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Estimation in Ecology - the Horseshoe Crab Census

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Overview

Estimation is a vague topic in most Math classes, regardless of the grade level. In most of the workbooks that I use with my sixth grade students throughout the school year, we see the word estimation in many of the questions. What we don't see however, is an explanation as to what estimation is, what it is used for, or how to use it. Too often estimation is thought of as merely a guess and not much more. Estimation is in fact not a guess, nor an educated guess, but rather an effective way of calculating an approximate answer to a question. Estimation is based on Math, not on a guess, educated or otherwise. There are several math concepts that students must be familiar with in order to effectively utilize estimation in the classroom including, place value, expanded notation, very round numbers, and order of magnitude. A familiarity with the powers of 10 would be beneficial as well. Since these concepts are ones that we teach in the sixth grade, I have incorporated teaching them into this unit.

At the beginning of each school year, I go through the same ritual with my Math students. I ask how many of them love Math. As a Math teacher, you probably already know where this is heading, but I'll tell you how it usually goes for me. If I get four confidently raised hands out of the 32 in the class, it is a lot. The next question I ask catches them by surprise. I ask how many of them hate Math. There are always the mischievous smiles on many faces, combined with a glance around the room to the other students looking for reinforcements before they commit to raising their hand. I tell them that yes, it is OK to answer the question honestly because it helps me know how I need to approach the class. Then my third question - Why do you hate Math? I usually get the same three answers each year; it's boring, I didn't like my teacher, or simply put, Math sucks! We can even thank Jimmy Buffett for writing a song of the same title, *Math Sucks.* By asking these questions, I get a good idea of the overall feel of the students' attitude towards the subject. My goal? That by the end of the year I can change their perception of Math and their preconceived notion that they cannot do well in it because they never have in the past. As teachers, if we can make the classroom environment lively and entertaining then we can hold their attention long enough to teach them Math and get them engaged in our lessons. We must also make our lessons as interesting as we can, which is why I chose the horseshoe crab (*limulus polyphemus)* as the focus for this unit.

Objectives

A recent fossil find in Manitoba places the horseshoe crab's origins at least as far back as 450 million years ago! [1] It is a unique creature that inhabits the Delaware Bay, amongst other places, and most of my students have seen them at the beach. Our school is located within a half hour's drive to some of the most popular horseshoe crab spawning beaches on the planet. Each year, I assist one of our 8 th grade teachers, Garth Stubbolo, on his annual overnight "*Green Eggs and Sand*" fieldtrip to the beaches to count the spawning horseshoe crabs. *Green Eggs and Sand* is a multi-state coordinated environmental education program designed to teach students about the horseshoe crab/shorebird connection in the Delaware Bay. [2] On the fieldtrip, it is always incredible to see the excitement and focus on a student's face as he/she holds a horseshoe crab for the first time and they can actually tell you about the different body parts and behaviors it exhibits. Its importance to the migratory shorebirds, commercial fishermen, the biomedical field, the major controversy surrounding its use and management, plus the fact that it lives right in our "backyard," - all these things combine to make the horseshoe crab a promising subject for creating an entertaining and educational math unit on estimation.

In 1990, the first annual horseshoe crab census was organized by the Delaware Sea Grant to estimate the number of spawning horseshoe crabs in the Delaware Bay. [3] This annual census continues to this day. However, with the implementation of the Atlantic States Marine Fisheries Commision's (ASMFC) fisheries management plan for the horseshoe crab in 2001, a more statistically rigorous scientific spawning survey was developed and has been in use since. [4]

Why the need to estimate the number of horseshoe crabs in the Delaware Bay? Many people are directly affected by the laws pertaining to the harvesting of the horseshoe crab, especially the eel and conch fishermen. They use the horseshoe crab to attract conch and eel to their traps. The male horseshoe crab does not attract the eel the way the female horseshoe crab does, therefore, the eel fishermen are mainly interested in harvesting the females. Since the females are the ones who lay the eggs, many environmentalists worry that the harvesting of the females could be, if it has not been already, detrimental to the future of this species. To ensure future generations of this ancient mariner, many environmentalists want a complete moratorium on horseshoe crab harvesting. This occurred in Delaware, though the decision was reversed in 2001, mainly due to the fact that the livelihood of the fishermen would be greatly affected, and the fact that the horseshoe crab estimates do not warrant a moratorium at this time. [5]

Another group of people who are interested in the laws affecting the harvest of the horseshoe crab are those who are concerned about the plight of the different species of shorebirds. There are many species of shorebirds that briefly stop over in the Delaware Bay in May to gorge themselves on the eggs of the horseshoe crabs, which are laid in the sand of the beaches. The horseshoe crab nests are usually just a few inches deep and are often disturbed by other nesting horseshoe crabs, thereby scattering many eggs near or on the surface making it easy for the shorebirds to eat them. One female horseshoe crab can lay up to 100,000 eggs in one season, so there are billions of eggs on the beaches during the peak mating season. [6] Although this is a huge number of eggs, with 1,000,000 or so shorebirds arriving in the Delaware Bay to feed on the eggs, some people believe that these numbers need to be sustained for the survival of the birds. The shorebirds are on an annual migration from the southern tip of South America to their nesting grounds in the Arctic, a trip that covers 10,000 miles for many of them. Some people feel there is a direct correlation between the number of horseshoe crabs and the number of shorebirds. This would mean that a decline in the number of horseshoe crabs will result in a decline in the number of shorebirds. (One species, a subspecies of the red knot, the most long-distance migrant to visit the Delaware Bay, has declined dramatically in population over the last decade and is now being considered for addition to the federal government's Endangered Species List. [7]) The shorebirds also bring in an estimated annual income between $30 to $70 million combined, for Delaware and New Jersey, due to eco-tourism. [8] There are many people who come to the Delaware Bay each May to see the shorebirds and their visits generate money for the local economies. This money would certainly be missed if the shorebirds numbers decline and they no longer offer the current "Wow" appeal based on their sheer numbers.

The third major group that is interested in the laws governing the horseshoe crab is the biomedical industry. The coastwide biomedical industry currently has a harvest of approximately 500,000 horseshoe crabs, which are bled, then returned to the wild. It is estimated that 10% to 15% of these crabs do not survive the bleeding process. [9] Why is the biomedical field interested in harvesting horseshoe crabs? Actually we are all benefited by the horseshoe crab's contributions to the medical field and to our own lives. Horseshoe crab blood is used to test that medicines (those that are injected) are free of contamination from bacterial endotoxins. [10] This is the main current biomedical field use of horseshoe crabs. However new research is suggesting promise of even more benefits to human health, including discoveries in anti-viral and anti-cancer activities in the proteins of the horseshoe crabs' blood. [11]

Mathematical Background

Now that you have a better understanding as to why estimating the horseshoe crab population in the Delaware Bay is important to many different groups of people, let's teach the students estimation using the place value system, expanded notation, very round numbers, order of magnitude, and the powers of 10.

Each of the math concepts listed above is directly connected to the others when teaching estimation. If you don't teach your students place value, how can you expect them to understand the relative sizes of different numbers. To understand that one number is 10, 100, or 1000 times larger than the other, the student must have a familiarity with the names within the place value system. How do you teach place value without a discussion leading into expanded notation? Expanded notation often helps students make sense of the place value system by showing them how the large numbers are broken down into smaller numbers, based on the place value system. It is almost impossible to teach expanded notation without a discussion on the concept of base ten. All of the topics are intertwined and each of them must be taught. However, the order in which they are taught may be more of an individual decision. Each school district, and perhaps even schools within the district have their own way, and order, of teaching these concepts, so do not feel obligated to teach them in the order that I have set forth in this unit. As long as the topics are taught and the students are able to use those topics with little teacher assistance, then the students will be able to accurately answer estimation questions.

For this unit, we will begin with a look at the place value system, also called the decimal system. It is important to get your students to realize the decimal system is the place value system because too often when they think of decimals, they only think of the numbers that are to the right of the decimal place. What I find to be the easiest way to begin is to simply ask the students what they remember about the place value system. In each of my classes there have always been a few students who were able to lead a discussion on the organization of the place value system. If you don't have any students volunteer for this, then a good place to start may be asking the students if they know why we write commas in our numbers. This is a good way to introduce the concept of periods in the place value system. You can also use money as a way of explaining the place value system. They always seem to have a good grasp of money related conversations. I like to begin by putting a decimal point on the board and then asking them what place value is in the first place to the left of the decimal. Once they give me the correct answer of ones, I like to organize their notebooks for this lesson. I have them turn their books sideways so the lines are now vertical instead of horizontal. I tell them to write a decimal point somewhere between two of the lines near the top of the page, but to make certain they have at least ten places to the left of the decimal, so we can go out to the billions place. If you want your students to go out further than the billions place, then have them organize their notebooks accordingly. At this time, I have them write the word "ones" in the first column to the left of the decimal. Then I ask what is the place value of the second column, after an answer of tens, I have them write "tens" in the proper column in their notebooks. After we get the correct answer of "hundreds" for the third column, and they write it in their notebooks, I tell them that they have just completed the first "period" of the decimal system. This is when I also explain to them that we write the commas in our numbers to separate the different periods. In their notebooks above the words, hundreds, tens, and ones, I have them write the word "units" to label the first period. We continue in this way out to the billions place. We also label each of the periods, units, thousands, millions, billions. Once we have made the place value system, we practice saying the numbers correctly, starting with the smaller, more familiar numbers, like those in the thousands. When teaching my students to say a large number correctly, we refer back to the periods. I tell them to say the number they see within a period, for example for 523,000 they would say five hundred twenty-three, then they just say the name of the period, thousand. We practice this quite often at this point and we continue it throughout the school year. Once I teach this, I don't want them to forget it. Teaching them to say the number correctly will assist them in writing the number correctly. This will also help them with decimals and fractions as well. I find it to be of great assistance when we begin converting decimals to fractions. At this time, I like to read numbers to them aloud to see if they can write the digits of the number in the correct place values in the chart we made in the notebook. It may be a good idea to make certain your students are using pencil as there may be a bit of erasing as they learn to write the numbers early on. If you don't want them to use the charts they made in their notebooks, create a handout and let them do it on the handout. Once they have are able to write the numbers without too much trouble, or assistance from me, I like to play a game with them. I write the digits 0 through 9 on large index cards, one number per card. I make three sets, using a different color for each set. These are something that you will use quite often in the future, so you may choose to laminate your sets after you create them. The way you play the game will depend on the total number of kids in your class. Here's how I play it with my students. I divide the class into three groups of equal numbers. Then each team is given a pack of the index cards. I call the groups by whatever color they were given, for example the red team, the green team, the purple team. Each student takes one card from the team's pile. Once they have that number, they remain that number unless I tell them otherwise. The game is simple. I call out a number to the class and the first team that gets in the correct order first wins. This is great for getting them out of their seats, working as a team, and practicing a math skill. I play this way for a few rounds, and then I change the rules. For the next round, the teams are not permitted to talk. This helps me determine whether everyone on the team understands the concept, or if there has been a team leader who has surfaced and is getting his teammates into place quickly for the win. The idea is not for one person to lead the team to victory, it's for everyone on the team to actively participate and the no talking rule helps ensure that this is happening. After I teach the right side of the decimal system, I add a decimal to each team. This takes the game to the next level. The decimal person may think he/she has it easy because he/she is just listening for the word "and." Remind the class that the whole number that you are calling out is not simply a matter of listening for "and," but rather the whole number is a combination of the sum from all of the places. You may even choose to leave out the word and to see if they are truly understanding the placement of the decimal. If you want to add yet another level of complexity to the game, once they are playing it well, purposely leave out digits from the numbers that you are calling out. See if they noticed that the person holding the four is not used on this number and therefore should be sitting down. This may cause some confusion on the first round, however, in the subsequent rounds it will actually cause the groups to increase their attention and as a result they will begin to play the game better than before.

Once they have a basic familiarity with the number names, I like to break the numbers down for them into the expanded form of the number. This is critical for getting them to understand that the leading digit of a number is the largest and therefore the most important digit in the number. As we go through this, they learn that the digits to the right of the leading digit are smaller and therefore a less significant part of the number. This is an important concept when we discuss rounding. Because the digits to the right of the leading digit are smaller, when we round a number, we are only changing the overall number by a small amount. If I give them a number, let's say, 4,235,681, I like to break the number down into its base ten expansion (expanded notation):

4,235,681 = 4,000,000 - millions

200,000 - hundred thousands

30,000 - ten thousands

5,000 - thousands

600 - hundreds

80 - tens

1 - ones

When a number is written in expanded notation, I call the individual components *very round numbers*. By writing the numbers out in their expanded notation, it is easier for the kids to see that place value is simply a way of writing the sum of a group of numbers of a special sort. For example, 4,235,681 is really the sum

4,000,000 + 200,000 + 30,000 + 5,000 + 600 + 80 + 1.

If your students are ready for it, you can also use this opportunity to talk about how these special numbers are written in terms of powers of ten. For example, 4,000,000 is written as 4×10^6. I find that they quickly understand the concept of the powers of ten. I start by teaching them 10 can also be written as 10^1. The one tells them how many zeros to write after the one, or the number they are using for a particular problem. For 10^2 I tell them that it simply tells us how many times we multiply 10 to itself. Make certain they understand that they are multiplying 10 by itself, not multiplying 10 times 2. This is a common mistake they make as they are learning the powers of ten. To practice base ten expansion (expanded notation), give them different numbers and ask them to write the numbers out in their base ten expansion. I find that they catch on to this quickly with relatively little assistance.

As the students improve upon their skills with base ten expansion and with converting those numbers to the powers of ten, it is a good time to discuss relative place value. What is meant by relative place value? It actually goes hand-in-hand with the powers of ten. Relative place value is simply the way of explaining how much larger one very round number is than another one. If we go back to place value, any given digit represents units that are ten times larger than the units of the digit to its right. So, as we start in the ones place and work our way to the left (towards the millions) each place value is ten times larger than the digit before it. The tens are ten times larger than the ones place, the hundreds place is ten times larger than the tens place, the thousands place is ten times larger than the hundreds place, and so forth. If we start at the billions place and work our way back toward the decimal, then we can say that each place value to the right of another is 1/10 its value. For instance, the hundred millions are to the right of the billions place, therefore the hundred millions are 1/10 the value of the billions. This holds true for all of the place values going from left to right through the decimal system, including the place values to the right of the decimal point. Make certain your students understand this relationship and that they don't just memorize the place values without understanding the relative place value. It might be useful to illustrate this point with your students in various concrete and pictorial ways, including using the number line to place numbers by means of their decimal expansions. They should also be able to deduce that a movement two places to the left on the place value system would be one hundred times larger than the other digit. Three places would be one thousand times larger, and so forth. To help them understand this concept a little better, I like to use a number such as 8,888,888. By using a large number with all digits the same, I can make certain they understand the relative place value of the numbers as opposed to the absolute value of the number of each digit. Then I can ask them questions such as "Would you rather have a summer vacation that lasts for eight days, or one that lasts for 80 days? Why?" or "Would you rather share a pizza with eight of your friends, or with 80 of your friends? Why?" and with that one I can also include, "What happens to the size of your slice if you share your pizza with 80 friends as opposed to 8 friends?" I am looking for answers that tell me the students understand that 80 is ten times larger than eight, or that the slices would be 1/10 as large with 80 friends. Money is also a good way to discuss this relative size comparison. Starting with a penny, you can ask them how much larger is a dime compared to a penny. They are all well aware that a dime is 10 cents, and therefore, ten times larger than the penny. Then go out to the $1 bill and ask them how much larger is it than the dime. Again, they will quickly realize that it is ten times larger than the dime. Continue in this manner to the $10, $100, $1000 bills. This is a real world example they can relate to in order to understand relative place value. Dr. Howe mentioned that he once read in a newspaper that "$100.00 to a billionaire, is like a dime to a millionaire." That may be something you can use with your students to help them visualize the size difference between these large numbers. As you continue with this exercise, remember to also go from larger to smaller so they can see that when it is getting smaller, the place value is 1/10 the size of the one to its' left. You should skip around to make certain they grasp the idea that a jump of two spaces is one hundred times larger (or smaller) number, and a jump of three place values would be one thousand times larger (or smaller) number no matter which place they are currently at.

This is also the time to discuss order of magnitude. As we increase by 10 we are increasing by one order of magnitude. Remember that to begin with order of magnitude, we must have a certain unit to work with for our problems. In this case, let's say a penny is our unit. Therefore, a penny has an order of magnitude of zero. Your base unit will always have zero magnitude, by definition. And remember that we only increase our order of magnitude when we multiply by 10. Therefore, any pennies from one penny to nine pennies will have an order of magnitude of zero. Ten pennies, or a dime, will have an order of magnitude of one, because a dime is 10x's larger than the penny. So, to get to an order of magnitude of three, we will have to multiply the dime by 10 to get 100 pennies, or one dollar. That means pennies 10 - 99 will have an order of magnitude of one. It is not an order of magnitude two until we reach 100. Again, this is just the definition. It allows us to talk about size in a qualitative and still precise way. As you see here, and hopefully your students will see, that the order of magnitude are bounded by successive powers of ten. An easy way to remember order of magnitude is to take the number of digits in your number and subtract one. [12] For example, 8,421, has an order of magnitude of three. There are four digits, so four minus one is three. Don't just give your students this shortcut without explaining the meaning, otherwise the shortcut will be useless because they won't know why they are getting the answer, or what it means.

OK, so we have covered place value, expanded notation, relative place value, base ten and order of magnitude. Let's put it all together now to complete some estimation. Now you can see that estimation is not a guess, but rather a culmination of all of the other math concepts described thus far. One point you will want to stress with your students and that is that estimation is not a precise answer. When we estimate, we want to try to get an answer that is between powers of 10, or increments of 10, 100, etc. [13] In other words, we first try to find out the order of magnitude of the number. If we can do that, the next job is to try to find out the leading digit of the number or which very round numbers it is between. According to Dr. Howe, a good analogy for the students is to tell them to think of goal posts. A kicker puts the ball between the field goal posts, not on them. We want the students to do the same thing with estimation, they need to get an answer that is between two acceptable powers of 10, or between the right two very round numbers. Sometimes if they are lucky, they may be able to find the next digit, but usually they should not try to expect to do better than knowing the first one or two decimal places. They should not be worried about exact answers or that their answer is different than someone else's in the class. As long as they are both within the same "posts," or they are within the same order of magnitude, then they are essentially in agreement. Again, I stress that this is not about exact answers. The reason for this is what we talked about earlier and that is the importance of the leading digit of a number. We won't take the space to justify it here, but it can be shown that the leading digit of a number alone gives us at least 50% of the whole number, the leading digit, and the next digit combined provide at least 90% of the whole number. [14] The first three digits provide over 99% of the whole number. So what does this mean? Let's take a large number like 12,435,694,392. The 1, the 2, and the 4 provide more than 99% of the size of that number, which means that the 3 must not be too important when we are discussing the accuracy of this number. And the 5 is even less important. In general, the importance of the numbers, as well as the likelihood that they are accurate, continues to decrease as we continue to the right. Why does this matter? Many times when your students are working on estimation problems, they may be dealing with rather large numbers. Don't let them worry about anything past the first two, or perhaps first three, digits of their answers. Let's look at our example number above once more. If a reporter gave us that number about, let's say, the number of dollars spent in the malls in the United States in a year. If the first three digits of a number provide us with 99% accuracy of a number, then can we even be sure about the accuracy of the rest of that large number? So, in truth, do we really need the rest of it? No, and to be as accurate as possible, we should probably leave it off and simply say that, "Malls in the United States made $12,400,000,000 last year." Even "made over $12 billion last year" would convey enough information for most people.

In terms of horseshoe crabs on the beach, this means that, for the estimate on Slaughter Beach (discussed below), we should be more than happy to replace the reported figure of 23,800 with 24,000. For many purposes, it might be satisfactory to report just the first digit: 20,000. We lose some horseshoe crabs here, but if on another beach there are 26,000 and we report that at 30,000, we get them back. The figure 24,000 is so close to the reported estimate (less than 1% more), that we can't be sure that the actual number might not be 24,000, or even a little more.

Strategies

Now it's time to start applying some of the ideas expressed thus far in this unit. The states of Delaware and New Jersey conduct a horseshoe crab census every year to estimate the number of spawning horseshoe crabs in the Delaware Bay. The numbers collected by this census have been used to create laws, some of which have had a significant impact on many people's livelihoods. Some of these laws have even been overturned due to a lack of convincing evidence, including the use of these estimated population numbers.

So how do they do it? What are the procedures that are in place for determining these population estimates? The main tool used for these estimates is a one meter quadrant. That means it is a large square, made out of pvc pipe that is one meter in length on each side. This is the device that is used to actually count the crabs in a given area. Volunteers are given a tally sheet to assist them with their count and to help keep the data organized. Without getting too involved in the process (we will get to that later) the overall idea is this; volunteers place the meter quadrant in the sand and count the number of horseshoe crabs that are within the quadrant. They have specific guidelines as to what constitutes a crab that is considered within the quadrant. After one quadrant has been counted, the volunteers move 20 meters down (or up) the beach, lay the quadrant down again and count the crabs at this location. They continue in this fashion until they have covered at least 1km. These surveys can only be conducted during the new and full moon phases, and perhaps a day or two before or after, so it is important that the volunteers follow the procedures correctly the first time. Due to the limited amount of time available, and sometimes unpleasant weather, there aren't many opportunities for second chances. So, now let's say the volunteers have their data table loaded with information. It lists how many males and females were found within each grid. Those numbers are then averaged for the total number of grids that were counted. Let's say that for Slaughter Beach, DE, one of the most popular horseshoe crab spawning sites, a group of volunteers counted an average of 7.96 horseshoe crabs per square meter. If they covered a total distance of 3 km during that count that they would find their total of horseshoe crabs for that beach by multiplying the average number of horseshoe crabs by the total length of beach surveyed. So in this case they would multiply 7.96 horseshoe crabs by 3 km to get a total of 23,880 for Slaughter Beach for that one night. These are the actual numbers taken from survey results from Slaughter Beach on May 5 th of 2001. [15] After going over all of this with my students, I would take a few minutes to ask them what they think about the techniques that are being used as well as the numbers that are being reported. Remember the further to the right that we move down a number, the less and less accurate the digits are likely to be. Therefore, I would hope that the students would be able to tell me that they believe these numbers are accurate to the leading digit of the number, 20,000, and perhaps even to the second digit, the 3,000. However, make certain your students think about the 880. It's supposed to be an estimate and we like to see estimates to the powers of ten, preferably to the leading digit for accuracy. Whether the survey reports the number of horseshoe crabs to be 23,000 or 24,000 is not realistically going to change the count by a significant margin. But if we think about 24,000 horseshoe crabs crawling on a beach, can anyone really think they can estimate the number down to the tens place? I couldn't, not with confidence. The number is much too large to attempt to estimate the number to the tens place. In fact, 23,880 is only .5% less than 24,000.

One activity that would be a fun learning experience for the kids would be to replicate this estimation process on a small scale in the classroom. This was recommended to me by my seminar leader, Dr. Roger Howe. To recreate the meter quadrant I will use four popsicle sticks glued together in the shape of a square. Instead of a 20m distance between each counted quadrant, we will change it to 50cm. However, since this is a scale model, our 50cm will represent 20m. You will need to find something in your classroom that can represent the horseshoe crabs, which technically could be anything that can be counted. You are going to want as many as possible, the more you have the better the survey will be for the students. Whatever you choose, just remember one thing: you need to know the total number of them prior to beginning the assignment so the students can compare their estimates to the exact number - something that is not possible in the real world of counting horseshoe crabs. Do not give the students the accurate number of "horseshoe crabs" until after everyone has completed their estimates. Have the students find the average number of crabs per square meter, then multiply that number by the length of the "beach" in meters. Make certain your students remember to give their estimates as *very round numbers*. Have them present their answers, and the methods that led them to their answers, to the class. This way they can learn from each other, as well as from you. Once everyone has presented, give them the actual number of horseshoe crabs and discuss your findings.

Now that they have a good idea of how to get accurate estimates, here are some ideas for problems that you can give them.

1. If a female horseshoe crab can lay between 80,000 and 100,000 eggs in a year, [16] can you estimate the number of eggs that were laid on one beach in one night based on the census report for that night? Accuracy is going to be an issue here, so the students shouldn't be attempting to report any number past the leading digit. They will need to do a little research however, to find out how many nights per year a female may lay eggs on the beach. In Delaware a horseshoe crab egg has an average diameter of 0.7mm. [17]
2. The Dover Air Force Base is home to a fleet of C-5 Galaxy cargo planes. Based on the dimensions of the cargo hold, estimate the number of male horseshoe crabs that will fit in the cargo hold. Do the same for the female horseshoe crab. The dimensions of the C-5's cargo hold are height 13.4ft, width 19ft, and the length is 143ft 9in. [18] The average size of the male horseshoe crab is 7-9in across, 2.5 inches high, and 13-16in long, the female is 9-12in wide, 3.5 inches high, and 16-20in long. [19] Keep in mind that one third of the horseshoe crab's length is its tail. Make certain to point out to the students that the cargo hold is a three dimensional object. Can they determine the number of C-5's it would take to transport the entire horseshoe crab population of a certain beach, or perhaps several beaches?
3. Let's say that Slaughter Beach has an area of 1.5 square miles shaped like a rectangle with a base of 3mi and a height of .5mi. Estimate the number of male horseshoe crabs that would fit on the beach. You cannot stand them on top of each other. Parts may not overlap each other. Do the same for females. Use the measurements of the horseshoe crabs from problem #2. Keep in mind when completing this problem that horseshoe crabs do not typically cover an entire beach when they spawn, we are only using that idea for this questions, for the sake of estimation.

Hopefully you can now see how the students will actually be using specific strategies to complete these estimation problems. These questions are not complete in the manner I have them written here. They are more for you to get idea of the types of questions you can ask your students. If you use these questions, then I would spend a few minutes asking the students what other information they feel they will need to know, or research on their own, to solve these problems. The students are not guessing the answers, they are not using "educated" guesses, they are using several mathematical applications to get as accurate an answer as is possible in the form of a very round number. There is more to estimation that most teachers realize and it is time we offer our students the necessary strategies to solve estimation problems with more than a guess.

Activities

The activities for this unit are not long, they are not in any way complicated, and they are certainly able to be interpreted or changed as you see appropriate for your students or for your teaching style. They are ideas that Dr. Howe and I discussed in class, though they are by no means the only activities you can do with this unit. At first, just read through them and see if they are appropriate for you. If they are not, feel free to alter them, or perhaps reading them gave you an idea for a better activity. No matter what you do, remember to make it enjoyable for the kids. If you change the activity, or create a new one, please let me know, I would love to hear from you and it would be beneficial for any teacher who is interested in teaching this unit. I may have written this unit, but I know that teachers are always able to improve upon lessons, so I look forward to hearing from you.

Activity 1

For this activity we will create a simulated horseshoe crab census for the classroom. Please keep in mind however, that the techniques described in this activity are not necessarily the way the horseshoe crab census is conducted in the real world. The way it is done now is a bit too complicated for the kids, and wouldn't serve us well for the sake of estimation. So, the main focus here is estimation, not a perfect simulation of the real world horseshoe crab census.

First you are going to need some materials for this activity. You are going to need lots of pennies and nickels. The pennies will represent the male horseshoe crabs in our activity and the nickels will represent the females, therefore, you will need more pennies, than nickels. Why? Because there are more male horseshoe crabs on the spawning beaches than there are females, so we would like to keep this as realistic as possible. Now, before you spread out the coins, you need to know the exact number of them, prior to starting the activity. Write down the total number of pennies, the total number of nickels, and the overall total of all coins. Do not share this number with the students at this time. You will not give them these numbers until the end of the activity. It is the control that we will use to determine the students' estimation numbers. The number of coins you need depends on the location you have to work with. The larger the work area, the more pennies and nickels you will need. If you do an image search for horseshoe crab spawning, or horseshoe crab beach, you will see the way the horseshoe crabs come to the beach to spawn. This is what you are attempting to replicate in the classroom. Horseshoe crabs come up the beach, just like the water does, and they can be 30 crabs wide, sometimes more, and can extend for a kilometer or further, depending on the weather. A few hundred coins would be good - the more the better. But don't forget, you MUST know the exact number of pennies and nickels prior to starting the activity. Once you have all of your coins, spread them out in a long line across the classroom floor, at least 1000cm would be best, and perhaps 20cm wide. You may find it helpful to spread out all of the pennies first, then go back over them with the nickels, just make certain you spread them out a little more sparingly. It is perfectly fine for the coins to overlap each other, which is exactly what horseshoe crabs do when spawning on the beaches. This part of the activity can get messy, so I recommend spreading out the coins yourself, or having some type of frame so the students don't get the coins all over the classroom.

The other thing that you will need for this experiment is a scaled down model of the meter quadrant that is used in the horseshoe crab census. Four popsicle sticks are all that you need for this. Just glue the four of them into the shape of the square. This is what the students will use to count the number of horseshoe crabs. You will need one popsicle stick quadrant per group of kids. Most teachers have popsicle sticks in their rooms already, so it is usually an easy item to use for this activity. If you would like to substitute something else in place of them, feel free. Again, there is a lot of flexibility in these activities.

You will also need a length of string per group to measure the distance between each quadrant. The real census measures out a distance of 20 meters between each meter quadrant, however, to scale ours down, 50 centimeters may be the best length. You may increase or decrease the measurement, based upon the total length of your "beach" - the total distance from the beginning of your paperclips to the end of them. If you were able to spread out your paperclips over a total of at least 1000cm, then using the 50cm distance will ensure the kids are able to get at least 10 quadrant counts recorded. If you have a lot of students in your class, and if you have enough coins, you may find it beneficial to make two or three rows of horseshoe crabs. More than one group can work comfortably on one row; however, too many may make it difficult for the students to maneuver.

So your male and female horseshoe crabs are spread over the floor, hopefully over a distance of at least 1000cm, you have one popsicle stick quadrant and one 50cm length of string per group, now what? Choose a different starting point for each group, so that you have random quadrants being counted. Tell the students to place their quadrant directly on top of the coins (horseshoe crabs) and to count how many crabs are in the quadrant. At this point, you will run into two problems: first, what counts as being "in" the quadrant? Is it half of the crab, all of the crab, any part of the crab in the quadrant? That decision is up to you, but here's an idea. For the first trial, have them only count those crabs that are entirely within the quadrant, nothing else counts. For the second trial you can change the criteria to include the ones that are at least half in the quadrant, and for the third trial you may include any crab that has any part of them inside of the quadrant. This way you will have three sets of data that can be compared within each group, and across the classroom. The other problem that you will run into is when the kids realize they do not have anywhere to record the data they are collecting. Instead of providing you with a template, I like for my students to create their own data tables for the different projects we are working on. Different projects require different data tables, so I like to see what they come up with own their own. What they will need for their data table however, is a column for males, females, and total crabs. They will need a separate row for each quadrant that they counted as well, with at least 10 rows for 10 quadrant counts. At the top of their data table I would have them include space for their name(s), the date, the time, and the starting position for their group. Remember that you will be starting each group at a different random place. For instance, if group 1 starts right at 0cm, they would mark that on their chart, but if group 2 starts at 11cm, they should mark that as their starting point.

Now that you have solved their problems, after they count the crabs in their first quadrant, have them stretch out the 50cm string from the beginning of the first quadrant, until it is tight. This is the place they will lay down their quadrant for the second count. They will continue in this fashion until they have completed the number of trials that you request. It does not have to be 10, nor does it have to be the same number of trials per group. Change the number of trials for each group to see if that changes the estimates that the groups come up with. For each quadrant, the groups only need three pieces of information, the number of females, the number of males, and the total number of crabs for that quadrant. At the end of the data table, they should have a place for the total number of males, females, and all crabs for all of the quadrants combined.

Once they have all of the data collected, the group will have to come up with a way to determine the estimated number of crabs on their beach. They should be able to give you an estimate for the males, another estimate for the females, and if that went well they should easily have an estimate for the total number of crabs on their beach. Have each team also record the largest number of crabs they counted in one quadrant and the smallest number of crabs they counted in one quadrant. Then tell them to use the largest number to find the estimated number of total crabs, then the smallest number to estimate the total number of crabs.

They can compare their estimates after using the largest number of crabs, the smallest number of crabs, and the average number of crabs. This part of the activity will provide the students the opportunity to see how uncertain this process can be for estimating populations. Answer questions as they arise, some groups may need you more than others. If necessary, have groups redo their counts if their numbers are not close to where they should be. If this happens, tell them they should not be discouraged since this happens in the real population studies as well.

After all of the groups have reported to you that they are finished and happy with their answers, have them report their findings to the class. Have them show their data on the board, and have them show the math they used to determine their estimated number of crabs. This is why it is important not to share the total number of coins used at the beginning. The total number of coins used is the correct answer for this activity, however, we are only looking for estimates, and any number after the leading digit here may not be necessary. After you try the activity once, change the criteria, using the suggestions described above (5 th paragraph in this section), but if you are going to do that, then still do not share the correct answer with them. You don't want to influence their math.

Activity 2

First have the students choose the animal of their choice, in case they have had enough of the horseshoe crab. Once they have chosen a favorite animal, they need to do a little research and find a characteristic of that animal that can be compared to humans. If you have the time, you can work on the research in class (computer lab), or if necessary you can assign the research for homework. They should only need a one night, perhaps two, to complete the research. The animal and the characteristic they choose are really up to them, and don't worry about more than one student choosing the same animal, or characteristic. If they do, you have answers that you can compare in class, which can lead to some interesting math discussions if the answers are different for the same problem.

One example of something they could research could be the number of teeth an adult shark loses, compared to the number of teeth an adult human loses in a lifetime. If they can't decide on something that can be compared to humans, they can choose two animals and make the comparison. For example, they could compare the estimated number of sea turtle eggs to chicken eggs it will take to fill up a bathtub.

Another area that may be fun for the students to study is the population of humans compared to the population of their favorite animal. What is the estimated total weight of the human population compared to the estimated total weight of the animal that they chose? It would be interesting to see how much more humans weigh as a group, than say all of the blue whales remaining in the wild. It may surprise the students!

At this point in the unit, teacher intervention should be at a minimum, as the students' ability to estimate effectively should have improved to a level that allows them to work independently. You will need to give them some advanced guidance for their research. Things like the size of the above mentioned bathtub with have to be determined to answer that question. Also, after the students have chosen their characteristic, they need to decide what question it is they are trying to answer. This is the area that they will most likely need the most guidance from you.

Once everyone has finished this activity, it would be most beneficial to have them present their answers, and the math that got them their answers, on the board. This way the rest of the class can see how other kids are doing the math. This is good for students to learn new methods, and may help those students who are having difficulty grasping the idea of estimation. Remember that you want their explanations to include the concepts that you taught them in this unit. The more concepts they are able to utilize and discuss, the deeper their level of understanding.

Activity 3

For this activity you will need to access the following link, http://www.ocean.udel.edu/mas/bhall/hsccensus/2001%20season%20report.pdf

This is the 2001 horseshoe crab spawning survey report for the Delaware and New Jersey beaches on the Delaware Bay. Starting on page five you will find lists of beaches, the dates they were surveyed, and the average number of horseshoe crabs per quadrant for those beaches. We are going to use those numbers for this activity, but not all of them. We want to give the kids the total length of the beaches that were surveyed, but please do not feel as though you need to use all of them. I wouldn't use more than a page worth of this data with most of my sixth graders, so use what you feel is in accordance with your students' ability.

In the first column of the data, you will find the name of the surveyed beach and next to the name in parentheses you will find the total length of the beach that was surveyed. The length of the beach remains constant for each night the beach was surveyed. In the subsequent columns you will find the date of the survey, the number of horseshoe crabs per meter, and the estimated total of crabs for that particular beach for the dates listed. Give the students the list of beaches, including the beaches length and the average number of crabs found in each quadrant for that beach. That's all the information the students receive. Now the problem.

Based on the numbers that you gave to them, do they have the ability to estimate the total crabs found on each beach? Can they estimate the total number of crabs found on all of the beaches for one night's survey? They will need to add the total distance of all of the beaches to begin working on this problem. Once they determine these answers, have them find the average number of crabs per square meter, for all of the beaches combined, based on their answers. Then have them compare their answers to the data from the actual survey from those beaches on those dates. Are they close? What type of math did they do to determine their answers? This activity is again focusing on estimations, so remind the students that the leading digit of their answer should be enough, they should not have exact answers. An estimate to the leading digit will still be accurate enough to see if the students' answers are within two powers of ten, and therefore close enough to the actual survey results recorded in this data table from 2001. Presenting their answers to the class will be useful to stimulate group discussion about the problem, their methods, and their answers.

Teacher's Bibliography

Adam, John A. and Weinstein, Lawrence. *Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin.* Princeton, NJ: Princeton University Press, 2008.

The authors give an explanation of how to estimate the answers to problems with nothing more than basic math skills. The explanation only takes up the first section of the book. The rest of the book is dedicated to estimation problems, as well as their answers. This is a good starter reference for teachers who are new to teaching estimation.

Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra." (January 2008).

http://www.maa.org/pmet/resources/PlaceValue_RV1.pdf (accessed July 09, 2008).

This paper was written by the Estimation seminar leader, Dr. Roger Howe, of Yale University. This paper provides the background of teaching estimation the way that I do in the unit. There is much more detail provided in this paper than was provided in my unit. This is a must read for any teacher looking to teach estimation.

Student Bibliography and Resources

Air Mobility Command. "C-5 Galaxy." (2008). http://www.af.mil/factsheets/factsheet.asp?id=84 (accessed July 12, 2008).

This is the United States Air Force website, which I used to get the measurements for the C-5 Galaxy cargo plane. The measurements for all different aircraft can be obtained through this site.

Delaware Department of Natural Resources and Environmental Control. "Green Eggs and Sand, Tri-State Horseshoe Crab/Shorebird Education Project." (2003). http://www.dnrec.state.de.us/fw/neware/Program%20Overview/Program%20Overview.htm (accessed July 11, 2008).

This is the State of Delaware's site for the programs provided through the Aquatic Education Resource Center. A listing and description of the programs is provided at this site.

Ecological Research & Development Group. "The Horseshoe Crab. In the News: Delaware Conch Fishermen Successfully Repeal Moratorium on Horseshoe Crab Harvesting." (2006). http://www.horseshoecrab.org/news/pdf/Pressrelease050807.doc (accessed July 11, 2008).

One of the most comprehensive horseshoe crab information websites available. This website provides all matters of things pertaining to horseshoe crabs. This particular link is to one of the articles that was found on the site, however, the main site, www.horseshoecrab.org is a great resource for any teacher looking to teach this unit.

Hall, William. "Horseshoe Crab Census Information." (2008). http://www.ocean.udel.edu/mas/bhall/hsccensus/index.html (accessed July 10, 2008).

This is a great place to find detailed descriptions of how the horseshoe crab census is conducted. Survey results from past years, a list of beaches, and a tally sheet can also be found on this site.

Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra," http://www.maa.org/pmet/resources/PlaceValue_RV1.pdf (accessed July 9, 2008).

This paper was written by the Estimation seminar leader, Dr. Roger Howe, of Yale University. This paper provides the background of teaching estimation the way that I do in the unit. There is much more detail provided in this paper than was provided in my unit. This is a must read for any teacher looking to teach estimation.

Maryland Department of Natural Resources. "Horseshoe Crabs: A Living Fossil. Life History." (2008). http://www.dnr.maryland.gov/education/horseshoecrab/spawn.html (accessed July 09, 2008).

This site provides the reader with detailed information about the horseshoe crab. Similar to the site provided by EDRG, though has some information specific to the state of Maryland.

Mid-Atlantic Sea Grant Network, "Horseshoe Crab History and Biology." http://www.ocean.udel.edu/horseshoecrab/History/index.html (accessed July 11, 2008).

Detailed descriptions of the horseshoe crab are provided including, the history and biology, shorebird connection, human use, research, fisheries management, as well as a listing of prime horseshoe crab beaches that can be found on the east coast of the United States.

Mid-Atlantic Sea Grant Network, "Horseshoe Crab. Research. Biomedical. Eye Research." http://www.ocean.udel.edu/horseshoecrab/Research/eye.html http://www.ocean.udel.edu/horseshoecrab/History/index.html(accessed July 11, 2008).

Detailed descriptions of the horseshoe crab are provided including, the history and biology, shorebird connection, human use, research, fisheries management, as well as a listing of prime horseshoe crab beaches that can be found on the east coast of the United States.

Miller, Mark. "The Issue: Too Close for Horseshoes." (2007). http://www.delaware-bride.com/ME2/dirmod.asp?sid=&nm=Archives&type=Publishing&mod=Publications%3A%3AArticle&mid=8F3A7027421841978F18BE895F87F791&tier=4&id=2DCD13F1B22344D39C6D309AC17D8DE5 (accessed July 10, 2008).

This article was found in the Delaware Today online magazine. It is a discussion on the effects of a harvesting moratorium. Topics include the effects on the eel and conch fishermen, as well as the effect on the migrating shorebird populations, and the resulting effect on the local tourism economies.

Norris, Scott. "Oldest Horseshoe Crab Fossils Found in Canada." (2008). http://news.nationalgeographic.com/news/2008/01/080131-oldest-crab.html (accessed July 17, 2008).

This article was just written this year and has changed the original estimate of the horseshoe crab's age - 300 million years old. Due this fossil find, the age of the horseshoe crab is now dated back to 450 million years ago, and some think it may be older than that.

Swan, Benji, and Hall, William, and Shuster, Carl Jr. "The Delaware Bay Horseshoe Crab Spawning Survey 2001 Season." http://www.ocean.udel.edu/mas/bhall/hsccensus/2001%20season%20report.pdf (accessed July 10, 2008).

This article provides detailed survey reports, beach by beach, for the 2001 season. This is the data that was used for the activities in this unit.

U.S. Fish and Wildlife Service. "News Release: Red Knot named candidate for Endangered Species Act protection." (2006). http://www.fws.gov/news/NewsReleases/showNews.cfm?newsId=A26DAA75-DFC1-18FC-1DF52CD3E63D886F (accessed July 09, 2008).

A news release describing the issues surrounding the red knot and its potential listing on the Endangered Species List.

United States Geological Survey. "Population Studies of Horseshoe Crab (*Limulus polyphemus*) in Delaware Bay." (2007). http://www.lsc.usgs.gov/aeb/2065 (accessed July 18, 2008).

Updated detailed information is provided on this site including population studies of the horseshoe crab. Projects, papers, reports, presentations, research, and a history of the horseshoe crab are also included. There are links to other horseshoe crab websites.

Wikipedia the Free Encyclopedia. "Horseshoe Crab." (2008). http://en.wikipedia.org/wiki/Horseshoe_crabs (accessed July 09, 2008).

A brief description of the horseshoe crab is provided. Some of the topics include, physical description, life cycle and behavior, evolution, and conservation.

Endnotes

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1. Norris, Scott. "Oldest Horseshoe Crab Fossil Found in Canada."

2. Delaware Department of Natural Resources and Environmental Control. "Green Eggs and Sand, Tri-State Horseshoe Crab/Shorebird Education Project."

3. Hall, William. "Horseshoe Crab Census Information."

4. USGS. "Population Studies of Horseshoe Crab in Delaware Bay."

5. Ecological Research & Development Group. "The Horseshoe Crab. In the News: Delaware Conch Fishermen Successfully Repeal Moratorium on Horseshoe Crab Harvesting."

6. MDNR. "Horseshoe Crabs: A Living Fossil."

7. U.S. Fish & Wildlife. "News Release: Red Knot named candidate for Endangered Specied Act protection."

8. Miller, Mark. "Too Close for Horseshoe Crabs."

9. ERDG. www.horseshoecrab.org

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12. Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra."

13. Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra."

14. Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra."

15. Swan, Hall, and Shuster. "The Delaware Bay Horseshoe Crab Spawning Survey 2001 Season."

16. MDNR. "Horseshoe Crabs: A Living Fossil."

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18. Air Mobility Command. "C-5 Galaxy."

19. Wikipedia. "Horseshoe Crab."