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2010 Volume IV: The Mathematics of Wallpaper

Math and Design

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Overview

Can we engage our students in active learning by applying math in an art classroom? This form of active learning seems to be lost in our academic world. I am always looking for ways to incorporate math into other subjects and taking math from the classroom into other activities, including art. In this unit we will look at symmetry and the transformation of designs through rotation, reflection, and translation. We will also look at the less well-known transformation of glide reflection. After studying these concepts we will explore frieze patterns (a pattern that repeats in an infinite strip) to see what kind of symmetries we notice and apply these symmetries to create our own geometric pattern.

Objectives

In this unit I will have my students understand the importance of symmetry. My students will be expected to develop two different but related skills. First they will be able to recognize and describe symmetries of designs by finding centers of rotation and axes of symmetry. The second is to make their own design with one or more specified symmetries. Students need to have experience in both identifying symmetries in existing designs and making designs using symmetry transformations. They will also be expected to perform reflections, translations, and rotations to create a frieze pattern to be used in an art project.

Rationale

Background

I am an 8th grade teacher at Pittsburgh South Brook Middle School and I am writing this unit for my students. We are an inner city neighborhood school located in the south hills area of the city of Pittsburgh. Currently, around 400 students are enrolled in 6th, 7th, and 8th grades, coming from three different grade schools, all following the same curriculum. Most of our students are average to low academically coming from families that are economically disadvantaged. In order to give our students the math advantages that they need, we currently provide them with 9 math classes a week. Next year we will increase that to 10 math classes a week. In the process of adding more math to their day, they are losing something in return. This always ends up being a related art class. The one period a day that we have available to provide them with related arts classes is divided among art, music, gym, and Spanish. Classes like building trades and home economics are no longer part of the middle school curriculum. It seems that the classes that provided the practical application of math are no longer available to our students. In my "wonderful world of math" classroom, my students think that math lives and stays within that room, never to be seen anywhere else. This unit is a way of providing my students with the opportunity to take math out of the math classroom and into the art classroom. So much of art incorporates math and I hope that through this unit, my students will make that connection. By connecting frieze patterns into my math classroom, I want my students to see friezes at a deeper level and see understanding symmetry as a form of art appreciation as well as math.

At Pittsburgh South Brook, the higher achieving students are in algebra class, so the pre-algebra classes (my classes) are usually students of lower achievement. Although we have an excellent curriculum, these students sometimes struggle. Our curriculum CMP 2, "Connected Math Project", emphasizes the process of launch, explore, and summarize,(which I will explain in my strategies). Using this process can be challenging in a classroom which does not include the highest achieving students. It isn't until the 8th grade that these students are separated, so some of my pre-algebra students struggle with being the new leaders in the classroom. The students they normally look to for math guidance are no longer with them. The year usually starts out slowly, but in time, new stars start to shine in the class. By seeing these students nine times a week, I am able to give the attention needed to our current curriculum. But once they leave the room, the math is left behind and the application of this math in other classrooms is non-existent.

CMP 2 is a discovery-based curriculum. It consists of interesting math problems embedded with mathematical concepts. The focus is on developing understanding rather than processes to apply. As the students explore problems, they start to develop a deeper understanding of relevant mathematical concepts. The students apply skills that they have learned in previous lessons, as well as skills that they have learned in previous years. It helps the students to recognize math as a way of thinking; making sense of situations rather than seeing math as a series of unrelated events.

The CMP approach helps to keep students engaged who struggle and usually end up with a dislike for the subject. Because of the connection that is made between math concepts and problems, the students begin to enjoy the process. They are forced to figure out their own strategy for solving the particular problem. They see a relationship between real life situations and math. However, not all the students are able to relate to the life circumstances in the book and need other situations to experience the importance of different mathematical concepts. These concepts focus on algebra and sometimes many of my students have a hard time relating these situations to what they plan on doing in the future. They never get the chance to see how math is related to their existence whether it be work or play. And much more, they never see math as an important part of life and other non academic subjects.

The concentration on math in the 8th grade is for the purpose of getting kids ready for algebra in the ninth grade; and to provide them with a foundation that will help make the transition into ninth grade algebra a little easier. Within the Connected Math curriculum, there are a number of different text books to do just this, each stressing a different algebraic concept. One of these books is devoted entirely to symmetry and transformations. With so many of the books devoted to algebra, and so little time to teach them, this particular book on symmetry never gets to be explored. They spend less than a week on this content in the sixth grade and no time at all in the seventh. As a result, my students spend very little time on both symmetry and transformations, a concept that appears on their PSSA (Pennsylvania State Standardized Assessment). Base on the Pennsylvania State Standards, by the end of the eighth grade our students are expected to use simple geometric figures (e.g., triangles and squares) to create, through rotation, figures in three dimensions; generate transformations using computer software; analyze geometric patterns (e.g., tessellations and sequences of shapes) and develop descriptions of the patterns; and analyze objects to determine if they illustrate tessellations, symmetry, congruence, similarity and scale.

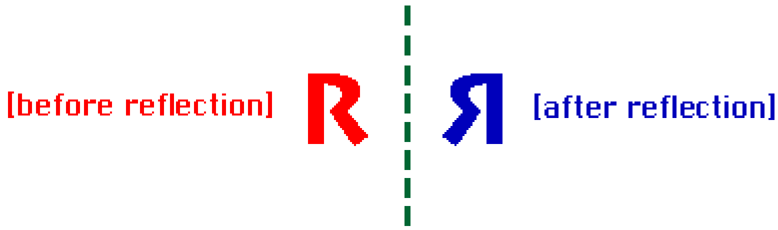
Even though we don't get to the book on symmetry (and even if we did, it wouldn't be until after the test) we are still expected to incorporate symmetry and transformations into our curriculum as a review in preparation for the PSSA. This is usually done through warm-ups at the beginning of class or via homework. As a result this ends up being a series of unrelated events with little retention. Through my unit on symmetry which will incorporate the most important points from the text, the students can make that connection in a series of related activities and a strong connection to art.

Symmetry

At an early age students do begin to recognize symmetry which is usually described in terms of transformations. A *transformation* of an object is the relationship between one position of an object and another position of that same object. This transformation of the object should preserve the structure of the object. This condition determines the symmetries of the structure. These symmetries may include the transformation of reflection, rotation, translation, and glide reflection. (The informal terms used by students in elementary grades are flips, turns, and slides.) All of these produce congruent figures (preserving both the size and the shape). I'm hoping to sharpen students' awareness of symmetry and to help them develop their understanding of the underlying mathematics of transformations.

A *transformation* of a set of points is a one to one correspondence from the set of points onto itself. The name given to any transformation that preserves distance is *isometry* which comes from the Greek *isos* (equal) and *metron* (measure). If points x and y are 3 cm apart, then after the transformation they should still be 3 cm apart. If point z is half way between x and y its image still will be halfway between the image of x and y after the transformation. Isometry preserves the distance of the points; it also implies preservation of the basic geometric shape. All of our main examples (reflection, rotation, translation, and glide reflection) are isometries.

The symmetries of an object form a system, much like addition and subtraction operations in arithmetic. The different types of symmetry can be seen by themselves or as products of several different symmetries. A motif has reflection symmetry if you can draw a line through it that divides the motif into two halves or mirror images. This line is called the *line of symmetry* or *axis of symmetry*. As a child you may have made ink blots where you drop ink on a sheet of paper, fold the paper in half over the ink and unfold to reveal your design. The fold in the center of the paper becomes your line of symmetry for the ink blot. If you fold the paper back along the line of symmetry and place it up against a mirror, you will see half of the design in the mirror and with its reflection you will have the original design. In general, a figure with reflection symmetry may not have any central point. For example, the figure below consisting of the R and its reflection, has no central point.



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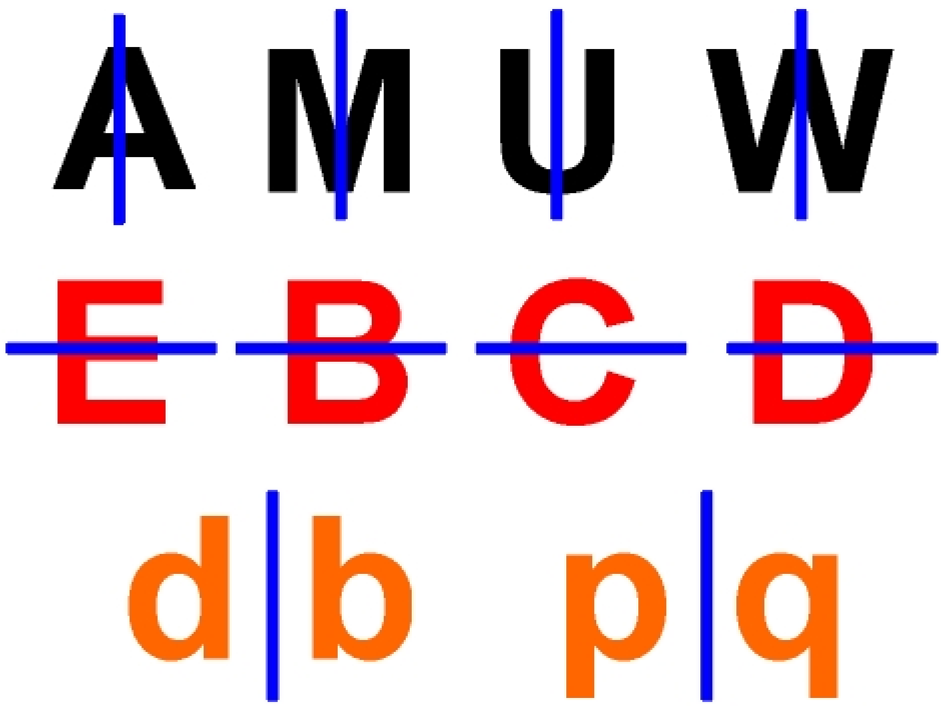
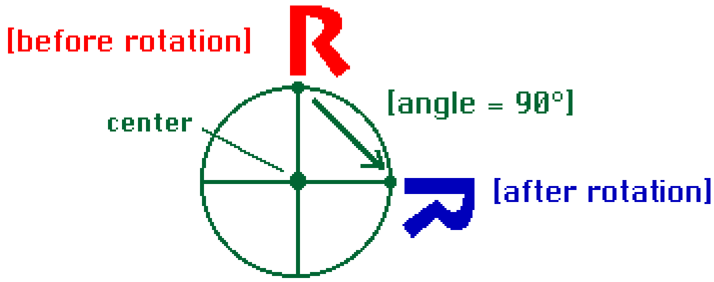
If you look at the letter "A", it has reflection symmetry because if you drew a vertical line through the peak of the letter, each point on the left half will match up with a point on the right half. This vertical line through the "A" is the line or axis of symmetry. Each point on the line of symmetry is unchanged by the reflection. The image of the one side of the line of symmetry is the same as the image on the other side of the line of symmetry. An *axis of reflection symmetry* divides a figure or motif into halves that are mirror images. Each point of one side of the line of symmetry and its corresponding point on the other side is the same distance from the line of symmetry. You can create a design with reflection symmetry by starting with a basic design and then drawing the reflection of the design across a straight line. The original design along with its reflection show reflection symmetry.



Another transformation is rotation. A rotation is also an isometry. A design has *rotational symmetry* if it can be turned or rotated around a point less than a full turn to a position in which it looks just like the original design.

A rotation is specified by:

i. the center of rotation
ii. the directed/oriented angle of rotation.



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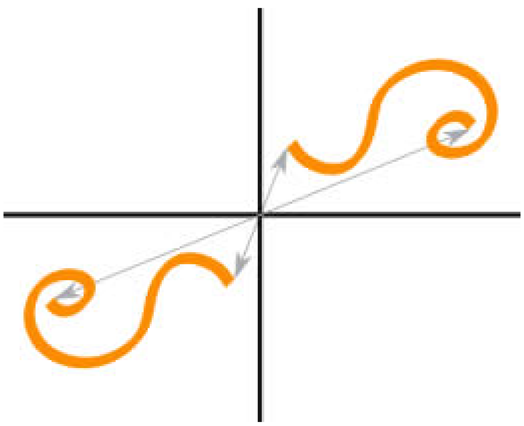
To completely specify a rotation, we must give a *directed* or *oriented* angle. The normal convention is to call the counterclockwise direction as positive. Thus, a rotation of 45 ◦ means a counterclockwise turn of 45 ◦ . All rotated designs will be equidistant from the center point of the rotation and travel on the arc of a circle. The points of the design and its image points travel around the point of symmetry in concentric circles.

The distance from the point of rotation in the original motif is the same distance from the point after the rotation. Think of a pinwheel with 4 lobes, it has a rotation symmetry of 90 ◦ . This means that it can be rotated 90 ◦ , or any multiple of 90 ◦ , about its center point to produce an image that exactly matches with the original. 90 ◦ is the angle of rotation for this pinwheel, it is the smallest angle through which the lobe of the pinwheel can be rotated to match the original lobe.

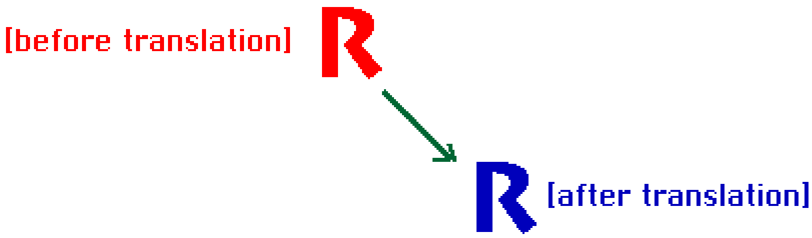
The following image has is an example of rotational symmetry of 180 ◦ around the center point.



In order to create a design with rotational symmetry, start with a basic motif and divide 360 ◦ by *n*, (*n* being the number of times you would like to rotate the motif around a center point starting from the previous rotation). This will give you *n*-1 copies of the original design, and *n* times to get the design back to the original place. The *n*-1 images and the original makebb a motif displaying rotational symmetry.

One of the most basic of all transformations is translation symmetry which is also isometric. If you can slide a design to a position in which it looks exactly the same as it did in its original position it has *translational symmetry*. In order to describe translational symmetry, you need to specify the direction and the distance of the translation. Think of the pattern of a brick wall (where there are many different possibilities), bricks running up and down and side to side.

We can indicate translational symmetry by using an arrow (a directed line segment) to show the direction of the translation and the distance the original motif has moved. Any two directed line segments that are equal and parallel define the same translation. A translation moves each point in the motif by the same amount. Each line segment is moved to a segment of equal length and parallel to the original.



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To create a motif with translational symmetry, start with your original motif and make copies of your design by displacing each copy in the same direction and the same distance apart from the previous design. Repeat thru an infinite number of repetitions, and also in the opposite direction. The original motif with all its translated images is a design with translational symmetry. With your motif repeating in both directions, a slide of a determined amount to the right or left would match each point of the motif with a corresponding point on another motif. The distances between the corresponding points are all the same. Think about using the number line. We normally think of numbers in terms of quantity, but numbers are more than that, we need to think about them in terms of distance and orientation. Translations of the number line amount to adding a fixed number to any number. Thus, sending x to x + 1 translates the number line one unit in the positive direction. Adding another 1 would repeat the motion. When you start at zero and move to the number one you are moving the distance of one unit, when you move to the number two, you are moving the distance of one more unit. The same holds true if you translate on the number line in the other direction (to the left). Moving from zero to a negative one moves you the distance of one unit also, but in the other direction. The numbers on the number line in this case do not represent quantities, but distance from zero. The sign of the number tells you the direction from zero. (Discussing this can provide a good review of absolute value). You need both direction and orientation. With translations, you add a fixed number to a general number.

$x, x + 1, (x + 1) + 1 = x + (1 + 1) = x + 2$

The associative property says that two translations by one unit makes a translation by two units.

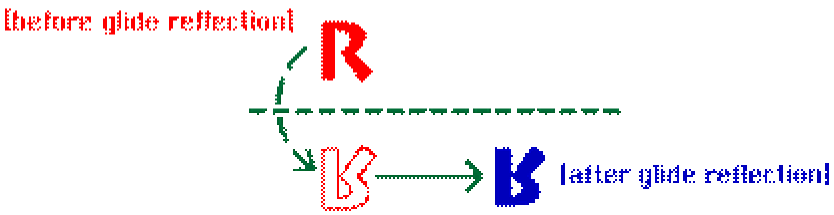
For some transformations, some points remain fixed the image of the fixed point is the point itself. The fixed points for a reflection are the points on the line of reflection. The only fixed point for a rotation is the center of the rotation. There are no fixed points for a translation; all points have images with new locations.

We can use combinations of transformations to move a motif to create a pattern. A given combination of transformations result in a single new transformation. If two different sets of transformations end up having the same out come, the ending results are equal and the routes they took are irrelevant. Only the final positions matter. The simplest example is when we translate a motif and then translate that motif a second time. The ending result is still a translation. It doesn't matter how many times it took us to translate the original motif to get there. The resulting outcome is a translation.

We can also combine a reflection with a reflection. If we reflect over line *r* and then reflect it again over line *s*, and *r* and s are parallel lines, then the result is a translation of the design in a direction perpendicular to the reflection lines for a distance equal to twice that between the lines. (Take the time to try that before moving on!) If the two lines, *r* and *s*, intersect then the effect of the two reflections is different. The net result is the same as rotating the original design about the intersection point of the lines of rotation for an angle measurement equal to twice that formed by the reflection lines. If you reflect a motif and continue reflecting that motif, the ending result will be translation of the original motif if you reflected the original motif an even number of times. And if you reflect a motif and then continue to reflect the motif, the ending result will be a reflection if the total number of times you reflected is an odd number. This is a nice exercise to do with your students, to see the result of continuing to reflect an original motif or object. Let them come up with their own conjecture about the process.

A reflection across even number of parallel lines is a translation of the original. A reflection across odd number of parallel lines is a reflection of the original.

A *glide reflection* is nothing more than a reflection followed by a translation parallel to the reflected line. Although we have described it as a combination of two transformations, it is often appropriate to think of it as a single motion. We can only accomplish this by performing both types of transformations and not by doing a translation, rotation, or reflection by itself. It doesn't matter whether you reflect and then translate or whether you translate and then reflect.



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When a combination of transformations result in all points of the design ending back at their original places, so that x ends up back at x, the ending result is called the *identity transformation* and is denoted by "i". This identity transformation may be the least exciting of all transformations but important for your students to understand. All designs are unchanged by identity transformation. The identity transformation is obviously isometric (preserving the distance). Transformations that do not equal the identity are called *non-trivial transformations*. If a rotation does not equal the identity transformation then it is considered to be a non-trivial rotation. This is an important term when classifying polygons.

Frieze Patterns

The term "*Frieze*" is from architecture. A frieze is the repeating decorative pattern commonly seen around the tops of older buildings. A "frieze pattern" is an idealized version of a frieze, in which the repetition goes on forever. Frieze patterns are also commonly found as wallpaper border. In school we use frieze patterns everyday in the form of borders around our bulletin boards.

A *frieze group* is a mathematical concept used to classify frieze patterns according to the structure of their symmetries. It has one direction of translation symmetry. There are seven different groups of frieze patterns. These can be distinguished by the particular symmetries they contain.

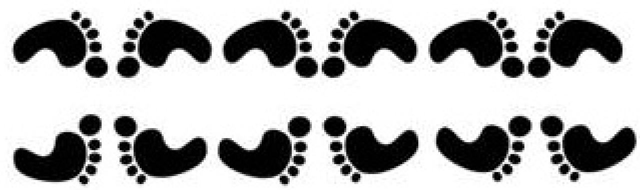
All frieze patterns have translation symmetry. We will always assume that the pattern is oriented so that the translation symmetry is in the horizontal direction. They can have other symmetries and may be distinguished according to those symmetries. They can contain elements of reflection across the central axis of the strip. They may also contain glide reflections along the axis. They could contain reflections across vertical lines (which are perpendicular to the strip). Other possible symmetries are 180 ° rotations. There are only seven different ways these transformations can be combined to form the system of symmetries of a frieze pattern. (Such a system is called a *group*.) We only show a finite portion of the design but assume that it extends infinitely in both directions. The distance between the repeat patterns is called the *translation length*. Mathematician John Conway named each of the seven frieze groups after patterns of footsteps:

Class I - Frieze pattern contains translation symmetry only. It is called the "HOP". Can we see why?

Class II - contains translation and reflection across the horizontal axis. It is called a "JUMP".

Class III - contains translation and vertical reflection - It is called "SIDLE". Can you locate the vertical axes of reflection? Do you see that they are spaced only half of the translation length?

Class IV - contains translation and rotation (by a half-turn) symmetries. It is called "SPINNING HOP"



Class V - contains all symmetries (translation, horizontal & vertical reflection, and rotation). It is called a "SPINNING JUMP". Locate the vertical reflection axes and the points of 180 ᵒ rotation. Do you see the centers of rotation lie on the reflection axes?

Class VI - contains translation and glide reflection - It is called a "STEP". It is the pattern your feet make when you walk normally. Notice that there is no purely reflectional symmetry of this pattern, only glide reflections (aside form translations).

Class VII - contains translation, glide reflection and rotation (by a half-turn) symmetries. It is called a "SPINNING SIDLE". Again locate the axes of reflection and the centers of rotation. Do the centers of rotation lie on the axes of reflection for this system

There are other ways of displaying the different classifications of frieze patterns but these footsteps make recognition easy.

Class I shows a symmetry of translation. This group is generated by a translation by the smallest distance. There is no line of symmetry, no point of symmetry and not fixed by a glide reflection.

Class II shows horizontal reflection. It is generated by a translation and the reflection in the horizontal axis.

Class III shows vertical reflection. It is generated by a translation and a reflection in a vertical axis.

Class IV shows translation and 180 ᵒ rotations. It is generated by a translation and a 180 ᵒ rotation.

Class V shows translation, glide reflections, and reflections in both axes and 180 ᵒ rotations. This is the largest frieze group and requires you to translate and then reflection across the horizontal axis and reflection across a vertical axis.

Class VI shows glide reflections and translations. It is generated by a single glide reflection. Translations are obtained by combining two glide reflections.

Class VII shows reflections across certain vertical lines, glide-reflections, translations, and rotations. It is generated by a glide reflection and a rotation.

Note that, both class IV and class VII contain both reflections in vertical lines, and rotations of 180 $^\circ$ around centers on the central axis. The separation between lines of reflection, and between centers of rotation is one-half of the translation distance. The difference between the two symmetry schemes is that, in class IV, the centers or rotation lie on the vertical reflection axes, while on class VII, the centers of reflection lie halfway between the vertical reflection axes.

Art

In art class we will talk about Shepard Fairey, an American contemporary artist, street artist, and graphic designer. His paintings can be seen on buildings throughout Pittsburgh neighborhoods. His is known for his Barack Obama "Hope" poster used in the 2008 U. S. presidential election. The original version featured the word "PROGRESS" and after its release the campaign requested that he use the word "HOPE". It was also requested that he create one with the word "VOTE" and one with the word "CHANGE". Fairey has been criticized for the use of his images from social movements. He will be the inspiration for our art project.

Strategies

In my classroom the students are seated around tables. There are four students at a table which makes for rich classroom discussion and the opportunity to talk freely. However, it sometimes make for a noisy classroom. But with the right combination of students it can work very well. The students are assigned to tables based on ability. I will use heterogeneous grouping around my tables in order to provide help for lower level students. With the lower end of the eighth grade students and full inclusion it can be challenging.

I will continue with the process of launch, explore, and summarize. In the launch phase, I am setting the context for the whole problem to the class. I do this by making sure the students understand that the situation and that the mathematical challenge is clear. This is where I connect the problem with past experiences. This is also when I decide if the students will work in their group, work in pairs, or work individually. In the launch process it is important to make sure the task is left intact so not to lose the integrity of the lesson. It's very easy to give too much away and lower the challenge of the task.

The second phase is the exploration phase is where the students actually work to solve the problem by sharing ideas, make conjectures, or develop other types of problem-solving strategies. As a teacher I move around the classroom to encourage on-task behavior and observe the students as they work towards solving the problem. This is the time to ask the students questions about their work and redirect where needed.

The final phase of instruction is the summarize phase. It occurs when most of the students have had enough time to solve the problem. This is where the students share and discuss the strategies that they used to solve the problem. This is the time to help students develop a deeper understanding of the mathematical ideas. Students are encouraged to question each other, refine their strategies, and make connections. During the summarize phase, I try securing the students' mathematical knowledge.

Besides having graphing calculators available, I also provide a variety of tools to help my students be creative during math class. I have both line paper and graph paper set up at a supply station that my students have access to during class along with colored pencils and markers. I find that by having these supplies readily available for my students encourages them to be neat and work creatively. They are more likely to take their time to graph a situation with colored pencils than just using a black lead pencil.

During part of the lesson the students will work in the computer lab using "Geometers Sketchpad" software that will be downloaded on the computers. There the students will have an opportunity to create their own polygon and through a series of transformations filling in the plane. They will also have an opportunity to create their own frieze pattern.

Activities

Activity 1: Reflection Symmetry

In the first activity, introduce the students to symmetry by providing them with a series of pictures in which to identify symmetries. Use pictures that are relevant to your students (i.e. heart, star, square). These pictures can be found in books, on the internet, or around your classroom. Your students will be focusing on reflection symmetries or mirror symmetry. Ask your students to also find representations of reflection symmetry around them. Then post a large piece of chart paper in the room with the word symmetry and invite students to provide meaning to the word. Write their meanings around the word symmetry and continue to have your students add to this poster as you continue with these lessons. Provide your students with the formal definition of reflection symmetry. You may want to provide your students with a worksheet which states the definition of "reflective symmetry and line of symmetry" and a series of objects and designs in which your students could find all lines of reflective symmetry. You could incorporate the alphabet using capital letters and have your students find the lines of symmetry within the letters.

Activity 2: Rotational Symmetry

In the second activity we will focus on rotational symmetry. Again provide your students with pictures or designs which display rotational symmetry (i.e. windmill, snowflake, hubcap). See if after investigating pictures and designs with rotation symmetry, your students can come up with their own definition of rotational symmetry and add it to the poster. Discuss with your students the angle of rotation and let them investigate to come up with an equation they can use for finding the angle of rotation. Again you may want to provide your students with a worksheet that displays different pictures and designs showing rotational symmetry. Let them indicate the center of rotation and figure out the angle of rotation. On the worksheet you may want to provide your students with a formal definition of both "center of rotation and angle of rotation".

A nice way to display rotation symmetry is to have your students make snowflakes. Real snowflakes have hexagonal symmetry, but for our purposes, we will create snowflakes that have other than hexagonal symmetry. Provide each student with two square sheets of paper. With the first piece have the students fold the paper in half and the in half again the opposite way (so that all four original corners are meeting in the same corner). Now fold the paper again the same way. Without cutting through where the center of the folds intersect (center of rotation), have your students cut into the sides creating unique designs. When they open up their paper they will have a snowflake with rotational symmetry of 90°. With the second sheet of paper, have your students fold their paper in half one way and then again in half and again continuing like they did the first time. Now have them continue folding their paper one more time. The corners will not meet with this fold. Again think of folding the paper as you would for creating a snowflake. Round out the top part of the paper by cutting off the corners. Now continue as you did with the first by cutting into the folds to create a design. After the students open up their snowflake, have them figure out the angle of rotation for their second snowflake.

Activity 3: Reflection and Rotation Symmetry

Continue with reflectional and rotational symmetry by drawing a coordinate grid with an *x*- and *y-axes*. In one of the four quadrants, draw a design or figure. If you prefer, give your students *(x,y)* points to plot to form a triangle or some other closed polygon. Have them label the coordinates of the vertices of their design if you didn't provide them with the points. Now have your students reflect over the *x-axis* and label the coordinates of the vertices of the reflected design. Next have your students reflect their design over the *y-axis* and again give the coordinates of the vertices of the reflected design. Then with the another copy of the same coordinate grid and design have your students rotate the design 90 $^{\circ}$ over the *x*- and *y-axes*. Again have your students give the coordinates of the rotated designs.

Now draw an equilateral triangle and have your students find all the lines of symmetry. This will divide the triangle into six parts. Shade in one piece of the triangle. Name and mark each line with a letter *(r, s, & t)*. Give your students a list of commands for reflecting the shaded piece over the lines of symmetry (i.e. reflect of *r*, reflect over *s*, reflect over *t*). After your have practiced that with them, see if they can reflect that shaded piece using just two on the lines of symmetry *(r & s)* and end up in every piece of the triangle. Have them list the commands that they use to get there. You may want to construct a worksheet for this activity.

The equilateral triangle shows that when you have reflection symmetry in two axes in different directions, you have rotational symmetry. You can have rotational symmetry without reflectional symmetry. Draw a simple flag on a flag pole. Use the bottom end of the flag pole as your point of rotation. Now rotate the flag 120 $^{\circ}$ around the center of rotation and draw another flag on a flag pole, not rotate the original design again 240 $^{\circ}$ around the center of rotation and draw another flag and flag pole. This new motif represents a design with rotational symmetry without reflectional symmetry. Now lets look again at the capital letter "A". Draw a line of symmetry vertical through the center of the "A". This represents a design with one reflectional symmetry without any (non-trivial) rotations.

Activity 4: Reflection and Rotational Symmetry using Polygons

Your students will be recognizing both reflection and rotational symmetry in 2-dimensional figures. Provide your students with a worksheet containing the polygons. Be sure to include a square, equilateral triangle, non-square rectangle, isosceles triangle, non-rectangular parallelogram, isosceles trapezoid, and a non-square rhombus. Have your students draw all lines of symmetry. If the polygon has rotational symmetry have you students identify the center and angle of rotation.

Ask your students the following question:

- If a triangle has a reflection symmetry, must it be isosceles?
- If a quadrilateral has 90 $^{\circ}$ rotational symmetry, must it be a square?
- If a quadrilateral has a line of reflection through two vertices, must it be a kite?

Activity 5: Translation Symmetry

Discuss translational symmetry by providing your students with different types of frieze patterns. Again check

the internet for interesting patterns. Have the students find as many symmetries as they can within the frieze pattern. Then discuss how the patterns continue in either direction forever, even though they only have a small portion of the strip. Again this would be a good time to add their definition to the Symmetry poster. Provide your students with a formal definition of "translational symmetry" and discuss how translating a design isn't always from left to right but can translate up and down and on an angle. You may want to provide your students with copies of the seven different combinations of symmetries within frieze patterns. Have your students continue looking at samples and identify as many symmetries as possible and then try sorting the patterns into groups. There is an interactive web-site that allows your students to look at all seven different frieze patterns and allows them to move the motifs in the patterns around.
http://illuminations.nctm.org/ActivityDetail.aspx?id=168

Activity 6: Creating a Frieze Pattern

Your students will now create their own frieze pattern.

Create a worksheet with the following commands:

1. Create a work sheet: Have your students create a simple design. Have them perform the following transformations on their design.

a. Translate - slide design to the left using the same distance 3 times.
b. Reflection - reflect the original design over a horizontal axis and translate the design including the reflection.
c. Reflection - reflect the original design over a vertical axis and translate the design including the reflection.
d. Reflection - reflect the original design over a vertical line and then reflect the new design over a horizontal line and translate the entire design.
e. Rotation - rotate the original design 180 ° about a point on the midline of the strip and then translate the new design.
f. Glide reflection - reflect the original design over a horizontal axis and then translate the reflection. Then translate the entire new design.
g. Reflection plus glide reflection - reflect the original design over a vertical axis and glide reflect the new design by reflecting the new design over the horizontal axis and then translate the entire new design.

2. If your school has "Geometer's Sketchpad" on their computers have students draw polygons and use different form of transformations to fill the plain.

3. Have your students pick one of the frieze patterns to use in their art project (if you are working with the art teacher). Have your students take their design to art class where they will paint a portion of their frieze pattern on a 10" x 12" canvas (or what ever is available to your students).

Activity 7: Frieze Pattern with Shepard Fairey (optional)

In art class discuss the contemporary artist Shepard Fairey. Show examples of his art work including one of his most famous pieces, the "HOPE" poster used in the 2008 U. S. presidential election. The students need to discuss an issue that affect them and they are passionate about. This may surprise you as to the issues that they are concerned about. They need to think of a single image that would show how they feel about their particular issue. Or have them think of an object that they feel represents their generation. Have them draw a silhouette of the image over their frieze design and paint it black. Display their art work around your school.

Another option is to have your students decide on their silhouette first and then use that as their motif in their design. They can use an number of transformations mentioned to create their background design and then paint it in two colors. Then they can put the larger version in black on top of the background.

Endnotes

1. The Math Forum Drexel
2. The Math Forum Drexel
3. The Math Forum Drexel
4. The Math Forum Drexel

Bibliography

Armstrong, M. A.. *Groups and Symmetry*. New York: Springer-Verlag, 1988. This book can be read without the support of a course of lectures but on a higher level. Not suited for eighth graders.

Barker, William; Howe, Roger. *Continuous Symmetry. From Euclid to Klein*. Providence: American Mathematical Society, 2007. This is a geometry text book designed with it's primary focus on transformations of the plane. The approach is concrete with complete explanations.

Farmer, David W.. *Groups and Symmetry A Guide to Discovering Mathematics*. *Mathematical World*. 5, Providence: American Mathematical Society, 1998. A very easy book to read and use for transformations.

Lappan, Glenda; Fey, James T.; Fitzgerald, William M.; Friel, Susan N.; Phillips, Elizabeth Difanis. *Connected Mathematics 2. Kaleidoscopes, Hubcaps, and Mirrors. Symmetry and Transformation*. Boston: Pearson Hall and Person Prentice Hall, 2009. An excellent text for eighth grade students on Symmetry and Transformation.

Martin, George E.. *Transformation Geometry. An introduction of Symmetry*. New York: Springer-Verlag, 1982. This book is used for graduate mathematics courses designed for secondary teachers. This is a good book for explaining different forms of transformations and giving definitions.

Ranucci, E. R.; Teeters, J. L.. *Creating Escher-Type-Drawings*. Palo Alto: Creative Publications, Inc., 1977. This book gives a step-by-step explanation of the basic geometric ideas and drawing techniques in the creation of Escher tessellaltions. It is a good book if you want to go further into creating tessellations.

Seymour, Dale; Britton, Jill. *Introduction to Tessellations*. Palo Alto: Dale Seymour Publications, 1989. A great book for teaching tessellations to children.

Websites

http://enwikipedia.org/wiki/Frieze_group

http://euler.slu.edu/escher/index.php/Frieze_Patterns

http://illuminations.nctm.org/ActivityDetail.aspx?id=168

http://mathdl.maa.org/images/upload_library/4/vol1/architecture/Math/seven.htm

http://mathforum.org/sum95/suzanne/symsusan.html

Materials List

- Calculators
- Square plain paper
- Centimeter Graph Paper
- Dot Grid Paper
- Rulers
- Scissors
- Colored Pencils
- Geometers Sketchpad

Standards/Appendices

Pennsylvania State Standards - 8th Grade Mathematics

Pennsylvania's public schools shall teach, challenge and support every student to realize his or her maximum potential and to acquire the knowledge and skills to:

2.3.8. - Measurement and Estimation

C Measure angles in degrees and determine relations of angles.

2.5.8. - Mathematical Problem Solving and Communication

- A Invent, select, use and justify the appropriate methods, materials and strategies to solve problems.
- C Justify strategies and defend approaches used and conclusions reached.

2.8.8. - Algebra and Functions

C Create and interpret expressions, equations or inequalities that model problem situations.

2.9.8. - Geometry

- A. Construct figures incorporating perpendicular and parallel lines, the perpendicular bisector of a line segment and an angle bisector using computer software.
- C. Classify familiar polygons as regular or irregular up to a decagon.
- D. Identify, name, draw and list all properties of squares, cubes, pyramids, parallelograms, quadrilaterals, trapezoids, polygons, rectangles, rhombi, circles, spheres, triangles, prisms and cylinders.
- F. Distinguish between similar and congruent polygons.
- I. Generate transformations using computer software.
- J. Analyze geometric patterns (e.g., tessellations, sequences of shapes) and develop descriptions of the patterns.
- K. Analyze objects to determine whether they illustrate tessellations, symmetry, congruence, similarity and scale.

2.11.8. - Trigonometry

C. Continue a pattern of numbers or objects that could be extended infinitely.