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Nanotechnology and Quantum Mechanics: Bringing High School Physics into the 21st Century

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Introduction

We are well into the 21 st century, and yet, remarkably, in high school physics classes we only teach material that was developed by the end of the 19 th century! How can this be true when physics underlies all scientific disciplines? There may be many answers, but a simple answer is that Newtonian mechanics, when dealing with physics on the human scale (the macroscopic world), is considered close enough! As a conceptual and theoretically minded person I don't think that this answer is adequate; we KNOW from modern physics that the world is different than we are used to encountering. However, on large scales the "weird" consequences of quantum mechanics, (which indicates that energy is not continuous but instead comes in small "packets" known as quantum and that the reality is really probabilistic instead of deterministic) are averaged out and, therefore, undetectable. However, at the nanometer (10^{-9} meters) scale, quantum mechanics cannot be ignored and in fact begins to dominate.

The physics behind quantum mechanics, which has been developed over more than a century now, is essential. We must address these developments in high school physics because science is all about presenting the best current understanding. So I am committed to understanding physics as well as I can and sharing our best CURRENT understanding of physical reality! Nanotechnology literally opens up a world of possibility in that it offers an opportunity to give a tangible explanation and use for quantum mechanics.

In first year physics classes we mostly learn about Newtonian mechanics, which is how macroscopic objects behave in the absence and presence of forces. The underlying assumption is that objects are discrete masses that can be manipulated by forces that can be applied in any amount. There is also a conviction that physical reality is deterministic, which means that we believe that if we measure all of the pertinent information about an object that we can determine its future physical parameters and interactions.

These are assumptions that have been disproven over the past hundred years. Matter is not discrete: instead, it is quantized. This means that it does not have a definite size or shape because it is quantized, which means that energy is not continuous and that there are only certain incremental amounts of energy that are allowable. Zero energy is not an allowable state so all particles must have some vibrational energy. In fact, matter and energy are interchangeable and matter is dualistic, exhibiting characteristics of particles and waves. This has been demonstrated by the dual slit experiments for electrons, which exhibit interference wave properties that result in diffraction and interference, unless the electrons are "observed," in which case their wave function disappears and they act like particles. Now much can be said about the phenomenon of being "observed," but it has been theoretically determined that any and all means of interacting with the electrons (or photons in light, or any type of matter, in fact) causes them to behave like particles.

We have a fundamental lack of understanding about the nature of matter: neither the wave model nor the particle model can explain the essence of matter or radiation. In the macroscopic world, quantum mechanical and Newtonian mechanical predictions of behavior appear to agree. However, at the nanoscale their predictions diverge and it is quantum mechanics that proves to be the most accurate. Consequently, quantum mechanics provides a more comprehensive understanding of how nature really is, so we are compelled as teachers to understand and impart this knowledge. Quantum physics is the best model of reality that we have and I believe that we as teachers are not teaching it because we do not understand it, and are ourselves, unable to fully grasp and engage quantum mechanical implications.

Quantum mechanics is not always consistent with our everyday experience of the macro world and in fundamental ways is contradictory to it. Consequently, it is counter to our intuition. According to J.L Martin, "The basic rules of quantum mechanics are not at all complicated: the main problem is the hurdle of unfamiliarity… it has only been relatively recently that physicists have been forced to accept that 'everyday' mechanics must be modified to deal with phenomena on the cosmological or on the atomic scale. In the main, quantum mechanics is concerned with the atomic scale. Moreover, quantum mechanics is *statistical* in nature: it deals with questions of probability. This is in itself a psychological barrier for many people: even Albert Einstein was inflexibly opposed to a theoretical scheme which leaves so much to chance." [1] Einstein was famous for his tremendous success pursuing his intuition, but the success of quantum mechanics over the past century seems to indicate that we must accept that reality is not the way that we would expect, or often, even for which we would hope. We must acclimate to this quantum mechanical world in order to progress in our fundamental understanding of how nature works.

As society delves further into the nanoscale, and as nanotechnology provides more and more opportunities for our students and more challenges for all citizens, it is no longer possible to accept Newtonian mechanics as being "close enough." Quantum mechanics forces us to see reality in fundamentally different and challenging ways, but it is our best understanding and it is imperative as teachers of future scientists and citizens that we impart this knowledge, even though it may be beyond our comfort zone. Nanotechnology, by getting into the scale where quantum mechanics begins to dominate, requires that we address the "weirdness" of quantum mechanics (because it behaves unlike the macroscale world), of REALITY, as best we can comprehend it!

Another important aspect of matter is that it is probabilistic in that there is a defined amount of minimum uncertainty in the product of the momentum and position, as defined by the Planck constant. This observation is called the Heisenberg Uncertainty Principle and it is a fundamental characteristic of matter. Simply stated you can only know the position and the momentum within a given amount, known as the Planck constant. The more accurately you know the position the less you know its momentum and vice versa. A corollary is that we can only know the energy and time within the same limit of maximum accuracy. In fact, we can only know the probability of specific outcomes, so nature is not deterministic!

Quantum mechanics dictates a level of uncertainty and breaks with determinism of classical mechanics. "Uncertainty has entered physics and replaced the determinism of Newtonian mechanics." [2] As is usual in quantum mechanics, we must look to experiments for answers, and, "experiment at the atomic level apparently suggests that the physical world is *not* deterministic… Thus Newtonian mechanics needs to be modified." [3] As scientists faced with the consequences of quantum mechanics, we must reacclimate to an unfamiliar, unexpected and often bizarre new reality.

I will be teaching this subject to a variety of levels of physics classes in the urban Pittsburgh Public School district. I will address this unit to general mainstream first-year physics classes, gifted first-year physics classes and algebra-based Advanced Placement second year physics classes. This represents a broad range of mathematical and conceptual skill levels. However, I believe this unit is accessible on a variety of levels. Conceptually, I believe this unit is accessible to eleventh and twelfth grade students. Some of the specific scientific concepts might be too difficult for the general physics class, but adaptations can be made to the unit by varying the level of explanation of the most difficult concepts. This is also true mathematically. The general physics students will be introduced to the formulas whereas the second year physics students will be expected to manipulate the formulas. This unit is intended to be broad enough to be applicable to any physics course.

Student Demographics

My high school has approximately 1350 students with 400 of them being designated as gifted. The school is roughly 58% Caucasian, 39% African American, 4% Asian and 4% multiracial. There is a range of socioeconomic backgrounds as well, with approximately 35% of the students receiving free or reduced lunch. My classes are more homogeneous with a majority of Caucasian students, a lower percentage of African-American students, and fewer economically disadvantaged students. However, all students are required to take physics and all seniors who have already taken physics are encouraged to consider taking a second year of physics.

Nanotechnology Background

I believe Richard Jones' assertion, in the book *Soft Machines*, that nature and evolution-over billions of years-have created soft machines that are nearly inconceivably well constructed and adapted for their purposes. Although we can attempt to design nanostructures, we would be well served to model our endeavors after the effective biological structures that already exist in nature. We can look to living organisms to see all sorts of sophisticated machines which have evolved over billions of years. "At the sub cellular level, nature's designs are in fact highly optimized. Only our experience of designing things in a realm in which physics is very different prevents us from recognizing this". [4] So we must embrace the physics at the nanoscale and the example of nature to successfully design nanomachines.

The fundamental concept that I would like to get across to my physics students is that the physics that is relevant is scale dependent! This means that classical mechanical physics that we utilize to design and construct in the macro world, which is heavily reliant on momentum and inertia, may not be true or useful in the nanoworld, which is dominated by Brownian motion, stickiness, fluid dynamics, viscosity, heat, friction and quantum mechanics. Brownian motion is the result of microscopic collisions that are temperature dependent and which result in a "random walk" of larger particles. It turns out that Brownian motion, which translates to the bulk property of diffusion, is an effective means of moving molecules over short distances but a terrible means for long distances, because the distance covered varies only with the square root of the number of "steps". In the nanoworld, a bacterium is small enough to rely on diffusion to mix molecules within it. [5] The only way to eliminate Brownian motion, which depends on the temperature, or the average kinetic energy of the surrounding molecules, is to eliminate all kinetic energy which means to achieve absolute zero and to isolate the object from its surroundings. This is impossible, but even to reduce the Brownian motion, which varies as the square root of the absolute temperature, by a factor of ten, you have to lower the temperature to around 3K (-270 C). Additionally, since the desire is to have the nanotechnology interact with the surrounding environment, this is impossibile. Brownian motion at the nanoscale must be accepted.

Stickiness is another fundamental aspect of the nanoworld. Materials, especially those made of the same material or oppositely charged (but even neutral ones), tend to stick together. Of the four forces (gravity, electromagnetism, the strong nuclear force and the weak nuclear force), electromagnetism dominates over the other forces even more at the nanoscale because although EM forces are much weaker than the strong nuclear force their range of action is much greater. Quantum mechanical effects become unavoidable!

The optical properties of very small pieces of matter, known as quantum dots, are determined by their size because photons are only absorbed at allowable energies and only certain wavelengths can achieve a "standing wave". According to the Uncertainty principle, quantum dots restrict electrons to spaces that limit their position, and consequently must have higher momentum, which is known as confinement energy, and results in higher frequencies for smaller nanodots. This means that the color of a quantum dot is determined by its size! Similarly, gold particles of different sizes in the nanometer range, when in solution are able to produce the entire spectrum of visible light. [6] As bizarre as all of this physics seems at the nanoscale, these are inevitable consequences of quantum mechanics, which has been demonstrated to be true at all scales and supplants Newtonian mechanics as a better explanation of reality.

Advances in quantum mechanics have allowed for incredible technological advances. One major advance is in our comprehension of electricity which has led to tremendous technological advances, primarily in information technology. A great success of quantum physics has been in understanding the way in which different types of solids conduct electricity. In solids it is the flow of electrons that results in electrical currents. "It is a great triumph of quantum mechanics that it can explain what makes different substances metals, insulators or semiconductors. It is no exaggeration to say that it is this quantum mechanical understanding of materials that has led directly to the present technological revolution, with its accompanying avalanche of new and cheap consumer goods ranging from stereo systems and color TVs to computers and mobile phones." [7] Quantum mechanics has enabled the information explosion and technological advances of the last half a century.

Self-Assembly

One of the fundamental physics principles, which we must comprehend thoroughly, is entropy, embodied in the Second Law of Thermodynamics. The Law of Entropy states that all systems will tend to go from states of low entropy (or high order) to high entropy (or greater disorder). Self-assembly is a process through which a system becomes more structured or ordered. Self-assembly would seem to violate the law of entropy because self assembly seemingly results in greater order; however, this is only an apparent paradox. Decreased entropy in one area of a system can be achieved by the increase of entropy in another area of the system of equal or greater size. Another intriguing consequence of entropy is that a decrease in temperature leads to a decrease in entropy. However, the information also increases because the consequence of Planck's constant is that a decrease in momentum of a system leads to greater knowledge of the system.

In order to achieve self assembly, we need a balance between stickiness and Brownian motion and these structures are dynamic. Self assembly is soft and self-healing. It is soft because the contents are continually in motion and self-healing because although the parts of the structure fall out of place their tendency is to regain their organization.

Soap is a good example of a system in which molecules create complex structures at the nanoscale. Soap molecules have hydrophilic ("water loving") and hydrophobic ("water hating") ends. When placed in water, which is a polar substance, the soap will arrange its molecules with its hydrophilic ends towards the water and its hydrophobic ends away from the water. One way this is achieved by soap molecules is by staying on the surface with the hydrophobic ends sticking up into the air so that they can get as far away from the water, like oil on water. Or if the soap is forced into the water the soap will form structures that make it as "happy" as possible. This can be done by forming pockets or spherical bubbles called micelles, where the hydrophobic ends face each other and the hydrophilic ends face the water!

Lipids, which are even more complex than soap molecules, but also have hydrophilic and hydrophobic ends, are centrally important building blocks of biological membranes. [8] Bilayer lipids are more complex structures where two layers, like the one layer on the surface of the water, have their hydrophilic ends pointing out and their hydrophobic ends pointing toward each other. This makes a membrane layer that is flexible and can become any shape, including the self enclosing cell membrane! These bilayer lipids, which can make up membranes, and surprisingly are in an ordered state which have higher entropy than a disordered state! [9] Although this seems counter-intuitive, it is so because some entropy is associated with the degree of order of the packing of individual molecules while another part of the entropy is determined by how much room each individual phospholipid has to move around, and bilayering allows for the tightest packing and therefore the most independent motion of each phospholipid! This is maximal spacing and maximal entropy.

Self-assembly is an essential aspect of designing many nano machines. It is the process of assembling molecules in bulk rather than one at a time. "What we need for self-assembly to work is for the surface forces, that stick the units together, and the Brownian motion that shakes them apart, to be roughly in balance. So our self assembling units can be small molecules or large known as polymers or macromolecules. Macromolecules are the building blocks of biology." [10] So when this balance exists the building blocks of biology can exist. Jones suggests that this is like a wood puzzle with sufficiently strong glue on the appropriate edges placed in a bag and shaken. Eventually, if shaken just hard enough and with the glue only sticking in the appropriate places, the puzzle might self-assemble.

Proteins

Jones suggests that the best way to make nano-machines is to model nature. Proteins are 3D structures and the structure determines the function. Proteins are key examples, because they demonstrate the construction of specific 3D shapes by the process of self-assembly. The fact that linear proteins self-assemble into specific 3D shapes is central to creating complex organisms from simple linear code! [11] Genes encode linear sequences that become 3D proteins. These proteins achieved highly specified shapes through the process of evolution! So proteins self-assemble from the code of genes. Bits of genes evolved because they were useful for catalyzing reactions and gave the organism an advantage.

In order for organisms to develop, it is essential that bilayer lipids form membranes that create internal, protected spaces from the outside environment. This is fundamental to life! This membrane defines the cellular unit. However, it is not sufficient to simply enclose an interior space because there must be traffic across the membrane and once again it is proteins that make this possible. Membrane proteins create pores that allow specific substances to pass through. In the most elegant example of utilizing a membrane protein across a differential to produce usable energy, the pore for hydrogen evolved in such a way that it produces ATP, the energy source used for virtually all of the processes of life!" [12] This is the source of energy that allows for lower entropy for the existence of highly ordered organisms and life. The mitochondria use the driving force of osmotic pressure for rotation because the gradient in bacteria outside and inside of hydrogen ions create ATP from ADP. Remarkably this is mechanical. [13] This is a truly incredible example of evolution of a self-assembled structure.

Evolution explains the variety of life. However as we look to the cellular level and further to the molecular level, we find "less variety in life, not more! The nanoscale soft machines on which life depends are remarkably similar, and the more fundamental they are to the business of living, the more similar they seem to be. The ribosome, the machine that puts protein together and the ATP—synthase, the fundamental machine of energy conversion— these are recognizably the same in the simplest bacteria as in ourselves." [14] This is stunning!!! Evolution is crucial, but we need to understand how molecules themselves evolve. There is virtually no trace of how this occurred. Consider the breadth of the 3D design space in which evolution works: a typical protein of 100 amino acids has $\sim 20^{100}$ possible linear configurations… making it seem statistically impossible to create or find useful proteins. Our understanding is crude, but Jones suggests that finding small useful proteins that reliably fold at all would be difficult. However, those proteins that did fold reliably would perpetuate themselves because they would be useful. This is the theory. It is very exciting that our understanding of the physics of the nanoworld and our comprehension of chemistry and biology can unify so remarkably into a greater understanding and appreciation for the existence of living organisms.

The advance of science in all of these disciplines indicates that there is a fundamental unity of the physics, chemistry, and biology at the nanoscale. The descriptions are either mechanical or they delve into the bizarre aspects of quantum mechanics and Brownian motion that is essential to appreciating the difference of the nano- and macroscopic world. I am captivated by the relevance of nanotechnology at this level, the accessibility of the physics, the awesome beauty of our evolutionary mechanisms that make life possible, and a desire to explain how all of this is related to physics.

Nanotechnology is our future. In The New Quantum Universe, Paul Davis is quoted as predicting that, "The nineteenth century was known as the machine age, the twentieth century will go down as the information age. I believe the twenty-first century will be the quantum age," and the authors go on to say that, "In the course of the next decades we will see how far this vision will be realized. Certainly, we believe that the influence on our society of this coming nanotechnology revolution, underpinned by quantum mechanics, will be at least as substantial as the fall-out from the present bio-informatics explosion. We hope that this book will assist in stimulating the imagination of a new generation of quantum engineers." [15] It is my goal as a physics teacher to prepare students to become part of this revolution. In the 1960's Richard Feynman challenged the scientific community to make significant advances in nanotechnology. Hey and Walters believe that we must continue to pursue Feynman's vision and they indicate that as Feynman indicated in his famous lecture "There is Plenty of Room at the Bottom" that there is a whole world at the nanoscale and they recognize that life at the bottom is fundamentally quantum mechanical. [16]

Students, who live to use the newest and best cell phones, iPods, and information technology, may not be aware of Moore's Law, that computing power will double every 18-24 months, but they certainly have grown up with that expectation. Their ingenuity and understanding of quantum mechanics may be necessary for this progress to continue. "At the level of integration envisaged by the semiconductor industry in 2010, there will still be many thousands of electrons participating in the storage of a bit or in the action of a transistor. The techniques described above make it possible to envisage devices that work with very small numbers of electrons. This in turn will enable the number of transistors on a chip to continue to increase without excessive power generation. To avoid problems with fluctuations in the number of electrons participating in such devices it will be necessary to use the principle of Coulomb blockade to control individual electrons. There are still many technological problems to be overcome but quantum engineering of new semiconductor devices may be able to keep Moore's Law true for another 35 years." [17] The nanotechnology revolution is here and our students must understand quantum mechanics to participate in its development.

Explanation of Quantum Mechanics

The double-slit experiment with light enables us to illustrate many of the implications of quantum mechanics and illustrates the limitations of classical mechanics. In the experiment, two slits are made in an otherwise impenetrable screen. Another screen with light sensitive detectors is set up parallel to the first screen. A light source is placed outside the first screen centered between the two slits so that the light emitted is equal distant from each slit. The light is turned on and the expectation according to classical physics and the photoelectric effect, which states that light is quantized in packets called photons, and is that the photons will go through the two slits like bullets. The photons that go through each slit will produce a pattern with greatest intensity across from the slit and diminishing in intensity away from the peak symmetrically. The pattern of the second slit will be the same. These two patterns will overlap and according to the classical model of the interaction of particles, the patterns will add linearly as $I_{1+2}=I_1+I_2$. However, this IS NOT what happens! Instead, the light forms an interference pattern! The pattern that is formed creates maximal intensity in the middle by constructive interference and bands of dark and light alternating out from the maximum intensity with decreasing intensity depending on whether they are in phase (constructive interference) or out of phase (destructive interference). Therefore, light, which has been shown to be made up of quantized packets of energy called photons, behaves like a WAVE! The light incident on the two slits, diffracts and interferes like a water wave.

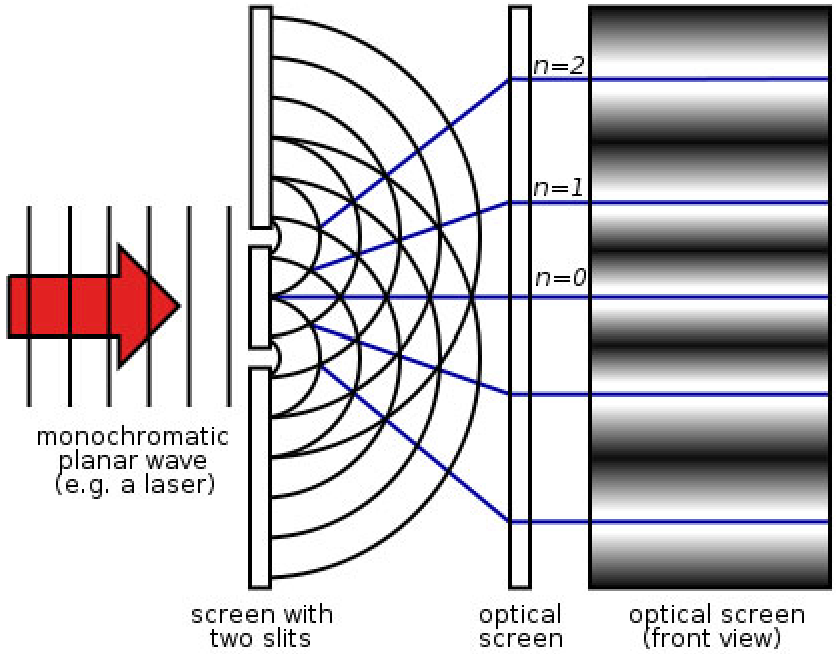

Diagram 1

The Duality of Light

The results of this experiment are very odd indeed. But it gets stranger. Is light a particle or a wave? It turns out that we can continue with this experiment. What would happen if we close one slit at a time? It turns out that if we restrict the light from going through one slit and then the other that we get a pattern that is consistent with the model of light as a particle! There is no interference and the intensity can be represented as $I_{1+2}=I_1+I_2$. We will discuss the explanation for this later, but for now it is sufficient to say that if we restrict the slit that the light can go through, it will behave like a particle.

But we are not done. There is more weirdness. What would happen if we leave both slits open but we reduce the intensity of the light that is emitted so much that only one photon is released at a time? Now, the photon is released and it goes through one of the slits and is detected when it hits the screen. From careful measurement of the detector it has been demonstrated that the light that is received always hits as a full "packet". The photons are never split into pieces. It takes a while but we allow the light to accumulate at the detector screen. Can you guess what it will look like? It turns out that even though only one photon passes through at a time, and the photons do not break apart, they still create an interference pattern! Somehow, the photons interfere with themselves and create a wave pattern consistent with $I_{1+2}=I_1+I_2$! Light behaves like a wave.

So, an ingenious experiment was performed. What if we place a photon detector (some type of EM radiation emitter with a short enough wavelength that it will scatter off the photon and indicate which slit the photon came through) inside the slitted screen? Now, the photon can go through either slit, but we will be able to tell which slit it came through. Will the light behave like a particle or a wave? It turns out, that no matter how low the frequency of the detector is, that simply by being detected, the photon having been detected, behaves as if one of the slits was shut. It acts like a particle and there is no interference pattern!

How can this be? Sometimes light acts like a particle and sometimes it acts like a wave. This is the duality of light. In fact, light is neither, but we do not have a better explanation. If photons are constrained, they will behave like particles. If they are allowed to pass through either slit unconstrained, their trajectory is indeterminate and then in some way they are able to pass through both! In this case, light will behave like a wave and produce a probability amplitude wave with interference!

This is not only true of light. In fact, the experiment has been done with electrons and it has been demonstrated that they also interfere and produce a probability interference wave pattern! So all particles actually have wave characteristics and are probability waves! So deterministic trajectories do not exist. If all particles have wave properties then why don't we see them and how has classical Newtonian physics survived for over 300 years? According to R. Shankar in *Principles of Quantum Mechanics*, if we were able to perform the double-slit experiment with 1 gram objects moving at 1 cm/s, the wavelength of each particle would be 10^{-26} cm! This is based on the formula Λ=2Πk=h/p. So the wavelength would be 10^{-13} times smaller than the radius of the proton and for any reasonable values for the width of the slit and the distance to the screen our instruments will only detect the smooth average $I_{1+2} = I_1 + I_2$.[18] So on the macroscopic scale, matter appears to behave like particles!

The explanation of quantum mechanics only explains what happens not why. "We conclude that electrons show wave-like interference in their arrival pattern despite the fact that they arrive in lumps, just like bullets. It is in this sense that we can say that quantum objects sometimes behave like a wave and sometimes behave like a particle. You may find this all rather mysterious. It is! We cannot do more to explain the magic of quantum mechanics — all we can do is describe the way quantum things behave. This description is quantum mechanics." [19] This is a crucial departure from classical mechanics. Hey and Walters attempt to explain particle-wave duality by indicating that, "It is as if the electrons start as particles at the electron gun, and finish as particles when they arrive at the detector, but the arrival pattern of electrons observed at the detector is as if they travelled like waves in between!" [20] J.L. Martin suggests that quantum reality is complex and that while some people suggest that a quantum particle is sometimes a particle and sometimes a wave that he "likes to add: 'and sometimes a clock'. However, the proper answer is 'none of these': to think of an electron as a *classical* particle or a *classical* wave, or even as some kind of paradoxical mixture of the two, is thoroughly misleading. The best description we know how to give is through the Schrodinger equation and the usual rules of quantum mechanics. It may be convenient to think of 'it' as a particle if we happen to be measuring its position **r**, or as a wave with wavevector **k** if we happen to be measuring its momentum **hk**-or for that matter as a clock with frequency **v** if we are measuring its energy hv (precisely the example of the 'atomic clock')… Is it now clear how very much the discussion depends on what we happen to be interested in at the moment?" [21] Obviously, the implications of quantum mechanics are profound and comprehensive and indeed it defies description.

The quantum wave function involves the probabilities regarding trajectories of particles. The quantum wave function also predicts the probability of arrival at different positions for an electron. The position of arrival of a single electron is thus inherently unpredictable: we can only make statements about relative probabilities of arrival for the electron. When the arrival of an electron is detected at one of the detectors, the previously spread-out probability wave function of this electron then collapses down to the region defined by this detector. "How this collapse happens is **not** governed by the Schrodinger equation. The collapse or 'reduction' of the wave function is **the** mystery of quantum mechanics." [22] In order to understand how strange this is, compare the behavior of a classical particle, which follows a classical trajectory all the way to the detector. Just before the particle arrives we could observe it heading for the chosen detector. But this is not true in quantum mechanics. Before the electron arrives at a detector we cannot say anything definite about it position or where it is headed. One of the fundamental difficulties for quantum physics is how classical quantities such as particle trajectories emerge from the probabilistic wave.

The Photoelectric Effect

The quantization of light, known as the photon, was demonstrated by the photoelectric effect experiment and explained by Einstein in 1905. When high frequency light is shone on certain metals, electrons are ejected. Classical physics could not explain why there is a cut-off frequency below which no electrons are emitted regardless of the amplitude of the light. The kinetic energy of the emitted electrons is determined by the frequency of light not the amplitude and the electrons are ejected almost instantly, if they are emitted, even if the intensity is very low. The explanation is that photons are quantized packets of energy. The photons have to have sufficient energy, based on the frequency, to overcome the atoms binding energy. When this minimum energy is supplied, the electron that is struck by the photon will be released. The amplitude of the incident light, which indicates the number of individual photons, determines how many electrons will be struck and emitted based on the equation $E_{photon} = hf$ (h is Planck's constant). An increase in the frequency results in kinetic energy of the ejected electron. Consequently, the concept of light behaving like a particle and a wave are not mutually exclusive! [23]

The Uncertainty Principle

A fundamental principle of quantum mechanics is that we can only know the position and momentum of anything within a given amount. Stated differently, there is always uncertainty about where a particle is and its momentum which is the product of these quantities and is proportional to Planck's constant (which is extremely small, $h = 6.626 \times 10^{-34}$ J's). The exact equation is $\Delta x \Delta p \geq h/4\Pi$. Consequently, the more accurately you know the position the less you know about the momentum and vice versa. This is the reason that when we measured the photons "position" in the dual-slit experiment that the wave characteristic collapsed. Based on this principle we never know what is going to happen, or when it will happen, even under a given set of conditions, only the probability of specific outcomes. [24] Uncertainty is a fundamental aspect of quantum mechanics and reality.

Schrodinger's Wave Function

There are challenging implications of Schrodinger's wave function, which indicates that matter can be explained by a wave function. This means that matter has a multitude of possible states that it can inhabit, and all of those states have a reality when the system is isolated from observation. The most familiar example is Schrodinger's cat. The thought experiment is that there is a cat in a box (with plenty of food and water) so that it cannot be observed, and it is present with a substance that emits a poisonous gas when it decays with a known half-life. The cat will die when the substance decays, but the half-life is a probabilistic event. So is the cat alive or dead in the period before we open the box? The answer is that there is a wave function of possibilities for the cat but until we open the box and make an observation, the cat is BOTH alive and dead! "Quantum mechanics seems to assert that it is in a quantum superposition. We cannot imagine how classical objects can be in a quantum superposition of two different states at once, let alone a living object like a cat. Can it really be that quantum mechanics says that it is the act of observing the cat that causes its wave function to collapse to a dead or alive cat?" [25] This wave function has been tested on the nanoscale and has proven to be true, so it is not theoretical and the implication is that quantum mechanics is continuous in its applicability... it does not just affect the atomic level, but is in fact relevant at all scales! We have been able to ignore the implications at our macroscale because the affects are "averaged out" over our billions of molecules. However, this "bizarre" reality has implications that are also useful for quantum computation and potentially for quantum computers, which would be able to take advantage of this feature of reality that there are actually a multitude of states that matter exists in that are probabilistic, until the system is "observed" and the wave function collapses into a single state.

The Many Worlds Interpretation of Quantum Mechanics

In the Copenhagen viewpoint, when an observer uses some classical measuring apparatus to make a measurement on a quantum superposition, only one of the many possible results is actually realized. The measurement process somehow mysteriously collapses all the different possible outcomes to the one observed outcome. Everett and DeWitt addressed this problem in a 'breathtakingly audacious way" by suggesting that all possibilities are realized, but each in a different copy of the universe. DeWitt indicates that each of these copies of the universe is itself constantly multiplying to allow for all possible outcomes of every measurement. In this theory there is no collapse of the wave function because the universe is replaced by a "multiverse" of parallel universes. This creative solution was dismissed by many prominent physicists. Feynman said "These are very wild speculations, and it would be little profit to keep discussing them" and Bell concluded, "if such a theory were taken seriously it would hardly be possible to take anything else seriously." [26] Quantum mechanics continues to pursue explanations of how measuring can have such a profound effect on the wavefunction nature of quantum reality.

Decoherence

A seemingly more reasonable attempt to solve the measurement problem goes by the name of "decoherence". According to this approach, quantum systems can never be totally isolated from the larger environment and the Schrodinger's equation must be applied not only to the quantum system but also to the entire coupled quantum environment. In *The New Quantum Universe*, the authors indicate that "Zurek claims that recent years have seen a growing consensus that it is interactions of quantum systems with the environment that randomize the phases of quantum superpositions. All we have left is an ordinary non-quantum choice between states with classical probabilities and no funny interference effects." [27] So if Hey and Walters are correct, there is a growing sense that interactions with the environment result in the appearance of particle behavior on the macroscopic world.

Quantum tunneling

Quantum 'particles' do not behave like classical objects! An electron "can tunnel through the forbidden region and appear on the other side! This 'barrier penetration' or 'quantum tunneling' is now a commonplace quantum phenomenon. It forms the basis for a number of modern electronic devices" [28] , including the Scanning Tunnelling microscope. How can such tunneling occur? Based upon Heisenberg's uncertainty principle we have discussed the uncertainties in the measurements of position and momentum, but there is an equivalent relation exists between uncertainties in measurements of time and energy $\Delta E \Delta t \approx h$. According to classical mechanics we can never change the total amount of energy without violating the conservation of energy, however, in quantum mechanics, if the time uncertainty is Δt, we cannot know the energy more accurately than an uncertainty $\Delta E \approx h/\Delta t$. So in a sense, we can 'borrow' an energy ΔE to get over the barrier so long as we repay it within a time $\Delta t \approx h/\Delta E$. If the barrier is too high or wide, and based on probabilities, tunneling becomes extremely unlikely and all the electrons will be contained. The phenomenon of tunneling is a general property of wave motion and since all quantum 'particles' have wavelike properties all matter is capable of quantum tunneling.

Scanning Tunneling Microscope (STM)

Rohrer and Binnig came up with the idea of using the "vacuum tunneling" of electrons to study the surfaces of materials. The fundamental idea is very simple. According to quantum mechanics, electrons in a solid have a small, probabilistic chance of being found just outside the metal surface of the microscope "needle". The probability for this to happen is predicted to fall off rapidly with distance from the surface. Based on quantum mechanics, if we can bring a sharp, needle-like probe close to the surface, and apply an electrical voltage between it and the metal, a tunneling current will flow across the gap, even in a vacuum. Since the electron wave function falls off rapidly, the magnitude of this tunneling current is extremely sensitive to the distance between the probe and the metal. If the distance of the tip of the probe from the surface can be controlled accurately, we can use the magnitude of this current to create an image of the surface features. [29] The STM has been incredibly useful in nanotechnology engineering and has actually enabled the placement of single molecules on metallic surfaces.

Fusion in Stars Resulting from Quantum Tunneling

For decades physicists have been attempting to solve the issues related to harnessing the immense energy available in nuclear fusion. Fusion reactions are the basic process by which the stars generate their energy. This is all the more surprising because as it turns out the temperatures in stars correspond to kinetic energies much lower than the Coulomb barrier. The Coulomb barrier is the energy required to overcome the electromagnetic forces within the atom. "Fusion in the stars is only able to take place at these low temperatures because of quantum tunneling through the potential barrier. It is not an exaggeration to say that we owe our very existence to this ability of quantum particles to penetrate classically forbidden regions!" [30] It turns out that quantum tunneling is an essential process for the production of the energy source that makes life possible.

Nuclear Fission

The nucleus of the atom is made up of protons and neutrons confined within a very small space. We would expect this to be an unsustainable situation because the protons repel each with tremendous force when in such close proximity. However, the strong nuclear force dominates at very small distances allowing for the stability of the nucleus. However, if we keep adding protons and neutrons to make heavier and heavier nuclei, the range of the electric force, which is vast compared to the strong nuclear force, is enough for all the protons to repel each other. For the strong nuclear force, on the other hand, the nucleus is now so big, and the nuclear force so short ranged, that any given nucleon only feels strong attractive nuclear forces from nearby nucleons. Because of this, the weaker Coulomb repulsive force, which acts between all the protons in the nucleus, can become comparable or stronger than the attractive nuclear forces. This is the reason that stable heavy nuclei have more neutrons than protons: the excess neutrons give more attractive binding energy without any Coulomb repulsion. What happens if we start with a radioactive, large nucleus and change one of the protons into a neutron by beta decay, or eject an alpha particle? We end up with a new nucleus with fewer protons and less Coulomb repulsive energy. The new nucleus is therefore more strongly bound and more likely to be stable. Using these principles we can form the basis for the nuclear stability curve. "This process of nuclear fission can be thought of as a tunneling process, similar to the others we have described in this chapter. The energy of the fissioning nucleus can be pictured as a roller coaster potential...this shows two valleys minima of energy one of which is lower than the other. Classically, a particle at rest in the upper valley will stay there forever. Quantum mechanically, such a state is not completely stable the system has the possibility of tunneling through to the true lowest energy state. The 'false' minimum is called a 'metastable' state and fission may be imagined as a tunneling process from a state such as this." [31] Once again quantum tunneling is responsible for an understanding of the fundamental processes of physics that we normally never address in high school physics classes even when we teach nuclear processes. Therefore, quantum mechanics provides a unifying theme for divergent physical phenomenon. This is the goal of scientific explanation and a validation of the need for this unit.

Advanced Quantum Mechanical Concepts

Quantum mechanics is indeed difficult to grasp and the implication is that our intuition is incorrect and that experiential knowledge gained from experiment is the only access we have to reality. In fact, much of quantum mechanics is so paradoxical that we can only indicate how things behave but can't even postulate on a mechanism for why reality behaves the way that it does. "Theorists whose prejudice inclined towards determinism were troubled, and they began to ask what was 'really' happening behind the scenes. Quantum mechanics not only gave no answer, but even declared that such questions ought not to be asked! Henceforth, intuition was to be regarded as a very fallible guide, and ideas which used to be regarded as obvious were to be viewed with reserve until experience either vindicated of invalidated them." [32] So in quantum mechanics we are bound to attempting to understand how matter and energy behave in a probabilistic manner without the ability to address why it behaves that way. In addition, measuring has a profound effect on reality. In regard to determining the polarization of a photon, until we decide what direction to set the measuring polaroid "it appears that we cannot talk of the photon having any definite polarization. It seems that our choice of the direction of the 'measuring apparatus' has an influence on the polarization of the photon! According to quantum mechanics, until we make a measurement, the direction of the polarization of the photon is apparently not just unknown but really indeterminate. Pascal Jordan, an author of some of the earliest papers on quantum mechanics, went so far as to say that 'observations not only *disturb* what has to be measured, they *produce* it.'" [33] Consequently, the act of measurement, or even interacting with the world in some fundamental way, creates or defines the reality!

Einstein's objection

Because the objects we see around us have a comforting, solid, reality, they do not seem to be conjured into existence because we choose to look and make a measurement. According to Hey and Walters, "It is not surprising that Einstein would have none of this. He disliked Bohr's denial of an underlying physical reality and believed passionately that physical objects had real, physical properties whether or not we were there to measure them. In a conversation with Abraham Pais, Einstein highlighted what he felt was the absurdity of the situation by asking Pais: 'Does the Moon only exist when you look at it?' It was to attack Bohr's view of the world that Einstein devised his famous 'thought' experiment with two young colleagues at Princeton, Boris Podolsky and Nathan Rosen." [34] John Bell consolidated these apparently philosophical questions into a mathematical formula that could be tested, which is known as Bell's inequality.

The most disturbing consequences of quantum mechanics are philosophical in nature. The classical notion that was formulated by Einstein, Podolsky and Rosen says that "once two systems have been in contact—no matter how long ago, and no matter how far apart they may be now-observing one system *drastically alters* our view of the other in ways which are difficult to understand. Such correlations are clearly and unambiguously predicted by conventional quantum mechanics; experimental evidence is in their favor; and yet we are unable to comprehend how they can happen. There is nothing wrong with correlation as such: life is full of it. A universe without correlation would be chaotic, uneventful and dull. But quantum mechanics appears to yield *too much* correlation, more than we can intuitively stomach without feeling compelled to ask what is really going on." [35] There is a profound problem, and no-one has yet provided a satisfying resolution. Even if an acceptable mechanism were found, the amount of correlation would almost always have to be less than that predicted by standard quantum mechanics based on Bell's inequalities which need to be satisfied if our natural intuitions are right. "Conventional quantum mechanics violates Bell's inequalities in theory, and there is consequently a number of experiments designed to test if they are violated in real life. On balance, available results suggest that violation does indeed occur. It is therefore inevitable, for reasons of philosophical prejudice, that attempts to rebuild quantum mechanics will be made, to make any violation appear more 'reasonable. Conceivably, however, we may have to give up the riddle." [36] Potentially, the underlying mechanisms of quantum mechanics may defy our comprehension or not exist at all.

Bell's response to the explanation of the wave collapse of the Copenhagen interpretation was oppositional. The experiment involves correlation. Two photons are released in opposite directions with the same undetermined polarization. Upon one encountering a polarizer or Polaroid, "this photon is in neither the V (vertical) or H (horizontal) state but a superposition of the two and 'has to make a choice' at the Polaroid and 'jump' into either V or H. The seemingly innocuous phrase, 'has to make a choice', is at the heart of the problem. How is this choice made? The photon cannot make the choice the quantum superposition evolves according to Schrodinger's equation and this does not describe a collapse to one state or the other. It must somehow be the act of observation with the polaroid that causes the collapse into one of the two photon polarization states. But exactly how can some 'classical' measuring apparatus like the piece of polaroid cause this 'collapse of the wavefunction'? After all, any so-called 'classical' measuring apparatus is actually made up of atoms and electrons and these are subject to quantum mechanics and Schrodinger's equation just like the photon. This is the nub of the 'measurement problem' of quantum mechanics and it troubled many of the founders of the theory." [37] It was in response to these challenges that the orthodox Copenhagen interpretation of quantum mechanics was carefully put together by Niels Bohr and his colleagues. "The Copenhagen interpretation offers a very austere and abstract view of the world. Bohr believed that the language of classical physics was inadequate to describe phenomena at the quantum level of reality. Ordinary words are incapable of giving us a satisfactory and unambiguous definition of a quantum superposition. Bohr offers no mechanism to explain the collapse of the wave function on measurement. Instead, to obtain results from quantum theory to compare with experiment, Bohr instructs us to split the experimental system into two parts — one part a classical world containing classical measuring devices and a second part containing the quantum system under observation. This Copenhagen distinction between classical and quantum systems is sometimes called the 'Heisenberg split'." [38] Such a split seems ambiguous, but it is clearly sufficiently unambiguous for physicists to use in practice with great success. Regardless, some physicists, especially John Bell, found such a "cookery book" approach to our most basic theory of matter unsatisfactory. John Bell detested this "shifty boundary" between Schrodinger's "wavy quantum states" on the one hand, and Bohr's "classical apparatus" on the other and insisted that, at its heart, quantum theory was "rotten".

However, Bell's Inequality was eventually shown to be violated and quantum mechanics' prediction proved to be accurate. "These experiments confirmed that Bell's inequality was indeed violated and that the predictions of quantum mechanics agreed with the data…most physicists now accept that quantum mechanics has passed this test. What do these experiments tell us about the nature of reality? The observed violation of Bell's inequality means that no hidden variable theory — without some explicit or implicit unpleasant action-at-a-distance property- can agree with experiment. Whilst Einstein would probably have preferred some underlying, deterministic hidden variable explanation for quantum mechanics, he would certainly not have wanted to accept the existence of such 'spooky action-at-a-distance' effects. [39] This still does not leave quantum mechanics in a very comfortable state. As modern scientists we might have to accept this degree of discomfort.

Strategies

In this unit on quantum mechanics, I will utilize a constructivist approach. Constructivism suggests that learners take in knowledge superficially and apply it to their existing framework even if that contains fundamental misconceptions and contradictions known as assimilation, or a learner can actively utilize the knowledge to reframe their world view which is known as accommodation. The goal is for the student to construct as comprehensive and consistent a world view as possible. This approach recognizes that the learner comes with a vast set of preconceptions about their world. In order to have some influence over how the information that we are presenting as educators is processed, we must actively attempt to understand the students' preconceptions and misconceptions. Then we must actively engage the students world view if we are to have a lasting impact on their thinking process. My main emphasis in teaching physics is to address my students' critical thinking skills. Physics attempts to address the fundamental nature of our reality, but our experiential knowledge is often deceptive or contrary to the "Laws" of physics, especially quantum mechanics. Consequently, it is apparent to me as an educator that we must address the learner as a comprehensive being with a tremendous amount of inclination to sustain their current world view. This is natural in human beings. But the scientific pursuit requires that we are open to accepting the experimental evidence about the world around us and to have a more comprehensive understanding of the world we inhabit even when a large part of the knowledge is conceptual.

So how do we begin to affect our students' world vision? Well, it is clear that first we must make learning an active process. It is essential that we engage our students as deeply and profoundly in the educational process as possible. How can this be done? It is my belief that we must stimulate their inherent desire to learn. It is only if we activate this passion for learning that students will invest enough of themselves to begin the process of allowing their learning to transform their vision of how the world is constructed. So students must be forced slightly beyond their comfort zone, known by Piaget as the "zone of proximal learning". This is the place where students are emotionally invested in what they are learning and in the space in which learning matters. It is my goal for my students to evaluate their views and to attempt to make their understanding as comprehensive and consistent as possible, and to incorporate at least parts of quantum mechanics.

How do we get students to engage at this level? I believe that constructivist, hands-on learning is primary. Students must engage on a multitude of levels including a physical level. They must be presented with problems that are meaningful, relevant and which they are compelled to attempt to resolve. Thus the teacher must act as a facilitator acknowledging the role of prior knowledge to stimulate students to incorporate new learning into altering their world view. The students must be encouraged to construct their new world view and this is strongly based on motivation. It cannot occur passively.

Students are most motivated by topics that are relevant to them, that stimulate their imagination and enable them to further their understanding of the world around them. I am convinced that the challenge that grasping and incorporating quantum mechanics into their world view is inevitably active. It allows me to frame many realms of physics knowledge in terms that are unexpected, relevant and pertinent.

Classroom Activities

Lesson Plan 1— Strength vs Weight and Self-Assembly

Demonstrating the implications of nanotechnology using classical mechanics, I will have the students conduct a lab on strength vs. weight. This will demonstrate the reason that structures are much stronger at smaller scales because their strength is dependent on the cross-sectional area, L^2, whereas the weight that is supported is related to the volume which is, L^3.

Another important implication of nanotechnology is the explanation of self-assembly. If the conditions are right, then complex arrangements of molecules can self-assemble. This is the greatest hope for nanotechnology because it allows for the production of large scale "assemblies" that it is not possible to currently create molecule by molecule. I will demonstrate how self-assembly works when there are the right constituent parts (molecules such as proteins that tend toward certain encoded shapes and have sufficient "stickiness") and the correct environment (the correct amount of thermal energy- Brownian motion). I will attempt to demonstrate this by objects in water minimizing the surface tension, which is a method of finding the lowest energy state; the separation of oil and water into hydrophilic and hydrophobic regions either along the surface in a lipid layer or in the formation of micelles if the oil is dispersed by mixing in the water; and lastly, by the formation of soap bubbles, reducing the surface area-volume ratio.

Lesson Plan 2— Quantum dots and Nanotechnology Design

We will study quantum dots and how the color change of nanoparticles can be explained. We will also consider methods of nano engineering. This involves discussing using the method of self assembly, building one atom at a time, or attempting to design like natural soft machines. We will discuss the differences between designing like nature and traditional engineering methods. The discussion will lead into quantum effects at this scale and how it affects design. We will consider the implications of decreasing scale where eventually individual electrons are flowing through "wires". This will introduce us to computation and information technology and will naturally lead to a discussion on what is meant by quantum computing.

My classes will attend a field trip to the university about nanotechnology and visit a lab to see quantum dot effects. We will set up the nanoparticles of gold at nanoscale sizes that will allow for the entire spectrum to be displayed. We will also discuss the progress of nanotechnology and see the relevant advances that are being made in the field including nanoscale electronic boards that are built in very cold temperatures less than 1K. We will also discuss the quantum computer endeavor to create computers without wires and bits that involve single atoms or molecules instead of thousands of electrons per current bit of information. The theoretical aspects of quantum computing using spin will be explored.

Lesson Plan 3— Quantum mechanics— double slit experiment

Using a laser, I will set up the double slit experiment and demonstrate the diffraction pattern that results. I will also demonstrate the "particle" like pattern without interference that results from opening a single slit at a time. (This will require a screen that can record the received photons over time). We will have a class discussion about the implications of the duality of light and particles.

The class will have a debate about whether light is a particle or a wave. The class will be divided into three groups. The first two groups will research the nature of light. One group will discuss all of the known information supporting the idea that light is a wave. The second group will research and present all of the information that light is a particle. The debate will last for twenty minutes. Afterwards the third group which has researched the duality of light will present to the entire class our best understanding of how light is both particle and wave or sometimes like either depending on the situation and whether there is observation taking place.

Once the presentation and debates are over the students will write a paper on their understanding of the duality of light and the best explanation that they can provide for the "weirdness" of the implications of duality of light and quantum mechanics. They will also be encouraged to suggest what the implications are for the fact that electrons and all matter actually have the same duality and have wave properties as well. I will present information about the Schrodinger Wave equation and some students may chose to discuss its implications as well.

Endnotes

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1. Martin, *Basic Quantum Mechanics*, v
2. Hey, et al *The New Quantum Universe*, 17
3. Martin, *Basic Quantum Mechanics*, 1
4. Jones, *Soft Machines*, 87
5. Ibid, 62
6. Ibid, 85
7. Hey, et al *The New Quantum Universe*,117
8. Jones, *Soft Machines*,97
9. Ibid, 98
10. Ibid, 101
11. Ibid, 108
12. Ibid, 116
13. Ibid,150
14. Ibid,120
15. Hey, et al *The New Quantum Universe*, xi
16. Ibid, 185
17. Ibid, 193
18. *Shankar, Principles of Quantum Mechanics*, 112
19. The New Quantum Mechanics, 15
20. Hey, et al *The New Quantum Universe*, 14
21. Martin, *Basic Quantum Mechanics*, 221
22. Hey, et al *The New Quantum Universe*, 160
23. Rogers et al, *Nanotechnology: Understanding Small Systems*, 65

24. Ibid, 75
25. Hey, et al *The New Quantum Universe*, 174
26. Ibid, 176
27. Ibid, 177
28. Ibid, 75
29. Ibid, 79
30. Ibid, 95
31. Ibid, 98
32. Martin, *Basic Quantum Mechanics*, 221
33. Hey, et al *The New Quantum Universe*, 164
34. Ibid, 165
35. Martin, *Basic Quantum Mechanics*, 235
36. Ibid, 235
37. Hey, et al *The New Quantum Universe*, 164
38. Ibid, 164
39. Ibid, 172

Resources

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Appendix- National Standards (NSES)

Different energy levels are associated with different configurations of atoms and molecules PS-EIM 9-12

Waves come in many forms (e.g. light, sound, seismic, in water), all exhibiting similar behavior. PS-EIM 9-12

The acceleration of charged particles produces electromagnetic waves. PS-EIM 9-12

The behavior of matter at the nano-, atomic, and subatomic scales cannot be predicted exactly. Only the probability of an event occurring can be determined. Benchmark 11c/7 9-12, 11A/4 9-12