Curriculum Units by Fellows of the National Initiative
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Rice to Feed the World- Estimations on Rice Consumption and Production

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Overview

In this unit, there are four main components that relate to the topic of rice and estimation. In working with the population of the world, the main math topics deal with place value of large numbers, notation in powers of ten, rounding and order of magnitude. In the second section, students use the Chinese rice tamale (zong) as a basic unit to estimate for the amount of rice needed to feed a population by using scaling and proportion skills. With the amounts of rice determined, students will find rice storage solutions through exploration of area and volume in the next section. A final section is to apply the proportion skills to find the land that is needed to produce the amount of rice needed. Unit conversion skills are practiced and applied throughout the unit.

School Design

This unit is written for 6 th grade math classes in a San Francisco public K-8 school that offers Chinese language immersion. To put it simply, the immersion education model creates a school environment in which students learn a target language in a natural setting. About 75% of our 550 students are learning Chinese as a second language. All the core subjects, including math, social studies and science, are taught primarily in Cantonese during grades K-3. During 4 th grade to 6 th grade students continue to learn math in Cantonese. I currently teach the Cantonese component of 6 th grade, which covers math and Cantonese language arts. Even though I am writing this unit in English, the activities and lessons will be conducted in Cantonese in my classroom.

Our math curriculum and expectations are no different from other schools. The California math framework guides the implementation of our math classes. One aspect of curriculum planning we add to the material is language objectives and goals. One of the underlying goals in all our school and curriculum is to provide opportunities to use Chinese. The current math curriculums put more and more emphasis on communicating math thinking. By design of integrated curriculum units, students utilize the acquired language across curriculum areas. Talking about math ideas is an authentic and meaningful exercise that builds both math and language skills. This unit on rice relating to our cultural study unit on the Dragon Boat Festival works perfectly for this purpose.

I believe that hands-on experience is one of the best ways to learn. There are many opportunities for exploration for students to make discoveries, to test their assumptions and to affirm learned skills in the classroom. I like students to work in cooperative groups that are supportive and encourage building community and language.

It is my goal to write this unit in a way that another grain or rice dish can be substituted for the Chinese rice tamales (zong). I have chosen a topic and activities that are very specific to fit the needs in my school program. But I also hope the lesson and structures are set up so that the same concepts and similar activities can be adapted to other foods that are of interest to other teachers.

Rationale

Estimation is an important skill in developing math understanding and in real life. However, it is not effective to teach it in isolated exercises. It should be embedded in all problem solving and calculation. Estimation skills require logical reasoning, which goes hand in hand with justification of the answer. There are two areas of math learning/ teaching that I would like to improve through this estimation unit.

One area I find many mistakes from my students is in labeling and converting units. Many students know how to do the arithmetic to find the area of a given shape. Very often, the units are missing in their answers or they label the unit incorrectly. They assume one square foot is twelve square inches. They are surprised to know that one square meter is not one hundred square centimeters. Usually, they understand their mistakes after seeing a pictorial representation of the units. Volume is another tricky concept for students to conceptualize, even though many students know how to find volume using a formula. In this unit, I try to build better number sense and deeper understanding through explorations in area and volume relating to rice.

Another area I want to improve is to help student develop stronger problem solving and math reasoning skills and the ability to communicate the math thinking. When students approach a math problem, the capable students will try to get to the answer using mental math. These are students who often have a hard time to explain their work because they already figured out the answer. Other students attempt to solve the problems using the easiest way they can think of, without thinking through the logic of their solutions. Then, they ask the ultimate question, is the answer correct? Most students will automatically think that their answers are wrong if I ask them to justify their answers. Through this unit, I would like to help students to be more confident in their solutions of math problems by developing a strategy to approach the estimation problems and a routine to show work and justify their answer.

Why Do I Choose Rice as the Topic?

On the global scale, rice is an important food. Over half of the world's population depends on rice as a staple food. World organizations have been researching the trends of consumption, production and price of rice for many years. A major area of research is to increase rice production to meet the demands in population growth and fight against hunger. In this unit, students will have opportunities to estimate using some of the some of the statistics related to rice consumption and production. I hope this will help student to gain a wider perspective on a familiar food.

On a personal scale, rice is relevant to our lives since it is a food we often enjoy.

Rice appears in many delicious dishes around the world. There is congee from China, sushi from Japan, pilafs from India, risotto from Italy, risotto, paella from Spain. When food is mentioned or present in the classroom, motivation and interest seems to heighten naturally. I choose rice for the topic as it relates to a cooking lesson in one of our cultural units. The idea came to mind because I had to estimate how much of the rice to prepare for the cooking lesson.

What is a Zong, aka Rice Tamale?

(photos by me)

As part of the Chinese language arts component, the 6 th grade classes learn about the Dragon Boat Festival, which is the 5 th day of the 5 th month on the lunar calendar and usually falls in June. One of the traditional foods for this festival is zong, a kind of rice tamale, rice with other ingredients wrapped in bamboo leaves and cooked by boiling or steaming. The rice tamale, which is also eaten by many Southeast Asian cultures, is sometimes called a rice dumpling. In Mandarin, the official Chinese dialect, it is called zong or zongzi. In this paper, I will refer to it as zong.

Wrapping of the zong is traditionally a family affair. Every family would make their own and give them as gifts to friends and relatives. These days, a variety of the zong can be purchased from Asian markets. The ingredients, size and shape of the rice tamale may vary a great deal, representing the many different regions and subcultures of China. The savory kind of zong may have peanuts, red beans, mung beans, chestnuts, shitake mushrooms, red onions, pork, Chinese sausages, dried shrimps, dried scallops, salted duck egg yolks, etc. There are also sweet kinds of zong with dates, sweetened bean paste, lotus seeds, chestnuts, etc. However, rice is always the main ingredient. The zong usually calls for glutinous rice, but white rice is also used. In this unit, we will do estimations referring to information on the most common type, the long grain rice. The zong with rice, meat and other ingredients wrapped in bamboo leaves makes a convenient meal bundle. We will establish the zong as the portion to feed one person for one meal.

Objectives/ Math Background

Estimates of Population and Powers of Ten

One of the goals is for students to have a deeper understand of large numbers. For this unit, students will use population figures of different countries to learn about making estimations on the demand of rice to feed the people. It is important for students to be able to read, understand and manipulate the numbers that represent the population of different countries.

Expanded Form and Place Value

When students research for world population, they will come across large numbers with many digits. I think most of my students are able to read and write large numbers in both the compact form and expanded form. They are also somewhat familiar with rounding the numbers to various places. However, these skills are worth spending time to review so that all students are on level ground as we explore other ideas in the unit.

For the purpose of discussions in this section, I will use the estimated figure of the U.S. population 303,824,646 from the online World Factbook website [1] . 303,824,646 is the compact form of writing this number. The English place value names from left to right are hundred millions, ten millions (this number has a zero holding the place), millions, hundred thousands, ten thousands, thousands, hundreds, tens and ones. This number reads as three hundred three million, eighty hundred twenty four thousand, six hundred forty six. The expanded form is the sum of values of all the places. This number in the expanded form is written as:

303,824,646 = 300,000,000 + 3,000,000 + 800,000 + 20,000 + 4000 + 600 + 40 + 6

It may seem simple, but reviewing the language for place value is important for my students. There are language differences in the way the numbers are grouped in English and Chinese. In English, numbers are read in groups of three and in Chinese, numbers are read in groups of four. The most confusing places are the ten thousands place and the hundred millions place. The value of 2 in the ten thousands place in this number is twenty thousand. In Chinese, ten thousand is presented by the word "wan4" (wan is the pinyin, Romanized spelling and 4 defines the tone); the value of 2 would be read as "two wan", not twenty wan. Students often mix the English and Chinese ways of saying these places.

Relative Place Value and Powers of Ten

In addition to reviewing place value names, we will review the meaning of the places and the relationship between the places. This leads us to the idea of the relative place value. When we talk about relative place value, the place to the left of any digit is ten times bigger and the place to the right is ten times smaller. For each place we move to the left, the value increases ten times.

Each digit in a base ten number stands for a multiple of a power of ten. The power represented by a digit depends on its place in the number. This is the principal of place value. For example, the leading digit 3 means 300,000,000 (three hundred million) and is written as 3×10^{8}. The leading digit "3" is called the coefficient in this notation; "10" is the base and "8" is the exponent or the order of magnitude, which is the number of times the base is multiplied by itself.

3x10^8= 3 x (10 x 10 x 10 x 10 x 10 x 10 x 10 x 10) = 3 x 100,000,000 = 300,000,000.

The next 3 stands for 3,000,000 (3 millions or 3 x 10^6). If we write the expanded form of our number using the exponent notation, we have:

303,824,646 = 3 x 10^8 + 0 x 10^7+ 3 x 10^6 + 8 x 10^5 + 2 x 10^4 + 4 x 10^3 + 6 x 10^2 + 4 x 10^1 + 6 x 10^0

Each place value is now showing its order of magnitude. The exponents or the orders of magnitude are sequential as the places move to the left or right. Each increase or decrease of one in the order of magnitude means ten times bigger or smaller. This makes it very important to be sure that each digit is written in the correct place. That is why we put a zero as a placeholder when there is no component for that magnitude. Doing this lets us make sure each digit is in the correct place. For example, we can see that a zero is holding the place of the ten millions (0 x10^7). When we write all the terms out and show all the exponents, we do not actually have to write the terms corresponding to a zero in the number, because they do not contribute anything to the sum. However, when we write the usual compact form of the number, it is essential to write the zero, to make sure that each digit is in the place corresponding to its correct power of 10. In other words, we could drop the term 0 x10^7from the expanded form without changing the sum, but we could not leave the zero out of 303,824,646 without changing its meaning.

The orders of magnitude are useful for making comparisons. There are two 3s and two 6s in different places; they represent drastic differences in value. The first digit 3 on the left represents 300,000,000 = 3 x 10^8 (three hundred million) and the other 3 two places down presents 3,000,000 = 3 x 10^6 (three million). They are two orders of magnitude apart (8-6). The value of the first digit is one hundred times that of the other. Even though most students consider 3 million a large number, it is only one percent 1% of the leading digit 3, relatively small comparing to three hundred million.

There are two six's in this number, representing six and six hundred. Again, since the difference in places is two, one value is 100 times the other value. Students may consider 6 as a small number and six hundred a bigger number. However, comparing six hundred to three million, it is a relatively small part, and to three hundred million, it is an even smaller part of that number. The last three digits of the figure are not significant to the whole number in terms the value of the whole number.

Rounding and Orders of Magnitude

Comparing the value of the digits leads us to think about the significance of the digits in the numbers we use. In fact, population numbers are usually rounded to a few significant figures in powers of ten. Instead of using all specific digits, the U.S. population is usually rounded to 300,000,000 (three hundred million). What does dropping the other digits mean? If we divide the part that was dropped by the specific number, it only amounts to about 1.3% (3,824646/ 303,824646 = 0.0127). This is not a big part of the number.

More importantly, population numbers are not static numbers. There are many factors that affect the actual number of people in a country. Death, births and immigration are the three main factors that change the U. S. population numbers constantly. Probably no one actually knows what the population is, to within hundreds, or perhaps even thousands or ten thousands. The U.S. Census Bureau website [2] estimates one birth every 7 seconds, one death every 14 seconds and one migrant every 30 seconds. In summary, the U.S. population is increased by one person every 10 seconds, which is about 9,000 every day. According to this figure, the U.S. population will increase by about three million people each year, about 1% increase annually.

(1 person/ 10 seconds x 3600 seconds/ 1 hour x 24 hours/ 1 day x 365 days/ 1 year

= 3,153,600 persons)

One can find many different figures representing the U.S. population on the Internet. The July 2008 estimates from the World Factbook website provided the figure of 303,824,646 for our earlier discussions. Information on the Wikipedia web page projects the U.S. population to be 304,706,028. [3] The discrepancies in the figures reflect the difference in the calculations of the estimates, not the actual population number. The only part that is consistent, at least for now, is the first two leading digits 3 and 0 representing three hundred million. So it is safe to say that the population of the U.S. is about three hundred million. Rounded numbers can be easily notated in powers of ten. The U.S. population can be written as 3 x 10^8.

The idea of relative value helps to explain using rounded numbers in estimations. In general rounding rules, we round up to the next place value when a digit is 5 or more. If we are rounding in a place two or more to the right of the leading digit, the added value to the number is relatively small. Similarly, in rounding down, we are giving up a relatively small part of a number. In many situations, the acceptable relative error of estimation is within ten percent of a number because it is a relatively small amount compared with the whole number. If the rounding error is less than 1% of the whole number, we usually can ignore it, and if it is less than 0.1%, we can almost always ignore it.

One benefit of rounding is that the numbers are easier to work with in calculations. It would be cumbersome to do calculations using all the specific digits. With this in mind, in this unit, we will use population numbers of one to two non-zero digits.

Comparing rounded numbers expressed in powers of ten notations can be quite easy. The order of magnitude makes it clear right away which number is bigger. For example, compare the population of Hong Kong (7 x 10^6) and the population of Taiwan (2 x 10^7). Taiwan's population with a magnitude of seven tells us that the number is in the ten millions place. Hong Kong with a magnitude of six only has population in the millions. The order of magnitude (7 and 6) tells us that Taiwan has a lot more people than Hong Kong.

The order of magnitude also makes calculations of comparison easy. If we go on to ask how much bigger is Taiwan's population, students usually will resort to subtraction and find the difference of the numbers. Students may say that Taiwan is more populous than Hong Kong by thirteen million people (20,000,000-7,000,000 = 13,000,000). However, they can be challenged to go deeper by thinking about how many times bigger or smaller are the two numbers in comparison. We can rewrite the numbers to the same order of magnitude to make comparisons less complicated. Taiwan's population can be rewritten as 20 x 10^6 . If we divide two numbers, the powers of ten (10^6) will cancel out and we will have twenty divided by 7: (20 x 10^6 / 7 x 10^6 = 20/7), which is about 3. By this, we can see that Taiwan's population is about three times that of Hong Kong. In this unit, one of the goals is to train students to think about the difference in terms of multipliers or percentages when comparing two values. This way of thinking would be also useful when we compare the rice demands for countries with similar rice eating habits.

Scaling and Proportional Calculations

This kind of comparison leads us to our next topic of scaling and proportional calculations for finding estimates. Many children's books on estimation show clump or group counting strategies to find estimates. However it is presented, the core ideas are scaling and proportions. If we know a ratio of certain quantities, we can scale up for a larger quantity or scale down for a smaller quantity using proportions.

Unit Rate- Zong, a Meal Wrapped Up

For this unit, we will establish the zong as the portion to feed one person for one meal. The amount of rice that goes into one zong will be used for estimations throughout the unit. The unit of measure I have chosen is the gram. The metric units will work nicely in our calculations with the powers of ten when we scale up to large populations.

In the classroom, I would have the student do an activity to find the average weight of rice in a zong. Below is a description of a couple of ways I tried to determine how much rice goes in a zong without doing the student activity.

The first method is to look at recipes to see how much rice goes into one zong. I took the average of the quantities, which comes out to about 55g for each zong. (100g + 50g + 40g + 30g)/ 4 = 220g/4 = 55g

Another method I used is to work backwards from a previous experience. Last school year, I prepared a total of 10 pounds of rice for the zong cooking lesson. 10 pounds is approximately 4540 grams. (10lb x 454g/lb = 4540g) I estimated that we made about 100 zong. The average for each zong was about 45g. (4540g/100 zong = 45.4g)

The average of the two numbers comes out to 50g of rice for each zong (55g + 45g) / 2 = 50g). For the rest of the discussion and calculations here, I will use 50g of rice for each zong.

Scaling Up- Ratios and Proportions

When students have the quantity of rice in one zong, we can use this ratio to scale up to make estimates for our cooking lesson and other situations of rice demands. A proportion is set up with two equal ratios. We have one known ratio of 50g per zong. The other ratio will consist of the unknown part that we want to estimate. If we were given a population number, we can estimate how much rice we need. If an amount of rice is given, we can estimate how many people we can feed. For now, we will focus on how much rice we will need to feed people for one meal. We will look at two examples. First we will estimate for our cooking lesson, then we will estimate for the population of the U.S.

To find the amount of rice needed to make zong for our cooking lesson, we need to know how many zong to prepare or how many people to feed, assuming one zong for each person. We have 60 students in 6th grade and about 25 staff in the school, and we can expect about 10 parent volunteers to help in this activity. If each person gets one zong, the minimum of zong to prepare is 95. To be sure we do not run short, it is always better to have more rather than not enough. We should round up to 100 to be sure we have enough zong for everyone. It is also easy to multiply by 100.

To find the total or rice for the cooing lesson, we can simply multiply the unit rate by the number of people

(50g x 100). However, this may also be a good place to teach about setting up proportions. Below is the proportion to find the amount needed for the cooking lesson to prepare to feed 100 people. Labeling the units and correct placements of the ratios are important. The units of the ratios must match up for both sides. To solve for the proportion, we can cross multiply and then simplify.

x/ 100 people = 50g/ 1 person

x = (50g) (100) (cross multiply, units cancel out for people)

x = 5000g = 5kg (convert grams to kilograms)

This would be a natural place to introduce or discuss with the students about unit conversions. With 50g/ person, the amount for 100 people must be larger by one hundred times. Five does not sound like a reasonable solution. If students know that 1kg = 1000g, then the number 5 would make sense. (5000 g = 5kg) We cannot emphasize enough the importance of labeling the units in a proportion. It would be also appropriate to show students how we can use this estimate to help us make decisions in a real life situation. If possible, bring in bags of rice or advertisements with rice sale prices. Let students study the labels and decide how many bags or rice to buy. We want to buy the amount that is closest to what we need. However, sometimes our decision is also dependent on other factors, such as the size of the packages and the price of the goods.

Next we will estimate the amount of rice needed to make one zong for everyone in the U.S. We set up the proportion the same way with two equivalent ratios. Here I put the unknown value x on the top of the ratio. As long as the units in the ratios match on both sides of the equation, the proportion would work.

x /3x10^8people = 50g / 1 person

x = (50g) (3x10^8)

x = 150 x10^8g = 1.5 x10^10g (15 billion grams of rice)

The result is a large number in grams. The next natural step is to convert the number in to a more appropriate unit. Converting it to kilograms would still give us a very large number. We then convert it to an even bigger unit of measure, the metric ton. Students may not be very familiar with the metric ton, but they should be able to apply the methods of unit conversion for it is an important skill in 6 th grade. One of the goals of this unit is to help students develop understanding and skills in unit conversion.

Unit Conversions

The way of writing the conversions below is probably different from the presentations in our textbooks. I think this way would be clearer to the students. We take the number that we have and multiply it by the conversion rates of the desired units. Writing the numbers in the fraction form will help with dealing with the units. If the conversion multiplication equation is set up correctly, the units will cancel out as in reducing fractions, leaving us with the units of measure that we want. Students can also learn that it is easy to divide by powers of ten by subtracting the exponents. Convert the 1.5 x10^10g to kilograms results in fifteen million kilograms. It is still quite a large number. We can further convert it to a larger unit, the metric ton. One metric ton is equal to 1,000,000g or 1000kg. Convert the grams to metric tons:

1.5 x 10^10g/ 1 x 1kg / 1000g x 1t/ 1000kg = 1.5 x 10^4 t (fifteen thousand metric tons)

This is a very large number and may be difficult for students to imagine how much rice this number represents. To help students to visualize the amount rice, we will introduce the idea of storage space. How big a space does it need to hold this amount of rice? Another application of proportions in this unit is translating from the mass of rice to volume when we explore the storage space issue.

Rice storage

Mass and Base Area

The purpose of the explorations on area is to prepare the students to make estimations in storage solutions and land needed in the production of rice. The volume of a rectangular prism is the base area times the height of the prism. A review on finding areas is a natural start to this section. Students will start by comparing the areas of squares.

For example, compare the areas of a square with sides of 4cm and one with sides of 6cm. The area of the 4cm square is 16 cm^2 and the 6cm square is 36 cm^2 . In fact, with a difference of only 2cm in length on each side, the area of the 6cm square is more than doubled than that of the 4cm square. It is hard to see or predict the difference in size when we compare the sides of the squares by subtraction (6cm-4cm =2cm). However, comparing the sides of the squares in a ratio makes mathematical sense. The ratio of 6 cm to 4 cm shows that it is 1.5 bigger on each side. The square of this factor (1.5 x 1.5) shows that the area is 2.25 times bigger. An example with whole numbers may be easier for students to grasp the idea. The area of a 3 cm square is $9cm^2$. If the sides doubled in length, the larger square's area (6cm x 6cm = $36cm^2$) will be four times the original size. 36cm $^{m^\wedge 2}$ / 9cm^2 gives the ratio of 4 to 1.

The same kind of ratio of change would apply to the rectangle. The area of a rectangle is its width times the height. In similar rectangles, the ratio of the change can be determined by finding the factor of change of the length and multiply by the factor of change of the width.

Now, relate this to our rice estimates. We can start by spreading 50g of rice in one single layer in a rectangle. I used a picture frame to help with this. The edge of the frame was helpful to contain the rice in the rectangular shape. The width of the frame is 20cm. 50g of rice filled up to the 15cm mark. The area of the rectangle is 300cm^2. Now we have the basic ratio of 50g to 300 cm^2. This means that one gram of rice covers approximately 300/50 = 6 cm^2.

Students can use this ratio to calculate how much space is needed to spread out the rice for our cooking lesson or other populations. For example, we estimated 5kg of rice for our cooking lesson. Using proportions, we will get:

x / 5kg = 300 cm^2/ 50g

x / 5000g = 300 cm^2/ 50g

x = 300 cm^2 x 100 (We can also simply multiply by 100 for 100 people)

x = 30,000 cm^2 = 3m^2

It would be impractical to really spread out the rice. But this is a good opportunity for students to discuss the dimensions of the rectangle that would give us this area. If it's a square, the sides are about 173cm each (square root of 30,000 cm^2). A rectangle of this area can be 1m x 3m, 2m x 1.5cm, 600cm x 50cm, 500cm x 60cm, etc. This discussion is to start the students thinking about the base area of a prism as we move into relating the mass to volume.

Mass and Volume

We can apply the same idea of change in area ratio to volume by adding the height dimension. The volume of the rectangular prism will change according to the ratio of the change in the length, width and height.

We will compare cubes with sides of 4cm and 6cm. With only 2 cm in the length of the sides, how much bigger is the 6 cm cube? Again, looking at the ratio of growth of the sides, not by subtraction, would give us a better idea of the change in the volume. The base area of the larger cube is 2.25 times bigger than that of the smaller cube. We can multiply 2.25 by the growth in the height, which is 1.25, to find the change in volume (2.25 x 1.5 = 3.375). The volume of the 6cm cube is more than 3 times but smaller than 4 times the volume of the 4cm cube. The actual volume of the 4cm cube is 64cm^3 and the 6cm cube is 216c^3.

To estimate rice storage space, we need to first establish a ratio of mass to volume. In the classroom we will build rectangles or squares with Cuisenaire rods. The rods will form wells with the height of 1cm. Students can fill the wells and then measure the weight of the rice.

I poured the 50g of rice into a 4cm cube made out of paper. The rice filled up the cube to about 3cm high. The volume of the 50g of rice comes out to 48 cm^3 (4cm x 4cm x 3cm = 48cm^3). I think students will find the area and volume figures surprising (the area of 300 cm^3 and the volume of 48cm^3) since they both represent 50g of rice. However, if we think about the height of the rice in the box, the rice is stacked in layers. Dividing 300cm^2 by the base area of the 4cm cube, we see that there should be about 19 layers of rice (300cm^3/ 16cm^2 = 18.75). That would mean that there are about 6 layers of rice in a stack one centimeter high. Area and volume figures are two different things and cannot be compared side by side. This is an opportunity to reinforce the importance of labeling area and volume units correctly.

To estimate for volume, I rounded 48cm^3 to 50 cm^3. I then converted the ratio to larger units (50cm^3/ 50g = 1cm^3/ 1g = 1dm^3/ 1kg = 1m^3/t).

The amount of rice we estimated for the zong cooking lesson is 5 kg. The estimated volume using this ratio is 5000 cm^3 (1dm^3/ 1kg x 5kg = 5dm^3= 5000 cm^3). During the teaching of the unit, we can probably compare the actual volume of the rice with the estimated volume.

The ratio 1m^3/t is a little smaller than the standard hulled rice weight to mass conversion ratio. But I think it still gives a general idea of the size of the piles of the rice. I have big computer boxes that are close to $1m^3$ in size that will make a good visual for the volume of 1 metric ton of rice.

The mass and volume ratio of hulled rice is 1.328 cubic meters to 1 metric ton according to the online-converter.com website. We will round the ratio to 1.3. We have determined that we need 15,000 metric tons of rice to make one zong for each person in the U.S. Using 1.3m^3/ t, we set up this proportion to estimate for the U.S. population:

x/ 15,000 t = 1.3 m^3/ 1t

x = 1.3 m^3/t x 15,000t (this is the simple step without setting up the proportion.)

x = 13 m^3 x 1,500

x = 19,500 m^3

For the ease of calculation, we can round 19,500m ^{3} to 20,000 m^3. It is a 2.5 % change in value, which is a relatively small difference, but it is much easier to find the dimensions for our storage solutions. Now we will explore the dimensions of rectangular prisms that have this volume. Below are some examples of heights, widths and lengths:

Height	**Base Area**	**Width x Height**
200m	100m2	5m x 20m, 10m x 10m
100m	200m2	5m x 40m, 10m x 20m
50m	400m2	10m x 40m, 20m x 20m
40m	500m2	10m x 50m. 20m x 25m
20m	1000m2	10m x 100m. 20m x 50m, 40m x 25m
10m	2000m2	20m x 100m, 40m x 50m

It may be good to compare the dimensions on the list with the classroom or school building to make a visual connection. Some of these dimensions are not practical dimensions for rice storage, for example, the walls may not hold up to the weight of the rice if the footprint is small and the walls are tall. This is a good place to have a discussion with students on the criteria for deciding which dimension is best.

Rice Production- Land Area

The last idea we will explore is how much land is needed to produce the rice that we need. USDA's Rice Outlook report on rice production stated the global rice yield of 4.1 metric tons per hectare (rough-basis) [4] in the last few years. Polished rice, or white rice, is about 68% of the weight of rough rice. [5] The production figure is adjusted to 2.78 t/ ha. (4.1t/ ha x 68% = 2.78t/ ha) The change of 2.78 to 2.8 is less than 1%. The change from 2.78 to 3 is about 8%. This is noticeable but not too large. I choose to use 2.8t/ ha for the calculations below to estimate for the land needed to produce the rice for our cooking lesson. I thought it would be easier for students to figure out the possible lengths and widths of the rectangle area of 18m ^{2} . However, we can probably round to 3t/ ha for other calculations. Here is the conversion from metric tons per hectare to grams per square meter:

2.8t/ 1 ha x 1 ha/ 10,000m^2 x 1000kg/ 1t = 2.8kg/ 10m^2 x 1000g/ 1kg = 280g/ 1m^2

The land needed to cultivate the 5kg of rice for our zong cooking lesson is about 18m ^{2} of land:

28kg/ 100m^2 = 5kg/ x

x = (5 x 100/ 28)m $^{m\text{^}2}$ = 17.86m^2 = 18m^2

18 square meters could have several dimensions: (2m x 9m, 3m x 6m, 1m x 18m). It would be interesting for students to see how to fit this space in the classroom. An extension can be to find out how much rice we can grow in the classroom.

So far, all the estimation we made has been for one meal for one person. For extension and practice of the estimation concepts, we can have the students discuss and compare rice eating habits of their families and different cultures and what effects will that have on the demand of rice. We can add the time element to our estimations. Instead of estimating for one meal, we can estimate for a day, a week, a month, a year, etc. The students will have the opportunity to explore these estimations in their unit projects.

Strategies

Hands on experiences are the most memorable for students. In each lesson, I will start with an exploration activity that relates to the skills or concepts as a motivation activity. Explorations and investigations allows for students to make discoveries of the new math knowledge. In many of the investigations, having students come up with a working plan will encourage them to be active participants and thinkers. In this unit, many of the activities require students to synthesize their prior knowledge of different math areas. Students may discover the mass of rice can be related to area problems and that powers of ten can be used in area and volume calculations.

Students will solve three general types of problems in this unit:

1. Given a demographic location, find the population and use that figure to find the mass of rice needed to feed the people, that is, make them one zong each.
2. Given an amount of rice in weight, find the volume and dimensions of a storage container to hold that the amount.
3. Given an amount of rice in weight, find the area of land to produce this rice.

Problem solving is usually not a one step exercise. Students need to utilize many skills: they need to understand the problem, identify the relevant facts, know what is missing, devise a plan, recall or research information, etc. in order to find a solution. Also, finding the correct calculation does not end the process. They need to organize the information and share it with the class.

In each lesson, we will define the specific situations for each of the general questions above. The lessons will include direct teaching using the zong cooking lesson to demonstrate the process and core contents. We will practice the steps and skills with the U.S. populations through guided practice. Students can practice the skills further by choosing a different country to do estimations. We start with the more concrete and tangible situation of the zong cooking lesson related problems and move onto more abstract situations of the U.S. population. I want students to see that the strategies and steps are the same, even though the numbers or situations may be more complicated. They can apply the strategies of the simpler situations to more complex situations by breaking down the problems to smaller problems.

One of the key focuses in my school district to improve teaching is to differentiate instructions to meet the needs of all students. Through investigations and explorations, I will challenge more advance students with complex problems and break down the problems even further for basic students. This unit provides a variety of activities that will meet students where they are. There are also many extension possibilities relating to the topic of the unit.

The cumulative project at the end of the unit can serve as an assessment for the unit. This is an open-ended project where students can revisit and demonstrate the skills they have learned in the unit.

Classroom Activities

Each lesson of the unit will take two to four class sections of 50 minutes to complete. If all the lessons are done together, this estimation unit will probably spread over about four weeks to five weeks. There are many essential 6 th grade math topics involved, so I will likely spread out the lessons as we come across them through out the semester. The cumulative project may be assigned closer to the end of the school year as we prepare for the zong cooking lesson and celebrate the Dragon Boat Festival. I hope the students will remember the things they have learned about estimation when they enjoy the zong.

Lesson 1- Population

Exploration/ Motivation

Play a game of hangman using the U.S. population figure 303,824,646. Students have to use the correct place value names to make a guess of the digits. After they have guessed the number, have a short discussion about how accurate this number is. What are the factors that may change this figure? We want the students to understand that this figure is only an approximation. It is nearly impossible to know the number of people in the U.S. precisely at any given moment due to births, deaths, migrations, etc. An option is to incorporating a quick computer search lesson by looking up the information online with the students in class.

Direct Teaching

Rewrite 304,583,860 in the expanded form and in the order of magnitude notation. Explain the notation and ask the students to describe the pattern they see in the notations. Compare the values of the two 3s and 6s in the number. Lead students to compare the places by thinking in multiplication terms.

Now introduce the rounded figure of the U.S. population (3 x 10^8). Go over the parts of the notation and their meaning. Ask students if it makes a big difference if we take away each one of the places starting from the largest place. The effect is huge with the hundred million place. Eliminating the ones place does not matter much. Furthermore, Show students how to write 3 x 10^8in different magnitudes.

Guided Practice

Show the population of Hong Kong (7 x 10^6) and Taiwan (2 x 10^7). First ask them to decide which place has more people just by looking at the numbers. Next, have the students examine the numbers more closely. How are the coefficients different? How are the orders of magnitude different? How do the numbers compare to each other? Again, encourage them to think in multiplication, not in subtraction. Rewrite the numbers to the same order of magnitude with the students and show them how to divide by subtracting the exponents.

Reinforce the understanding of the powers of ten by giving data for other places and have students compare the numbers. Provide pairs of numbers with either the same coefficients and/or the same order of magnitude. Also, compare other pairs of populations with a larger multiple. The difference may be nearly the same as the larger country. The smaller country might even be lost in the rounding error of the larger country if we round to a large place. The 20 million population figure for Taiwan is rounded from 23 million. The amount lost in rounding is nearly half the population of Hong Kong.

Independent Practice/ Assessment

As a conclusion to this lesson, give students a list of population figures and country names. Have the students work cooperatively to put the population numbers in sequence from smallest to largest and match the country names with each population. Encourage them to think about what knowledge do they have that would help them to make the match ups. A list of countries and population figures is included in the appendix. An extension activity can be to have students find the top ten countries of rice consumption and or production.

Lesson 2- Ratios and Proportions

Exploration/ Motivation

Working in small groups, let students dissect a zong to discover the ingredients in it. Each group must discuss and device a plan of action and have it approved from the teacher before starting to dissect the zong. The plan should contain their steps in sequence and each person's responsibility. If it is possible, find zong in different sizes to show the variations. That will provide the opportunity to discuss about finding an average and what it means to estimate. Each group will make a poster and give a report at the end of the activity that must include these information: the net weight of the rice, the weight of the other ingredients, the total weight of the zong and whether this figure includes the bamboo leave wrapping and string, and the percentage of the rice compared to the total weight. The class can compare their findings to recipes. At the end, the class as a whole will determine an average amount of rice to go into one zong.

Direct Teaching

After the class finds the weight of the rice needed to make one zong, we will learn how to to do estimations by using proportional reasoning. The problem solving question is how much rice to prepare for our zong cooking lesson. Ask the students to think about what information do we already know and what other information do we need. We will know the amount of rice for one zong from the dissection activity. We need to determine how many zong to prepare or how many people to feed. Show students how to set up proportions correctly and how to solve it using cross multiplication. Some students may figure out that we can simply multiple to get the quantity of rice. However, some students may be confused about when to multiply or divide to solve these problems. Setting up the proportions gives us a way to trace out steps and check our thinking.

Guided Practice

Practice the proportion skills using the U.S. population. The new problem solving question is how much rice do we need to make one zong for each person in the U.S.? This time, let students come up with the information that they already have and the information that is missing. Guide them through the steps to set up the proportion. Ask students to explain how it works. Talk about the possible mistakes in the proportions and how to check if it is correct.

Independent Practice/ Assessment

Students can choose a country from the list of populations from the first lesson to find the rice quantity needed. An extension may be to include the frequency element. Instead for preparing one zong for each person, how does the estimates change if two people would share one zong or eating zong for three meals a day?

Lesson 3- Area and Volume Investigation

Exploration/ Motivation

Have students use Cuisenaire rods to build squares for various areas. Fill the square with one layer of rice and measure the weight of the rice. To help make sure the rice is layered one grain thick, I suggest using a ruler or paper to press down. Repeat this with squares of several sizes. Ask students to record their measurements and make observations of the data. Discuss how they can predict the difference in the mass of rice to fill the different squares. Again, lead students to think about the change in multiplication terms. After investigating with area, perhaps in another session, students can make cubes out of cardboard and centimeter paper to explore about volume of different sizes.

Direct Teaching

The problem solving question for this lesson is what is the volume requirement of the storage place to hold the rice for our zong cooking lesson? We already know that we need 5 kg of rice for our cooking lesson. We need to know the volume to weight ratio to complete a proportion for our estimation. We can either use information from research or we can use figures from our volume investigation to write the proportion to solve this problem. This is also the good time to show students how to decide on the appropriate units and how to do unit conversions.

Guided Practice

Again, here the problem solving question relates to the U.S. population. The question is what is the volume of a storage shed to hold 15,000 metric tons of rice? Since the number is so huge, we may want to help students break down the problem to more manageable parts. We can first find out how big is the volume for 1 metric ton of rice. Then, we can use the new ratio to calculate for the 15,000 metric tons.

Independent Practice/ Assessment

Here, students can use the volumes of rice they estimated earlier in lesson 2 to practice finding volume of the storage spaces. A variation of this exercise is to put the rice into boxes. Determine the volume of the box and how many boxes of rice we have. Use the volume of the boxes to determine the total volume. Or convert the usual 50lb bags of rice to metric measures and determine how many bags of rice we would have. Then calculate how many bags will fit in our classroom and how may classrooms we need to store all the bags. We can also find the volume of a storage shed in our school and calculate how many sheds we need and how big is the land to fit all the sheds.

Cumulative Rice Project

Students will work in small groups to complete a project on estimations relating to rice.

They will create their own problem solving question, define the smaller questions, devise a plan, solve the problem, organize the information and present it in a report to the class. Each group will customize a rice problem by selecting a place in the world, choosing a recipe to use and deciding on a time criterion for the estimation. They must estimate for the rice quantity needed in weight for the population and devise a storage solution for the rice.

Alternatives for basic students may include allowing them to use the rate of the rice usage in the zong activities, choosing a more manageable size of population to work with and/ or estimating for a more manageable time frame. Options for proficient and advanced students may include researching other elements relating to rice, such as water consumption, comparing rice yield and land usage of different states or countries. For students who are highly interested, they may expand to other grains or other types of foods.

Resource

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Reading List For Students

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Materials for Classroom Use

Calculators

Centimeter paper

Cuisenaire Rods, classroom set

Metric Rules

Metric Scales and weights

Paper for making rectangular prisms, may be preprinted with templates

Uncooked Rice (short grain, long grain or glutinous rice), 10lb or 5kg

Zong (rice tamales), available in Asian markets

Appendix

Population

Countries	Population	Rounded to One Significant Digit	Rounded to Two Significant Digits
World	6,671,226,000	$7x10^9$	$6.7x10^9$
China	1,324,891,000	$1x10^9$	$1.3x10^9$
India	1,135,273,000	$1x10^9$	$1.1x10^9$
United States	304,575,000	$3x10^8$	3.0x108
Brazil	187,242,000	$2x10^8$	$1.9x10^8$
Japan	127,690,000	$1x10^8$	$1.3x10^8$
Mexico	110,009,988	$1x10^8$	$1.1x10^8$
Egypt	75,046,000	$8x10^7$	$7.5x10^7$
United Kingdom	60,587,300	$6x10^7$	$6.1x10^7$
Italy	59,619,290	$6x10^7$	$6.0x10^7$
Spain	46,063,000	$5x10^7$	$4.6x10^7$
California	38049,462	$4x10^7$	$4.8x10^7$
Canada	33,315,700	$3x10^7$	$3.3x10^7$
Taiwan	22,990,000	$2x10^7$	$2.3x10^7$
Greece	11,147,000	$1x10^7$	$1.1x10^7$
Israel	7,282,400	$7x10^6$	$7.2x10^6$
Hong Kong, SAR	6,963,100	$7x10^6$	$7.0x10^6$
Ireland	4,339,000	$4x10^6$	$4.3x10^6$
San Francisco	824525	$8x10^5$	$8.4x10^5$
Macau	538,100	$5x10^5$	$5.4x10^5$
Bahamas	331,000	$3x10^5$	$3.3x10^5$
Bermuda	65,000	$7x10^4$	$6.5\ x10^4$
Monaco	33,000	$3x10^4$	$3.3\ x10^4$
Cook Island	20,200	$2x10^4$	$2.0\ x10^4$
Vatican City	800	$8x10^2$	$8.0\ x10^2$

Unit Conversions

Mass

The standard metric unit of mass is 1 gram (g).

1 kilogram (kg) = 1000 g

1 metric ton (t) = 1,000,000 g =1000 kg

Length

The standard metric unit of length is 1 centimeter (cm).

1 decimeter (dm) = 10 cm

1 meter (m) = 100 cm

1 hectometer (hm) = 100 m

1 kilometer (km) = 1000 m

Area

1 m^2 = 100 cm x 100 cm =10,000 cm^2

1 hectare (ha) = 100 m x 100 m = 10,000 m^2

1 km^2 = 1,000,000 m^2 = 100 hectares

Volume

1 dm^3= 1000 cm^3

1 m^3= 1000 dm^3

Weight to Volume

1 metric ton = 1.328 cubic meter

Implementing District Standards

The San Francisco Unified School District adapts the Math Contents Standards from the Mathematics Framework for California Public Schools, Kindergarten Through Grade Twelve (2005). This unit addresses concepts in three main strands. [6]

Strand: Number Sense

Standard 1.0 Students compare and order positive and negative fractions, decimals, and mixed numbers. Students solve problems involving fractions, ratios, proportions, and percentages: 1.2, 1.3- Students set up proportions to estimate mass, area and volume relating to rice.

Standard 2.0 Students calculate and solve problems involving addition, subtraction, multiplication, and division: 2.1, 2.2, 2.3- Students use the rate of rice quantity to estimate for larger populations and convert to appropriate units of measurement.

Strand: Measurement and Geometry

Standard 1.0 Students deepen their understanding of the measurement of plane and solid shapes and use this understanding to solve problems: Students explore scaling factors of area and volume to find rice storage solutions.

Strand: Mathematical Reasoning

Standard 1.0 Students make decisions about how to approach problems: 1.1, 1.2, 1.3- Students learn to understand a problem and how to break the problem into smaller parts. They determine the relevant and missing information and device a plan to solve the problem.

Standard 2.0 Students use strategies, skills, and concepts in finding solutions: 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7- Students apply strategies from simpler problems to more complex problems by using logical reasoning and arithmetic techniques. They learn to use appropriate math notation to express solutions and communicate the methods and the reasonableness of their calculations. Students approach problems more than one way and explain the mathematical reasoning for the differences and similarities of the methods.

Standard 3.0 Students move beyond a particular problem by generalizing to other situations: 3.1, 3.2, 3- Students will design a problem and a plan to solve it by utilizing the skills throughout the unit.

Endnotes

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