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Math and Consequences: Environmental Context in Math Instruction

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Introduction and Rationale

It is my fundamental belief as a teacher that I am duty bound to make my instruction applicable to tangible, important, and emergent issues in the communities which our students are part of. As a teacher of mathematics, I have been continuously challenged to make the content of study more accessible to students by providing significant context to their studies of algebra and geometry. Of all contexts available, no issues are more pertinent, to the well being of the current and future generations of students, than those of environmental impacts of human action. If we are not preparing students to apply mathematics in defining and solving real problems in their environment, we might as well not trouble them with the effort to learn the skill sets of algebra or higher mathematics.

Perhaps this point could be put more simply with the question: What are we trying to accomplish as teachers of mathematics? The obvious answer is that we are trying to guide our students in learning mathematic concepts and skills. Yet we are all too often disappointed in the outcomes of our efforts as reflected in the achievement of our students. On the many, many occasions when students ask, "Why do I have to learn this?" we should resist the urge to take a defensive or authoritarian attitude. Our students' learning and our own sanity will be much better served by authoritative responses that neither avoid nor artificially pacify our students' questions. In fact, what we should be doing is asking our own form of the question, "Why do I have to teach this?"

This flies in the face of conventional mathematics instruction, which tells us that math is an absolute staple of academia. However, while the esoteric appreciation of the pure form is something that those of us who teach it can enjoy, it is not a valid approach to teaching the majority of our students. Our intent might best be focused on more immediately improve the lives of our students and the communities of which they are members, than on enforcing an aesthetic of mathematics. And, at our given time in history there is no greater exigency than the environmental impact of human actions. This is the most legitimate context we have available for the type of strong problem solving skills that can be developed in mathematics study. But, recognizing this is only a start.

It is invaluable to encourage students to ask, "What is, and has been, the impact of human energy use and industrialism on the natural world?" The connections that exist, between how we live and the state of the world in which we live, are vast and can quickly become overwhelming. However, for our students today, avoiding thinking about the ecological state of their world is much like procrastinating over completing a homework assignment - the longer it goes untouched, the more likely it is that there will be too little time to do anything about it.

This is cause for both pause and action in how we should proceed. In our capacity as teachers, the enablement of others to use the skills that we teach takes precedence over running out into the world to solve every problem by employing these skills ourselves. Whether or not our students become scientists, industrialists, economists, or policy makers who solve the world's problems, it is to us to educate them for any of these potentials as best we are able. We will be better able when we provide them the perspective to articulate and attack problems - which are exemplified best, and at a high-level of rigor, by environmental concerns.

To implement environmental contexts in mathematics curricula does not require that we become experts in the field of environmental science. Indeed, as a comparatively new field of study, which requires multidisciplinary knowledge and research, there are relatively few experts on the subject in the world. [1] There is such a high degree of knowledge necessary to navigate any one of the sub-specialized areas of research that synthesizing them altogether is still a process-in-development for even the most knowledgeable researchers. What is required is that we have a general acceptance of the scientifically grounded reality of environmental impacts, cumulatively called "climate change," and that we familiarize ourselves with some of the complexities of what, thus far, I've referred to as environmental issues in a very general way.

My lack of specificity, on the one hand, is a conscientious choice and, on the other hand, it is out of necessity. That is to say, it is necessary to be a bit general in order to broach the subject. What we may now commonly refer to as climate change requires some abstraction if I am going to write about it in a paper that is somewhat less than the size of a telephone book.

To fully understand environmental issues, which no one "fully" does, goes beyond having a mental image of melting polar ice caps. Although this is a legitimate part of the overall picture, it is far too simplistic to represent what is truly an enormous, variable, and still increasing catalog of issues. It is an important objective for us, before we set objectives for our students, to tackle a broad notion of what climate change means.

What Climate Change Means

A search query on Google generates forty six million three hundred thousand hits, even when "climate change" is entered in quotation marks, which force the search to only find the exact phrase. [2] While this is currently outshined by the two hundred sixteen million hits for "michael jackson," it still represents an enormous amount of data.

Besides being vast, climate change has remained a politically contentious issue for longer than a rational length of time. To get past this contention, let me start by saying that, if a teacher is conflicted about the politics or supposed debate on climate change, there are several options open to her or him: explore some of the multitude of reportage in the popular media based on correlations and causation; read some of the overwhelming academic papers that verify human actions as a perpetuating influence on climate change - the best of which may be the coordinated research findings of respected, independent scientists contained in the Intergovernmental Panel on Climate Change report of 2007 [3] ; directly consult scientific experts who are open to sharing their findings; or, if you are either truly skeptical until more obvious effects of climate change become part of our everyday lives or find yourself politically polarized on the issue, there is always the option to go on in denial.

In actuality, even if we exhaust the research in order to satisfy our skepticism, retirement will arrive before we will be able to make up our minds. There is so great a spectrum of knowledge, with more continuously being developed, that no one can take it all in over any short period of time. However the knowledge monumentally points to the industry of humanity as a primary catalyst for climate change. The reasons why this has been so thoroughly resisted in political debate is open to any number of accusations - from ignorance to malicious corporate conspiracy - a topic beyond my scope here. All confrontational jibes aside, and to invoke President Obama's diplomatic manner of speech, "Look… the point is not to argue whether climate change is somebody-in-particular's fault, the point is to consider the realities of it and find solutions that will provide a better future for all of us."

To consider the realities, briefly, a note on usage of the term "climate change" is appropriate. We have lived with the term "global warming" for a significant amount of time, but we will likely hear it less often in the future. The warming of oceanic surface temperatures, overall average temperatures, and a myriad of warmer temperature consequences are a big part of the climate change puzzle. However, other consequences arising directly from human activities and energy use, as opposed to coming about via global warming, are also hugely important issues in the realm of environmental and human health. These include water pollution from industrial effluence, soil degradation and erosion from homogenous mega-farming, and species loss from human encroachment - among many other issues. There are also issues that arise as chimeras, monstrous combinations, of environmental impacts - some of which are relatively well understood and some of which are not yet fathomed. Air pollution, to which I will give some attention later, is an example of climate change that results directly from the spreading of airborne particulate matter in fossil fuel exhaust and is indirectly exacerbated by the effects that warming air temperatures have on urban weather patterns. [4] Furthermore, even those consequences resulting from the rise in average temperatures do not always "feel" like "warming."

The earth's average global temperature has risen by 1.13 degrees Fahrenheit since the beginning of the industrial revolution. [5] A strong analogy can be made for why this is so important, and why it might lead to such serious consequences, by considering an elevation of about one degree in internal temperature for a human being. With a rise of 1.13 degrees in body temperature, a person would be at, on average, 99.73 degrees, nearly 100. This also serves as an analogy for the variance in environmental outcomes. The symptoms range from flush dry skin to skin drenched with sweat, from severe thirst to an inability to eat or drink comfortably, from unbearable warmth to painful chills, from restlessness to exhaustion - all, seemingly different, symptoms of an overheated system or fever.

The power of altering the terminology away from "warming" is quite serious. In a positive sense, the word "change" is more inclusive of the other grave concerns, not necessarily directly related to warming, which we should also be giving our attention to. "Change" also more accurately represents the effects that many people will observe as a result of the rise of average global temperatures (which will actually mean cooling temperatures in some regions and which is certain to mean erratic changes in many regions). In a negative sense, taking away the ominous note that is rung by "warming" may also take away exigency from popular perception of the issue. [6] By any other name, the problems - of human health, population and sustainability of our ecological and economical existences - are still as serious. The goals I propose are meant, through a regular and sustained practice, to help students build the math skills to solve them.

Objectives

Writing in 1945, the eminent mathematics professor G. Polya had a prophetic vision of the study of mathematics as much more than an esoteric interest of academics.

> The future mathematician should be a clever problem-solver; but to be a clever problem-solver is not enough. In due time, he should solve significant mathematical problems; and first he should find out for which kind of problems his native gift is particularly suited. [7]

I particularly like Polya's choice of words in referring to math skills as a "native gift," because I most certainly believe that the "kind of problems" that these skills are suited for are those of the environment that we are creating for ourselves to live in. This will become more relevant with each passing year, as, according to the most sophisticated climatologist predictions, human influences on weather patterns will surpass all other natural influences by the year 2050. [8]

Another guiding principle from Polya is, "The teacher can make the problem interesting by making it concrete." There may be an unequalled educational opportunity to make math interesting in the climate change issues that we are facing. In fact, it may very well be, through our concerted efforts, that the popular focus on climate change and its myriad consequences can be rendered into a vehicle for inspiring our students to achieve in mathematics. The guiding principle and foundation for each objective I will put forward below is that, by implementing concrete context in math instruction. I expect to encourage students to gain a greater understanding of how mathematics applies to their environment and to become agents of change for their communities, present and future, through their application of mathematical tools.

Not every useful application of mathematics in a student's life will necessitate ecological topics. The application of basic math skills to their personal finances and basic economics, as well as other career fields, whatever they may be, is obviously extremely valid. Assuming the basic mathematical operation and notation skill sets are in place for students, however, the real reason to teach them the advanced fields of algebra, geometry, and above is to prepare them to manage bigger conceptual constructs and to apply problem solving skills in complex scenarios. By giving them the contextual experience of valid, applied mathematics we can not only prepare students for future "out of the box" performance in a more effective and more rigorous way, but we can improve their focus and engagement throughout the course work, and thus their attainment and retention throughout their academic mathematics experiences. The best of contexts will be focused on the issues that are most poignant and, as I make the case above, there are few concerns that could be considered more all encompassing for this generation than the environmental issues that comprise climate change.

I will use four major objectives, each based on the expectation that it can be achieved by bringing environmental contexts to mathematics instruction. These goals are to: 1) Engage students in a sense of urgency about their achievement in mathematics; 2) Improve students' application of mathematics skills; 3) Improve students' decisions about which mathematics skills to apply; and 4) Improve students' abilities to apply skills in pragmatic ways to solve unique, unscripted problems.

The first point, of engaging students, goes beyond the obvious need to engage students in the class work of their mathematics course. I expect that by providing environmental contexts it will be possible to instill a new sense of urgency in their process of thinking about mathematics and its utility as a tool for dealing with climate change. This is a rather large order to fill in a time when the decline and inadequacy of mathematics education in the United States is widely acknowledged and despaired. While every student will not wish to become an environmental scientist, the cohort of adolescents, who are typically of quite a diverse class in the Pittsburgh Public schools, have a strong general aversion to the idea of being a victim of ignorance. This may not be readily apparent in the aversion to learning mathematics among these students, but that reluctance to invest their thought and energy is a product of devaluation of the course content. By applying their study of mathematics skills toward solving issues of environmental consequences and human induced climate change in accessible, observable ways - these same students can be won over to recognition of and investment in their course work. I expect that this investment will extend to students culturing esteem for mathematics as they become more versed in its uses, by further means of environmental context.

An appropriate critique of my expectations as stated here might be, "Do we really expect students to become significantly interested in the minutiae of ecology?" I have been trying to honestly work this question out for myself, whenever this reasonable doubt creeps up. I think, that in the midst of this doubt, both my skeptical colleagues and I are stuck again with that earlier, vague image of a melting ice cap and very little specifics or tangibility about climate change. This bleak landscape, while evocative, isn't necessarily one that rouses a person to action. We might advance the mental picture by introducing a polar bear, adding a factor of cuteness and humane perhaps, but will students be able to connect with a creature that lives far away and might eat them?

I believe that they can actually, though there may be some steps between students' initial awareness of climate change and their initiation into the conscientious stewards of their world. These steps to environmental enlightenment might be better trod toward an animal that lived in the other direction though, to a Central American rainforest amphibian that no longer exists. The golden toad was the first documented creature to have suffered extinction as a result of climate change. [9] Its habitat on mountain hillsides in Costa Rica was severely affected by loss of moisture that had come in the form of mists and fogs. An average temperature rise pushed cloud levels higher, to the point that these animals lacked the environmental means to survive. This story goes further than the loss of one beautiful, rare creature in nature's spectrum however.

The trick is that toads, frogs, and small lizards can often serve as harbingers for the types of problems that we ourselves might also become susceptible to. In a roundabout fashion, the golden toad guides us, like a totem spirit animal might, to a crisis that is dear to, if not particularly well understood by, many students in schools across the country - the crisis in Darfur. Of the many complexities of the human condition in sub-Saharan Africa, which is actually being slowly transmuted into the Sahara proper, is the fact that a dramatic change in weather patterns has led to the loss of annual rains across a swath of the continent. This shift has driven those who were dependent on life-giving rainwater to find resources in other regions, where, in conjunction with many other complications, conflicts have arisen between refuges and former inhabitants.

In this sense, climate change is a major contributor to the desperate situation in the Sudan. [10] A human crisis that students can emotionally sympathize with and intellectually come to understand as a problem that may be articulated and, hopefully, be mitigated through the use of mathematics, among other skills. I am not promoting horror as a motivator for students to better learn math, but a responsible and thoughtful examination of these kinds of interconnected environmental contexts can establish a valid urgency in their learning.

The second objective I am offering is the expectation that the use of cogent, valid environmental contexts will improve students' comprehension and application of mathematics skills, beyond the abilities that would otherwise be learned statically or in less meaningful contexts. G. Polya made the case for using the very basic context of the walls, floor, and ceiling of a classroom, which is typically cubic, or hexahedron-shaped, as an environmental context for students to better understand the geometric principles that applied to a parallelepiped of the same shape. [11] Seeing is believing, and students are better able to learn about what they believe in and trust with their own senses. This basic fact can be lost on us as adults because we have already made the leap into abstract thought so long ago. Environmental contexts are, by definition, around, and even within, us at all times. Even in the seemingly intangible air, there are opportunities to engage students, provide them some touchstone to ground their work upon, and thus improve their application of skills. This goal is valid for the improvement of students' mastery of skills in the abstract concepts that fill both algebraic and geometric studies.

My third objective is to improve students' abilities in choosing the most appropriate math skills to employ in defining, evaluating, and solving problems and problem solving. To separate the "skill to choose the right skills to use" from the previous objective of "using the right skills properly" is more than a slight nuance. It is, in fact, the division between independent mathematical competency and mere, though still profoundly important, ability to follow through algorithmic processes. Providing environmental contexts can vastly improve upon students decision making in how to go about solving problems because these contexts offer valid examples of situations that don't come with "how to" guidance. What these situations do provide are a finite set of clues, which students can begin to build interpretive skills to manage in parallel with the mathematical processing skills that form their toolkit to choose from.

An example of the important ability to recognize factors, which are not immediately or explicitly stated, can be found in another zoological note. Victoria Fabry, an oceanographer and expert in pteropods, ran into a problem during a 1985 study of how these tiny animals' form their calcium-based shells. She found that if she placed too many of the creatures in one container they would not grow shells - rather their shells would dissolve and the pteropods would die. At the time, Fabry did not fully appreciate that it was the concentration of carbon dioxide in the containers of seawater, building up from the animals' waste, which was killing them. [12] But the situation would allow her to consider the math skills she might employ and find the limit of how many creatures she could place in one container for a given amount of time without killing them. If she had pursued this modeling with inequalities she might have been able to calculate a rate of the animals' processing of oxygen into carbon dioxide. This is an example of the kind of problem solving that can help when unexpected situations arise. In other words, how to figure out… how to solve a problem.

The fourth main objective takes yet a step further by seeking to improve students' abilities in applying mathematics skills in pragmatic ways to non-scripted and, as yet, not fully understood problems. After some relative success in establishing the earlier goals of engaging students in the urgent need to build math skills, improving their abilities in applying math skills, and improving their decision making about when and how to apply skills to a given problem, there is a new frontier. This grand adventure hits on the highest cognitive levels of analyzing, synthesizing, and evaluating problems. Actually it involves nothing less than asking students to go out and look for trouble. I say this somewhat tongue-in-cheek, but in all seriousness the premise of solving unscripted problems is for students to critically observe an environmental context and find the problematic issues inherent therein. This is the ultimate objective that I am proposing - legitimate, versatile mathematical competency of students. It is very important that this high cognitive goal comes as number four of four objectives that I am outlining here.

In past school years I have asked students to simply indentify, in a few sentences, a problem that they saw in the world around them and a way in which math might be applied to solve it. Despite some minor successes, these instructional efforts have been a bit floundering, quite honestly. Students require a fluency in mathematics in order to interpret and communicate a real life situation in terms of math. Asking students to make mathematical sense of what they see around them can only be a fruitful activity if students are already performing with some proficiency in a variety of math skills that they know how to apply. As with language skills, which cannot be developed in isolation of the world which they are used to articulate, define, and communicate about - math skills must first be developed in environmental contexts if students are expected to later call upon their skills in new, untested, and novel problem solving scenarios.

Assessing Environmental Context Objectives

Measuring the achievement of these four objectives may seem an even more challenging task than introducing environmental contexts as a whole. This concern can be allayed by recognizing that each objective above is, essentially, a general enhancement of existing, more specific math objectives (e.g. students will be able to accurately draw a graph of the given linear equation in the first quadrant of a coordinate plane). For the more specific objectives there are established state standards and many existing and/or easily developed forms of assessment (e.g. provide students with a linear equation in slope-intercept form and rate their ability to accurately produce the graph). Therefore, the assessment of how well students' skills and abilities are improving, given environmental context in instruction, can be facilitated through the assessment of existing skill objectives.

The objectives I have outlined may also be directly translated to existing requirements in Pennsylvania's Math Standards. The section of standards for eleventh grade math proficiency in Problem Solving opens with a statement that is a perfect paraphrase of my objectives: "Select and use appropriate mathematical concepts and techniques from different areas of mathematics and apply them to solving non-routine and multi-step problems." [13] From this broad objective, it is possible to move forward by assessing objectives from the standards for math proficiency in Algebra and Functions [14] and Geometry [15] on a more specific, per lesson basis.

If students are meeting the existing state standards for the given mathematics curriculum, after receiving instruction that is rich with environmentally valid context, then a basic correlation can be established between the context and achievement. To go deeper, it is also possible to ask students to self-assess their efficacy and ability with math skills - in relation to the environmental context that has been part of their instruction. This is a particularly good method for assessing the second objective (to improve application of skills) and third objective (to improve choice of skills), because students will, in effect, be evaluating their own application and choices. This can also serve as an additional reinforcement for student retention of those skills that are being considered. At worst, students will receive the core content of their mathematics courses, with the bonus of some environmental perspective added. At best, students will exhibit improved proficiency levels for all skills that have been adapted with some degree of valid environmental context.

For either the first objective (to engage students with urgency) or the fourth (to improve skills in novel problem solving) additional tasks or techniques may help teachers assess student success. I would particularly recommend the use of, and will personally make regular use of, formative or summative writing assignments that are sequenced into course work to create a regular feedback loop for students' affective and conceptual achievements. This is a topic that will develop in the strategies below.

Ultimately, I hope to both enrich and improve my students' learning of mathematics skills by showing them that they are the means of understanding and solving problems in the environmental in which they live.

Ideally, I want students to be motivated by the sense that mathematical literacy, like the consequences of climate change, is a subject that can be neglected only to our own peril.

Strategies

The most important thing to have in mind about using environmental context is that it is exceptionally scalable. As teachers we can make many different adaptations to our instruction of math, and we can make them short-and-sweet or deep-and-embedded. We can use a little or a lot of context, and it may make more sense to do one or the other at specific times.

For instance, at a very basic level, we can validate a lesson full of raw skills practice in calculating the area of polygons by providing one example that briefly references human expansion into forests. On the other end of the spectrum, we could assign a long-term project in which students continuously develop geometry skills by modeling urbanization and deforestation of a region of the earth (e.g. Irian Jaya, the Indonesian state on the island of New Guinea). So, there are many ways in which, and "levels" at which, climate change issues can be added to instruction, or used as organizing themes, to make math lessons more meaningful for students. To keep this manageable, I focus on three levels - simply referred to as low, middle, and high.

Levels of Environmental Context

The first, low level of environmental context is to provide simple, limited reference to an issue that falls within the broad realm of climate change. Examples might include representing the exponential growth or decay of a species population in equation form, graphing the rate of change in crop yields for a given region, or calculating the average number of commuters per vehicle entering the downtown area of your city during a weekday morning rush hour from given information on the estimates of automobile, bus, and other traffic. These contexts, while valid, merely give a backdrop to the raw mathematics skills that students are developing within the curriculum. They might be employed in warm up exercises to quickly set a tone of environmental context or be used to build up a basis of tangibility in specific moments anywhere throughout a lesson. To maintain their integrity, these contexts should be drawn from real data as much as possible, and creating overly contrived scenarios should be avoided.

The next, middle level that can be employed might take on many appearances. Short reading excerpts from scientific journals, or articles of environmental interest from the local newspaper, might be used to broach a mathematical concept and/or skill set that the class will focus on that day. Video clips of research activities in progress may serve an identical purpose. More significantly, a sequence of word problems might be written to use the numeric data from an environmental contextual source in order to practice skills, and the context referred to continually throughout the tasks that comprise the lesson. This context can also be referred back to as a reflection of "what do these numbers mean?" when the computations and other tasks are completed to provide numeric results.

Providing problem scenarios that require students to creatively apply mathematics skills to find solutions may necessitate the inclusion of more writing exercises. Such writing can effectively be placed in advance of, in parallel with, or following after students' application of math skills (as I mention above in reference to facilitating the teacher's assessment of objectives). Student writing may also fit in at multiple points throughout coursework, with reference to and building from their earlier observations of math skills practice.

Expanding the role of organized group discussion at this moderate level will also strengthen students' conceptualization of the connectivity between the mathematics skill set they are practicing and the given environmental context that is being implemented. Such discussion can be used in both small and whole class arrangements, with serious thought toward the objectives of improving students' skill application and decision making relative to which skills are best to apply in a given situation.

The final, high level of context is something that I envision as a project-based series of activities. The overarching structure of such projects (the format and actual skills applied) may rely heavily on which of many environmental subjects are providing the context. The timing of introducing this high level, robust context is very important. It is imperative that the content works with students' prior knowledge as well as with the current curricular priorities. In general, I would make any such project-based course work inclusive of a series of readings, discussions, and mathematics skills application based on real data from the chosen context. A project will be more valid for students' learning if it involves ongoing data collection, communication, and management on the part of students themselves - to the end that they actually discuss and document findings in their own words. [16] Ideally projects can also include high level features such as field studies and guest experts, who can give anecdotal and practical examples of how they use mathematics skills in their fields.

The choice of context level may depend more on student preparedness and prior achievement, or more on the content to which the context will be applied. The best implementations of environmental context to a mathematics course will incorporate some composition of all three levels. From the outset of a math course, an understanding should be established, that the students and the teacher are working together toward a valid and meaningful set of educational objectives. As the course progresses, day-to-day lessons can regularly use low and moderate levels of context to relate all of the different, and seemingly disparate, skills and concepts of the course toward some central environmental theme (this is key because a single math curriculum can contain a wide range of concepts and skills). A high-level project structured over these daily activities can provide continuity between subtopics of the given curriculum so that students can see how the many different math skills they learn within a course are actually linked to one another. In this way, the context works symbiotically with content.

Before truly bringing any level of environmental contexts to my mathematics instruction, I have to first face one serious reality check. In the Pittsburgh Public Schools, mathematics curricula have come to contain increasingly scripted materials, which math teachers are mandated to follow on a strict basis. As this is also the case for many of my colleagues, the guidelines to adapt lessons that I have developed above require some creative manipulation. The key to this for my purposes will be the direct variation of levels of context to scripted materials.

I will use low level contexts to build some validity into the math exercises and activities that my curricula mandate and most thoroughly script. In this way it is possible to remain compliant with district guidelines and still provide the urgent engagement in real application for students. In the instances where the curricula expressly give the teacher options to choose problem sets or sections of the course textbook to draw assignments from, I will use middle level contexts to fortify validity in my students' skill practice and reflective exercises. Middle level contexts can also add a dimension of legitimacy to my teacher created assessments, such as periodic quizzes and take home assignments that contribute significantly to measuring students' independent proficiency at applying math skills. I see the greatest opportunity for high level contexts, again, in multi-part projects that tie together the various skills that sum up to a course.

Choosing Valid Environmental Contexts

As to choice of environmental contexts, a teacher should look to her or his own interests as well as the interests of the students. In making choices of context that are most specifically valid, a holistic consideration of the school, community, and city can inform the process very well. Our choice of context for mathematics can actually be guided by a metaphorical consideration of the principles of green design. With a nod to Frank Lloyd Wright's architectural philosophy of using local, indigenous materials in building, I recommend favoring the use of environmental context that is immediately visible and accessible for students. It is largely to the discretion of individual teachers to interpret the "environment" of their city just as much as the curriculum of their course. It is also important to be cautious because the first interest of the teacher may not always be the "greenest choice for development" in developing lessons.

For instance, I would personally love to focus context on wind and solar power as the future of renewable energy sources, however, significant examples of these technologies are largely lacking in the urban environment of Pittsburgh. Wind and solar technologies will most certainly enter in my use of low-level context, but for the sake of moderate to higher-level problem solving I will guide my students to work with topics that they can "reach out and touch" on an experiential level. Given my surroundings in Pittsburgh this will include two major topical foci: air quality and green building.

Pittsburgh is historically associated with rust belt pollution and has gained infamy in the last year with a worst national air quality ranking for short-term (daily levels) particulate matter, and a second worst national ranking for long-term (annual levels) particulate matter from the American Lung Association. [17] This situation is all the worse for my particular school because it is based in our downtown area, amid the exhaust of two interstate highways and several other major traffic routes. However the real story of our air quality is highly impacted by coal burning power plants to the west of the city. The small particle pollution, PM 2.5, which is emitted from their smoke stacks wafts for miles, over relatively flat land, to converge in the steep river valleys that form the primary topography of Pittsburgh. [18] Many mathematical principles and skills of the math courses that I teach will be applicable to this context.

On a more progressive note, there are several opportunities to implement the environmental context of intelligent design and green building in Pittsburgh. Immediately adjacent to my school is the LEED Gold certified Pittsburgh Convention Center. In our urban center we also have the Pittsburgh Children's Museum and the national headquarters of PNC (Pittsburgh National Corporation) Bank, each of which is LEED certified and may offer models for students to study the relevance of logical argument and geometric principles in green architecture. Also, as of late July 2009 the Pittsburgh City Council is holding votes on implementing LEED standards on all new construction and development within the city. Even more compelling than all of these contextual resources is the fact that my school is currently undergoing the final phases of major renovation for the 2009/2010 school year - making it possible for my students to observe our own school building through the lens of environmental health and green construction.

These two topics are the best starting point for me. The best opportunities for other teachers, in either other Pittsburgh schools or other cities, may well be different. For instance, some school districts, or individual buildings, may have visible solar arrays and in a variety of places around the nation wind farms have become prominent. In these cases there is every reason to create context around these technologies and to organize valid mathematical exercises around these observable structures and industries. In areas with significant agriculture a focus on the use of pesticides, herbicides, and fungicides may be more appropriate environmental context for math teachers. In areas where significant construction is ongoing, where alternate forms of transportation exist, or where other specific industries are prevalent, different choices should probably be made to offer the most valid and immediate contexts for students.

Of course there are also ubiquitous contexts that will work in most any American city to create connections between mathematics studies and the environment for students. Water use, electrical infrastructure, the materials science and life cycle of consumer products, food distribution, and many other environmental contexts are common frameworks for any of us to teach mathematics from. There are also social climate change issues, as alluded to above with reference to Darfur. Most any teacher may use such dramatic climate issues to engage students and validate math instruction for them.

Classroom Activities

The following two activity outlines (Algebra 1 - Air Quality and, more briefly, Geometry - Green Building) are meant to be amenable to adoption by other secondary teachers of mathematics, in either Pittsburgh or in other districts, or at other grade levels with appropriate alterations to the sequence of content, levels of context, and/or levels of rigor, depending on the needs of those teachers and their students. These outlines may also serve as mere notions of how a teacher may apply other environmental contexts that are more valid for math instruction in their region.

Algebra 1 - Air Quality Project

Functionality is arguably the most important, central concept in algebra. You put one value "into a function" and you get one value out - with a unique output for each input of the function. This simple core can quickly revolve outward to the concepts of substitution, direct and inverse relationships, linearity, quadratic and exponential behavior, etc., but functionality is always at the origin point of understanding algebra. The skill sets of algebra can all be differentiated and categorized in terms of tasks and actions performed on functions: interpreting, articulating, applying, analyzing, changing, manipulating, synthesizing, creating, comparing, and, ultimately, evaluating.

Climate change is a highly complex set of concepts, but similar to algebra in that when it is sifted down to its observable, measureable parts, it is also fundamentally about functionality. For each topic within climate change, correlation and causation become the main story that environmental science has to tell us. Therefore, opportunities to demonstrate functionality from environmental context are innumerable.

Air quality is a strong example for me to choose from the topics of climate change, for the reasons of validity and regional concern that I have mentioned above. I will apply it in the form of a series of introductions and an overarching project, which are primarily aimed at the first two (of seven) units in my Algebra 1 curriculum. [19] However, to help students connect the disparate forms of math skills they will study in Algebra 1, all subsequent units of study will use some, ongoing low or middle level environmental context of air quality.

Unit 1: Patterns in Numbers and Figures

This first unit is meant to both enrich student learning of preliminary algebra skills, as well as prepare students for their second unit of study. The three keys given for this unit are: students will be able to 1) use algebraic expressions, equations and inequalities; 2) model real-world problems; and 3) apply algebraic properties. This is a ripe opportunity to begin students in practical expression of their environment, of which the air that they breathe is a primary element.

There are three steps to the basic methodology that I will use to introduce air quality context in this first unit. First, to have students articulate and express functionality and patterns that they can observe in their environment, using common language and terms of expression. Second, to have students translate their linguistic expressions into number sentences and symbolic expressions that capture the meaning of their original observations. Third, to have students practice making direct number sentences and symbolic expressions from novel observations - thus guiding students in use of a triangular representational skill set between observation, language, and algebra. This model draws from numerous pedagogical methods of associating concepts and the basic premise of language acquisition promoted by Piaget and other constructivist linguists.

Examples of student assignments will include: creating expressions for the volume of cubic air it takes to fill a classroom, school, or city block; creating equations that estimate how much carbon is emitted in order for students and faculty to arrive at school each day; creating inequalities for estimating the minimum percentage of oxygen necessary for healthy biological function; using calculations of environmental air quality to prove the importance of following the proper algebraic order of operations (improper work would lead to "life and death" consequences); and relating algebraic properties, such as the distributive property of multiplication, to distribution of air pollutants.

As the curriculum that typically introduces students to the high school mathematics continuum, the first unit of Algebra 1 also allows me the opportunity to refresh students' skills with the notation and use of fractions, decimals, and percentages. Air quality context is ideal as it provides the chance to consider the percentage composition of elemental and molecular gases in the air, differing percentages of composition and density between levels of the atmosphere, pollutants, etc. Because the air is a concoction, full of so many parts and types of gases and particles, it is also full of forms of fractional, integer representations (such as micrograms per cubic meter) to build students' sense of numeracy upon. A serious skill set for all future math study.

Unit 2: Patterns in Data

The two keys given for this unit are fundamentally related to basic statistical analysis and measurement of central tendency (i.e. averaging): 1) use single variable statistics to analyze and compare data sets; and 2) use graphical representations to analyze and compare data sets. This unit is a conceptual goldmine for considering air quality and, in turn, air quality is a powerful context from which students can improve their comprehension and practice of statistical reasoning. A specific advantage of using air quality as a context for this unit is that Pittsburgh has a wealth of locally relevant documentation. A wealth of studies in physical and social sciences, as well as public policy, are available to present true example applications of the skills that we are asking students to perform with statistical reasoning in this unit.

I will begin this unit by presenting students with several local newspaper and magazine articles that report on poor air quality issues in Pittsburgh. From these articles I will present a series of questions about the claims being made, which will include, "How do we determine what the 'best' or 'worst' air quality is?" and "How do we know what average air quality should be?" These questions will be designed to generate an objective inquisitiveness in the students' approach to interpreting statistics.

I will follow this segment of exploratory reading and discussion by having students generate a list of terms that they would need to understand in order to be able to make sense of the information on air quality. The terms "mean, median, and mode" will be ones that students have already had contact with in their middle school mathematics curricula. What will be new for most students will be the concepts of "standard deviation" and "margin-of-error." The ability to analyze data in these advanced terms will be invaluable for their future navigation of the statistics that are presented to them on a daily basis. Using air quality studies as a context for learning about standard deviation is a powerful way to reinforce student comprehension. Students will become legitimately competent at interpreting statistical information by being able to toggle back-and-forth between their own observations, which they will make of data in our activities, and their rationalizations of standard deviation, which they will be able to calculate with math. [20]

Early in this unit I will have students participate in a central activity to quickly and simply produce valid, observed data sets. I will have students relax for a moment and then count their own breaths over the course of 30 seconds. The class will then report their individual counts - allowing us to produce a cumulative data set, out of thin air. Although this data set will have been obtained so easily, from so little substance, it will hold much more meaning for students because they have made it as a function of their very living and breathing. In this way air quality as a context can be an innocuous tool to significantly strengthen attainment and retention of concepts.

Once this data set is established, exercises can be carried out with it, just as they would for any set of fabricated, contrived, or arbitrary numbers (in past years they have been generated by measuring height and femur length of students, while this has been valid to true context and creating a correlation, it has been limited in expansion to more meaningful application). Students will find the mean, median, and mode - as well as discussing which of these three kinds of average might be the most valid for estimating the average rate of breathing for the class. They will identify any outliers and interpret what these distant abnormalities might signify. And, from the mean, they will develop the measure of standard deviation and margin-of-error. These are exercises and content that any students would perform in this curriculum. However, placed in the context of air quality, students will address the fact that with every few moments their bodies are taking in particulate matter from the environment around them and, again, because the students will have been intimately involved in the measurement and collection of the data set they will be better engaged and invested in the work.

As the groundwork of student awareness and vocabulary in statistical modeling is established, I will extend these advantages by making use of data from the source reports that the news articles, read earlier, were based upon. Source materials will provide mathematical enrichment to the data students have learned about. They can also benefit greatly from the very process of moving from statistics published in popular media to directly examining the objective mathematically based documents for themselves. Allowing students to draw conclusions independently from real data is an ideal, which I will continuously work toward as I refine the process of introducing environmental context to my mathematics instruction.

These reports on air quality are also robust resources of the graphical representations that we want students to be proficient working with: histograms, box-and-whisker plots, scatter plots, and functional graphing. Primary to all of these are the same measures of central tendency that make the core content of the unit. Examples of primary documents that students will use are: the State Of The Air 2009 report from the American Lung Association (2009); Chapter 5 - Inhalation, of the Environmental Protection Agency's Exposure Factors Handbook (1997); Chapter 6 - Inhalation Rates, of the Environmental Protection Agency's Child Specific Exposure Factors Handbook (2008); and the Children's Exposure to Diesel Exhaust on School Buses report, from Environment and Human Health, Inc. (2002). These resources are excellent tools to show students that their algebra skills have real ramifications in understanding their health and the living conditions in their city.

Subsequent Units

Students will continue to refer to air quality, at least at a low level of context, as they work in many aspects of organizing, solving and graphically representing linear functions, which makes up the bulk of the Algebra 1 curriculum. Too often, because the skill sets "look different" to students, they have difficulty maintaining a link between the patterns in data that they begin the Algebra 1 course with and the extrapolation of functional significance that they end the course with. The resonance of air quality throughout the course, with frequent reference to prior knowledge from the first two units of study, will facilitate continuity and conceptual depth for students of all ability levels.

The most significant achievement of using air quality as context for Algebra 1 would be to have students autonomously conduct a facilitated study of the air quality. Such a study could borrow from the methodology of the Children's Exposure to Diesel Exhaust on School Buses report from Environment and Human Health, Inc. (2002) or take a far more modest approach to data collection and development of exposure findings.

Depending upon time and resources, this project could be conducted to measure rates of particulate matter in our school building, on the public city buses that the majority of our high school students use to commute to and from school, on the school buses that our middle school students use to commute daily, and on the downtown sidewalks where our students inevitably spend a significant part of their days - exposed to whatever the city's air holds for them. This project could begin with unit two of the course and be incrementally developed, so that the data could be applied to the skill sets that make up the content of the third through seventh units.

Geometry - Green Building

Geometry is built, for all practicality, on the fundamentals of building. The development of human civilization, in its most basic structure, has relied on at least rudimentary geometric reasoning since the beginning of ancient history. Now, on the eve of revolutions in the construction industry impacted by growing recognition of economic and climate change issues, geometry is no less essential.

What I modestly and briefly propose is to make consistent low and middle level context of LEED standards [21] and green building concepts for students in the first four units of our Geometry curriculum, and one, summative, high level context project in which students will assess our own school according to the documents for new construction and major renovations. This will consist of me reducing the complex certification document checklists down to basic geometrical principles that are applied within them. In this way I can present students with palatable notions of true application of what they are studying. It is my intention to include talks given by local designers and architects who are versed in green building codes, to take students to at least one LEED certified structure in the vicinity of our school, and to have students apply their geometry skills to a proposal for optimizing student and faculty usage of our school building.

This series of contexts can be applied to the core of geometry study (e.g. basic elements such as points and line segments, measure and properties of angles, midpoint and distance formulas, etc.) and can be expanded upon through the study of geometric proofs and logical argument. There are also many aspects of civil management of our downtown environment, such as the zoning of commercial property in proportion to public space, which might be explored via the linguistic tools of conditional statements such as the reflexive, associative, and transitive properties.

I will go on to reference some real health and social issues of our city. Ideally this will include some interaction between students and city representatives. Because ancient Mediterranean civilizations are so reputed in the study of geometry, I think students will also better understand how mathematics impacts their everyday lives by considering their connections to our city. This is most fitting for my students, who are focused in the arts.

> Athens was a place where the life of the individual and the life of the city were not split into separate places. Theater, sculpture, music, poetry, business, ath letics and philosophy were part of the city's life and part of the individual's life. A great temple like the Parthenon belonged to each citizen and to the city as a whole at the same time. [22]

Annotated Bibliography

Flannery, Tim F. The Weather Makers. Melbourne, Australia: Text Publishing Company, 2005. This book lived up to the promise of making the massive topic of global warming and human induced climate change accessible. It strikes far more on the complexities of the 1.13 degree Fahrenheit than that simple number would suggest are possible.

Ayres, Ian. Super Crunchers. New York: Random House, 2007. The breadth of application for super crunching of data sets covered here is amazing. While the content does not delve into climate change, its examination of consumer markets, economics, policy science, medical industry, education, and a number of other subjects make it engrossing. The challenge will be to further examine how super crunching has been and can be applied to climate policy decision making.

Bernstein, et al, "Climate Change 2007: Synthesis Report." An Assessment of the Intergovernmental Panel on Climate Change. Report adopted section by section at IPCC Plenary XXVII, Valencia, Spain, November 12-17, 2007. This document cannot be overvalued as a tool for comprehensive, objective understanding of the best and worst case scenarios of carbon emissions on our global environment. The findings of the IPCC have been underutilized by media in the past two years and can become a primary resource for teachers of mathematics and other fields.

Polya, G. How to Solve It: A New Aspect of Mathematical Method. New Jersey: Princeton University Press, 1945. This book has been a continued inspiration for me and also informative to each paper that I have written within the Institute approach for the Pittsburgh Teachers Institute. It is an essential for math teachers.

Wargo, John. Green Intelligence. New Haven, CT: Yale University Press, 2009. The breadth of study and significance of content allows the reader to absorb a great deal in a short time, which is essential to make a foray into this ever-developing field of study. While the tone of the content is measured and objective, the emphasis given to dealing with climate change is sincere and could be summed up with Benjamin Franklin's adage, "an ounce of prevention is worth a pound of cure."

Doucet, Clive. Urban Meltdown: Cities, Climate Change and Politics as Usual. British Columbia: New Society Publishers, 2007. Though I make only a scant reference to this novel in my research unit above, it has done a great deal to inform my sense of connection between climate change and municipal politics. Doucet, as a city councilor in Ottawa, opens many avenues of thought as I consider my own city's governance and the lessons I can provide to students.

Endnotes

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[21] LEED 2009 for Schools New Construction and Major Renovations

[22] Clive Doucet, Urban Meltdown: Cities, Climate Change and Politics as Usual (British Columbia: New Society Publishers) 2007, 54.