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Estimation: What's the Big Deal?

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Objectives

This unit is about estimation and the benefits of being able to understand large numbers. Most of my students have only a vague idea of large numbers and if asked about one million, they would only think of the number of zeros that are after the one. They have heard of six figures in terms of money but would have a hard time figuring out how far one million inches is or how much time one million seconds is. It is not a common practice to think of distance in inches or to think of time in seconds because we are taught to use the largest unit. This system of using the largest unit is beneficial, but it makes students confused when they are asked to think in different units. It also puts them at a disadvantage when it comes to thinking of large numbers because the larger the unit, the smaller the number. This is one of the reasons that students do not think about large numbers.

I also have a fuzzy idea of large numbers, and I am looking forward to utilizing various strategies to help my students see the magnitude of these numbers and at the same time, gaining a clearer understanding for myself. I do not think of large numbers in terms of powers of ten, and I am pretty sure my do not either students. The idea of these numbers just being approximate digits times power of ten should be easy to understand once the students think about them, but I do not think that they have ever been taught to think in this way. I hope that this unit opens their eyes to the idea and simplifies the whole idea for them

This unit will be taught to students who need one more math class to graduate and are unwilling, or do not have the math background, to take either Calculus or Elementary Functions. They have below level math skills. They have taken Algebra 2, but they have not been recommended by their previous teacher to go on to the higher level class. These students have not had much luck with math in the past and are not very enthusiastic about math. I know that this is a common problem that teachers face, but the advantage is that these students need this class to graduate and they are mature enough to understand this. This does not mean that they are able to do the math; it just means that this is the last opportunity I have to change their ideas about math. I look at this as my responsibility to at least have them rethink math and have them realize that math does serve an important role in their lives.

Most students are willing and even excited to do the work if they are able, and when students are given the tools and know how to use them, the results are usually good. I usually begin the year with a quick review of common ideas and ask students what concepts are still fuzzy to them. The first thing that I hear is that the students cannot do fractions. Their lack of fraction knowledge never fails, so every year I begin with a quick review of the basic rules for adding, subtracting, multiplying, and dividing fractions. I have no problem going over very basic ideas to make my students comfortable. This sets the tone for the students to feel free to say that they do not remember how to change fractions into decimals or they never learned how to convert feet to yards. I have students who do not know how many feet are in a yard, and it is reassuring for the students when I just explain without judging.

I want us all to be on the same page when we begin our lesson on large numbers, and I want the problems to be interesting to the students. This is why I am creating problems that connect to their lives. I hope to engage my students and eventually have the students create some problems themselves. My objective is twofold. First, I want to change their attitudes and perceptions of math. Secondly, I want to show them that they are quite capable of answering interesting questions using math and really big numbers

I work in a Performing Arts High School and my students study various majors. The majors are dance, musical theater, technical theater, instrumental, vocal and art. I hope to have an even distribution of all majors so I can group the students and have them work on problems that pertain directly to their major. For example, I would have the dancers estimate the number of steps in a dance and then find the number for the entire company. Here are several examples of problems designed specifically with these students in mind. How many performances would the company have to perform before they have done one million steps and then one billion steps? I expect the students will be surprised by the difference between one million and one billion. The vocal majors will have to figure the length of a song and find out how long they would have to sing the song before they have sung for one million words and then one billion words. The art majors will have to estimate the number of hairs in a paint brush and then determine the size of a paint brush that has one million hairs and one billion hairs. This will be fun to see because now we are talking about area and the numbers will be different. I will have the instrumental majors estimate the number of hairs in a violin bow and then have them find the length of a bow that has one million hairs and then one billion. The theater majors will choose a play and find the number of times the play would have to be performed consecutively to have spoken one million words and then one billion words. Finally the technical theater majors will be asked to determine the size of a theater that holds one million people and one billion people. In each of these cases, I think the answers will surprise the students.

To help them think about large numbers, we will discuss the concept of grouping. For example, one million is ten groups of one hundred thousand and one billion is ten groups of one hundred million. This is quite a difference. If the students are having a hard time visualizing that, I will break it down further for them. I will ask them to picture one thousand, which should be easy to do and then one thousand more and one thousand more and so on. It will be hard to visualize one thousand groups of one thousand, but this visualization will certainly help them understand how large one million is. I wanted to have a visual and originally planned on using one million inches of yarn and running it through the building. Once I did the math, I realized that this was a very costly and dangerous idea. The conversion of one million inches into feet, then yards, then miles comes out to 15 miles, much to my surprise! Lesson learned! I hope the students have some surprise moments too. I will still have the students do the calculations and come to the same conclusion as I did. That is a lot of yarn, and wrapping it around the building might not be such a good idea, but asking the question who big the yarn would be might be a good question to ask here. Yarn is usually packaged with either weight or length, and either measurement could be used to ask this interesting questions.

Strategies

My first strategy will be to name and write the numbers, starting with one and doing powers of ten until we reach ten billion. I want the students to have a solid understanding of grouping because it is important for the understanding of large numbers. We will create a chart that will show this in words, numbers, expanded form and exponents. This will be a great time to discuss the importance of digits and their place value. Placing a large number at the end of a very large digit really won't have much impact on that number. This will be displayed and referred to throughout the unit. This chart will be an introduction or reminder, that the number one thousand is ten groups of one hundred and one million is ten groups of one hundred thousand and so on. This chart will also be a great reference when we use exponents in the future because students can look at the chart to see that one million can be written as ten raised to the power of six and surprise, that is the number of places after the digit! I am going to spend a lot of time talking about the power of ten and making sure that the students can explain large numbers in terms of powers of ten.

I believe that it is important to start small and work my way up, so I will ask questions that will force my students to think of the numbers as powers, and think of each successive power as ten times the previous one. For example, if I ask them how many students can fit in the theater, I will have them start with small numbers. Can ten people fit into the theater? Can ten more groups of ten, or one hundred people fit into the theater? This progression of multiples will be easier for students to visualize than just randomly picking a number. This way, the students will have a point of reference that they can use for all numbers once they begin to think of numbers as powers of ten, one hundred, one thousand and so on. I will continue to ask questions based on the idea of multiples. What if we take the seats out of the theater and we are all standing, then how many students can fit into the theater? How many students can fit in this classroom? Could I put more desks in to fit more students? If I take out all the furniture, how many will fit in now? Where in our school could you put one hundred, one thousand, ten thousand, and one hundred thousand all the way up to one million people? How long a row would one thousand people make? Could they fit inside our school in a straight line? How much area would one million people, in one thousand rows of one thousand ,, fill up? Could they fit inside the school grounds? I am going to reinforce the idea that one hundred is ten groups of ten and one thousand is then ten groups of one hundred and so on. I hope to have the students visualize each multiple of groups. This will result in my students developing a solid grasp of very large numbers.

It will be important to talk about notation to make sure that all the students understand the power rules. I am anticipating many diversions during this lesson because of the level of the students, but I would much rather spend the time up front to clarify things than have to go back, or worse yet, have the students just not understand. Preparation and clarification is always time well spent for any lesson.

I know that all of my students have been introduced to scientific notation but may not be comfortable with the operation of powers. This is when I will regress to teach addition, subtraction, multiplication, and division of exponents. Since Scientific Notation only concentrates on the significant digits and add the place values it should be a easy lesson. Once again, this short refresher lesson will serve my students well because it will remind them of what they already know or show them something that they may not be comfortable with. This is when I will refer back to our chart and using both the powers and expanded form, I will reintroduce the rules. This lesson will assure that we are all on the same page once again.

I will now begin to familiarize my students with the order of magnitude. The term will be unfamiliar to them but the idea is quite simple. The order of magnitude of a number is its largest non zero number single place component, or put simply the number of digits minus one. In the number 5279, the order of magnitude is 3 because 5279 would be $5x10^3$ since the five is the leading digit. The two is less important than the five and the seven is less important than the two and the nine is less important that the seven. This is a great opportunity to talk about the importance of place value because this concept will become very important when we begin estimation of large numbers. If you have one hundred dollars and lose ten, you may notice, but if you have one million dollars and lose ten, the difference is insignificant.

Since there will be many conversions during this lesson, I plan on spending some time going over simple conversions for the same reason as stated above. I had to look up exactly how many feet are in a mile, so I cannot expect my students to know this off the top of their heads. I will introduce them to conversion web sites that can help them, and we will do some problems to check for understanding. I want to make sure that my students have all the tools they need, and it is important to make certain the math is within their capabilities. It is also important to make sure that our calculations are correct and since we will be using a lot of rounded numbers, I will instruct my students on the percent of error. For example, the difference between using Π or 3.14 is about a 4% change in the answer, as is the difference between using 3 instead of 3.14. While 3.14 is accepted for most calculations, it is important to note that there is a difference in answers and to decide if that difference is acceptable.

Classroom Activities

This is going to be an ongoing unit and I would like to share it with the rest of the school, so I am going to use one of the bulletin boards in the common area to display facts that my students will have to update daily. This will not be hard to do once the first conversions have been made but it will be fun to see and I think the other students will find the information interesting. The students will be responsible for answering and updating the following facts.

You have been in school for _______ seconds.

You are __________ seconds away from the weekend (3:30 Friday).

You have spent _____________ seconds in your math class so far this school year.

Since Monday at 7:00 am, your heart has beat ________________ times.

CAPA students have flushed the toilets _____________times so far this year.

CAPA has used ___________ gallons of water just by flushing the toilets.

If you live in ________, you are _____________ inches away from home.

Christmas is _______________ seconds away.

You have only ___________ seconds left in this school year.

This bulletin board will be easy to update and will be updated daily, the number are constantly changing and some will increase as others decrease. Since my information will be time sensitive, I hope to have it up the first week of school. I do not expect my students to have a real grasp of the large number at this point, but this will simply require a quick lesson on conversions. Since we will be using conversions often during this lesson, this will be a quick check for understanding. I do not anticipate much trouble with conversions, but I will be prepared to assist if needed.

The ground work is now in place to begin the estimation portion of this unit. The students have been given a solid foundation of large numbers, a small refresher of conversions, and a reminder of the rules of exponents. I hope this will give them the confidence to attempt the problems I have designed for them. I call these problems "ball park problems" because the answers will not be exact numbers but rather educated estimations. I am very interested in watching the journey that the students will have to travel in order to answer some of the questions. I am also curious to see the different paths each group/student takes to get the answer.

I will guide the students on the first problem by listing the facts/questions they need to answer before they can begin. I plan on helping my students through every step of this first problem to make sure that they have a good understanding of the math and some of the methods we will use. It is only after that, that I will expect the students to work alone or in small groups.

Question 1.

How many bottles of water would it take to fill Heinz Field (Pittsburgh's Football Stadium)?

What do you need to know before you can begin? We will list all the information on the board.

1. What size are the bottles?
2. Are the bottles full or empty?
3. Does it matter?
4. How big is the stadium?
5. What shape is the stadium?
6. What formulas do we need, if any?
7. What are the dimensions of the stadium?
8. What are the dimensions of the bottle?
9. Do you think your answer will be more than one million?
10. Do you think your answer will be more than one billion?

Once we have agreed on the size of the bottle, we will talk about what else we need to know about the bottle. Since we are filling a space, we will need to find the volume of a cylinder. I hope that my students will be able to work through the panic of not knowing the formula of the top of their heads and remember that a cylinder is just a flat shape (the base),with height, and its volume is the product of the height and the area of its base. In this case, we will just find the area of the circle and multiply it by the height of the bottle. Therefore the math needed is $V = \Pi r^2$ multiplied by the height. Did I mention that we will not be using calculators or measuring tools! Do not panic, we are ballparking, or using the easiest acceptable number to get our answer, so we can want to make our numbers simple.

We need to find the diameter of the bottle. Here is a quick trick. Rip the label off the bottle and take an educated guess as to the length. Use this to work backwards finding the circumference and then find the radius. The bottle I used is a 20 oz bottle and the label is approximately 9 inches long. The Circumference formula is C = DΠ, so 9 = DΠ. I am merely making a quick estimation so I will use 3 for Π here to get a nice easy number to work with. Nine divided by three is three, but be careful because this is the diameter and we are really working to find the radius. Once we have found the radius (1/2 of 3 inches is 1.5 inches) we can now use the volume formula $V = \Pi r^2 (h)$ or $V = (3)(1.5^2)(9)$ or V = 3(2.25)(9) or V = 27(2.25) or V = 27(2) + 27 (.25) or V = 54 + 27(.25) - (think quarters and there are four in one dollar so now we have one quarter short of 7 dollars or 6.75). Our final answer is V = 54 + 6.75 or $V = 60.75in^3$. Please note that this once again is a Ball Park Figure and is in no way the actual volume of the tapered bottle.

We have now just found one of the necessary figures we need to continue, we now need to find the dimensions of the stadium. Thank goodness for computer at this point because, as with most of the problems we will be doing, we will have to do a Google search to find some of our facts. I was not able to get the volume of the stadium, but I did get the seating capacity. Let us compile some facts. Here is what I know.

1. The football field is 100 yards long.
2. The seating capacity is 65,050.
3. The shape is approximately rectangular.
4. The volume of a rectangular box is l × w × h.

I am making educated estimations for the rest of the information.

1. The length of the field is 100 yards and I am adding 50 to both ends, to make my total length 200 yards.
2. The width of a football field is 60 yards and I am adding 50 to both sides, to make my total width 160 yards.
3. The seating capacity is 65,050 and there is seating only on the length sides. If I divide 65,005 in half, I will have 32,525 seats on each side.
4. The space for each seat is 2 feet wide, so 10 seats/people will occupy 20 feet.
5. The length of the stadium is 200 yards, or 600 feet, so there is seating for 300 people in each row. Therefore, 30,000 people will require 100 rows.
6. Since each row will rise up by 1 foot, it will take 100 feet to seat the 30,000 people.
7. The length and width are in yards, so I need to convert 100 feet to approximately 30 yards. Remember we are using very loose approximations for now.
8. L×W×H becomes [(200yds × 160yds × 30yds)] = [(6000×160)] = [(6000 × 100) + (6000 × 60)] = [600,000 + 360,000)] = $960,000yds^3$. I can justify using one million here once I have pointed out our percent of error, which in this case is only 4%.

This is the time to stop and think about what we have just done. First we figured the volume of a bottle (60.75 in^3), then we determined the volume of the stadium (960,000 yds^3). The units of measurement are different and we need to address that but more important right now is to have the students begin to visualize the space the bottles will fill. I will start small; let's think of a 3 ft × 3 ft × 3 ft cube. How many bottles will fit inside the cube? The bottles measurements are in inches, so I am going to change the 3 ft length to 36 inches. The diameter of the bottle is 3 inches, so we can fit 12 bottles on the length and 12 on the width. The height of the bottle is 9 inches so we can stack the bottles 4 high. We can now use our mental math skill to determine the number of bottles will fit in a 3 ft cube (12 × 12 × 4) = (144 × 4) = [(100 × 4) + (40 × 4) + (4 × 4)] = (400 +160+16) = 576 bottles in each 3ft cube. We can fit 576 bottles in a 3 ft cube, or about 600 bottles. Again, this is 4% less than 600 bottles.

We now have to figure how many 3 foot cubes can fit into the stadium. We already know the volume of the stadium is 960,000 yds^3 and our crate is already a cubic yard, and there are 576 bottles in each crate. So, using very round numbers we can just 1,000,000 × 600 = 600,000,000 bottles of water needed to fill Heinz Field. Wow!

What happens if we just use the water that the bottles hold? Since our crate is in feet cubed and our bottles are in inches cubed, we need to change our cube into inches. This conversion is easy, 12 inches in a foot, 3 feet, so 36^3 will give us the volume of the cube in inches, or 46656 in^3. If we divide that number by the volume of the bottle (60), we get 777 bottles. If we simplify that number to 800, we can now take the number of bottles times the number of crates, we get 1,000,000 × 800 to get 800,000,000. Using exponents, we can express this by $[(1 \times 10^6) \times (8 \times 10^2)]$ or (8×10^8).

It seems like we have gotten two different answers. The first answer was 6×10^8 and the second answer was 8×10^8. What did we do wrong? The answer is nothing! We answered two different questions. Let's just think about our 3 ft crate again. To fill our crate with bottles we will need 576 bottles, but to fill our crate with water we will need 768 bottles. That is a difference of almost 200 bottles.

Think again about the number of crates we need to fill the stadium. We found out earlier that need 960,000, if we us simple math again (100,000,000 × 200) we get 200,000,000 and that is the difference that we found between the two answers. Using exponents it is easy to see - $(8 \times 10^8 - 6 \times 10^8 = 2 \times 10^8)$.

I will have the students figure out the reason for the difference between the two answers and spend some time noting the difference. The 600,000,000 represents the number of bottles while the 800,000,000 represents the water and the number of bottles needed to get that water, and the 200,000,000 justifies the difference between the two.

These problems can take many twists and turns, because even though we have answered the original question, there are many more questions we could still ask. What about the number of fans it would take to drink this amount of water? How many games would it take if each fan drank one bottle? How many minutes of football would that be? What is the area of space not used in the crates after we fill them with the bottles? The bottle is a cylinder and the space the bottle uses in the crate is a cube, the difference between the two will represent the difference between the number of bottles need and the water needed to fill each crate/stadium. This is another way to present this problem to the students.

The possibilities are endless, but I caution you to think all your problems through because they can take some turns - remember the yarn! Before you run out and buy the yard, think about the math and practicality of your idea.

I have just guided the class through a very lengthy problem with really big numbers and they did it all without calculators or rulers. This problem was an exercise in mental math, conversions, organization and a demonstration of their math skills. I hope to have built the confidence level of my students and shown them that they are much better at math than they give themselves credit for. I feel that they are now ready to tackle similar problems on their own. My role now will be as the observer. I will ask the students to redo the same problem using the exact figures. Remember we used 3 for Π, and determined that we have an error rate of 4%. Using the exact numbers will produce a different number and we can see if the error rate is what we predicted. I will have the students find the percent of change, in this case, the percent of error between the two numbers. This is just change over starting point and I will allow them to use their calculators for this portion. The students will see that the percent of change is very small because the numbers were so big. This may encourage them to think about ball park figures a little more because they will realize that exactness in not always the point.

This lesson, though lengthy, covers a lot of math and if broken down into manageable steps is easy to do. We have covered conversions, mental math, educated estimations and percent of change. I think that the students will be able to work in groups now to work on some "Ball Park" Problems on their own.

I will break the class up into the various majors and assign problems pertaining to the appropriate major. I mentioned the problems earlier but will state them again along with the beginning questions that each group needs to answer before they can begin.

Dance Majors

1. How many performances would a Dance Company have to perform to have danced one million steps/one billion steps?

Questions we need to think about before we begin.

(questions asked may lead to interesting discussions and more questions)

- What performance are we talking about?
- Does it matter?
- What are the numbers of steps performed in the dance?
- How many dancers are performing?
- Does every performer dance the same number of steps?
- Does it matter?
- What dancer would you start with?
- What type of dance should you choose?
- Does it matter?

2. If a dancer can jump to a height of 5 feet, how many times would he/she have to do the jump to have jumped a total of 1 million inches?

Vocal Majors

1. How many times would you have to perform the same song continuously to have sung for one million minutes/one billion minutes?

Questions we need to answer before we begin.

What is the length of the song?

- Does the length matter?
- What type of song would you choose?
- Would adding more people change the answer?
- Do you need to include the pauses?

- Is the song length, the time you spent actually singing?
- Does the piano introduction count?

2. Choose your favorite opera and estimate the number of notes sung total for each performance. How many performance would you have to perform for the entire company to have performed 1 million notes?

Visual Art Majors

1. How big would your paint brush be if it contained one million/one billion hairs?

Questions we need to answer before we begin.

- What paint brush would you start with to find the hairs, a very large or a very small?
- What shape is your paint brush?
- Does the shape matter?
- If you start with a circular, can you use a rectangular later?
- Are there any formulas that you think you will need?

2. How many camels would it take to make your new paint brush?

Instrumental Majors

1. What size bow would you have to have to have one containing one million/one billion hairs?

Questions we need to answer before we begin.

- What kind of bow, for what instrument?
- Does that matter?
- Will our new bow be wider or thicker?
- Could we figure out the height of the violin/cello/bass that you would need for the new bow?

2. What is the vibration of a note? Estimate the number of times you would have to play the note to have created 1 million vibrations.

Musical Theater

1. How many performances of *Ragtime* would you have to do to have performed one million/one billion minutes?

2. How many words were spoken total for each performance. Estimate the number of performances would have to be performed to have said 1 million words total.

Question we need to answer before we can begin.

- What is the length of Ragtime?
- Does it matter the number of actors that are in the play?
- Are the performances running consecutively?
- Does that matter?

Technical Theater

1. What size would a theater have to be in order to accommodate one million/one billion people?

2. What would the dimensions of the new theater be?

Questions we need to answer before we can begin.

- Will they be sitting or standing?
- What space will each person need?
- Will the height of the people matter?
- What is the shape of the theater?

Students will be grouped by majors (dance, theater, vocal, instrumental, technical theater) and given a problem that pertains to their specific major. I will allow calculators to work on this problem. Once they have completed their own problem, I will assign one of the other major problems to each group. I will not allow calculators for the second problem. I want them to do two problems so that we can compare the different answers. It will be interesting to see if the group assigned the problem pertaining to their major, using calculators got a different answer than the group that didn't use calculators. I am guessing that they will be in the same "Ball Park" and if we use scientific notation to write them, they will be the same order of magnitude. I will have the students present their answers and explain how they got their answers. They will need to share any formulas they used and be able to explain exactly what they did to arrive at their answers. When each group has presented and the answers have been written on the board, I will have the groups share and write the answers to the second problem on the board also so we can talk about the differences. We will calculate the rate of change or percent difference between the two answers and talk about the answers. Remember that the same problem has been done by two different groups, one with calculators and one without. I am doing this to show two things. The first is that they have the skills to do math without a calculator and the second is that the order of magnitude will be the same. This will emphasize the importance of the place value of the digits.

I believe that the students have the skills and the curiosity now to create their own questions. I will have them think about "ball park" problems and then list all the skills needed to solve them. I will then have the students exchange the problems and solve them. I want at least two people/groups working on the same problem always so we can compare our answers. Since the first group of questions we designed with the student's majors in mind, I will have them create problems that would interest specific groups of students. The students interested in sports may have to find the number of blades of grass at the stadium. The activist may find the number of miles/inches that Hillary Clinton traveled during her campaign. These are just suggestions I will give them as a starting point.

The students will have to have access to the internet in order to get certain facts. The Google search will be the fastest way to find some of the information and I will assign certain "fact gathering" homework assignments.

- How many miles to New York?
- What is the route of the local bus in miles?
- What is the circumference of the earth?
- What is the height of the Statue of Liberty?
- What is the number of miles Americans drove last year?
- What is the depth of the Allegheny River?
- What is the length and width of the Mississippi River at specific places?

These are a few of the questions I will assign and once the facts have been gathered, we will create problems together that incorporates the facts into our problem. The possibilities are many and I think that the students will enjoy making the problems themselves They will have to think about the skills needed to solve the problems and that is what I want them to do. Yes, it is important to solve the problem but, if my students are thinking beyond the crunching of numbers, they are becoming problem solvers.

I will give the students some web sites I have found helpful, but I expect them to have no trouble gathering information using the computer. I will be using the internet myself to gather facts and have listed some relevant information in the appendix. I have also listed a problem bank that may be useful. It is always important to think the problem through before beginning to make sure the students have the formulas and math skills need to tackle the problem. The math is not that complicated but the organizations skills are important since most of the problems require students to think small and expand. I would encourage students to use mental math whenever possible and demonstrate it every opportunity I could. I would then have them do the problems again using accurate math and show the students that the difference is small or the same depending if you show your answer in expanded form or exponential form.

The final part of this lesson is a brainstorming session with the students to think of a way to share our knowledge of large numbers with the entire school. Since the students now have a much better idea of what one million really is, I will have them try to come up with an idea. I want this project to have some social value and want the student input but there is a practicality issue that must be remembered. There are many interesting websites that help to show the magnitude to this project and I don't think that there are many ways to display one million. I want the students to recognize this after checking several websites.

I have included a small question bank of questions that emphasized the space, amount or length of one million objects. The following questions can be altered or expanded into many interesting questions. Be creative and have fun.

Question Bank of Possible Questions:

If you line up one million $1 bills end to end, how long would be in feet, yards and miles?

If you covered a surface with one million $1 bills, how big a carpet would it be in square feet / square yard / square miles?

If you filled a space with one million $1 bills, how large a room would you need in cubic feet / cubic yards / cubic miles? Even one billion will be far less than a cubic mile. Do a rough calculation: how many cubic feet in a cubic mile? $(5000(1 + 5.6\%))^3 = 125{,}000{,}000{,}000(1 + 5.6\%)^3$. You can check that $(1 + 5.6\%)^3$ is about 1.18.

Find the population of the world and write in using exponents. (List your sources)

Find the population of China and write in using exponents. (Where did you get your information?)

List by order of magnitude the top ten billionaires. (List your source)

This lesson fits very nicely with the standards required by the NCTM. According to the Principals and Standards for School Mathematics: the standards are listed below.

Number and Operations - Grades 9 - 12

- Develop a deeper understanding of very large and very small numbers and of a various representations of them.
- Understanding meaning of operations and how they relate to one another - Grade 9 - 12
- Judge the effects of such operations as multiplication, division, and computing powers and roots of the magnitude of quanties.
- Compute fluently and make reasonable estimates - Grade 9 - 12
- Judge the reasonableness of numerical computations and their results

I feel confident that this lesson will comply with any state standards.

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Math Spoken Here (MSH)

www.mathnstuff.com

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"*tables, scientific notation, conversions of years to seconds*"

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