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Making Connections in Symmetry

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Introduction

Symmetry has always seemed an afterthought, when teaching geometry in the elementary grades. We have always stressed an importance on learning about the different kinds of polygons, without giving a complete understanding of how symmetry plays into defining the polygons themselves. In fact, I would venture to bet that most elementary school teachers do not fully understand, themselves, how symmetry is inherent to different kinds of quadrilaterals. When I realized that these two elements were connected, I thought that this would be a perfect opportunity for me to bridge this gap in understanding with my students.

This unit has been created with fifth grade curriculum in mind, but I think would be appropriate for third through sixth grade. Students will need to have an understanding of properties of quadrilaterals in order to get the most out of this unit, but some of the activities may be appropriate for students that are still learning about geometry at a more basic level. In my fifth grade class, we work with geometry for eight weeks. For the majority of the time, we investigate polygons and their properties. We spend a great deal of time exploring the properties of quadrilaterals and triangles. The students look at how the diagonals of a polygon, along with its sides and angles, define the kind of polygon it is. Another big focus of the unit is for them to make and test conjectures about triangles, quadrilaterals, and other polygons.

I teach at an elementary school in North Carolina. In my instruction, I follow the local pacing guide that is supplied to our school by the school district. The curriculum is based on the North Carolina Standard Course of Study for fifth grade. Most students come from homes with parents that have college degrees, and in some cases are professors at the local college. For the most part, the students are very high achieving and work hard every day. I also teach students with special needs, so I try to provide lessons that allow students to work with physical representations of the concepts, as well as use lots of examples of what we are working on. I also have to be careful to balance that with work that is challenging enough to stimulate the advanced students in my class. Through this unit, I hope to provide both of those kinds of learning opportunities to my students.

Rationale

Symmetry is everywhere. That may seem like a broad generalization, but it is true. People see reflections in the mirror every morning, wallpaper patterns as they sit and eat their morning cereal, traffic signs like the regular octagonal stop sign and the equilateral triangle yield, and frieze patterns on the buildings on their way to work. The students in my class are seeing the same things, and are probably paying more attention to how these kinds of patterns make them feel about what they are looking at than I do. Tapping into the child's natural curiosity about the world around them, and creating a deep connection with their life and what they are learning in school is a powerful teaching opportunity that I feel shouldn't be overlooked. This unit is designed to help make these kinds of connections with the real world, as well as make the subject matter more cohesive.

Symmetry, as a main idea, seemed very basic to me as an elementary school teacher. Before I participated in this seminar, my knowledge was limited to "line and rotational" symmetry. However, I feel that it is disadvantageous for me to approach symmetry in only these ways with my students. After learning how integrated symmetry is, not only to the understanding of the everyday world, but as it connects to science, art, and different aspects of geometry, I realized that I needed to increase my background understanding of this subject area to better serve my students. I also realized that it is imperative that I have an understanding of geometry as it relates to points and figures within a plane.

Before the unit begins, the students will have worked with geometry for several weeks. During this time I think it is important for them to build a working knowledge of the language of geometry. I have seen many instances in elementary school in which students learn mathematical concepts that are given more kid-friendly names for the sake of understanding. This allows them to work with an idea without having to understand the complicated language that may be attached to it. I believe that this works for awhile, but when the students are expected to become more sophisticated in their level of comprehension, sometimes this difference in language actually impedes their understanding. For my students, fifth grade is the appropriate year for them to start using more formal mathematical terms.

Therefore, I find that it is appropriate to maintain a level of expectation in my classroom in which students are expected to use the standard mathematical terms consistently. At first some of my students are apprehensive to do this, but as time progresses I find that this vocabulary starts to come naturally to them, and that students actually begin to correct each other when their peers do not use the more sophisticated words. In my experience in the classroom, setting up this level of expectation not only helps them understand the concepts better, but also makes the language of math more than just a vocabulary lesson.

Background Knowledge

Polygons

Triangles and quadrilaterals are the plane figures that we will work with in this unit. A triangle is a figure with three sides (line segments) that connect at three vertices. A vertex (plural form: vertices) is the point where two sides intersect. Where the sides intersect, an interior angle is formed inside the triangle. A famous fact of

Euclidean geometry is that all three interior angles of the triangle add up to 180 degrees. A regular polygon is a figure with all sides the same length and all angles the same measure. For triangles, the equilateral is the regular polygon.

There are three types of triangles that are determined by the lengths of the sides of the triangle. An equilateral triangle has three equal sides, and three equal angles that measure 60 degrees each. An isosceles triangle has at least two sides of equal length (we can also call these sides congruent). This triangle will also have two interior angles congruent, because of the Isosceles Triangle Theorem. This theorem states, "If two sides of a triangle are congruent, then the angles opposite these sides are congruent." (1) A scalene triangle has no two sides congruent to each other, and thus no interior angles congruent.

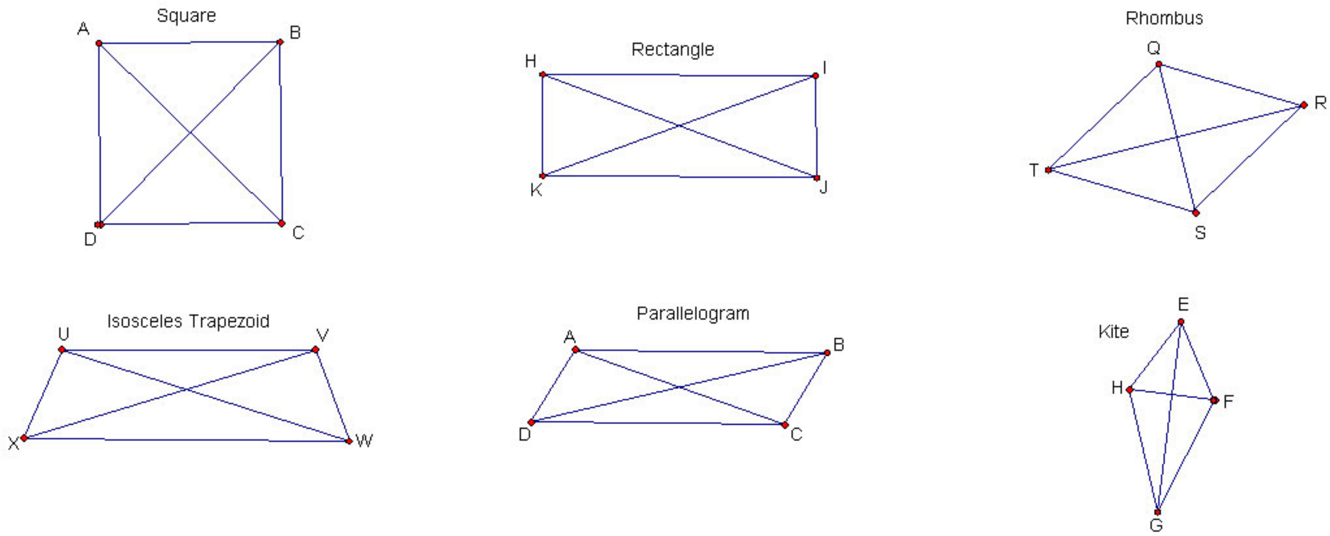
A triangle can also be named for the kinds of interior angles it has. An acute triangle has all interior angles acute (measure less than 90 degrees). If a triangle has one obtuse angle (an angle that is more than 90 degrees, but less that 180 degrees), then it is called an obtuse triangle. A triangle cannot have more than one obtuse angle because the sum of two obtuse angles will yield more than 180 degrees, and this is not possible in a triangle , since the sum of all three angles is only 180 degrees. A right triangle has one angle that measures 90 degrees (a right angle).

Quadrilaterals have four sides and four vertices. Two sides end at each vertex. The sides should not cross each other. The sum of the interior angles of a quadrilateral is 360 degrees, as one can see by dissecting the quadrilateral into two triangles using a diagonal. There are several categories of quadrilaterals that are determined by factors such as side length and parallelism and perpendiculars of the sides. Different kinds of quadrilaterals are parallelograms, kites, and trapezoids. A parallelogram is a quadrilateral with two sets of opposite, parallel sides. There are three special kinds of parallelograms: rectangle, square, and rhombus. A rectangle is a parallelogram whose adjacent sides are perpendicular to each other. A rhombus is a parallelogram with all four sides congruent. A square is the combination of the properties of both of these quadrilaterals. It has four congruent sides, and four right angles. The square is the regular polygon for the quadrilaterals.

A kite is a quadrilateral that has two pairs of congruent, adjacent sides. Trapezoids have at least one pair of parallel sides. There are a few specific kinds of trapezoids. The right trapezoid has one side that is perpendicular to the two parallel sides. The resulting adjacent angles formed both measure 90 degrees. (*Adjacent* describes angles or sides that are next to each other. *Opposite* angles or sides are across from each other.)

Figure 1.1

Diagonals of Quadrilaterals



A diagonal is the line segment connecting a pair of opposite vertices of the quadrilateral. The diagonals of each of these figures can be used to identify the quadrilateral. When examining diagonals, we can look at how they intersect, as well as if they are congruent to each other. For example, the square has diagonals that are congruent to each other, bisecting (bisect means that they intersect at the midpoint), and perpendicular (they intersect at a 90 degree angle). (see Figure 1.1)

Isometries Within a Figure and Symmetry Classifications of Triangles and Quadrilaterals

Polygons can be transformed within a plane. The kinds of transformations we will discuss are called **isometries**. "Isometry" means distance preserving: the transform of every line segment by an isometry will have the same length as the original segment. There are two kinds of isometries that can be a symmetry of a bounded figure: rotations and reflections.

A figure has rotational symmetry when it can be rotated around a center point, and match up exactly with itself before it has rotated 360 degrees. If the figure rotates and only matches up with itself at 360 degrees, that is called the identity. When we refer to the other rotational symmetries of the figure, we call those non-trivial symmetries. When students are asked if a figure has rotational symmetry, it is assumed that they are being asked about the non-trivial symmetries.

Reflectional symmetry (otherwise known as "line symmetry") is what students most commonly think of when they hear about symmetry. This is the kind of symmetry in which a side can be reflected over an axis of symmetry to produce the same image on the other side. Essentially, if you could fold an image in half and both sides match exactly, that fold line is an axis of symmetry for the figure.

Figures can have both reflectional and rotational symmetry. In fact, if a figure has two lines of reflectional symmetry, it will also have rotational symmetry. This was something that I never knew, but think that it is important to articulate to my students. By illustrating this relationship, the students can make more connections within the content that they are studying. This can be illustrated by looking at a rectangle. A rectangle has two reflection symmetries, across either of the lines through the center that are parallel to one of the pairs of opposite sides. It also can be rotated 180 degrees to match up with itself. We can also use the equilateral triangle to illustrate this point. This triangle has three reflections, and can be rotated 120 degrees and match up with itself twice.

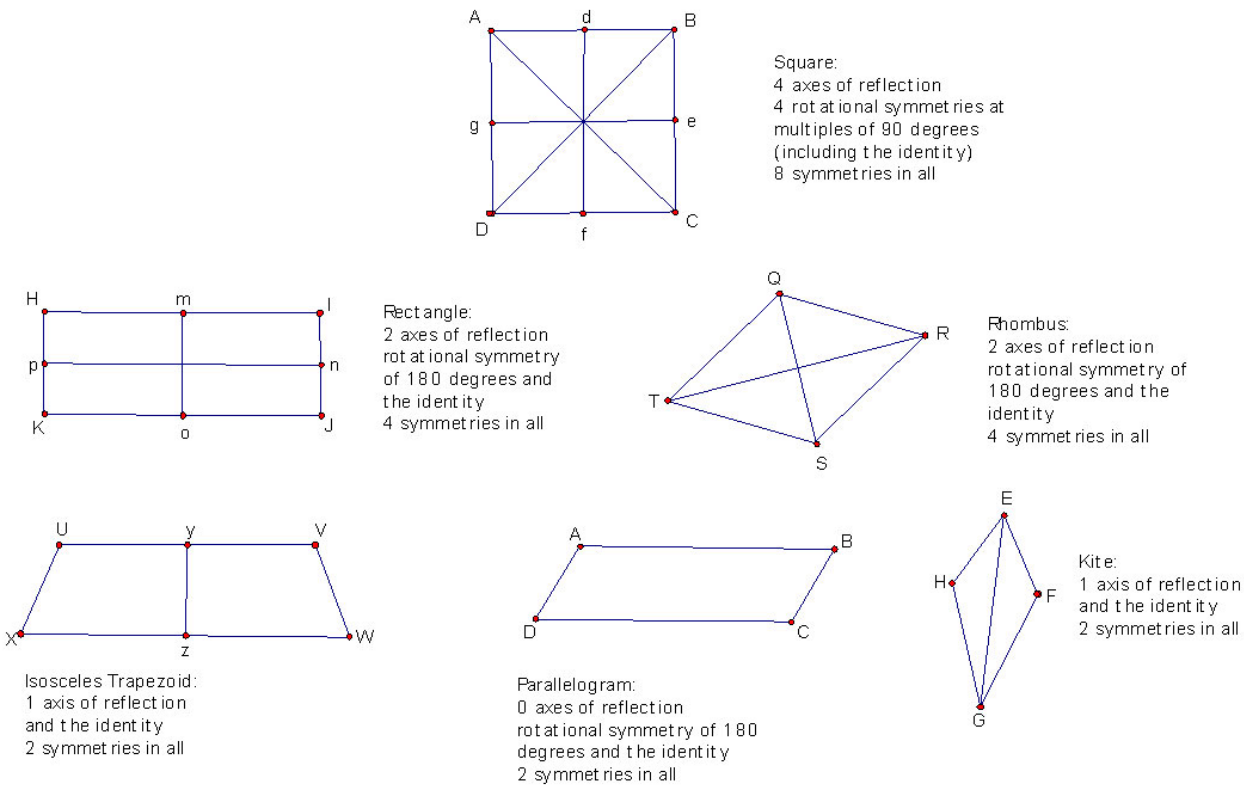
In geometry, we learn that polygons can be classified using properties like number of sides, length of sides, angle measurement, and diagonals. We have mentioned several examples above of this kind of classification. Triangles and quadrilaterals can also be identified using their symmetries. Not only can we identify a triangle using its symmetries, but also it is specifically defined by its symmetries. For example, if we are looking at a triangle that has only one reflection symmetry and no rotational symmetries, what kind of triangle must we have? We can't have a scalene triangle, because all of the side lengths are different, so none of them would match up if reflected. We can't be looking at an equilateral triangle, because it has rotational symmetry, and three lines of reflection. Therefore, we must have an isosceles triangle. Why does this make sense? Because two sides are equivalent in length, these will be the only sides that when reflected, will mirror each other. Thus there will only be one line of reflection, and no rotational symmetry.

At this point, it may be important to point out that getting used to the idea that the identity as it is considered a symmetry, can take some getting used to. I know that this idea goes against most everything I have been taught about symmetry as a child, and how I learned to teach it to my students. I found that the best explanation of this idea came from Marcus du Sautoy when he had the same curiosities about this property of symmetry and said "But I soon saw that if symmetry meant anything you could do to the triangle that kept it inside its outline, then not touching it at all- or, equivalently, picking it up and putting it back in exactly the same place- was also an action that had to be included." (2)

An equilateral triangle has all equal side lengths and all interior angles congruent, thus it will have three axes of reflection that go through each vertex and the midpoint of the opposite sides. Since this kind of triangle has more than two reflections of symmetry, it will also have rotational symmetry. In this case, it has rotational symmetry every 120 degrees. So, in total every equilateral triangle will have five non-trivial symmetries and the identity, for a total of six. Since a scalene triangle has no equal sides or angles, it won't have any non-trivial symmetry.

Figure 1.2

Symmetry Classes of Quadrilaterals



Quadrilateral classification can also be determined by the symmetries of the figures. We'll start by explaining the square, since it is the most special kind of quadrilateral. Since a square has four equal sides and angles, it will have reflectional symmetries across four axes. Two of those axes of symmetry travel through opposite vertices. (See Figure 1.2) In square ABCD those axes are labeled line segment AC and line segment BD. The other two axes go through the midpoints of the opposite sides, as illustrated in by line segments bf and ge in square ABCD. Because there are more than two reflectional symmetries, the square must have rotational symmetry. The square has rotational symmetry at 90 degrees, thus there are three non-trivial rotational symmetries. Including the identity, every square will have eight symmetries in all.

The next most symmetric quadrilaterals are the rectangle and the rhombus. Both figures will have two axes of reflectional symmetry. The rectangle's axes go through the midpoints of the opposite sides, while the rhombus' go through the opposite vertices. Please refer to rectangle HIJK and rhombus QRST for a visual representation of these line symmetries. Both of these figures have rotational symmetry at 180 degrees. So, in all each of these figures will have four symmetries, including the identity.

The last three quadrilaterals to classify are the isosceles trapezoid, kite, and the parallelogram (not a square or rectangle). Each of these will have one, non-trivial symmetry, along with the identity, for a total of two. The isosceles trapezoid has one set of non-congruent, parallel sides, and one set of congruent sides that are not parallel to each other. This figure will only have one reflectional line of symmetry across the axis, which runs through the midpoints of the parallel sides, and is perpendicular to them. (See Figure 1.2) This is labeled by the line segment yz in trapezoid UVWX. It will not have any rotational symmetry besides the identity. The typical kite is composed of 2 pairs of adjacent, congruent sides. This figure will have a reflectional symmetry over the axis that travels through the vertices where the congruent sides intersect. (See Figure 1.2) Line segment EG in kite EFGH illustrates this axis of reflection. There will be no rotational symmetry for this figure, except for the identity. Finally, a typical parallelogram will have no reflective symmetry, but will have rotational symmetry at 180 degrees, as well as the identity.

There are a few other things to make note of that will help in proving to that these symmetry classifications hold true. In all the cases that have been described here, if a quadrilateral has symmetries of the type belonging to a given class of quadrilaterals, then it belongs to the class. For one, a 180-degree rotation always takes a line parallel to itself, if the center of rotation lies on the line. This fact can be used to explain the rotational symmetry of parallelograms. The 180-degree rotation of a parallelogram around its center exchanges pairs of opposite sides, and therefore must also exchange opposite vertices. The center of rotation must lie on both diagonals. Since the opposite vertices are exchanged, they must be at equal distance from the center of rotation. The general parallelogram, the rectangle, the rhombus and the square all share this symmetry characteristic. Another thing worth considering is that rotational symmetry will also preserve the diagonals and the intersection point of the two diagonals, around which the figure rotates. If your students study the diagonals of quadrilaterals, then this would be worthwhile for them to consider. Lastly, take into account that a line that reflects to itself through an axis of symmetry must cross that axis at 90 degrees. This is shown in the axes of symmetry that pass through the midpoints of the sides, in the cases of the trapezoid, rectangle, and square.

Figure 1.3

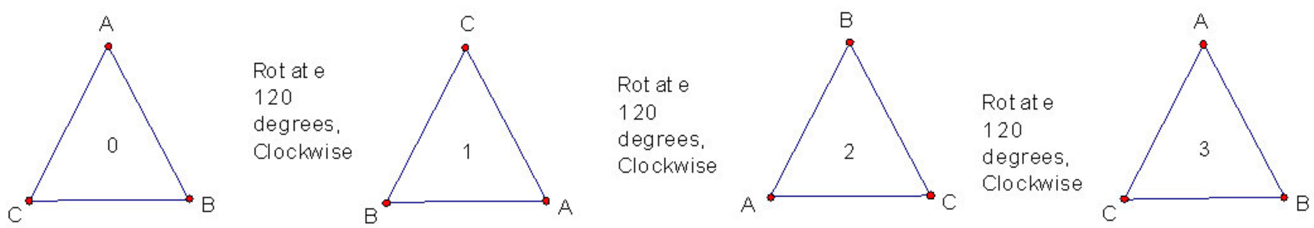
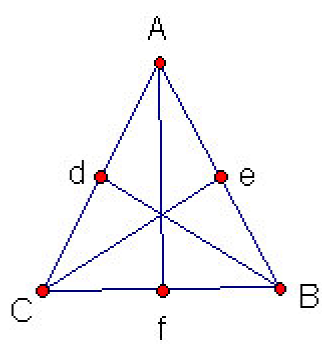


Figure 1.4



If it is difficult to picture these symmetries, do what we would have our students do; draw an example. By drawing an example of the figure you are investigating, and labeling its vertices, you can perform the reflection and rotations to prove to yourself that they are indeed possible. For example, let's take a look at the reflections and rotations in an equilateral triangle.

Figure 1.3 illustrates the rotational symmetry of the equilateral triangle. You can see that as the figure rotates 120 degrees clockwise, the shape and size remains the same. We can see that rotation has occurred because the locations of the points A, B, and C have all changed. Notice that when the original triangle (Triangle 0) is rotated 120 degrees clockwise the vertex labeled A moves to point B. Point B moves to where Point C was, and Point C moves to where A was. Triangle 3 shows the final rotation, which is the figure's identity. Figure 1.4 shows the reflectional symmetry of the equilateral triangle. From every midpoint of a side to the opposite vertex is a line of symmetry.

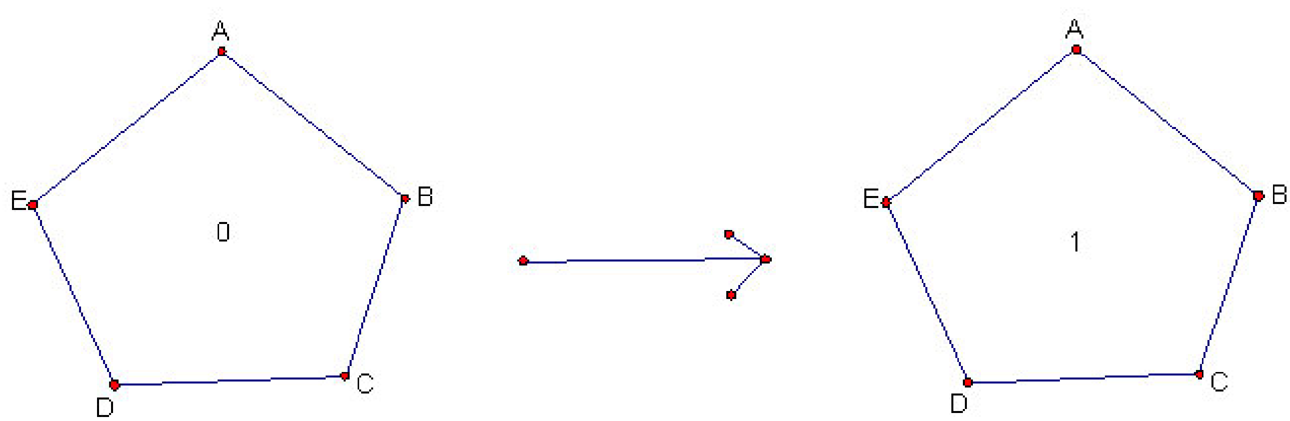
Isometries of Plane Figures

In the early grades transformations of polygons are most often referred to as "flips, turns, and slides." Again, I believe that it is important for me to recognize that my students will most likely come to me with this prior knowledge of the content without being comfortable using the most formal mathematical language. Thus, it is my job to connect the standard mathematical terms with their prior knowledge, and then be sure to insist that they use the proper terms for these different transformations.

There are four types of isometries that are to be understood as transformations of the whole plane: translations, rotations, reflections, and glide reflections. A translation is commonly known as a "slide" in the early grades. When an object is translated, it moves over a distance while maintaining its original shape and size, and orientation. All isometries preserve shape and size. The special thing about translations is that all lines are moved parallel to themselves, and with the same orientation. To illustrate a translation it may be helpful to label the sides, so that it can be clearly shown that the object has not changed in orientation, but has just moved in a linear path. (see Figure 1.5)

Figure 1.5

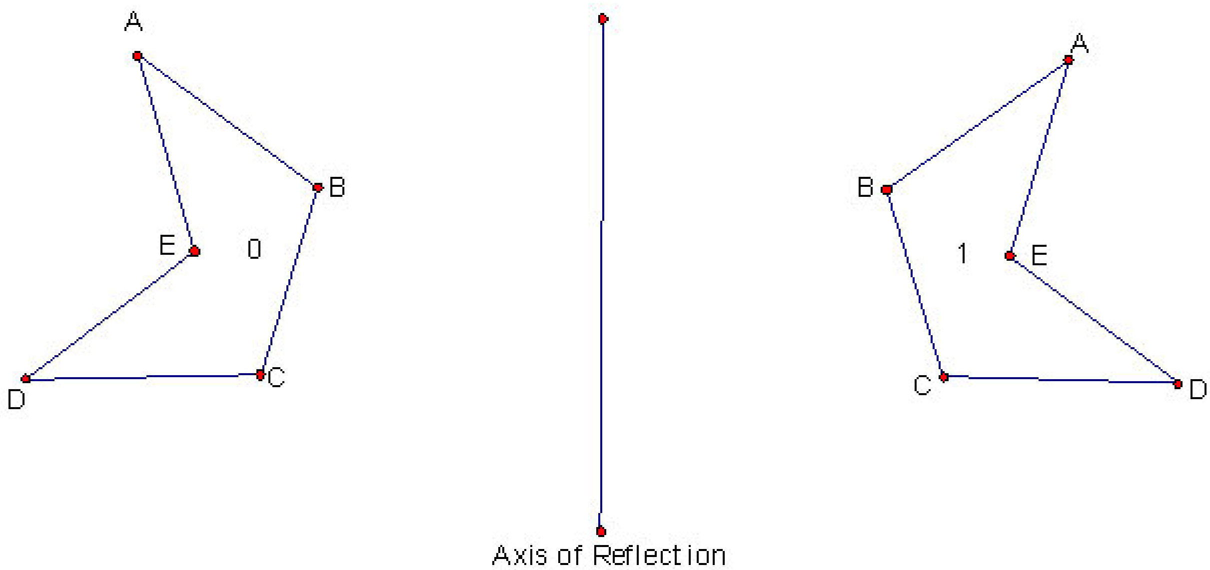
Translation



A reflection of a figure is the transformation that produces reflectional symmetry when it preserves the figure. In the early grades this is usually referred to as a "flip." A reflection flips over a line (called the axis of symmetry or line of reflection). The figure will maintain the same size, shape, and angle measurements after this transformation. It can be best illustrated with real world examples such as a picture of a mountain reflected in a lake or other body of water. In a reflection we can identify the line over which the flip was made. This is called the axis of reflection. Each point of the transformed figure will be the same distance from the line of reflection, but on the opposite side. (see Figure 1.6)

Figure 1.6

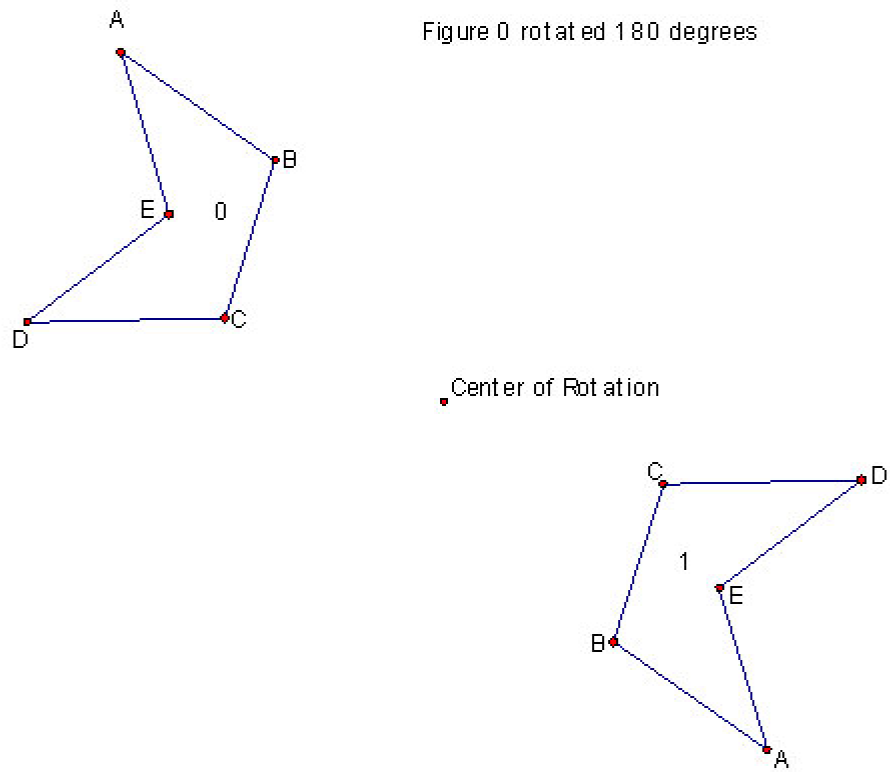
Reflection



The third transformation covered in the elementary curriculum is what students will call a "turn." These turns are rotations. These are the same rotations that determine a figure's rotational symmetry, but instead of a figure being rotated around its center, it is being rotated around another fixed point and can be rotated through any angle. This point is called the center of rotation. Along with a center of rotation, the figure needs an amount and direction of rotation. Figures can be rotated clockwise or counterclockwise up to 360 degrees. (see Figure 1.7)

Figure 1.7

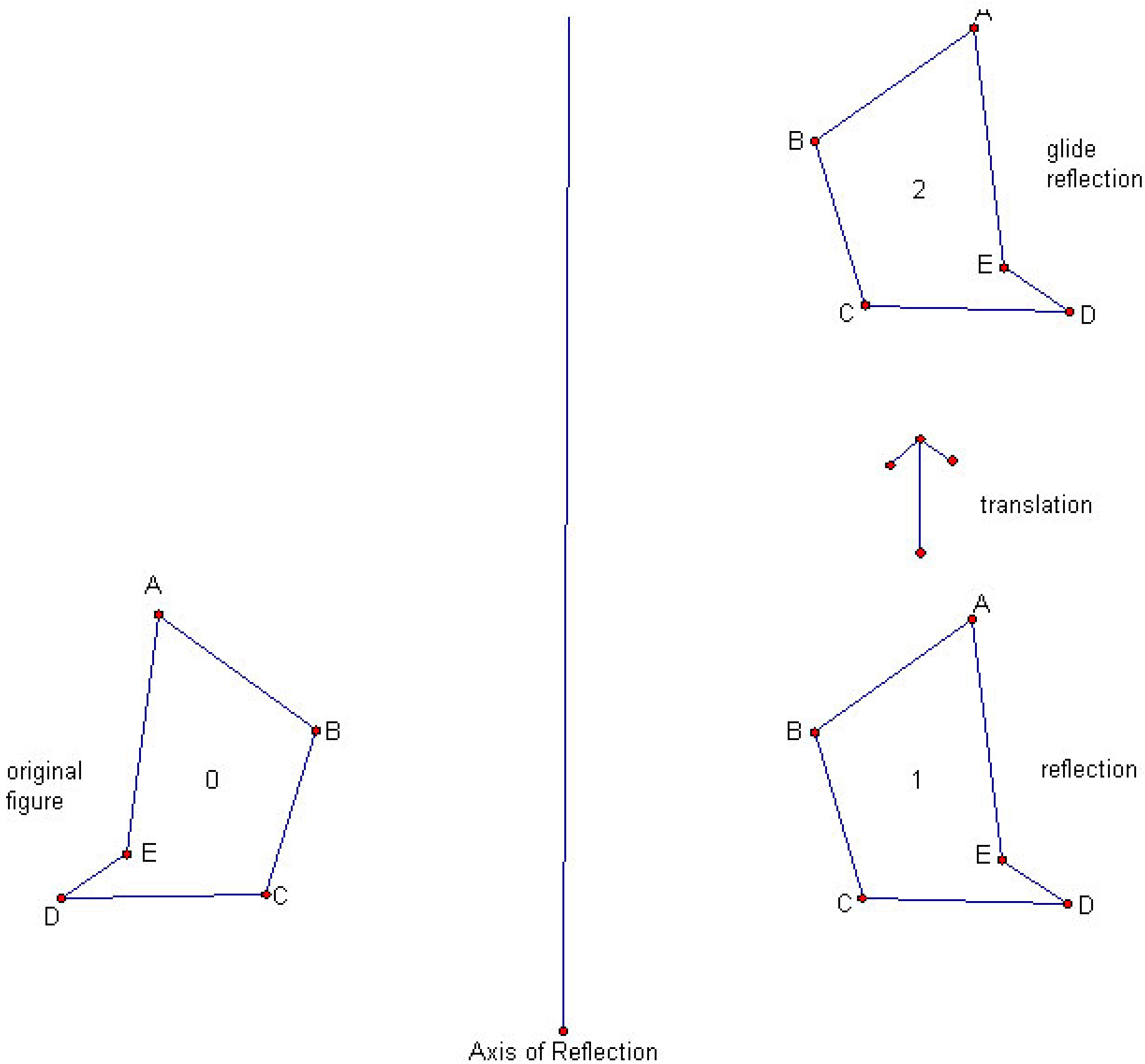
Rotation



A glide reflection is not usually covered in elementary school curriculum, but since students will be familiar with what it looks like, it is one that I think students would enjoy learning about. A glide reflection is a combination of a translation and a reflection. Since the figure is reflected and then translated, the transformed figure will have the opposite orientation from the original. The best illustration of a glide reflection in real life is a picture of footsteps in the sand. The images of the feet are reflective in nature (as one is a mirror image of the other), and in a natural gait the two footprints will be separated. Each footprint is a glide reflection of the previous one. Glide reflections are also commonly found on decorative borders.

Figure 1.8

Glide Reflection



Some isometries won't change the orientation of the figure. For example, in a rotation, the position of the points on a figure does not change once the transformation has occurred. The same holds true for translations. These kinds of isometries are called direct transformations, or orientation preserving. If the transformation changes the images of the points on the figure, as in a reflection, then this is called an opposite isometry, or orientation reversing. (3)

Strategies

Journaling

Geometry involves a lot of new vocabulary for most students. By using a math journal, I want to encourage my students to write, retain, and synthesize the new words they will learn during this unit. The journal will be interactive, as we will be adding to it and referring to it during the unit. Most of the time, when the students are expected to add vocabulary to their notebooks, it will be important for them to draw diagrams to illustrate the meanings, as well as write the definitions. Vocabulary that will be included in this unit can be found in the Background Knowledge section.

Using Graphic Organizers to Classify Polygons

"Graphic organizers are powerful ways to help students understand complex ideas. By adapting and building on basic Venn diagrams, you can move beyond comparison and diagram classification systems that encourage students to recognize complex relationships." (4) This kind of document will be a work in progress, so as the students learn more about the characteristics of the different kinds of polygons, they can add more detail to their graphic organizer. It will also provide a resource for them to refer to as they work through questions on their own.

The first kind of graphic organizer I will use with my students is a Venn diagram. A Venn diagram is used to show similarities and differences between information. In math we can use a Venn diagram to show how polygons share characteristics.

When I begin to diagram quadrilateral classifications, I will start with a giant circle or oval that will encompass all of the other circles within it. This will be labeled "quadrilaterals" as all of the figures we will put in the diagram will be four-sided polygons. Within this circle I am going to have the students draw some non-specific quadrilaterals. Then we will go through each category of quadrilaterals.

I want to start with the trapezoids. This circle or oval will not overlap with any other figures or circles. The students will draw a circle for the trapezoids and label it with the name and defining characteristic (one set of parallel sides). The students will draw out three different kinds of trapezoids within the circle and label them correctly. There should be an isosceles trapezoid, right trapezoid, and a general trapezoid that does not have congruent sides or a right angle. As we will move on to parallelograms next, I would take this time to point out another way that trapezoids are related to symmetry, which is that each trapezoid can be obtained by cutting a parallelogram in half by a line through its center. The corresponding trapezoids created will be congruent under a rotation around the center of the parallelogram.

The next section the students will create is for the parallelograms. This circle or oval will need to be rather large in size because there are several different kinds of parallelograms. The circle or oval should be labeled and the defining characteristic of the parallelograms should be written underneath the heading (two sets of parallel sides). Then the students will draw an example of a general parallelogram that is not an example rectangle, or rhombus. For the curriculum used in North Carolina, a parallelogram is not considered a trapezoid. For this unit, when a trapezoid is discussed, it is considered a quadrilateral with only one pair of parallel sides.

Within the region parallelograms, the students will create two more overlapping ovals. One will be labeled rectangle and the other rhombus. The defining characteristic for the rectangle is that it has four right angles (or you may want your students to write down that the sides are perpendicular to each other). The rhombus category has all sides congruent. For each circle the students will draw an example of each of these kinds of parallelograms. In the section where the two circles overlap, the square is located. Its defining characteristics are that it exemplifies both rectangular and rhombic qualities. The students will then draw a picture of a square to complete this section of the Venn diagram.

The last section is for the kite. This section will need to overlap with the rhombus, as a kite with two sets of congruent, adjacent sides can also be a rhombus. The students will write that the kite is defined by having two sets of adjacent sides congruent. The students will draw an example of this kind of quadrilateral and complete their Venn diagram on quadrilateral classification.

The Venn diagram that the students created will be a document that they will use in this unit and for the remainder of the geometry section. Students will continue to add more information to their Venn diagram when they explore the different diagonals of each figure and how that defines that kind of quadrilateral.

The students will also use their diagrams to create written relationships between the different quadrilaterals. The students will write comparison sentences that begin with sentence starters such as; "the rhombus is like the rectangle because ___." They will also create true and false statements about the figures to share with their classmates. An example of a true or false statement that they might create would be, "all rhombi are squares." In this case the students would have to prove or disprove this statement. By drawing an example of a rhombus that is not a square, the other students could prove that that statement is false.

Student Exploration

In order to meet the needs of all of the varying abilities in my classroom, I like to provide activities that promote student exploration. Students work with manipulatives and in groups to work through problems. I will use this approach when investigating reflectional and rotational symmetries of quadrilaterals.

After learning what it means for a figure to have rotational or reflectional symmetry the students will be given the task to find how many symmetries there are in the different kinds of quadrilaterals. Students will work in pre-selected partners for this task. I believe that students should be paired up with someone that complements their learning style. For a task that is exploratory in nature, it does not benefit the students to have a partner that will go too fast or too slow for them. In their partner groups, the students will be given several different kinds of material to complete the task.

Each group will have construction cut outs of each different kind of quadrilateral (square, rectangle, rhombus, isosceles trapezoid, kite, and parallelogram). They will also get overhead sheets labeled with the varying degrees of a circle. They can use these overhead sheets to draw their figures to test for rotational symmetry. The students will work together to prove the kinds of symmetries each figure has and write down how they know. When the groups are finished, they will share together what they discovered. When the class has come to an agreement on the symmetries of all of the figures, I will share with them the diagram (see Figure 1.1), and they will record the information in their journals.

Utilizing Technology

An integral part of my mathematics program is the integration of different technologies. For this unit, I want to incorporate the use of a geometry computer program. Geometer's Sketchpad is a licensed computer program that allows students to create geometric shapes, as well as perform transformations. (5) If this program is not available at your school, as it is not at mine, you can download a free version of a similar program called GeoGebra. (6) My students will be using this program to create symmetry patterns using quadrilaterals.

Another technology that is available in my classroom is the use of the Promethean Whiteboard. (7) We will use this to have students share what they have created using GeoGebra.

Activity One

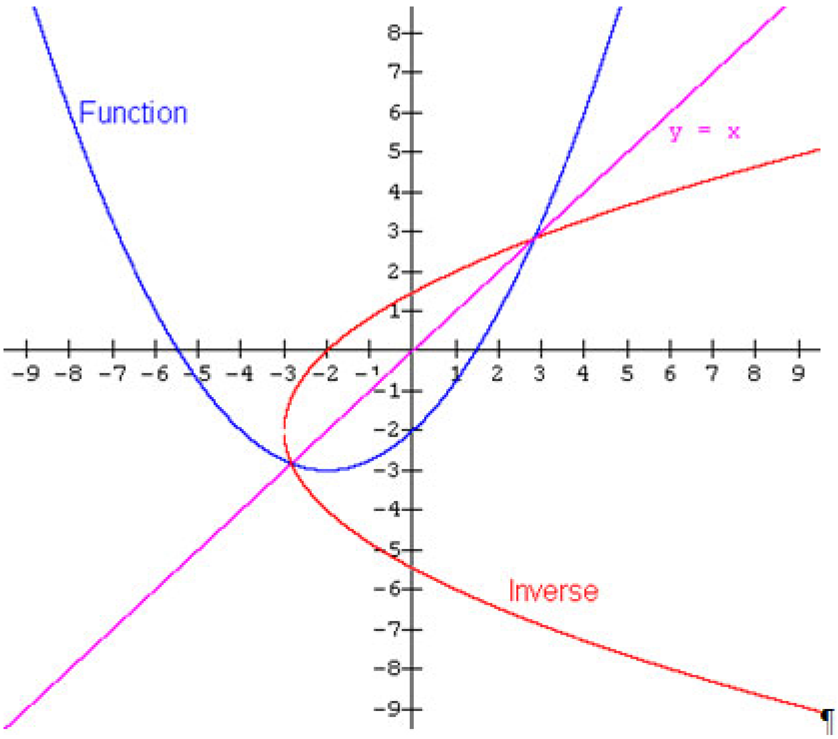
Objective

In this activity students will use a geometry computer program to create a tessellation using a specific quadrilateral as the fundamental domain, and then analyze the symmetrical properties of the tessellation to determine if these properties match the properties of the quadrilateral from which they were built.

Procedures

Before this lesson, students will have had some time to get familiar with the geometry program. Since my school does not have Geometer's Sketchpad, I will be using a program that can be downloaded for free called Geogebra. Even though the students will have had some time working with Geogebra before this lesson, I will provide an instruction sheet to help them construct their tessellations.

Figure 1.9



Each student will be assigned a different parallelogram (rectangle, square, rhombus, or a typical parallelogram, that is not any of the more special types). This will be the fundamental domain they are going to create for their tessellation. This is illustrated in Figure 1.9 with a rectangle labeled with the number one. On each side of the quadrilateral, they will then construct a square (using the side length as the length of each side of the corresponding square). This is shown above in the areas labeled with a two. Then, using the side lengths of the squares, create another quadrilateral to fill in the remaining area of the fundamental domain, as shown by the areas labeled with a three.

After they are finished constructing the students will get with the other students that used the same quadrilateral as the center of their fundamental domain and discuss the following questions.

What symmetries define the quadrilateral that you started with?

What do you notice about fundamental domain that was created?

What are the symmetries of the entire picture you have created?

Can you make any connections to the symmetries of the quadrilateral you started with to the figure that you created?

Does this pattern that you created show up in your everyday life? Where might you see this pattern?

The students will discuss and write down their ideas about the answers to these questions. Then the students will form groups in which everyone started with a different quadrilateral. The students will share their patterns and share their ideas, and then discuss if there is a conjecture that they can come up with that explains the symmetry of these kinds of patterns based on the quadrilateral that they were created from.

As an extension to this activity, have students start with any arbitrary quadrilateral. Then have the students find the midpoints of the sides and rotate by 180° around those points. Then have the students use the same questions to examine this tessellation. In this case the students will find that some of the tessellations they have created will not yield four-fold rotational symmetry, as the parallelograms did.

Assessment

Students will be assessed on the different components of their work in class. Each student will be responsible for creating a pattern (even if they worked in partners on the computer for this part of the activity). Each student will also need to have written down his/her ideas about the questions they discussed in their groups. For homework, the students will look for these kinds of patterns in the real world. They can take pictures of places in their house or around the town or in the school to find examples. They can also use the internet to find pictures of these kinds of patterns.

Activity 2

Objective

Students will use their understanding of transformations to create patterns using specific movements of their bodies.

Procedures

Students will be given the task of using a few limited resources to create the different transformations they have been studying. Students will be split up into groups of three that are predetermined by the teacher based on the similar learning styles and working pace of the individual students. In my classroom, this kind of group setting allows the students to work together more effectively so that all of the students have a fair chance to participate and understand the concept.

Each group will be given pencils, lined paper, white construction paper, large pieces of butcher paper, washable poster paint, and brushes. Since this activity will involve paint and present the opportunity for students to get messy, I will be sure to have students dress accordingly, as well as try to set up an area outside for them to work. Each group will also get the following information presented to them to complete their task.

You have been learning about how points can transform within a plane to create different patterns. Today your task is to use your body to illustrate these different kinds of transformations. To create your patterns, you can use your hands and/or feet as stamps with the paint and paper provided. Before you make your final product, you must make a plan using the lined paper and pencils. Your plan should answer the following questions for each transformation.

How can you use your hands or feet to illustrate this transformation? Where must you position your hands or feet? Draw a sketch of what you think this should look like. Why does this illustrate this kind of transformation? How can you justify it? Be sure to use the mathematical reasoning that we have discussed to support your argument.

The transformations that you are to illustrate are as follows: a translation, a reflection, a rotation, and a glide reflection. Each transformation should be done on a separate piece of construction or butcher paper. Complete each kind of transformation as many times as you'd like, but in only one direction. You should end up with a linear path of transformations for each one.

When the students are finished creating their transformations, they will go to another group to classify the pictures that group made. Finally, we will mount the patterns on the walls of the classroom by the type of symmetry it represents.

Assessment

Students will be assessed on their final products, as well as the written plans that were used in creating their patterns. I will be looking for accuracy in the final product, as well as an understanding of how the transformation was created. Teacher observation of groups discussing other patterns will also be used as an assessment tool. For homework, students will be asked to go home and teach a parent or another adult how to create one of the transformations that they created in class. Using what their child explained to them, the parents will create their own patterns that the students will share the next day in class.

Activity 3

Objective

Students will connect symmetry to their own lives, as well as to real world applications using prior knowledge and independent research of the topic. Students may choose to explore symmetry in nature, architecture, science, art, clothing design, or advertising. Students will create a final product that will explain to their classmates what they discovered through their research.

Procedures

As a warm up for this activity, I will have pictures of different symmetrical items. Pictures of things symmetrical items in nature, frieze patterns on buildings, symmetrical pieces of art, art created by MC Escher, pictures of symmetry in advertising (i.e. logo designs), pictures of symmetry in fashion, etc. I will also have pictures of these same kinds of items that are not symmetrical.

First the students will get a set of pictures of items that have symmetry and those that do not, and they will discuss in partners things that they notice about these pictures. They will use guiding questions such as, "What do you like about this picture? What do you not like? What do you notice about the image? Can you make a connection between any of the pictures and designs? Is there anything that you would want to change about what is in the picture?"

After the students are done with these discussion questions, I will ask them to look at the pictures again and separate them into two categories; item has symmetrical properties and item does not have (non-trivial) symmetry. Then the partners will discuss what role they think symmetry plays in our everyday lives. In what parts of our lives do we see symmetry? Was there any difference between how you felt when you looked at symmetrical items versus those that did not have symmetry?

This warm up will provide an opportunity for students to brainstorm different areas of their lives that they encounter symmetry. Together as a class we will compose a list of the different places we find symmetry, making sure to include the areas listed in the objective above.

The students will then be given the task to choose one of these areas to research. As they are researching they are to find examples to share with the class. The examples can be photographs they take of the item, the actual item itself, pictures from a book, magazine, or off the Internet. They are to research how symmetry is used in the particular area they have chosen. They must also provide at least three examples of symmetry and be able to identify what kind of symmetry the item has. They will be responsible for presenting this information to their classmates using a method of their choice (i.e. PowerPoint presentation, written essay, pamphlet, poster, video, etc.).

Assessment

Students will be assessed on their final product based upon the research they have done. They will be graded on the accuracy of their evaluation of the type of symmetry the objects have. I will also be looking for complete, thoughtful research using the guidelines the students were given for completing the task. That includes providing examples, as well as a final product with which they can present their research to their peers.

Materials for Classroom Use

For this unit students will need a spiral notebook to keep as a math journal. I have the students use a three-subject notebook so that the students can have a section for vocabulary, notes, and practice problems. For the student exploration, polygon cutouts as well as overhead transparencies are needed for each group of students. Computer access is needed for Activity One. Each student computer will need the geometry program Geogebra, which is available online. For Activity Two students will need construction paper, butcher paper, and washable paint. Pictures of symmetrical and non-symmetrical objects, as explained in Activity

Three, will be needed for that warm-up activity.

Bibliographies

Annotated Bibliography for Teachers

Barker, William, and Roger Howe. *Continuous Symmetry*. Providence: American Mathematical Society, 2007. This textbook is much more in depth than what elementary students need to know, but it provides comprehensive background information for teachers. It also contains pictures of wallpaper and frieze pattern groups.

Britton, Jill, and Dale Seymour. *Introduction to Tessellations*. New York: Dale Seymour Publications, 1986. This book explores elementary tessellations with polygons and other kinds of tessellations. It includes instructions on how to create your own tessellations.

Britton, Jill. Symmetry and Tessellations (Investigating Patterns, Grades 5-8). New York: Dale Seymour Publications, 1999. This is another great instructional book for teachers by Jill Britton using symmetry and tessellations. It goes through the basics of symmetry and has a lot of great activities that teachers can use with their students, including reproducible activity sheets.

Chebotarevskii, B. D., and S. V. Duzhin. Transformation Groups for Beginners (Student Mathematical Library, Vol. 25) (Student Mathematical Library, V. 25). Providence: American Mathematical Society, 2004. This textbook is a college level text that can provide more in depth background knowledge for teachers.

Farmer, David W.. Groups and Symmetry: A Guide to Discovering Mathematics (Mathematical World, Vol. 5). Providence: American Mathematical Society, 1995. This book goes through explaining symmetry in a logical progression that is very reader-friendly. Teachers that aren't very familiar with symmetry would find this book easy to read with a lot of great background information about the subject area.

Libeskind, Shlomo. *Euclidean and Transformational Geometry: A Deductive Inquiry*. 1 ed. Boston: Jones & Bartlett Publishers, 2007. This textbook provides a great deal of detailed background information about the theorems in geometry as well as pictures that help explain vocabulary used in geometry.

Sautoy, Marcus Du. "August: Endings and Beginnings." In *Symmetry:A Journey into the Patterns of Nature*. New York: Harper Perennial, 2009. This novel gives some background information about symmetry in the context of a personal narrative about the author's life.

Schattschneider, Doris. M.C. Escher: *Visions of Symmetry (New Edition)*. 2 ed. New York: Harry N. Abrams, 2004. This book can be used to provide pictures of Escher's drawings.

Annotated Bibliography for Students

Hohenwarter, Markus. "GeoGebra." GeoGebra. http://www.geogebra.org. This website is where students can download the geometry program to complete Activity One.

Libbrecht, Ken. *Ken Libbrecht's Field Guide to Snowflakes*. 1st ed. Stillwater: Voyageur Press, 2006. This book explores the symmetry found in snowflakes. Students could use it for their research during Activity Three.

Kalman, Bobbie. What Is Symmetry in Nature? (Looking at Nature). New York: Crabtree Publishing Company, 2010. Students can use this book for research on symmetry in nature for Activity Three.

Moskal, Greg. *Modern Buildings: Identifying Bilateral and Rotational Symmetry (Powermath)*. New York: Rosen Publishing Group, 2004. Students could use this book for research on symmetry in architecture for Activity Three.

Implementing District Standards

The standards for the Charlotte Mecklenburg School District come from the North Carolina State Standards for Grade Five. All of the math objectives addressed in this unit fall under Competency Goal 3, which states, "The learner will understand and use properties and relationships of plane figures."

Objectives

3.01 Identify, define, describe, and accurately represent triangles, quadrilaterals, and other polygons.

Throughout this unit students will define, describe, and be asked to accurately represent triangles and quadrilaterals as they create transformations of these figures within the plane.

3.03 Classify plane figures according to types of symmetry (line, rotational).

Line and rotational symmetry are the main focus of all the activities and involved in many of the teaching strategies in this unit. Students will be asked to not only classify polygons according to these types of symmetry, but also use those types of symmetry to create figures.

3.04 Solve problems involving the properties of triangles, quadrilaterals, and other polygons.

Students will be given patterns and figures and will be required to solve problems based on their understanding of the properties of triangles and quadrilaterals. They will also be accountable for finding real world examples of these polygons to assess their understanding of these properties.

The following English Language Arts Standard for Grade Five is also addressed within this unit.

3.06 Conduct research (with assistance) from a variety of sources for assigned or self-selected projects (e.g., print and non-print texts, artifacts, people, libraries, databases, computer networks).

Students will be responsible for conducting research about symmetry as it relates to the use of symmetry in art, science, and/or the media. This research is done during Activity Three in the unit.

Works Cited

1 Libeskind, Shlomo. *Euclidean and Transformational Geometry: A Deductive Inquiry*. 1 ed. Boston: Jones & Bartlett Publishers, 2007.

2 Sautoy, Marcus Du. "August: Endings and Beginnings." In *Symmetry: A Journey into the Patterns of Nature*. New York: Harper Perennial, 2009. 6-7.

3 Libeskind, Shlomo. *Euclidean and Transformational Geometry: A Deductive Inquiry*. 1 ed. Boston: Jones & Bartlett Publishers, 2007.

4 Walbert, David . "Higher order thinking with Venn diagrams." LEARN NC. http://www.learnnc.org/lp/pages/2646.

5 KCP Technologies. "The Geometer's Sketchpad Resource Center." The Geometer's Sketchpad. www.dynamicgeometry.com/

6 Hohenwarter, Markus. "GeoGebra." GeoGebra. http://www.geogebra.org

7 "Promethean Interactive Whiteboards, IWB and Classroom Technology : Promethean.." Promethean Interactive Whiteboards, IWB and Classroom Technology : Promethean.. http://www.prometheanworld.com/