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The Power of Estimation

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Overview

This curriculum unit deals with estimation. It is designed for the fifth grade Math (English as a Second Language) curriculum. It can also be modified for native speakers of English. The main parts of the unit are the following: place value and order of magnitude, relative place value expanded form in relation to rounding off numbers, using area models, and sampling to produce estimates. Benchmarks will be established throughout the year in order to help students meet the promotion standards.

Estimation will strengthen my teaching in mathematics and reading (literature), which are two subjects that I enjoy. Being a teacher in the upper elementary grade level, I like integrating mathematics with literature. Literature presents math concepts in a "friendly" way for my second language learners as well as the few native speakers in my class. When math and literature are combined, I feel that students learn better. Visuals such as pictures and graphs will improve students' understanding of a math concept. Once a math concept is introduced, finding the right book that my students can understand, enjoy and attract their interest will definitely get them hooked. I will also use interactive bulletin boards. I will make one of these for the Estimation unit. Using both mathematics literature and interactive bulletin boards will make teaching mathematical concepts easier.

Since my students have difficulty using their estimation skills in solving mathematical word problems in everyday situations and with their English language skills, I will incorporate the literature books: *Take a Guess, Great Estimation, Greater Estimation, Betcha!, and One Million Dots* which will enable them to grasp the different concepts of estimation.

I will use the book *Take a Guess* to introduce estimation. The students will understand that estimation is more than just "taking a guess". It is reasoning to find a useful estimation. I will be introducing estimation using the visual presentation of 10s, 100s, 1000s and 10,000s. The next book I will use is *Great Estimations* for guessing the order of magnitude of the number of objects in various pictures. The third key concept using the same book will compare pictures with the same order of magnitude. I will use *Betcha!* to introduce an estimation technique for arrays. It involves using the area (length × width). After that, I will connect *Greater Estimations* with *Betcha!* by using grids to think of an area as an array. Finally, the student will learn the expanded form of numbers and estimating relative sizes using single place components.

My estimation unit will be taught the first nine weeks (first quarter) of the school year to find out how much the new fifth graders learned in the previous grade. Then on the last nine weeks (fourth quarter) I will review the estimation concepts again to prepare them for the Texas Assessment of Knowledge and Skills (TAKS) standardize testing. The TAKS test will consist of some estimation word problems. The learning focus of my unit is on place value of whole numbers. Estimation will be taught in conjunction with the following key concepts: place value, representation of number and expanded notation, comparison, estimation, ordering, rounding and compatible numbers. I will teach the unit for two to three weeks incorporating it into the math curriculum. Estimation will be taught in compliance with the state standards. One of the main objectives in this unit will be for the students to model, read, write, compare, order, add and subtract whole numbers.

The math and reading (ESL) integration will include hands on activities for the students. They will engage in lessons that will incorporate the skills of estimating, calculating, and reasoning. They will do this through projects, stories, activities and games! Lastly, the fifth grade students will have enough knowledge to be successful on our state standardized math test. When given at the end of the year, the hope will be that all students will pass and in turn be promoted to the next grade level.

Demographics

Houston Independent School District is the largest school district in the state of Texas. Tinsley Elementary school is populated with approximately 60% Hispanic, 37% African American, 1% Caucasian and 1% Asian students in grades first through fifth. 88% of our students are eligible for free lunch and 6% of the students' population is eligible for reduced lunch. Thus, the school is considered as Title I. Located in inner city Houston, Tinsley Elementary is surrounded with low income housing, particularly apartment style. There is high mobility because 93% of the students live in this type of housing. The school has the bilingual and the English as a Second Language (ESL) Program from first to fifth grade. There are 6 classes in each grade level. In the first through third grade, the Spanish speaking students go to the bilingual classes. Then from fourth to fifth grade, these bilingual students are placed in the ESL classes. The school is labeled as a Title III because the school services students who are Spanish speakers and students with limited English proficiency levels.

Rationale

Presently, our students are exposed to a lot of information through media, television and other electronic gadgets. Statistical numbers are presented to them daily. For example in science, students are informed that global warming has affected the population of the polar bears. The population of the Eurasian Lynx is below fifty thousand. There are approximately 100 pairs of Bearded Vultures left. The Amur leopard has a population of 40 individuals. It is important to teach the students estimation strategies because it will help them understand the magnitude of small and large numbers. They will be able to take a guess with common sense when they fully grasp the concept. Thus, estimated numbers will be meaningful to them, and they should use the skills learned as often as possible in their everyday life to maintain them.

Students must understand that estimation is beyond guesswork. Estimation is an important skill because it

helps students make sense of what is around them. Carlow (1986) describes estimation skills as making intuitive judgments within a holistic framework. Taylor - Cox (2001) called this the range-based estimation. The rationale for finding a range of numbers rather one definitive number is based on the belief that if an exact answer is required then individuals should be doing something other than estimating, e.g., mental math maybe more appropriate. According to Taylor-Cox estimation should be used when it is not important to find an exact answer, or it is too difficult to calculate one quickly.

Some people believe that the person who could estimate to the closest number is the best estimator. Research explains that when adults and children were asked to provide estimate in the inquiry, the adults estimated to the closest number while the children provided the researcher with wild guesses. Leutzinger, Rathmell and Urbatsh explain for that for a young child, there is little difference between 35 and 1000; they are both large numbers (Leutzinger et al 1986.p82)

Children should be taught the estimation process clearly so that as the children mature their mathematical thinking matures and develop a clear understanding of estimation. Thus, estimation has to be taught in a continuum. For example, the students in third grade begin to round off numbers to the nearest 10 and 100 to approximate reasonable results in problem situations. When the students move up to fourth grade they are expected to round whole numbers to the nearest 10, 100, and 1000 to approximate reasonable results in problem situations. In fifth grade, the students are using strategies to round and finding compatible numbers to estimate solutions to addition, subtraction, multiplication and division problems.

Carlow (1986) used the term "perceptual anchor", e.g. what might 10, 20, or 100 look like. Siegel, Goldsmith and Madson (1982) used the term "benchmarks" to describe nonstandard units, such as, "How many glasses of water are there in a bucket?" Perceptual anchor, referent, or benchmark refers to the number of chunks in a whole. Reys, Suydam, Lindquist and Smith (1998) refer to the refinement of chunking as unitizing.

I mentioned above that children should learn estimation in their early childhood years. One concrete example of estimation activities that can be taught to the children is putting 30 jelly beans in a jar. I will work with them to find out how about how many jelly beans are there in a whole jar. I will record their different estimated numbers. As a whole class, we will remove the jelly beans from the jar and put them in arrays to find out the total number. After that, we will compare the numbers found with their responses. Allow the students to explain their answer choices.

Another example of an activity to show the range of numbers is the number line. Give the students any four numbers within the range of 11-20. Let them use sticky notes to find the numbers in the number line. When the students can visualize that a gummy worm is the size of their finger, then strategies of using referents will help them understand the concept of estimation and lead them to be better thinkers. Every year students make progress in their estimation skills. The estimation problems become more complex in terms of quantities and analysis.

What is estimation?

The fifth graders that come to my class have difficulty with estimation. There is a mathematical gap that began in third grade when they had difficulty rounding whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations. When the students were in fourth grade they had also missed the rounding of whole numbers to at the nearest ten, hundreds, or thousands to approximate reasonable results in problem situations. These students moved up to fifth grade with a math deficiency in using their estimation strategies. They are challenged with rounding and understanding compatible numbers to estimate solutions in addition, subtraction, multiplication and division problems.

The curriculum unit will enable them to develop the ability to estimate and work out how big or how small numbers are. They will be able to figure out a reasonable approximation of how many objects are there in all without computing for the exact value. For instance, they will improve the ability to estimate the number of jelly beans in a jar. Also, when ask to estimate the volume of water, given glasses with different shapes which glass will hold more, the students will be able to estimate the number of ounces or liters.

There are several definitions of the word estimation. According to the definition of estimation, it involves working out a rough answer to a calculation. The most common way to estimate a solution to a calculation is to round the numbers up or down to numbers which are easier to calculate with (Wikipedia). For example estimate the answer to 47×11. One can easily see that that 47 rounds to 50 and 11 rounds to 10. So when one can replace the calculation as 50×10 it is easier to calculate. When using estimation one wants the answers to be close to the reasonable value. The number has to be close enough in order for it to be useful.

Another definition of estimation is to be able to find the upper or lower bounds of a quantity that cannot be readily computed precisely. A third definition is an educated guess. I have mentioned earlier that "taking a guess" is using reasoning to find a useful approximation. The dictionary furthermore explains the Estimation Theory. It is a branch of statistics and signal processing that deals with estimating the values of parameters based on measured empirical data. An estimator attempts to approximate parameters using the measurements.

What is the order of magnitude of numbers?

Howe defines the order of magnitude as the next lowest power of ten, which is the same (for whole numbers) as the number of digits minus one. The orders of magnitude of numbers are used to make approximate comparison. If two numbers differ by the order of magnitude, one is on average about ten times larger than the other. If they differ by two orders of magnitude, they differ by a factor of about 100. The power of ten is important because it makes us understand the world around us using the concept of a scale. With such understanding we have the ability to comprehend and locate the relative distance of objects in our world. The power of ten enables us to have the perspective of seeing the relative sizes of objects. Also, it gives us a sense of place and the expansion of our thinking in different ways in the world.

What is a place value?

The number 7,452 has four digits. Each digit has a different place value. The value of each digit increases ten times more you move to the left. The first digit is the thousands' place. It tells you how many sets of one thousand are in the number. The number 7,542 has seven thousands. The second digit is the hundreds' place. In this number there are four hundreds in addition to the seven thousands. The third digit is the tens' place which is 4 in this example. Thus, there are seven sets of thousand, four sets of hundred and five sets of ten. The last digit is the ones place which is two in this example. Therefore, there are seven sets of one thousand, four sets of one hundred, five sets of ten and two sets of ones. The number 7,452 is a sum of all these numbers: 7,452= 7,000+400+50+2. This is called the expanded form of 7,452. The individual terms 7,000 and 400 and 50 and 2 are called the *single place numbers.* Expanded for shows a number as a sum of single place numbers. This means that single place numbers are the building blocks of the base ten system.

Relative place value refers to the relative value of the units of single place numbers. As you move to the left in the base ten expansion, the units get larger by a factor of ten at each step. 10 is ten times larger than 1; 100 is ten times larger than 10; 1000 is ten times larger than 100; and so forth. This relationship between neighboring places continues no matter how far out you go. One billion is ten times one hundred million. If you move the other way to the right, the value of the units of the places gets smaller, by a factor of ten at each step. 100 is only 1/10 of 1000; 10 is only 1/10 of 100; 1 is only 1/10 of 10. This progression can continue to the right, leading to decimal fractions. 1 is 1/10 of 1; .01 is 1/10 of 1 and so on. Relative place value can also be applied to pairs of places that are not next to each other. For example, places that differ by two represent quantities that differ by a factor of 100=10×10. Thus 1000 is one hundred times as large as 10. A billion is one thousand times as large as a million. The main point about relative place value is that the leftmost places contain most of the size of the number and the places further right are smaller and smaller details. This means that they are least important. Then often can be ignored when doing estimation.

What is the area sampling method?

The area sampling method is a method for estimating numbers of irregularly scattered objects. It works by dividing a region into sub regions, for example by imposing a grid and then counting or estimating numbers in a random sample of sub regions. One of the primary applications of cluster sampling is called area sampling where the clusters are townships, city blocks, or other well-defined geographic sections of the population (Britannica).

One of the reasons for writing this unit is to be able to help my second language learners develop their mathematical skill in estimation. The students will learn the language of math while increasing their fluency and reading comprehension. Once students gain math concepts while developing their English in the academic area, they will be empowered with the knowledge and the ability to solve mathematical related problems in their daily lives. The key concepts are on place value, representation of numbers and expanded notation, comparison, estimation, ordering, rounding and compatible numbers.

Based on the State standards the following objectives will be met on the first nine weeks of school (first

quarter): Math 5.1A Use place value to read, write, compare and order whole numbers through the 999,999,999,999 in standard and expanded form. According to the National Council of Teachers in Mathematics Standards, students from Grades 3-5 should be able to compute fluently and make reasonable estimates. The students should be able to develop and use strategies to estimate the results of whole numbers computations and to judge the reasonableness of each result. They should be able to use strategies to estimate computations involving fractions and decimals in situations relevant to students' experiences.

Strategies

I mentioned earlier the integration of math and language arts/reading for this curriculum unit. According to Marzano's research information, there are certain kinds of strategies that consistently positively impacted learning. The strategies are ranked according to their impact on student achievement. They are as follows: similarities and differences, summarizing and note taking, reinforcing effort and providing recognition, homework practice, nonlinguistic representation, cooperative learning, setting objective and providing feedback, generating and testing hypothesis, cues, questions and advanced organizers. I will describe briefly each strategy that will impact student learning.

To identify similarities and differences, the students will use Venn diagrams or charts. They will compare, and classify. To promote comprehension skills, I will ask students to write their analysis in their own words. I will ask the students to clarify (ask questions) and make predictions. Students should take notes of important information. Research states that taking more notes is better than fewer notes. Effort and recognition tell about student's attitude and belief. Both are important. One way of doing this is to write their thoughts down, reflect on them and make a mathematical analysis. It is more important to recognize a student's achievement than to get awards. Homework and practice are strategies used to extend learning outside the classroom. Nonlinguistic representation is stimulating to brain activity. In order to present a relationship, I will use words, images and physical models. Cooperative learning is allowing students to work in groups to process, be social with one another and accountable to the completion of the given task. I set objectives or goals for the concept and allow the students to identify areas of interest to them. I ask students to make predictions/conclusions and explain them. Cues, questions and organizers help students further enhance their learning. When asking questions, I often like to pause briefly to allow students to analyze and explain their answers well.

This is an example of one of my strategies. I will use an interactive bulletin board. I like using interactive bulletin boards outside my classroom because they generate my students' curiosity. An interactive bulletin board is a board with pictures, questions and answers written on index cards. When a student reads a question, he/she can pick up a card and put the answer beside the question. A sticky note and a pencil are available at the bottom of the bulletin board. Students who respond to the interactive bulletin board write their names and homeroom teachers. At the end of the day I put the names in a box. After teaching the lesson or the concept, my students and I go through the questions and answer them. The students will explain why each answer choice is correct. After the explanation or class discussion I will pick a name from each class and check their answers. The winners will claim their prizes from the treasure chest. This is how I create my interactive bulletin board. Whenever I teach a new concept I work on the interactive bulletin board display outside my classroom wall.

When teachers have a linguistically diverse classroom, reading, aloud each problem or direction can allow the

students to draw or illustrate any key vocabulary word they might have trouble reading.

Another application that helps the students improve their estimation skills is writing in their interactive math journals. Students write on their journals after learning a new concept. They use two pages, the right and the left, for journal writing. The left page is for the student input and the right page for the teacher input. Student input's page is when students write what they learned on the new concept or use graphic representation to show their work. Students' thoughts are written down. Also, the math journal helps the students reflect or evaluate on their areas of success or difficulties. A teacher's input page is on the right side of the paper. The students write the important vocabulary words or the answers to the questions/problem. This is a guided activity with my students. The teacher writes notes down on the board and the students copy those notes on the right hand side of their interactive math journal.

Lesson Plans

Part 1 Introduction to Estimation (Take a Guess: A Look at Estimation)

The objective of this lesson is to give students the basic idea of estimation and how useful approximate answers can be, and how much easier is it to get approximate answers than exact answers. I will use the book *Take a Guess* to show examples of how estimation can be used. However, I will also emphasize that estimation is not really "taking a guess" and that it involves a lot of careful thought. Also, the calculations you do in estimation are usually easier than exact calculations, because they don't involve too many digits. I will use the book as an opportunity to introduce valuable vocabulary as well as the general idea of estimation.

We will read the book together as a class, and I will enact some of the scenarios mentioned in the book, such as comparing heights of students. I will ask the students to provide examples of situations where estimation can be useful.

I will explain one of my examples, someone's age. When comparing small numbers for instance, 15 years old from 50 years old, the age difference is 35 years. The margin of error is large compared with 15. It is more than twice as big. However, when large numbers are compared say for example a million years old, a 35 year difference would be insignificant.

Part 2 Guess Order of Magnitude (Great Estimations)

The objective of this lesson is that students will learn how to estimate the number of objects in pictorial representations and use the order of magnitude estimation technique to come up with a reasonable number. The students will learn how to tell whether a group of objects is closer to 10, 100 or 1000. According to the definition of the word, orders of magnitude are generally used to make very approximate comparisons. If two numbers differ by one order of magnitude, one is about ten times larger than the other. If they differ by two orders of magnitude, they differ about a factor of 100. Two numbers of the same order of magnitude have roughly the same scale: the larger value is less than ten times the smaller value. This is the reasoning behind significant figures: the amount rounded by is usually a few orders of magnitude less than the total and is therefore insignificant.

I will also teach the powers of 10. The students will understand that 1 raised to the 0 power, that 10 is raised

to the first power, that 100 = 10×10 is 10 raised to the second power, and so on. The chart below will help the students understand the power of ten and the order of magnitude.

In words	Decimal	Power of ten	Order of magnitude
ten thousandth	0.0001	10^{-4}	-4
thousandth	0.001	10^{-3}	-3
hundredth	0.01	10^{-2}	-2
tenth	0.1	10-1	-1
one	1	10^{0}	0
ten	10	10_{1}	1
hundred	100	10_{2}	2
thousand	1,000	10_{3}	3
ten thousand	10,000	10_{4}	4
million	1,000,000	10_{6}	6
billion	1,000,000,000	10_{9}	9
trillion	1,000,000,000,000	10^{12}	12

I will use colorful illustrations of objects or make the pictures myself with different groupings of 10s, 100s and 1000s. Another example that I can use to show the class order of magnitude is elbow macaroni. I will have a bag of 10 elbow macaronis, a bag of 100 elbow macaronis, and a bag of 1000 elbow macaronis. I will ask my students to estimate how large a bag of elbow macaronis do I need to make 10,000? 100,000?

We will read the book together as a class and discuss. I will prepare questions to check their comprehension. The students will give their estimations for the different pictures and explain how their answers. The next part of my lesson will be the visual presentation of 10, 100, 1000, and 10,000. I will write the numbers down on sentence strips. To get the students attention I will ask them to describe what is on the table. I will have 10 bags of M&Ms candy prepared in advance with 10 pieces in each bag and 10 more bags of M&Ms with 100 pieces in each bag. I will tell them that the first bag contains 10 pieces of M&Ms. the students will see through the first plastic bag so they can visualize the M&Ms. Then I will ask the question, "How many M&Ms do you think are there in the second bag?" I will write their responses on a chart paper in front of the class because we will go back to their answers after the activity. Students will now work in groups of 4. Each group will figure out how many M&M's are there in the second bag. They will refer to the first bag of 10 M&Ms as they figure out the answer. This activity will keep them engaged. I will be observing the class and allow each group to explain their estimation. I will reiterate to the students that a good estimation involves reducing a lot of guessing. Students will use the interactive math journal to write their reflections.

Part 3 Compare Pictures with the same Order of Magnitude (Great Estimations)

The objective of this lesson is that students will learn the estimation concept and will learn how to compare pictures with the same order of magnitude. This is a continuation of part 2. I will introduce the topic to my students and brainstorm what they already know about the topic. Then, the students will look at the book cover and make their predictions. Students' responses will be recorded on a chart paper and will be displayed on the board. I will use the book to emphasize that estimation will help them think clearly to figure out how many objects are there in each picture. Important vocabulary words will also be discussed. One of the activities for this lesson is using cereal Os.

I will put cereal Os in 6 plastic bags. The students will work in groups to show the different ways to arrange 10 cereal Os and 100 cereal Os. They will then glue each arrangement of the 10 and 100 cereal Os and put them together to show a group of 1000 cereal 0s. Then the next activity will be students working in groups of 4. Each group will estimate the number of cereal 0s in each box. They will answer the question, "Do you think there are more than 8,000 cereal 0s in the box?" The students will explain their answers. I will ask the students to explain how they came up with an accurate estimation. They will use their interactive math journal to reflect on how to compare pictures with the same order of magnitude and the estimation strategies they used to find the accurate answer.

Part 4 Estimation Techniques for Arrays (Betcha!)

The objective for this lesson is that students will learn that an array is a rectangular arrangement of objects in evenly spaced objects with the same number in each row. I will use *Betcha!* to teach arrays and introduce important vocabulary words. I will mention how two friends use their estimation strategies in everyday life. The most important estimation technique in this story is how both boys use the array to come up with their estimation. To find the number of objects in an array, multiply the number in each row by the number of rows. This is like the area formula, area = L×W for a rectangle. We will read the book together as a class and discuss the different scenarios. One of the scenarios is estimating the number of people in the bus. The first boy tells his friend that there are 4 people in each row and there are ten rows of seats. So this is how he did his estimation, 4 people ×10 people = 40 people seated then he added the three people standing up. His answer was 43. This is an example of estimating. The other friend counts all the people and gets 45. Our discussion on estimation will focus on the different scenarios in the book. I will explain further that the array method is for finer estimation than order of magnitude. Each boy's guess was actually quite close to the actual number.

The students will also be able to create mathematical games. Together the whole class will design a "Betcha!" game. The students will be given the task of thinking and picking something that is difficult to count, such as people in all the lines at the busiest supermarket during the rush hour, the number of soup cans on display at the supermarket, the number of cars parked at the ball game, and the amount of ice cream students in the whole school eat during the summer months. Help students to come up with different strategies for making their estimations. Then check to see how close their estimates are to reasonable numbers.

For the pair share activity, students will come up with a real life situation that requires estimation and explain it orally. For homework, to check for understanding the array model, the students will estimate how much pizza an average student eats and then multiply by the number of students. For six classes they can multiply the one class estimate by six. At the end of the lesson, the students will reflect and write their responses on the interactive math journal.

There are also extension activities that students can do to extend the concepts presented that are related to real-life situations. Estimate how many hamburger sandwiches each family eats in one month. How much will the family spend for the hamburgers? Students will keep track of their estimation and ask their parents to help them find out if their estimate is reasonable.

Part 5 Use Area Techniques (Greater Estimations)

The objective of this lesson is to reinforce estimation skills to find the area and the estimation technique by using the grid to gather information, counting and averaging. The student work focuses on developing a sense of the size of 10,000. The colorful illustration of the book will generate students' interest. Comprehension

questions will be prepared in advance. I will generate curiosity by asking the question how many rubber ducks are on the cover of this book. Are there more than 10 rubber ducks? Are there fewer than 10, 000? The visualization strategy will help the students estimate the number of rubber ducks. I will say to the class that counting every duck is difficult and using the grid as their estimation technique will enable them determine the answer. Therefore, to get a reasonable idea of how many ducks are there, the students should try to find the number of ducks in a small area then multiply by the ratio of the whole picture to the small area. To answer the question, "Are there fewer than 10,000?". I will elicit students' responses from the class and record them. The students will be in cooperative learning groups of four to explain their responses. Important vocabulary words will be discussed as we read.

I will explain to the students that estimation strategies will require eye training, finding the area and using the grid to group the objects then put them together to make a whole picture. At the end of the lesson, the students will write individual reflections on their math interactive journal.

Lesson 6 Relative Place Value (A Million Dots)

The objective of this lesson is for the students to understand relative place value. The student's work will focus on developing a sense of the relative size of 1000, 10,000, and larger powers of 10. I will use the book, *A Million Dots.* We will read the book together as a class. I will ask the students to determine the number of dots on a page that contain 7-by-9 array of blocks of 100 dots. They will use the handout to construct rectangles of 5,000 and 10,000 dots. I will ask the students the following questions: What is the relationship between thousands, hundred-thousands, millions and billions? Why are multiples of 10s, 100s, 1,000s "friendly numbers"? I will use base 10 blocks to provide students with opportunities to represent and build number in a variety of ways including 2-, 3- digit numbers to review the relationship between each place value.

To engage the students with the concept of estimating to the right of a decimal point, I will show them a whole square cookie. Then divide a large cookie into ten parts. Each part will be named by the student as 1out of 10 and will be written as .1, one tenth. Each one tenth will be again cut into 10 parts. The students will identify how many parts are there in all. Each part will be one hundredth. The students will discuss the hundredth's place value.

Since the fifth graders are doing estimation of whole numbers when they add, subtract or multiply I will start first with the explanation. They should look for patterns in addition, subtraction, multiplication and division of whole numbers. My first question is what will happen to 57 + 9=? Do mental math with estimation. You know that 57 is between 50 and 60, and 9 is between 0 and 10. So 57+9 is between 50+0=50 and 60+10=70. In this case, it is closer to 70. In fact 57 is missing 3 from 60, and 9 is missing only 1 from 10. So is all, 57+9 is missing 4 from 70, so it is 70-4=66.

Furthermore, we can use similar tricks for multiplication. How much is 9×45? The students should use mental math to multiply by 9 quickly: (10×45)-45= 450-45=405. I will explain to the students that the same method works for 99, which is 1 short of 100. So (100×number-number): 99×5= 100×5 take away 5=500-5=495. The next example will be 999 which will be (1000 × numbers -number): 999×5= 1000×5 -5=5000-5=4,995. These examples show that when numbers are rounded off to the nearest 10, 100, or 1000 finding the missing number is easier in addition or subtraction or multiplication or division. I will explain how to multiply single place numbers: for example: I will display…

40 x 600= (4x10) x (6x10) = (4x6) x (10x100) =24 x 1000= 24,000.

When integrating math and literature, we will read the book together and then teach the math concept. After brainstorming, the teacher will write student responses on chart paper. Students will work with their cooperative groups of 4 for the next activity. All the students will draw 10 dots and then100 dots. We will put the dots together in groups of 10, 100 and 1000. I will ask the students to go to the 10 dots corner, 100 dots corner and the 1000 dots corner. A speaker will report on each group.

A way to keep the students engaged is to prepare in advance a picture of one dot that will be magnified. I will explain to the class that to magnify is to enlarge it with a magnifying glass but magnitude is to enlarge a number by a power of 10. I think the students will relate to this magnified dot because it is the size of a large pizza. I will divide the dot into ten equal parts. Each part will be 1/10 of the whole dot. This number will be express as 1÷10=.1; Next, I will show another magnified dot divided into 100 equal parts. One part will represent 1/100 of the whole dot. I will write the math sentence as 1÷100=.01; I will ask the class to work with the same group and show how 1/1000; 1/10,000; 1/100,000; 1/1,000,000,000 would be represented in decimal form.

I will also represent decimal fractions using volume. I will bring to class a round watermelon to represent one whole. I will slice it into ten pieces so each one will also represent 1/10.I will explain that when cutting a watermelon I will not be able to make the pieces exactly equal, but will try to make them as equal as I can, by estimation. The activity will also be a good practice of measurement. The pieces will be weighed to see how close they are in weight, and will adjust them if they a little if they are very different. I will write on an overhead one out of ten slices, 1/10 and explain that after the decimal point, the place value is tenths. Moving away from the decimal point is the second digit which is the hundredths place. Therefore, if there is a one in the tenths place that is "one tenths". I will refer to .1 and ask the students the question what is after 1. The next digit will be a zero. I will explain to them that the number is now .10 which is 10/100 or 10 percent. The watermelon activity will continue. I will ask each group to chop one tenth of the watermelon into 10 pieces to show them the value of each piece which is1/100. I will use a second watermelon for this activity because I will not be able to make the pieces exactly equal. I will use measurement and explain to the students again that I will try to make them as equal as I can, by estimation. At the end of the activity the students will have to put all the slices together to get a whole watermelon. I will emphasize that it is alright to ignore 1/1000. Howe states that relative values less than 1/1000 are ignorable for most practical purposes.

Another way to illustrate the idea is to use 1000 cubes, and will discuss that if the cube represents 1, then each little cube represents 1/1000. To check if the students understand the idea I will use the next activity. I will ask students to estimate distances in terms of 10, 100, and 1000 steps. How many steps will it take you to go to the cafeteria from the fifth grade classroom? Do you think it will take you less than 100 steps or more than 100 steps? Will it take you 1000 or more steps to go to the school gate? Is it more than 1000 steps to go home or is it more than 10,000 steps? Students will work in groups and write their answers in their interactive math journal.

Lesson 7 Expanded Form and Relative Sizes of Single Place Components (A Million Dots)

The objective of this lesson is to teach the students expanded form and relative sizes of single place component. The students will develop a sense of quantities in the thousands, ten thousands, and hundred thousands as well as develop a sense of the size of 1,000,000. I will use the book *A Million Dots* to introduce the concept. I will emphasize the idea than when students use the expanded form they also have to compare which part of the number is the biggest, next biggest and the smallest. I will use the book to as an opportunity to introduce math and reading vocabulary words. To generate my students' curiosity I will review the lesson

prior to this on the order of magnitude of 10. I will prepare in advanced a handout that will show 10 dots, 100 dots, 500 dots and a 1000 dots. I will ask them to estimate how many thousand dots will make a million dots.

We will read the book as a class and will discuss the fact on each page. Thus, the students will get a new sense of how much a million is really worth. To start the concept of the expanded form, I will direct them to look at the picture of the mosquito and the Empire State Building on the first page of the book. I will ask, if the wings of the mosquito beat 612 times each second, how do you write the expanded form of 612? I will record the response on a chart which will show 612= 6 00 +10 +2. I will make it a point to ask all the time which part of the number is the biggest, next biggest and the smallest. The next question will be if a person must climb 1,860 steps to walk to the top of the Empire State Building how do we write the expanded form of 1,860? The students will respond to this question and I will record 1,860=1,000+800+60+0 on a chart paper. The next example will be the number sentence, if a person blinks 134,000 times each week, write the expanded form. Ask for a student volunteer to write the expanded form on the chart. The students will work with their group, choose a picture and work on the dot number of their choice from the six digit numbers. They will illustrate the number of dots on the picture and write its expanded form. The students will choose a reporter to show and explain their illustration and how they came up with the expanded form. At the end of the lesson student will reflect on their learning and write in their interactive math journal.

Towards the end of the estimation unit, I will create a record of students' work. Students will look back through their interactive math journal and write or revise what they have learned for each concept taught, what they remembered most, and what was difficult or easy for them. They will select two pieces of their work as their best and I will choose one or two pieces to be saved in their portfolio. One of the work samples could include students' written solution to an estimation word problem.

In conclusion, I hope that this curriculum unit will help all students achieve their mathematical goals. They will be critical thinkers and problem solvers; they will use their estimation skills to solve real word problems; they will develop various ways to express their mathematical thinking in oral and written form, thus enjoy and appreciate mathematics.

Works Cited

Teachers

Berge, Margaret & Gibbons, Philip. Help Your Child Excel in Math. *Easy Practical Methods*

That Make Learning Fun! (Life Time Books, Inc. Hollywood, Florida, 1992).

The book states bringing basic math concepts to preschoolers, reinforce an elementary school

curriculum with fun activities, and hardest of all, motivate kids to remember their facts.

Carman, Robert, & Carman, Marilyn. Quick Arithmetic. *A Self-Teaching Guide (*John Wiley and

Sons, Inc (3 rd Edition). New York, NY 2001). This self-teach book explains the quickest and

easiest way to understand and learn the basic math skills.

Howe, Roger. *Estimation Seminar Notes: A Pretty Round Number.* (Yale University. New Haven, Conn., 2008). The seminar leader of the Yale National Initiative explains how numbers are rounded off in one of his seminar discussions.

Kliman, Marlene & Russell, Susan et al. (Building on Numbers You Know. *Computation and*

Estimation Strategies. Dale Seymour Publications. White Plains, NY, 1998, I-1; 106-107).

This book is a complete K-5 mathematics curriculum that supports all students.

Kenschaft, Patricia Clark: Math Power How to Help your Child Love Math, Even If

You Don't. (Addison Wesley Longman Inc. 1997). This book explains how to help and

encourage children like math.

Stigler, James & Hiebert, James. The Teaching Gap *Best ideas from the World's Teachers for*

Improving Education in the Classroom. 1999. The Free Press. A division of Simon & Schuster

Inc. New York, NY. This book gives information on strategies to improve classroom teaching.

Kogelman, Stanley & Heller. The Only Math Book You'll Ever Need. (Facts on File. Inc.

New York, NY 1994). This book provides information on the basic mathematical concepts in

solving problems.

Paulos, John Allen. A Mathematician. A Mathematician Reads the Newspaper.1995. Bantam

Doubleday Dell Publishing Group. Inc. New York, NY. The book explains the importance of

math literacy.

Seiter, Charles. Everyday Math for Dummies. (IDG Books Worldwide. Inc. Foster City,

CA.1995). This book explains on how to improve basic math skills and how to solve word

problems.

Gullberg, John. Mathematics from the Birth of Numbers. (W.W. Norton & Company.Inc.

Fifth Avenue, New York, NY 1997). This easy to read book shows the history of numbers.

Carlow, C. D. "Critical balances and payoffs in an estimation program." In

Estimation and Mental Computation: 1986 Yearbook, edited by H. L. Schoen and M. J. Zweng,

pp. 93-102. Reston, VA: National Council of Teachers of Mathematics1986. The book explains

the importance of teaching children estimation skills in Pre-K.

Huff, Darrell. The Complete: *How to Figure it.* (Darrell & Frances Huff, Inc. W.W. Norton & Company. New York, NY 1996). This book shows easy techniques for calculating budget, retirement plan, buying a house, estimating travel and leisure expenses etc.

Lawrence Weinstein and Adams, John. *Guesstimation*. (Princeton University Press. 2008). This book shows mathematical approximations in real world problems.

Leutzinger, L. P., Rathmell, E.C., & Urbatsch, T.D. "Developing estimation skills in the primary grades." In *Estimation and Mental Computation: (1986* Yearbook, edited by H. L. Schoen and M. J. Zweng, 1986, 82-92. Reston, VA) National Council of Teachers of Mathematics.

Reys, Robert E., Marilyn N. Suydam, Mary M. Lindquist, and Nancy L. Smith. *Helping children learn mathematics* (Fifth edition). (Needham Heights, MA: Allyn & Bacon 1998). This book provides the readers how to teach children learn mathematics.

Ruedy, Elisabeth & Nirenberg Sue. Where Do I Put Decimal Point?*: How to conquer* Math Anxiety and Increase Your Facility With Numbers.(Henry Holt and Company Inc. New York, NY 1990). This book encourages those people who get anxious with math and for those who avoid math.

Siegel, A.W., Goldsmith, L.T., & Madson, C.R. 1982. "Skill in estimation problems of extent and numerosity." *Journal for Research in Mathematics* Education, 13, 211-232. This book is about children's development of ranged based estimation skills.

Taylor-Cox, J. 2001. "How many marbles in the jar? Estimation in the early grades." *Teaching Children Mathematics, 8*, 208-214. This article examines the known methods of estimation.

Students

Clements, Andrew. *A Million Dots*. (Simon and Schuster Children's Publishing Center. New York, NY 2006). This book contains "one million" dots that will give the readers a sense of what a million really means.

Goldstone, Bruce. *Great Estimations*.2006. (Henry Holt and Company LLC. New York, NY 2006). This colorful book offers great math lessons to build up estimation skills.

Goldstone, Bruce. *Greater Estimations*. .(Henry Holt and Company LLC. New York, NY 2008). This attractive book shows how to use estimation skills to find the length, volume and area.

Murphy, Stuart. *Betcha!* Harper Collins Children's Books. 1997. New York, NY. This is book about two friends who used their estimation skills to win a contest.

Scott, Janine. *Take A Guess*: A Look at Estimation.2003. Compass Point Books. Minneapolis MN. This is a picture window book showing how estimation can be used.

Websites

Area sampling Encyclopedia Britannica.2008. Encyclopedia Britannica http://www.britannica.com/Encyclopediacheckedtopic/333981 (online.03.Aug.2008) This website gives the definition of area and area sampling.

Array. Merriam Webster. http://www.merriam-webster.com/dictionary/array (online. 03. Aug.2008) This website gives the definition of the word array.

Number Operations Standards for Grades 3-5.National Council of Teachers in Mathematics. http://www.standards.nctm.org/document/chapter5/numb.htm (online.03.Aug.2008) This website provides the information for mathematics standards for Grades 3-5

Order of magnitude. 2008. Wikipedia. http://en.wikipedia.org/wiki/Order_of_magnitude (online.03.Aug.2008) This website provides the order of magnitude definition and chart.

Children's development of range based estimation skills: Far more than guesswork http://www.edu.uwo.ca/essofamilymath/Research/estimat.pdf (online.03.Aug.2008) This website is an article about estimation skills.

What works in the classroom instruction by Robert J. Marzano et al. http://www.middleweb.com/MWLresources/marzchat1.html (online 12 Aug.2008)

This website provides information on the nine instructional strategies that work in across all

content areas and across all grade levels.

Addendum

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