, product, difference, or quotient. [a8a] This version is quite challenging for fourth graders, but once they make the connection between the relative place value of the numbers and the function of each operation, they can become quite adept at playing.

Greatest/Least/What's Between Numbers: Use 1000's place value charts. Someone rolls a die, draws cards, etc. for four digits. Students model and write the greatest number that can be created with those digits and the smallest. Worksheets can be created with different questions that assess student understanding; for example:

What is a number between these two numbers?

Take away x - what number do you have now?

How many more tens would you need to change the value of the hundreds place?

Race to the Place: Body-kinesthetic activities are great for this age group and really help them understand abstract concepts through physical manipulation. For this activity, students are split into two teams, and every student has either a digit card 1 - 9 or a card with a set of zeroes: one zero, three zeroes, six zeroes, etc. The leader calls an equal number of names from each team, and those students race to the front of the room and line up to create the greatest number. The class determines which team has the greater and why; either the number of zeroes would be different, or if the zeroes were the same, they would have to compare the leading digits. Once students demonstrate working knowledge of these concepts, they write the "rules" for this game to teach another class. Ideally, they would determine the same sequence as below in their own words:

1. Look at number of digits. If different, greatest number of digits wins.
2. If the number of digits are the same, look at leading digit. Greater leading digit wins.
3. If number of digits are = and leading digits are =, look at successive digits after leading digit from left to right and compare digits in same place. First number that has a greater digit in the same place wins.

Although not explicitly stated, this game reinforces the rule that the leading very round component is larger than the entire rest of the components put together. For example, we might compare:

a) 1999 and b) 2111

Students would determine that 2111 is still greater than 1999 even though all the place value components (excluding the leading very round components) of (a) are the greatest they can be, and those in (b) are the least they can be. However, the leading very round component of (b) is greater than that of (a), which renders (b) the greater number regardless of the value of the remaining place value components.

Activity 4d: Application of Concepts to Computation and Self-Evaluation

At this point it would be appropriate to demonstrate the application of these strategies towards computation. *Lesson Plan C* demonstrates the use of estimation strategies as they relate to student self-assessment of their addition and multiplication computation.

Activity 4e: Measurement

At this point I will shift the direction of instruction to measurement in order to prepare students for the cumulative project. For the purpose of this unit, I will explain in detail the following activity on measuring length. Related additional instruction would deal with techniques used to find the area of a figure or space and determining volume of three-dimensional forms to the nearest very round or pretty round number. I would try to get students to understand that changing the length and width dimensions when figuring area does not result in an equal change in square area; i.e., doubling both dimensions does not double the area, but quadruples it. Focus points in teaching volume would demonstrate how volume increases exponentially with only slight changes in square area; i.e., doubling each dimension octuples the volume! Volume would be computed using multiplication as well as addition by adding "stacks" of area. These lessons would be demonstrated by modeling with manipulatives so the students can actually see these proportions.

To introduce the lessons on area, I'd have students gather the units together to show how a quantity of length compares to area. We would make predictions: "How much space on the floor could you cover up with these?" For volume lessons, we'd ask and predict "What size of box would you need to put these in?" and experiment to find solutions.

In its simplest form, this could be demonstrated easily by having students draw the two dimensional forms on centimeter graph paper, and a net of the cube which would then be folded into a box. Students could then fill the cube with centimeter blocks to see that the volume does octuple when the dimensions are doubled. For example:

Example a:

Length = 2 cm

Area with l = 2 cm and w = 2 cm

a = 2 cm x 2 cm

a = 4 sq cm

16 is *four* times greater than 4 because 4 x 4 = 16

Volume with l, w, h all = 2 cm

v = 2 cm x 2 cm x 2 cm

v = 4 cm x 2 cm

v = 8 cubic cm

Example b: (all dimensions are doubled)

Length = 4 cm

Area with l = 4 cm and w = 4 cm

a = 4 cm x 4 cm

a = 16 sq cm

16 is *four* times greater than 4 because 4 x 4 = 16

Volume with l, w, h all = 4 cm

v = 4 cm x 4 cm x 4 cm

v = 16 cm x 4 cm

v = 64 cubic cm

64 is *eight* times greater than 8 because 8 x 8 = 64

Activity 4g: Analyzing Units

Students will record the length of a variety of objects. The only available units to use for measuring will be cm unit cubes, tens rods, or meter sticks. We will all start by measuring the length of a book with unit cubes to the nearest whole number. Then I will ask them to measure it using tens rods. A problem exists in that the book length will most likely fall between two multiples of ten, so it will be a little bit shorter than one sum of tens, and a little bit longer than the next sum of tens. The phrase "rounding to the nearest (ten)" will be introduced. Since students are familiar with the idea of round units, they should make the connection that this process requires them to record the number as a *very round number*. Then I would ask them to measure the book with a meter stick to the nearest meter and ask, "Does this give us a good estimate? (No) Why not? (All of the books are less than one meter) Are they less than half a meter? (Yes) What does that mean? (It would round to zero) Does it make sense to say there are zero books? (No) What can we do about this?" Discussion will emphasize the importance of using appropriate units to measure, and that all units must be uniform; i.e., you cannot measure the length of a table using one meter, three tens rods and two unit cubes and obtain a sum of 6 units. Students will work independently to measure and record the classroom objects, determining the appropriate unit of measure and rounding to the nearest unit. Understanding the importance of uniform units is essential not only in measurement but also in understanding concepts of factors and multiples as the basis for working with fractions.

Lesson 5: Extend

After all of these techniques have been discussed and manipulated for some time and the parallel science unit completed, we will return to the problem of the trash. The students will be given the task of making a report on our school's trash management practices as explained in the introduction and rationale. First, we will create a list of questions that will need to be answered to develop a plan:

- What will we measure?
- What kinds of measurement will we be computing?
- What units should we use?
- Should we use *very round numbers, pretty round numbers*, or *exact numbers*?
- What computation strategies should we use?
- How do we check for accuracy?

We will then make what Liping Ma calls a "knowledge package" to guide our study. This is a web or "groups of knowledge" that students will use to solve the problem. [a9a]

Next we will explore various methods of mathematical reporting. A number of lessons could be inserted in here on the different kinds of charts and graphs used to present data. Students will then determine what kind of graphic would be best suited to reporting their data set.

To emphasize the role of voice and to give students opportunity to strengthen their voice in non-fiction writing, and strengthen the impact of their visual data, I will also give them the task of adding a comparative statement to one of their data groups. The non-fiction reading strategy called "Making Comparisons" demonstrates that authors help their readers make better sense of content, particularly quantities and size, by including pictures or text that compare the quantity to a non-numerical familiar object. For example, an elephant pictured standing on the ground next to a man shows the comparative size of the elephant.

To model how authors use words in making comparisons, I will read Helen Nolan's How Much, How Many, How Far, How Heavy, How Long, How Tall is 1000? and How Much is a Million? by David M. Schwartz. Both of these books give numerous examples of 1000 and one million expressed as comparisons, such as:

"1000 sheets of paper neatly stacked make a pile about as high as four thick books." [a10a]

and

"If a goldfish bowl were big enough for a million goldfish, it would be big enough for a whale." [a11a]

Students will attempt to write their own comparison statements using 1000 and one million. The idea of a "ballpark figure" could easily be integrated into this lesson by having the students write two statements: one designating what that number could represent, and one designating what that number could not represent. [a12a] For example:

"One million could be the number of people in a country, but it could not be the number of students in a class."

Lesson 6: Refine

This is the time for revision of student plans, evaluating progress, and determining if any help is needed. Students will develop their inquiry and proceed with gathering and evaluating their data and reporting strategies. Individual or group projects might have points of departure based on student needs, abilities, or interests. The teacher role at this point in the process is to give help and support student learning by giving guidance and feedback, but ultimately allowing the responsibility for learning to fall on the students.

Lesson 7: Perform

At last students will complete the journey by celebrating their learning and presenting their reports to disseminate the information to an audience. The final reports will have two components: the mathematical representation of the data, and the comparative statement. The initial audience would be the group, at which time we will compare results. Afterward, students will evaluate the results to make meaning by contemplating, "Do you still think this situation is a problem? Why? If you think about your results in combination with what you learned in the environmental studies unit, what can you infer?" Perhaps students will find out that our population is exceedingly responsible and does not in fact create a lot of trash. But, if students decide that our trash is a problem we will have additional problems to consider: What now? What should we do next?

A logical follow up to the unit would be to examine alternatives to the current materials being used and analyze those statistics in order to make an informed evaluation of the costs and benefits of potential changes, such as rewashing plastic trays, which consumes water (a scarce resource in our community), and biodegradable disposables, which are more costly to purchase and require community buy-in to manage. I hope to apply the model to examining other waste vs. conservation situations once students comfortable and fluent with the more complicated mathematics required. Other extensions would be for students to conduct their own examinations of their environment, and develop their own campaigns for environmentally and socially responsible choices in the school community.

Lesson Plans

Lesson Plan A: Understanding Relative Place Value Using OM Cards (See Appendix 1)

Students take out the cards for 1000, 200, 30 and 4. Have them separate the cards and name each number, and verbally name each number. Then ask for the operation we should use to "put this number together", and they write "+" between each. When students represent sums, the digits line up so that only the leading digit of each order of magnitude is exposed and the zeros are hidden, i.e., 1234. Then have students compose and decompose the number a few times. What does this mean?

1234 = 1000 + 200 + 30 + 4

The term "standard notation" means the number written as 1234, and "expanded notation" is the form with the orders of magnitude separated. Students will model the number with the cards in vertical and horizontal form as well as changing the order to demonstrate the commutative and associative properties. We will then experiment with combining the numbers in different ways, such as:

1000 + 200 = 1200 and 30 + 4 = 34 so 1200 + 34 = 1234

and

1000 + 4 = 1004 and 200 + 30 = 230 so 1004 + 230 = 1234

This model helps students make the connection between the digit and its corresponding value. The "3" in the above number does not represent the value 3, but 30. This process of repeatedly composing and decomposing numbers will strengthen student's conceptual knowledge so that the process of arithmetic makes more sense.

Lesson Plan B Part 1: Comparing Relative Place Value and Order of Magnitude Using Base Ten Blocks

Students will model the number 1234 using Base 10 Blocks. Generally "1000" is modeled as a 10 cm x 10 cm x 10 cm cube, but for this purpose I would have them break it down into ten 100's. Then students will decompose each order of magnitude into a quantity of tens (except of course the ones). Students should then have:

1000 + 200 + 30 + 4

100 tens + 20 tens + 3 tens + 4 ones.

They will model each order of magnitude as a length or "train", i.e., 1000 linear centimeters, 200 linear centimeters, 30 linear centimeters, and 4 linear centimeters. It is important in this exercise to have the first cube of each train aligned with the others to accurately compare lengths. I will ask," Why are there *so many* in this line (1000) - there's a one in that place!" (It's 1000, not 1, silly teacher!) "So even though the 4 is the greatest digit in this number it's not the most?" (No.) "Why?" (It's 4 ones, and ones are the smallest place value.) "So - tens are always greater than ones, hundreds always greater than tens, thousands always greater than hundreds?" (Yes)

I will then have the students keep the 1000 cm train intact but combine *all* the other trains into a second train representing 234, aligning the starting points. Next I will demonstrate this action by using the cards: 200 + 30 + 4. Then we will put the 1000 card with that train and 234 with the new train and have students compare the trains, asking: "Which is greater? (1000) How do you know? (It's longer)" I'll write: 1000 > 234, then ask "Is it half as long? (No) How do you know? (Half would be up here, six meters, etc.) So can we write 234 ½ of 1000? (Yes)" Then we will find the halfway point on the 1000 train, and I will ask students to figure out how much ½ of 1000 is. Next they will find "half of a half" by examining the train, and put markers at 250 and 750. We will observe that our 1000 is now split into four parts, and will name each as "one fourth". I will then ask them, "If each part is 250, what does that mean? (Four 250's make 1000, or 4 x 250 = 1000.) So, how does our 234 train compare? Is it between 0 and 500? (Yes) Greater or less than 250? (Less) So, 234 ¼ of 1000, right? (Yes). Depending on their level of understanding this process, I would repeat or extend this activity.

Lesson Plan B Part 2: The Importance of Leading Digits and Very Round Numbers

We will then contemplate: "Even though we split our 1000 train into four equal pieces of 250, is it still 1000? (Yes). Is it still greater than 234? (Yes) So, where is most of this number - in the 1000 train or the 234 train? (1000) Therefore, that first digit "1" that represents 1000 gives us most of our information about that number. We can tell that it is a little more than 1000. Is it close to 2000? (No) How do you know? (The second train would have to be almost as long as the first one.) We call numbers like 1000 "Very Round Numbers" - they have a leading digit and all the other numbers are zeroes; lots of zeroes makes a number very round."

The next part of the exercise requires a long hallway to model the entire number as one long linear "train". As base ten blocks are based on centimeter units, I would say that this train is "one thousand two hundred thirty four units long", writing "1,234 cm". The students will observe that the metric system is modeled on the decimal system by measuring the train using meter sticks, discovering that there are 10 tens in a meter. Then I'll ask, "If we know that a meter is 10 tens, do we need to use the meter stick to measure this train? (No) How else can we do it? (count the tens; every ten 10s is a meter)." After measuring 12 meters, students should observe that the remainder of the train is too small to be measured using meters. I'd start a line of questioning to reinforce the importance of the leading digit in understanding the relative value of a number, i.e., How much more do we have to measure? (34 cm) We already know that our number is greater than 1000, right? What if we wanted our number to be a little more accurate? How much do we have with our 12 meters? (1200 cm) Is it important to measure the rest, or do you think the number "1200 cm" is a pretty good number we could use to describe the length of this train? (Pretty good) We call these kinds of numbers *pretty round numbers*, because they're pretty round - they still have zeros, but not quite as many as *very round numbers*. We know that the first digit was very important - what additional information did we use this time? (The 2) So we used the 2 in the second place, which really stands for what number? (200) - to get us a little closer to our actual number, correct? (Yes) So when we moved from the thousands place to the hundreds place, which direction did we go? (Left to right) Just like reading, right? Let's review what we did. So when we want to evaluate a number, first we look at the order of magnitude. It takes four digits, or four places to name this number, so we know it is in the thousands - ones, tens, hundreds, thousands. Then we look at the leading digit to help us know how many thousands. We know 1234 is greater than 1000 because we looked at the first digit, a 1 in the thousands place and we know we have more than that. To get a better idea of how much more, we moved right to the next digit, which was a 2 in the hundreds place, or 200. So, we know that our number is a little greater than 1200, because we can see that 1000 + 200 = 1200 and we have a little more than that."

Lesson Plan C: Application of Estimation Concepts in Evaluating Multiplication and Addition Strategies

I will give students a multiplication problem using very round numbers:

200 x 30 = 600

Next, I'll ask students if they notice anything interesting about the problem that is related to our explorations with leading digits. Discussion will lead students toward noticing that the first digit of the product is equal to the product of the leading digits:

2 x 3 = 6

Then students will compare the number of zeroes in the factors with the number or zeroes in the product - they're equal!

2 + 1 = 3

3 = 3

We'll conclude that if you multiply the leading digit and add the total number of zeroes in all factors, you will have a product with the correct order of magnitude. Students will test this rule with a variety of Very Round Numbers. Afterwards, I'll pose the following question: "How do we evaluate

214 x 36?"

"Remember, we're not trying to get an exact answer right now, but an idea about the place value of the leading digit and the order of magnitude so we know if our exact answer is reasonable." Students should recall activity 4c; "Can we turn 214 into something closer to a Very Round Number?" (Yes, 200) What about 36? (Use 30 or 40 - it's closer to 40). So now we have:

200 x 40

and by our process we know that that is 8000. Therefore, 214 x 36 should be somewhere close to 8000. Is it?" Students would then solve the actual problem and determine the product to be 7704. Students will look at the leading digits of these two numbers and the order of magnitude: "Are 7 and 8 close? (Yes) Do they have the same order of magnitude? (Yes) Is 7704 close to 8000? (Yes) So if you had estimated that your product would be about 8000 and you got 7704, is your answer reasonable? (Yes)"

Another approach would be to demonstrate the concept of a "ballpark figure". In this instance, students round both factors down and multiply to get the lowest boundary, then round up and multiply to get the high boundary. The product of the actual number must be between these two boundaries. For example, in the same problem:

Rounding down:

214 rounds to 200

36 rounds to 30

200 x 30 = 6000; this is the lower boundary.

Rounding up:

214 rounds to 300

36 rounds to 40

300 x 40 = 12000; this is the upper boundary.

Therefore, we can determine that product (p) of 214 x 36 = 6000 p 12000

Although this ballpark is rather large, it still would help students analyze common errors that occur with the conventional algorithms, such as:

214 x 36 = 12624

12624 + 63120 = 75744

For the next activity I will then have students work in groups to try to determine how one would estimate to check accuracy when *adding* large numbers. Students will answer the following prompt: "My estimate is ___________. I think this is reasonable because _______..." [a13a] Each group will then report on their process for class discussion. In their description of the process I would look for evidence of using leading digits to estimate and if they realized that the subsequent number of digits of the larger number would tell them the number of places the sum would have. For example:

295 + 391 = ?

Based on our previous learning, an appropriate student response would be: "We estimate that the sum will be a three digit number greater than 500. The front digits are 2 and 3 in the hundreds place so 200 + 300 = 500, plus more for the rest of the number." Using the ballpark method, students could determine "We estimate that the sum will be between 500 and 700 because rounding down gives us 200 + 300 = 500, and rounding up gives us 300 + 400 = 700."

However, in the above equation, there is the problem that there are nines in both of the tens places. If no one brought up the issue in discussion, I would ask them if these numbers are close to 200 and 300 as they thought, or if they should use a different Very Round Number based on the information in the tens place, or perhaps use a Pretty Round Number Ideally, they would determine that a more appropriate response would be: "We estimate that the sum will be a little less than 700, because 290 is almost 300 and 391 is almost 400, and 300 + 400 = 700." The amount of instruction around this concept would depend upon student understanding at this point.

Notes

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11. David M. Schwartz. *How much is a million?* (New York: Lothrop, Lee and Shepard Books, 1985), 11.

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Appendix 1: How To Make OM Cards

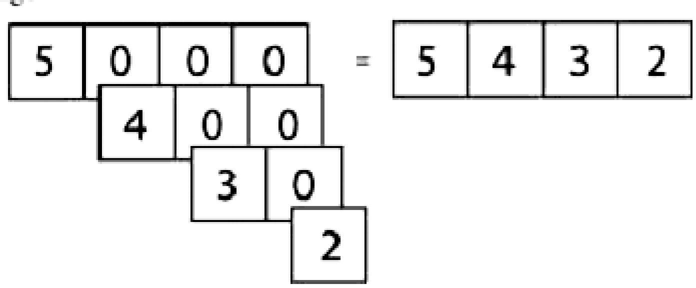
Materials: 1" Graph paper or templates and cardstock
Scissors
Marker

Create: Make a set of 9 cards for each order of magnitude from ones to thousands. Each card represents a Very Round Number with a different leading digit from 1 - 9 as shown below:

1000	100	10	1
2000	200	20	2
3000	300	30	3
4000	400	40	4
5000	500	50	5
6000	600	60	6
7000	700	70	7
8000	800	80	8
9000	900	90	9

To Use: Build numbers by stacking cards. Highest order of magnitude cards go on bottom. All cards must be aligned on right edge.

Example: 5000 + 400 + 30 + 2 =



Appendix 2: Implementing District Standards Mexico Content Standards

Mathematics Standards: Primary standards

Strand 1: Numbers and Operations: Students will understand numerical concepts and mathematical operations.

- Benchmark 1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.
- Benchmark 2: Understand the meaning of operations and how they relate to each other.
- Benchmark 3: Compute fluently and make reasonable estimates.

Strand 2: Algebra: Students will understand algebraic concepts and applications.

- Benchmark 1: Understand patterns, relations, and functions.
- Benchmark 2: Represent and analyze mathematical situations and structures using algebraic symbols.
- Benchmark 3: Use mathematical models to represent and understand quantitative relationships.
- Benchmark 4: Analyze changes in various contexts.

Strand 4: Measurement: Students will understand measurement systems and applications.

- Benchmark 1: Understand measureable attributes of objects and the units, systems, and process of measurement.
- Benchmark 2: Apply appropriate techniques, tools, and formulas to determine measurements.

Strand 5: Data Analysis and Probability: Students will understand how to formulate questions, analyze data, and determine probabilities.

- Benchmark 1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.
- Benchmark 2: Select and use appropriate statistical methods to analyze data.

Science Standards: Secondary standards

Strand 1: Scientific Thinking and Practice: Students will understand the processes of scientific investigations and use inquiry and scientific ways of observing, experimenting, predicting, and validating to think critically.

Strand 3: Science and Society: Students will understand how scientific discoveries, inventions, practices, and knowledge influence, and are influenced by, individuals and societies.

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http://fcit.usf.edu/math/resource/bib.html

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