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If You Can See It, It's Not Nano: Working with Numbers at the Extremes

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by Nancy Rudolph

Objectives

At my vocational-technical high school, students expect answers to the question, "Why do I need to learn this?" Using the field of Nanotechnology as a backdrop for the semester, I think my students will be excited and energized (every pun intended) to learn the mathematics related to this emerging field. As I read *The Big Ideas of Nanoscale Science* and Engineering published by the National Science Teachers Association (NSTA), I found several areas of overlap where science and mathematics support each other. Therefore, this unit will naturally integrate the two subjects to benefit student understanding as the two disciplines reinforce each other.

I am writing this unit for high school math students, primarily in 10 th - 12 th grade. I teach at a comprehensive vocational-technical high school where students spend up to one-half of each day in their chosen career area and the remainder of their day in academic classes. Career areas include auto repair, cosmetology, construction trades, culinary arts, business technology, dental assisting and laboratory, nursing, drafting, landscape design and pre-engineering. My school is one of four high schools in its own vocational school district. Each of the four schools pulls students from any school in the county so that students arrive having used a variety of curricular programs. The vocational schools are "choice" public schools and our students are held to the same academic standards as all public school students in the state of Delaware. Students choose our school for a variety of reasons. Some are focused on what they want to do when they finish high school and use the vo-tech school to get a head start; some have been moderately successful students and are looking for a route to success other than a four-year college.

Our math classes are generally grouped heterogeneously so we do find a wide range of abilities in each class. The district mathematics curriculum is somewhat restrictive. All students take four courses from the Core Plus Math Project (CPMP), an integrated program. We currently stretch the 3-year CPMP curriculum into four semester courses, completing four units (chapters) per semester, but we eliminate some CPMP units completely. The majority of our students complete the four courses in three years because freshmen take two semesters of math. After the required integrated courses, students have the option of taking a traditional Algebra course, followed by Precalculus, or a self-paced, computer-based math course to prepare for acceptance into a vocational Apprentice Program or for the Placement Test at the local community college.

Each year of the integrated program contains units in Algebra, Geometry, and Probability/Statistics, and builds on content from previous chapters and years. Despite this thorough preparation in prior coursework, I still see weaknesses when the students reach the upper level courses. Fortunately, the areas of overlap between science and math in Nanotechnology address the most common areas of weakness that I see: Number Sense (estimation, relative size); Exponents (applying properties, negative exponents); and Geometry, (composite area and volume, effects of scaling). This curriculum unit is designed to help students in these areas of weakness, by showing the relevance of these skills to an area that is scientifically important and also significant to society.

Background – The Science

Nanotechnology is an emerging field of science that is the study of controlling matter on an atomic and molecular scale. Nanoscale refers to objects with at least one critical dimension between 1 and 100 billionths of a meter. The properties of matter that most of us learned in school do not always hold true for materials at the nanoscale. I felt like my world shifted as I learned about nanoscience because the proportional reasoning that I relied on from my math background, and the intensive properties of matter that I studied and taught in science classes, may no longer apply. Instead, there are size-dependent properties that apply to objects on the nanoscale.

To understand nanotechnology applications, it is helpful to understand the basic (unique) chemistry of nanomaterials. As we learned in high school Chemistry (and probably other science classes, too), the basic unit of matter is the atom. The size of an atom varies from element to element, but most atoms have a diameter of at least 0.2 nm. Atoms consist of a nucleus, which contains protons and neutrons, and orbitals (often referred to as "shells") which contain electrons. The number of (positively charged) protons determines what type of element it is; the atomic number on the Periodic Table is equal to the number of protons in the nucleus. Electrons are negatively charged and are equal in number to the protons in a neutral atom. Electrons are in constant motion in the orbitals outside of the nucleus; their exact location is never known, but there are locations where they are more likely to be at a given time (this distributed location of an electron is sometimes referred to as an electron cloud).

It is the number and position of electrons that determine what atoms do when they come in contact with other atoms – whether they react or not, and how. Some atoms, such as the Noble Gases like helium (He) and neon (Ne), do nothing; they are more stable as individual atoms because electrons completely fill all of their outer orbitals. Other atoms—such as sodium (Na), chloride (Cl), carbon (C), and oxygen (O)—interact with each other to form molecules. This interaction occurs by ionic, covalent, or metallic bonding.

Some atoms can easily transform to charged particles, called ions, because their outer orbitals are either nearly empty or nearly full. Atoms form positive ions by giving up typically 1or 2 electrons, or negative ions by accepting 1 or 2 additional electrons. Positive and negative ions join together with ionic bonds in a way that creates a neutral molecule. Sodium Chloride, table salt, is a classic example of a molecule formed with ionic bonds. Sodium (Na) easily gives up one electron to become the positive ion Na^{+}. Chlorine accepts one additional electron to fill its outer orbital to become the negative ion Cl^{-}. The two ions join together in a 1:1 ratio to form the neutral molecule NaCl. Other atoms "do not want to" (energy/stability-wise) completely give up or take in electrons, but would "be willing to" share electrons with another atom in the formation of a molecule. This sharing of electrons between atoms is called covalent bonding. In covalent bonds, the electrons travel back and forth between the two atoms so that each one has completely full outer orbitals some of the time. Ionic and covalent bonds are the most common types of bonding between atoms; however, a third type of bond, metallic bond, allows electrons to float "delocalized" among several adjacent metal atoms that are in a lattice arrangement.

Ionic and covalent bonds, which join atoms together into molecules, are examples of intramolecular forces. Molecules also experience affinity to one another: the forces holding the molecules together are called *inter*molecular forces. When the forces between molecules are weak, molecules move easily relative to each other: these molecules are stable in the gaseous state. Liquids have stronger intermolecular forces, and solids even stronger, preventing as much movement. There are two major types of intermolecular forces – hydrogen bonding and London dispersion forces. While *intra*molecular forces are always stronger than *inter*molecular forces, hydrogen bonding is the strongest of the intermolecular forces and accounts for some unexpected properties. For example, water

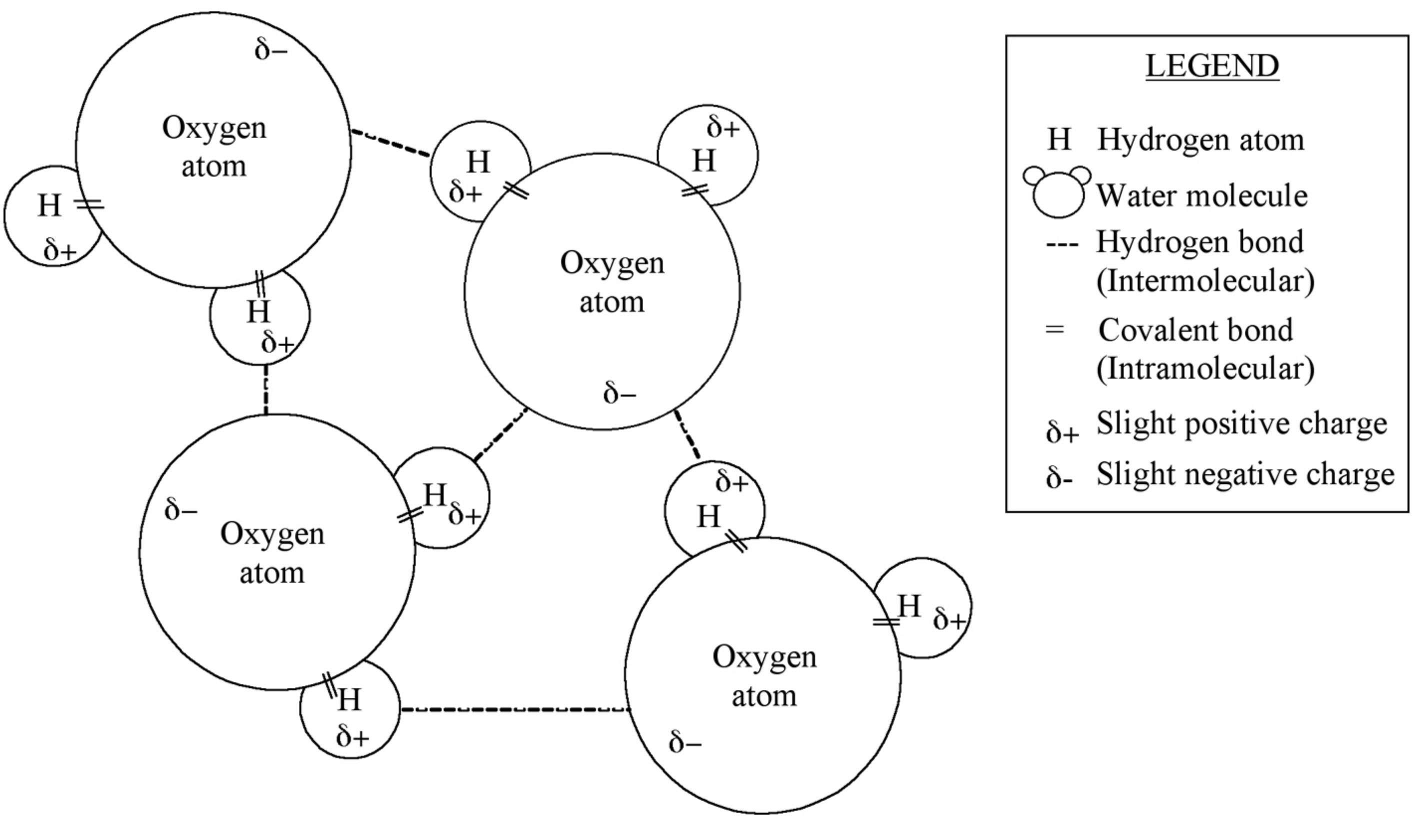

Figure 1 – Bonding in and between water molecules

has a high boiling point because its molecules (in the liquid state, of course) are held together by hydrogen bonding. Hydrogen bonds (refer to Figure 1) are formed between two polar molecules – molecules that have a partial positive region and a partial negative region because of an unequal sharing of electrons. Water is a polar molecule because the oxygen atom is so much larger than the hydrogen atoms that it has a stronger attraction for holding onto the single electron each hydrogen atom has to share. As a result, the electrons spend more time in the oxygen orbitals creating a partial negative region of the molecule, while the hydrogen atoms have no electrons in its orbital most of the time, creating a partial positive region. The intermolecular force is created by attraction between the partial negative region (oxygen) of the water molecule and the partial positive region (hydrogen) of another water molecule, thereby creating a hydrogen bond holding the water molecules together. London dispersion forces are weaker intermolecular forces between temporary polar molecules. Temporary polar molecules are created from fluctuations in electron density (location of electrons in the orbitals) or are induced to become polarized by other nearby polar molecules. London dispersion forces increase as molecular size increases because the electron cloud can become polarized more easily when electrons are farther from the nucleus.

These interactions between molecules lead to the properties of matter. There are macroscopic properties that include temperature, pressure, and viscosity that we can measure with common technology. On the other hand, measuring microscopic properties such as molecular size and shape, molecular velocity, and intermolecular forces would require sophisticated technology typically not available in high schools. Properties of matter are also split into two categories: intensive and extensive properties. Extensive properties such as mass, volume, heat capacity, energy, and entropy always depend on the amount of material present. For example, the larger the sample, the greater the mass and volume will be. Intensive properties, as I learned in school, were properties that do not depend on the amount of material present. Examples of intensive properties are color, temperature, density, pressure, viscosity, thermal conductivity, and electrical conductivity.

However, as I learned in this seminar, some intensive properties are size-dependent and may change when particles reach the nanoscale. For example, a chunk of gold metal is gold-colored but the color of gold nanoparticles (on the order of 10 nm, or 10^{-8} m) ranges from blue to red to yellow to colorless depending on their size! Electrical conductivity and magnetism also depend on particle size. A material that is conductive at the macroscale (i.e. things we can see) may be a semi-conductor or non-conductor at the nanoscale depending on the level of confinement of the electrons in the material. [1] As an example, metals are conductive, since metallic bonds allow electrons to float with little confinement; therefore, less confinement of electrons allows for increased conductivity.

Probably the most critical size factor affecting properties of matter at the nanoscale is the surface area to volume (S/V) ratio. As an illustration, consider cutting a sugar cube multiple times. With each cut, there is more surface area exposed but the total volume (and mass) of sugar remains the same. We know from experience that the physical property of rate of solubility of the sugar cube will increase if we break it into smaller pieces: rate of solubility increases as surface area increases. Some other examples of the effect of the surface area to volume ratio are:

> the adhesion property of powdered sugar ("sticking" to the sides) is greater than granulated sugar in a plastic measuring cup, the burning rate of a thick log is lower than the same mass of twigs, and the increased ability for the small intestine to get nutrients because of the millions of villi on the lining that increase its surface area. [2]

Mathematically we know that increasing/decreasing all dimensions of an object by some factor (i.e. multiplication factor) will increase/decrease the cross-sectional area by that factor *squared* since area is a 2-dimensional measurement. Likewise, the volume of the object will increase/decrease by the factor *cubed* since volume is a 3-dimensional measurement. Since the area and volume change at different rates when the dimensions change, the S/V ratio also changes. To be more specific, as particles get proportionally smaller, the ratio of surface area to volume increases because we are dividing by a smaller number (the 2nd object is a fraction the size of the first, and a fraction cubed is an even smaller number). As an example, consider two round balls of pizza dough, one with a 4-inch diameter and the other with a 2-inch diameter. The surface area of the larger ball (radius = 2 inches) is $4\Pi(2)^2 = 16\Pi$ in^2, and its volume is $4\Pi(2)^3/3$ in^3. The surface area and volume of the smaller ball (radius = 1 inch) are 4Π in^2 and $4\Pi/3$ in^3 respectively. That makes the S/V ratio for the larger ball 3/2, and S/V for the smaller one 3. Since 3 is greater than 3/2, this example confirms that the surface area to volume ratio increases when the linear dimensions (radius, in this case) decrease.

In our Nanotechnology seminar we spent a significant amount of time discussing "why size matters," also the title of a book by John Tyler Bonner. We considered the relationship of weight versus strength in that if a person grew 10 times taller, then his weight would increase accordingly, but his legs would not be able to carry that much extra weight; the bones would not have enough strength to support him. By comparison, an elephant's legs are shorter and thicker to support its own weight, and ants can carry many times their own weight because of the strength of their legs relative to their body weight. One of the most intriguing concepts I learned in this seminar is that the effects of gravity versus molecular cohesion are hugely dependent on size; for this reason, gravity has a much greater effect on humans than it does on very small creatures. A fly, for example, can climb a vertical wall without falling to the ground because, for its insignificant weight, the molecular cohesion forces between the fly and the wall are more significant than the force of gravity. Of course nature's design plays a role; a gecko can climb a wall because of its small size (and weight) and also because it has lots of tiny hairs on its feet to provide increased surface area for even greater cohesion.

The element carbon deserves special attention: I will use carbon as the basis for my instruction in this unit. The atomic structure of carbon allows it to form four strong, covalent bonds with many different types of atoms, including four other carbon atoms. There are different forms of pure carbon, called allotropes, which have very different bonding structures and, therefore, different properties. The most common

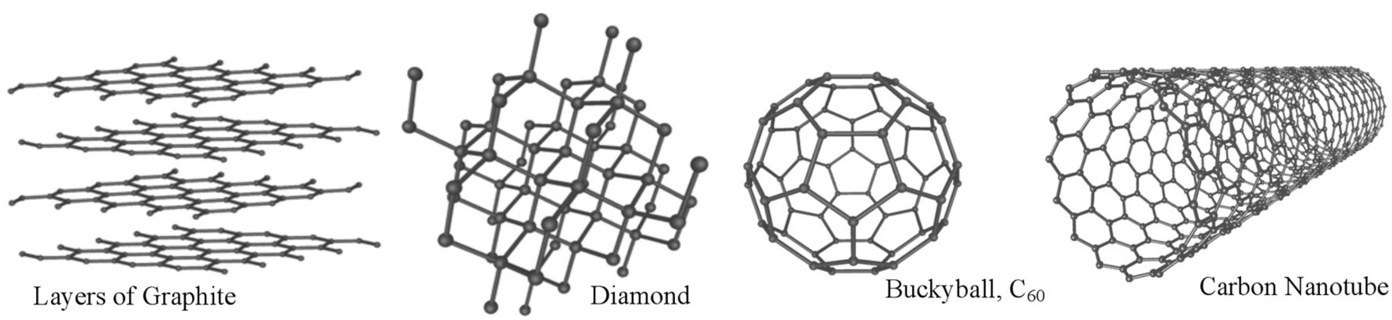

Figure 2[3] - Carbon Allotropes

forms of carbon are graphite, diamond, and carbon nanomaterials, called buckyballs (buckminsterfullerenes) and nanotubes. The strength of diamond comes from the tight arrangement of each carbon atom being bonded to four other carbon atoms in a tetrahedral shape extending to the edge of the sample. In contrast, graphite is softer because it bonds to only three other carbon atoms to form layers of hexagonal rings across a lattice, and the layers are held together by the relatively weak London dispersion forces. The "extra" electrons float as in metallic bonds. These floating electrons make graphite electrically conductive, whereas diamond is not. Buckyballs, discovered in 1985 by Harry Kroto, Richard Smalley and Bob Curl,[4] are spherical molecules formed by 12 pentagons and 20 hexagons that look like a soccer ball. The diameter of a buckyballs is approximately 1 nm! Whereas diamond and graphite structures can contain any number of molecules—because their structures are theoretically repeated—buckyballs contain exactly 60 carbon atoms, each bonded to three other carbon atoms. Their properties are not well-defined, except that they are able to bond on the surface with many other atoms and have the potential to trap smaller atoms on the inside. The discovery of carbon nanotubes was credited to Sumio Iijima in 1991. [5] Carbon nanotubes are basically graphite layers rolled into cylinders with a diameter of approximately 1 nm but have varying lengths. They come in different forms depending on their orientation [6] - imagine rolling chicken wire horizontally, vertically or on the diagonal to get different orientations of the wires. There are also single-walled and multiple-walled nanotubes, some with both ends open, and some with one end closed and rounded off. Nanotube properties are better defined than buckyballs. They are extremely strong (high tensile strength), conduct hot and cold, and can be either electrical conductors or semiconductors depending on their structure.

So, once we know about nanoparticles, it would be helpful to know how to produce them. One method is called "top-down" manufacturing, which involves starting with an object and removing part of it either mechanically or chemically (i.e. etching process) until we obtain material of the right size. Another production method is "bottom-up." Given the right conditions, nanoparticles will self-assemble, meaning that they will organize themselves into a stable orientation spontaneously. Self-assembly happens in nature all the time; the formation of cell membranes from phospholipids is one example. John Pelesko, in his book *Self-Assembly:The Science of Things That Put Themselves Together*, gives the example of a common cereal that self-assembles into hexagonal shapes when put into a bowl of milk, as long as the cereal pieces float, enough time passes, and perhaps the bowl is given a single shake. Self-assembly "provides a useful means for manipulating matter at the nanoscale." [7]

Scientists continue to find new ways to exploit the unique properties of nanomaterials. As more sophisticated instruments, such as a scanning electron microscope, were developed, scientists were able to "see" nanoparticles. At the same time, nanotechnology is being used to make tools and instruments that can sense changes in and the concentration of things such as contamination or disease. In my readings, I found dozens of career area and industrial applications of Nanotechnology. The medical field, the dominant setting for our seminar, is using nanoparticles for drug delivery and its controlled release, medical imaging and cancer therapy. The computer and electrical industries are using Nanotechnology to increase RAM and data transfer rates, for robotics and to make transistors, conductive wire, energy-efficient light bulbs, and high-resolution displays. Other applications include genetically modified crops, paint with greater adhesion, composites of greater strength and less weight, fuel cells, cosmetics, cryptography, ink jet printer inks, and food. An annotated list of websites and books for learning more about these applications is given in the Teacher Resources and Bibliography sections of the Appendix.

While this unit is geared toward mathematics, I would be remiss if I ignored the fact that Biology is steeped in Nanotechnology. All living organisms depend on DNA, which is a nanomaterial because the diameter of the double helix is approximately 2.5 nm. It is the understanding of nanoscale biological processes that is allowing scientists/engineers to make progress in areas like the delivery of cancer drugs to targeted areas in the body. In addition, some of the research being done in the field is called Biomimicry or Biomimetics - the process of imitating what nature does, but under controlled conditions in the laboratory. Physics also relates to Nanotechnology in the study of energy, Brownian motion, entropy, enthalpy, viscosity and quantum mechanics. As much as I would like to, I don't have time during the semester to go into depth on the science of Nanotechnology because of the breadth of my required mathematics curricula. However, I can easily envision integrating units with Biology, Chemistry and/or Physics teachers in my building in the future.

Background – The Math

Both the national mathematics and science standards reflect the need to integrate the two disciplines. The science standards speak directly about coordinating math and science programs so that students learn the necessary math skills for use in their science classes. The math standards state that students should be able to apply mathematics in contexts outside of math, and that "viewing mathematics as a whole also helps students learn that mathematics is not a set of isolated skills and arbitrary rules." [8] That's where Nanotechnology comes in! I will use content related to Nanotechnology to give students concrete examples that will help them overcome common mathematical weaknesses. At least some subset of the connections I am focusing on in this unit can be taught in nearly all of the math courses I teach. The following sections specify the key mathematical concepts I am including in this unit.

Scientific Notation

In every book and article I read about Nanotechnology, the concept of "Size and Scale" is of major importance because properties of nanoparticles are dependent on size. Unfortunately children and even many adults have difficulty with the conception of size. In my research, I learned that there is a continuum to conceptual understanding of size: ordering, grouping, number of times bigger or smaller, and absolute size. Younger children start by putting objects in relative order from small to large. The next stage in development is grouping objects of similar size, as in really big things, things we can see and maybe measure, really small things, things too small to see, etc. More sophisticated subjects in the studies specified more groups with narrower ranges of sizes. Most subjects had difficulty expressing the size of an object as a number of times bigger or smaller than another object. For example, the unit of measure to be compared could be the height of a person, and they were asked how many times longer a school bus is, or how many times smaller a ladybug is. Scale measurements were most accurate for objects close in size to human size, less accurate for very large (i.e. distance between cities) measurements, and even less accurate for very small (i.e. virus) measurements. [9] The more successful subjects explained that they estimated actual measurements of both objects and divided to calculate how many times bigger or smaller one was. The authors of the study determined that experiences of the subjects helped determine their level of success in estimating the scale factor between objects. [10] The final stage of development is estimating absolute size using conventional units such as meters, inches, etc. Again, subjects that could visualize changes in scale, or assign measurements and then multiply/divide them were most accurate in estimating absolute size. As nanoscience becomes more common, it is important for students to improve conceptual understanding of size and scale.

According to the math standards, students should develop a deeper understanding of very large and very small numbers and of various representations of them. In this unit, discussion of Nanotechnology necessitates representations of very small numbers, so scientific notation is a natural math topic to incorporate. Scientific notation is a shorthand notation for very small or large numbers. Numbers are written in the form n.pp x 10^{a} where n has a value between 1 – 9, a is an integer that represents the number of times needed to multiply/divide by 10 when the decimal point is after the 'units' place. The number of digits after the decimal (p's) depends on the number of significant figures (an important topic that will not be addressed in this unit). When a is greater than or equal to one, the number is greater than or equal to 10. When a is less than zero, the number is less than one, meaning we are dividing by 10 a times. The graphing calculators we use represent very large and small numbers in scientific notation using the format n.ppppp E aa, where n, p, and a represent the same as above. The exponent is shown following 'E' and is always written with two digits, using a leading zero, if appropriate. Often students ask what the 'E' means, but even more worrisome is that some ignore it, or don't look for it when answers are expected to be very small or very large! I will give instruction on scientific notation and equivalent decimals in all math classes, although students in the upper level courses may only need a 10-minute refresher while those in lower levels may require more.

Exponents

For reasons I do not understand, simplifying expressions involving exponents is a significant weakness of many of my students at all levels. Perhaps it is because we teach rules of exponents at the end of the Integrated Math 1 course and rush to finish so that students memorize rules without having a true understanding of them. Thus, when they reach the upper-level courses, their skills are weak. This unit will use the very small size of nanoparticles as a reference point, along with the concepts of scientific notation and place value in our base-10 number system, as an alternate means of learning/reinforcing exponent rules, especially the meaning of negative exponents. In addition, as I teach students to visualize the scale of nanoparticles relative to things they are familiar with, we will do repeated divisions to get from the macroscale to the nanoscale. I can demonstrate how to write the repeated division in multiple ways using both positive and negative exponents. I believe that teaching exponents in this context will have a tremendous positive impact on my students.

The following example illustrates several exponent rules. Start with a penny with a diameter of approximately 1cm. If we divide by 2, how many times must we divide until we reach a diameter less than 10 nm? To keep track of units, I would convert 1 cm to 0.01 m before starting to divide. Then, since 10 nm = 10×10^{-9} m = 1×10^{-8} m (this conversion could be a lesson in itself), our goal is to reach a number below 10^{-8}. A graphing calculator would be helpful to perform the 20 recursive divisions to reach $9.54 \times 10^{-9} = 0.00000000954$ meters, which represents about 9.5 nm. So far, I have shown two representations of the same number (scientific notation and decimal), but it can also be written as $0.001/2^{20}$ which shows the operations performed. Or, to demonstrate more exponent properties, it can be written as 0.01×2^{-20} or as $0.01 \times (2^{-2})^{10}$ if we only recorded every two divisions, repeated ten times to reach our goal. Of course, there are many other ways to demonstrate the meaning of exponents using the penny example. I think it also could be beneficial to describe in words what each numeric form means relative to a physical act of dividing by 2.

Geometry

Delaware math standards expect students to be able to use partitioning and formulas to find the surface area and volume of complex shapes. Because of the unique properties of nanoparticles, because of their increased surface area to volume ratio, students will have a reason to perform calculations for surface area and volume. That may sound simple, but I also want my students to understand the concept of measuring area, and not just use an algorithm to calculate it. A research article by Konstantinos Zacharos confirms what I have believed for many years – "students have difficulty in measuring area because of the emphasis on formulas without conceptual characteristics of the measurement." [11] Historically, in Euclidean Geometry, measuring area meant overlapping to compare the measured area of a surface to the area of a chosen unit. Thus, the measured area of the surface would be the quotient of the two, as in how many squares of a chosen size (i.e. 1 in by 1 in or 1 cm x 1 cm) it would take to cover the surface to be measured. The same article recommends that the tool used to perform a measurement should have the same dimensions as the object to be measured. Thus, length, having one dimension, is measured with a common tool such as a ruler. Area, having two dimensions, should be measured using 2-dimensional units such as squares or triangles to cover the surface. Volume, having three dimensions, should be measured using 3-dimensional units such as cubes to fill a space.

There is a geometry unit in our Integrated Math 2 course, but our students traditionally show a weakness on the Delaware State Test, so I would use illustrations of nanoscale materials with complex/composite shapes as warm-up problems for any other math course I teach to continue practicing these critical skills.

Logarithms

An additional math topic for precalculus students that comes from the study of nanoscale materials is logarithms. In essence, logarithms are exponents. The "log" of a number is the exponent that a base must be raised to in order to get the number. Logarithms and exponential functions are inverses of each other. The use of logarithms reduces multiplication to addition, and division to subtraction by applying exponent rules. Graphs that use logarithmic scales change a curved graph to a linear one (Figure 3). As an example, if students plot the surface area and volume measurements for varying scale factors (i.e. side length of a cube) the graphs are curves (SA L^2 , V L^3), whereas on a logarithmic plot the surface area increases linearly with a slope of 2, and the volume

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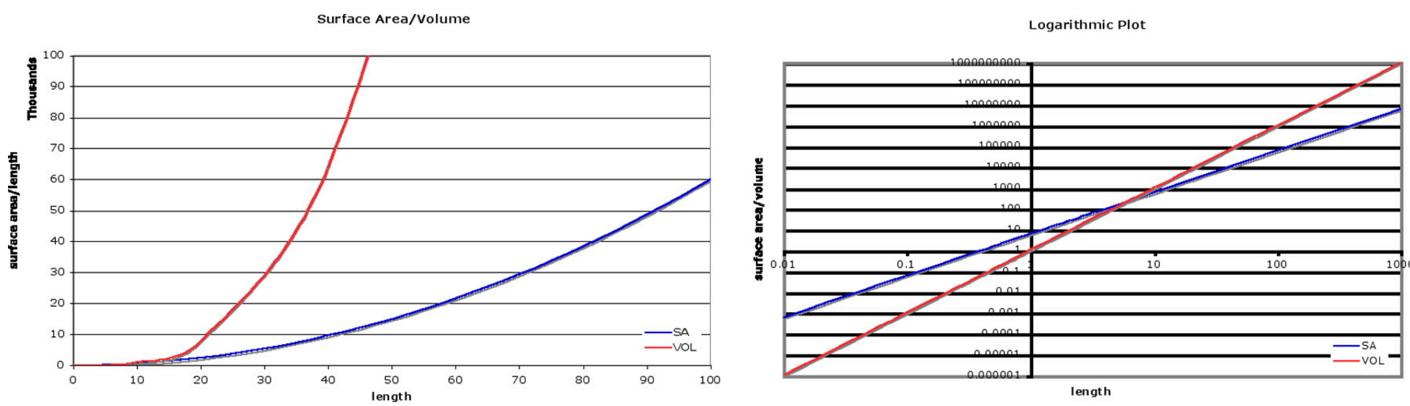

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Figure 3 – Surface Area and Volume of a Cube

increases with a slope of 3. In a typical Precalculus course, logarithm concepts are taught as skills with drills to practice them. Relating the skills to what students have learned about nanoscale particles will add interest to the concept. Creating graphs with logarithmic scales and adding images or labels of common objects for sizes represented on the graph and discussing the difference in scale will, again, reinforce students' number sense while keeping the ideas of Nanotechnology fresh in their minds.

Fibonacci Series

The Fibonacci series is an integer pattern that has intrigued my students in previous classes whenever it came up. It is not part of any curriculum in my school, but I am including it here as an extra for students that may be interested in further exploration. The series begins with the integers 0, 1 and continues by adding the two previous numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21…. The Fibonacci series has ties to nature as described in John Pelesko's book on self-assembly. It matches the breeding pattern of rabbits and the spirals in pineapples and pinecones and sunflowers. Counting the number of clockwise spirals and the number of counter-clockwise spirals, the pair of numbers are consecutive numbers in the Fibonacci series (unless outside factors interfere with growth). "They arise because of the relationship among the Fibonacci sequence, the golden mean, and optimal packing." [12] The golden mean, or golden ratio, has been studied since Euclid wrote about it in his book Elements in 303 BCE. It is an irrational number, , defined by (a+b)/a = a/b where a + b is the total length of a line segment and a is the longer of the two segments. Many ancient buildings, including the Parthenon, were built according to the proportion of golden ratio because it is aesthetically pleasing. [13] As each number in the Fibonacci series is divided by the previous number, the ratio approaches the golden ratio, approximately 1.618. [14] In nature, flowers often grow in spirals following the pattern of 360 * (golden ratio) because it is the most efficient use of space and allows optimal access to sunlight. [15] The connection to Nanotechnology cited in Pelesko's book is a self-assembly process which coats silver spheres with a layer of silicon oxide in either the spiral pattern described, or another natural (hexagonal) configuration.

Formulas

If I were collaborating with a science teacher to teach this unit, I would have him/her provide context and appropriate formulas related to Nanotechnology. I could include these already-familiar formulas with a gentle reminder of why/how they used them in science to practice solving, evaluating, and manipulating equations full of variables. However, I will not include formulas at this time.

Concentration

Concentration is another topic that I could potentially include. As we discuss size and scale, and students' conceptual understanding of very small things improves, it may be appropriate to review calculations for extremely low concentrations that would be expressed in parts per million (ppm) or parts per billion (ppb). In the past I found it helpful to refer to percent as parts per hundred, so that the calculations of ppm and ppb are set up in the same way, but with larger denominators.

Teaching Strategies

I teach three 90-minute block periods a day during each semester, for a total of 6 classes per year. I use an interactive SmartBoard in my classroom daily, and I prepare lessons using Notebook software. In a lot of ways my Notebook files are similar to Power Point presentations, in that the lesson flow is generally dictated by the slides. The difference comes from some of the interactive capabilities and links on the SmartBoard, but the strategies I describe in this section can be used anywhere there is access to the internet.

After the obligatory introductions for a new semester, and before introducing students to the Nanoscale World, I will do a Pre-Assessment Activity about nanoscale materials to assess their level of understanding of Nanotechnology. The next step is to get students thinking about how small nanoscale is. There are several interactive websites listed in the Teacher Resources section in the Appendix that can help students visualize relative sizes of common objects, from galaxies down to atoms, and I will let those images swirl in their minds for a little while.

To introduce the science of Nanotechnology, I will focus on the structure of carbon. My students should have some background in atomic structure and bonding, either from Earth Science or Chemistry, so I will start with a KWL activity. KWL evaluates what students already Know, what they Want to Know, and later, what they Learned. Based on the results, I will evaluate how much I need to review about the structure of carbon and the reasons it can bond covalently with four other atoms. I will use links to one or two websites (also in the Teacher Resources section) to show the structures of graphite, diamond, buckyballs and carbon nanotubes. From the visuals of the different structures, we will discuss the different properties of each carbon allotrope, and why they might be useful for different end products. It will be important for students to be familiar with buckyballs and carbon nanotubes since they are both commonly used in Nanotechnology research.

Since one of my objectives is for students to find the relevance of Nanotechnology to themselves, I will have them do some research outside of class to learn how it could be used in their (vocational) career area, or in any area of interest. Because Nanotechnology has a future in computers/internet, cell phones, cosmetics, medical and green energy applications, every high school teenager should be able to find something of interest to him/her. In addition, a large percentage of the students I teach in Precalculus are in Nursing, Electrical, Computer or Pre-Engineering career areas, all of which are areas with important connections to Nanotechnology. Students will create Power Point presentations, either individually or in pairs, of their findings to share with their classmates early in the semester. The presentations will be done at a rate of only one or two a day to leave adequate time for math instruction.

The mathematics lessons will be spread throughout the semester and will vary depending on the math course. In all courses, the first lesson will relate to size. To understand the significance of Nanotechnology, students need to grasp the scale of nanoparticles (10^{-9} meters, or approximately 1 billionth of a meter). To accomplish this, and to emphasize what they saw on the interactive websites at the very beginning, students will create 3-dimensional models having complex shapes. One example I can envision is attaching an empty toilet paper tube to an ice cream cone (cylinder plus cone). During the Geometry unit in the Integrated Math 2 course, I will have them first cover their models with appropriately-sized cut-out squares (in^2, cm^2, mm^2) and count the squares to measure surface area. If their models are too small for individual cutout squares, students can create a net to cover the model and draw a grid of squares on the net to count the overlap. After this activity, these models should look like mosaic masterpieces that we can display in the classroom to continually reinforce the overlapping concept of area, discussed in the Background section. For other classes, I can use the models on display to reinforce the overlapping/coverage definition of area. With the conceptual understanding of area, students can then use the combination of appropriate formulas (i.e. the sum of the areas of each part of their complex shape) to verify their surface area measurements. Next, I will have them fill their 3-dimensional models with cubes, sand, or water, if possible, to measure volume. If it's not possible, I can demonstrate the filling of classroom manipulative models made for this purpose. Students will also use a combination of appropriate formulas to calculate the volume of their models.

Again, using the models, I will have students address another math standard - the effects of scaling on surface area and volume of three-dimensional solids. They will enlarge or shrink the model and then build it, using their creativity, from whatever materials they choose. Finally, through discussion and calculation, students will determine how many repeated "shrinkings" by the same factor they used for their models, of their smaller model it would take to reach nanoscale. During this activity, they will be able to describe sizes in multiple ways: different units, decimals, fractions and scientific notation.

At this point, I will need to give some instruction about the importance of the surface area- to- volume (S/V) ratio in the study of nanoscale particles. As similar (same shape) particles get smaller, S/V increases. That is, there is more surface area available, compared to volume, for reactivity with the environment. This is a significant fact that affects the properties of nanomaterials: once again, students will know "WHY" they need to learn the effects of scaling in geometry. Students will use their two models to compare the changes in first, dimensions, second, surface area and third, volume by dividing to find the three scale factors. This hands-on activity will clearly demonstrate the fact that changes in dimensions (scaling) affect surface area and volume differently and will confirm that S/V increases as linear dimensions decrease. While these geometric concepts only apply directly to the objectives of one of our district's integrated math courses, I can reinforce the concepts via warm-up activities in upper-level courses.

Revisiting the student-created models, and multiple divisions, may be just the thing that students need to comprehend why negative exponents are inverses of the corresponding positive ones (i.e. multiplication and division are inverse operations). I will begin by multiplying and dividing by tens and relate the results to what they already know about scientific notation. As I discussed in the Background section, I can illustrate other exponent properties using this idea of repeated divisions, multiplications. Exponents are a topic in my district's Integrated Math 1 course, used again in Integrated Math 2, and then in Intermediate Algebra and Precalculus, so getting a solid foundation of understanding will go a long way. With all that I have learned about Nanotechnology, and my enthusiasm for it, I'm sure I will find other ways to interject more examples of the topic throughout all of the courses I teach!

Classroom Activities

Lesson 1 – What is a nanometer? What is Nanotechnology?

This lesson will take place on the first day of the semester, after all introductions and paperwork are complete.

Pre-Assessment: One side of the classroom will be designated as "True" and the other side "False." I will ask questions about nanomaterials and applications from Chapter 1 in the book *Nanoscale Science: Activities for Grades 6-12.*[16] Students will move to the side of the room that represents their answer to each question, and we will discuss the answers to each to introduce the field of Nanotechnology.

Instruction: Define nanometer as one billionth of a meter and ask students to name things that are that small. Show a video (refer to links in the Teacher Resources section) that illustrates relative sizes of common materials. There are several videos to choose from, and I will experiment with different ones to determine which my students prefer. Introduce and define Nanotechnology using another video or Power Point presentation with general background information.

Assignment/Assessment: Students will work individually or in pairs to research a Nanotechnology application of interest to them. Research will be done outside of class, and they will create a Power Point presentation to share with their classmates. Students will present their findings at a rate of 1-2 per day over several weeks. Presentations must have a minimum of three slides that cover:

1. Student Biography: name(s), career area, post-graduation plans, other interests
2. Nanotechnology Application: how it is used, how it works, fun facts/illustrations
3. Reason for choosing Application (above) and Source(s) of Information

Lesson 2 – Size and Scale

This lesson will cover more than one day, depending on the course and level of the students. It will begin on the second day of the semester to lay more of a foundation for the study of Nanotechnology, but the mathematics may be spread throughout the course.

Warm-up/Pre-Assessment: Several topics can be used as mathematics warm-up activities and repeated as often as necessary. A) Convert between decimals and scientific notation. B) Calculate area of basic and/or composite geometric figures. C) Apply scale factors and calculate surface area and volume of resulting objects.

Instruction: Set up discussion of size and scale by reading the poem "One Inch Tall" by Shel Siverstein or an excerpt from Gulliver's Travels and/or by showing video clips from old movies such as *King Kong, The Incredible Voyage*, or *A Bug's Life* (Teacher Resources). Discussion should focus on whether it is physically possible for the animals or people depicted to exist, followed by examples from nature where size matters (elephants versus geckos and ants). This discussion will lead into basic chemistry instruction about the element carbon. The amount of chemistry instruction will be determined by the KWL activity described in the Strategies section. I will show internet images of the four common allotropes of carbon, along with some of their properties, and give examples of how some properties change at the nanoscale (i.e. strength and conductivity).

Scientific notation and the geometry topics surface area, volume, and "scaling" (enlarging/shrinking by a scale factor) will be embedded within this lesson. At this stage, I want students to demonstrate how to convert between very large (millions) or very small numbers (billionths) and scientific notation by multiplying or dividing by 10 repeatedly, representing the number of repetitions with an exponent on 10. They will also use their calculators to add, subtract, multiply and divide numbers at the extremes to ensure that they recognize the notation using "E" followed by the exponent on 10, and that they look for "answers" shown in scientific notation on the screen.

Students will use materials of their choice to create "complex" 3-dimensional models made from two or more basic shapes. They will first measure appropriate (linear) dimensions for their models. Next they will measure the surface area in one of two ways: cover the model with cut-out squares of appropriate size or cover the model with a net having a square grid on it, and count the number of squares. Students will then compare their measured areas with the sum of the areas calculated by area formulas for each piece of their models. If possible, students will measure volume by filling their models with a known volume of water, sand, cubes, etc. All students will calculate the volume of their models using appropriate formulas. Finally, all models will be displayed in the classroom as visual reminders that surface area is a measure of overlapping squares.

To study the effects of applying scale factors on surface area and volume, students will enlarge or shrink their models by a factor of 10 or 100, depending on the size of their original models, by multiplying each of its original dimensions. They will build it and then calculate the surface area and volume of their second model. Finally they will compare the new versus original measurements by completing a table:

Scale Factor	Basic Shapes	Linear Dimensions	Surface Area (S)	Volume (V)	Ratio: S/V
1 (original)	#1: #2: #3 or more, if needed:	r= ; h=			
10 or 100	(Same)				
.1 or .01	(Same)				

Assessment: Students will complete additional rows in the table and summarize the relationship between the scale factor and the changes in surface area and volume. Students will also verbalize the pattern for the ratio of surface area to volume as the linear dimensions decrease.

Extension for Precalculus: Students will create one graph of both surface area and volume versus one of the linear dimensions in the table above. They will create a second graph using the same data on a logarithmic plot. Students will then verbalize and explain the differences between the two graphs.

Lesson 3 – Exponents

This lesson can be inserted into any math course when rules of exponents are being taught or reviewed.

Warm-up (sample): Rewrite each expression showing all factors individually.

1. x^4 x^5 x 2. p^8/p^4 3. $(4h)^3$ 4. $(k^6)^2$ 5. $(2d^3)^4$

[KEY: 1. xxxxxxxxxx, 2. (pppppppp)/(pppp), 3. 4h4h4h, 5. 2ddd2ddd2ddd2ddd, etc.]

Instruction: Relate exponents to nanometers and scientific notation by asking questions such as: "How many nanometers are in 5 meters?" "How many meters is 3 nm?" "Starting with a length of 5 m, how many times must you divide by 10 to get a length less than 100 nm?" Starting with a length of 5 m, how many times must you divide by 5 to get a length less than 1 nm?" If necessary, review how 100 nm or 1 nm will be displayed on the calculator screen. Discuss the meaning of the base (10) and positive versus negative exponents in scientific notation. Relate the concept of multiplication and division being inverse operations to the negative sign in an exponent indicating the inverse of multiplying repeatedly.

Emphasize the meaning of symbolic exponent rules by having students verbalize them. The following table gives examples of how to verbalize five basic rules when the exponents are integers. (NOTE: The symbolic rules apply for real numbers as exponents, but the meanings may not make sense.)

Symbolic rule	Meaning of rule
$b^n \bullet b^m = b^{(n+m)}$	(b multiplied 'n' times) multiplied by (b multiplied 'm' times) is equal to b multiplied ('n' plus 'm', or the sum of 'n' and 'm') times
$\frac{b^n}{b^m} = b^{(n-m)}$	(b multiplied 'n' times) divided by (b multiplied 'm' times) is equal to b multiplied ('n' minus 'm', or the difference of 'n' and 'm') times
$(a \bullet b)^n$	(the product of a and b) multiplied 'n' times is equal to the product of (a multiplied 'n' times) AND (b multiplied 'n' times)
$(b^n)^m = b^{(n \bullet m)}$	'm' sets of (b multiplied 'n' times) is equal to b multiplied ('m' times 'n', or the product of 'm' and 'n') times
$b^{-n} = \frac{1}{b^n}$	The inverse of b multiplied 'n' times is equal to dividing by b 'n' times

Assessment: Practice rewriting expressions with exponents in simplest form and verbalizing their meaning. Practice can be in the form of worksheets, matching cards with symbolic expressions to corresponding cards with written meanings (group activity or interactive SmartBoard activity), or displaying a written meaning on the screen and students writing the corresponding symbolic expression on dry erase boards or paper at their seats for the teacher to check.

Appendix A - Teacher Resources

http://www.ucsd.tv/getsmall/ is a long, detailed video, but the first 5 min. gives excellent visual examples for nanoscale.

http://nanozone.org/howvideo.htm has several videos relevant to nanoscale.

http://learn.genetics.utah.edu/content/begin/cells/scale/ has excellent sliding scale, especially related to biology (cells, bacterium, atoms, etc.), to show nanoscale

http://www.nanoed.org/concepts_apps/tool_for_nano_thermodynamics/Computer-based_Learning_Tool.html is a 3-part module that demonstrates nanoscale, the surface area to volume relationship and melting point depression at the nanoscale.

http://www.sciencentral.com/index3.php3?cat=3_5 gives an index to articles and videos showing applications of Nanotechnology.

http://www.sciencemuseum.org.uk/antenna/nano/ has great applications, including safety.

http://www.understandingnano.com/ has lots of applications, but it's a little "dry."

http://www.discovernano.northwestern.edu/affect/applications_content has lots of applications broken into categories: security, medicine, energy and environment.

www.nanooze.org links to articles for young readers about uses of Nanotechnology.

http://www.discovernano.northwestern.edu/whatis gives excellent overview of size, size-dependent properties, production methods, and instruments for "seeing" nanoscale.

www.youtube.com/watch?v=LFoC-uxRqCg adorable video/song about nanomaterials.

http://www.creative-chemistry.org.uk/molecules/carbon.htm shows excellent models of carbon allotropes and links to chemistry puzzles and games.

http://www.nanoed.vt.edu/links.htm has resources related to Nanotechnology.

http://cohesion.rice.edu/naturalsciences/nanokids/index.cfm has videos of basic chemistry concepts, done at an elementary level.

http://mrsec.wisc.edu/Edetc/modules/HighSchool/CNT_Vector_Activity/index.htm has an activity about different structures of carbon allotropes.

The following is a list of books and movies that can be used to prompt discussion about size-dependent properties:

A Bug's Life. VHS. Directed by Andrew Stanton: Disney/Pixar, 1998.

Fantastic Voyage [VHS]. Theater viewing. Directed by Richard Fleischer. Tucson: 20th Century Fox, 1966.

Honey I Shrunk the Kids [VHS]. VHS. Directed by Joe Johnston. Hollywood, CA: Walt Disney Video, 1989.

Innerspace. VHS. Directed by Joe Dante. Burbank: Warner Home Video, 1987.

Silverstein, Shel. "One Inch Tall." In *Where the Sidewalk Ends - The Poems and Drawings of Shel Silverstein*. New York: Harper And Row, 1974. 55.

Swift, Jonathan. Gulliver's Travels, Literary Touchstone Edition. Clayton: Prestwick House, Inc., 2005.

The Incredible Shrinking Man [VHS]. Theater viewing. Directed by Jack Arnold.

Washington DC: Universal Studios, 1957.

Appendix B - Bibliography

"Allotropes of carbon - Wikipedia, the free encyclopedia." Wikipedia, the free encyclopedia. http://en.wikipedia.org/wiki/Allotropes_of_carbon (accessed July 19, 2010). Excellent cartoons of carbon allotropes and description of different properties.

Bonner, John Tyler. *Why Size Matters: From Bacteria to Blue Whales*. Princeton: Princeton University Press, 2006. Great explanations from nature for size-dependent properties.

Booker, Richard D., and Earl Boysen. *Nanotechnology For Dummies (For Dummies (Math & Science)).* Indianapolis: Wiley Publishing, Inc., 2005. This book explains the science, the production process, the uses, the measuring devices, etc. of nanomaterials.

Delgado, C., S. Y. Stevens, N. Shin, M. Yunker, and J. S. Krajcik. "The development of students' conceptions of size." Paper presented at the National Association of Research in Science Teaching Conference, New Orleans, 2007. Excellent resource article about stages of development with relation to relative and absolute size.

"Golden ratio - Wikipedia, the free encyclopedia." Wikipedia, the free encyclopedia. http://en.wikipedia.org/wiki/Golden_ratio (accessed July 21, 2010). This site gives an explanation of the history of the golden ratio and its relation to the Fibonacci series.

Jones, M. Gail, Michael R. Falvo, Amy R. Taylor, and Bethany P. Broadwell. *Nanoscale Science.* Arlington: NSTA Press, 2007. This book has many activities for students in grades 6-12 categorized by math and science subject.

"Patterns in Nature." Cornell Center for Materials Research. http://www.ccmr.cornell.edu/education/modules (accessed July 19, 2010). This article presents a lesson with activities on the Fibonacci sequence in nature.

Pelesko, John A.. *Self Assembly: The Science of Things That Put Themselves Together*. Boca Raton: Chapman & Hall/CRC, 2007. This book defines and gives examples of self-assembly in nature and Nanotechnology.

Saunders, Nigel. "Creative Chemistry Interactive Molecular Models - Carbon Allotropes." Creative Chemistry - fun activities, worksheets, games and revision quizzes. http://www.creative-chemistry.org.uk/molecules/carbon.htm (accessed July 12, 2010).

"The Carbon Allotrope Group." Solar Power Facts: A Brighter, Cleaner Future. http://www.green-planet-solar-energy.com/allotrope.html (accessed July 12, 2010). This is another resource for carbon allotropes.

Stevens, Shawn, LeeAnn Sutherland, and Joseph, Krajcik. *The Big Ideas of Nanoscale Science and Engineering: A Guidebook for Secondary Teachers (PB241X)*. Arlington, VA: National Science Teachers Association, 2009. This is an excellent resource outlining how to connect all of the high school sciences and mathematics courses.

Tretter, Thomas R., M. Gail Jones, and James Minogue. "Accuracy of scale conceptions in Science: Mental Maneuverings across many orders of spatial magnitude." *Journal of Research in Science Teaching* 43, no. 10 (2006): 1061-1085. www.interscience.wiley.com (accessed July 10, 2010). The study described in this article also describes weaknesses in estimating relative and absolute size.

Tretter, Thomas R., M. Gail Jones, Thomas Andre, Atsuko Negishi, and James Minogue. "Conceptual boundaries and distances: Students' and experts' concepts of the scale of scientific phenomena." Journal of Research in Science Teaching 43, no. 3 (2006): 282-319. http://www3.interscience.wiley.com/journal/31817/home (accessed July 10, 2010). This article is very similar to the previous one.

Zacharos, Konstantinos. "Prevailing educational practices for area measurement and students' failure in measuring areas." *Journal of Mathematical Behavior* 25, no. 3 (2006): 224-239. http://www.sciencedirect.com/science/journal/ (accessed July 10, 2010). This article suggests that the use of algorithms for calculating area limits students' understanding of the concept.

Appendix C - Implementing District Standards

My school district standards align with state standards; the following is a list of Delaware Mathematics Standards for grades 9-12 addressed by this unit. As discussed in the Objectives section, this unit addresses some science standards, as well.

Content Standard 1– Numeric Reasoning: Students will develop Numeric Reasoning and an understanding of *Number and Operations* by solving problems in which there is a need to represent and model real numbers verbally, physically, and symbolically; to explain the relationship between numbers; to determine the relative magnitude of real numbers; to use operations with understanding; and to select appropriate methods of calculations from among mental math, paper-and-pencil, calculators, or computers.

This unit will prepare students to perform these grade level expectations:

- Represent and operate with very large and very small numbers to include various representations of them (grade 9).
- Simplify numeric and symbolic expressions involving exponents (grade 10).
- Simplify expressions with negative and fractional exponents (grade 11).

Content Standard 3 – Geometric Reasoning: Students will develop Geometric Reasoning and an understanding of Geometry and Measurement by solving problems in which there is a need to recognize, construct, transform, analyze properties of, and discover relationships among geometric figures; and to measure to a required degree of accuracy by selecting appropriate tools and units.

This unit will prepare students to perform these grade level expectations:

- Demonstrate an understanding of and apply formulas for area, surface area, and volume of geometric figures including pyramids, cones, spheres, and cylinders (grades 9 and 10))
- Demonstrate the effects of scaling on volume and surface area of three-dimensional solids (grade 10)

Process Standard 5 – Problem Solving: Students will develop their Problem Solving ability by engaging in developmentally appropriate problem-solving opportunities in which there is a need to use various approaches to investigate and understand mathematical concepts; to formulate their own problems; to find solutions to problems from everyday situations; to develop and apply strategies to solve a wide variety of problems; and to integrate mathematical reasoning, communication and connections.

This unit will prepare students to:

- Build new mathematical knowledge (all grades)
- Solve problems that arise in mathematics and in other contexts (all grades)

Process Standard 8 – Connections: Students will develop mathematical Connections by solving problems in which there is a need to view mathematics as an integrated whole and to integrate mathematics with other disciplines, while allowing the flexibility to approach problems, from within and outside mathematics, in a variety of ways.

This unit will prepare students to:

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

Notes

1. Shawn Stevens, LeeAnn Sutherland, and Joseph, Krajcik, *The Big Ideas of Nanoscale Science and Engineering: A Guidebook for Secondary Teachers*, 130.
2. Ibid, 39.
3. http://en.wikipedia.org/wiki/Allotropes_of_carbon.
4. Richard D. Booker and Earl Boysen. Nanotechnology For Dummies, 69.
5. Ibid, 73.
6. Ibid, 76.
7. Stevens, Sutherland, and Krajcik, *The Big Ideas of Nanoscale Science*, 43.
8. NCTM Mathematics Process Standard: Connections. http://standards.nctm.org/document/chapter3/conn.htm
9. C. Delgado, S. Y. Stevens, N. Shin, M. Yunker, and J. S. Krajcik. "The development of students' conceptions of size," 10.
10. Tretter, Thomas R., M. Gail Jones, Thomas Andre, Atsuko Negishi, and James Minogue. "Conceptual boundaries and distances: Students' and experts' concepts of the scale of scientific phenomena." *Journal of Research in Science Teaching* 43, no. 3 (2006).
11. Konstantinos Zacharos. "Prevailing educational practices for area measurement and students' failure in measuring areas." *Journal of Mathematical Behavior* 25, no. 3 (2006), 224.
12. John A. Pelesko. *Self Assembly: The Science of Things That Put Themselves Together*, 255.
13. http://en.wikipedia.org/wiki/Golden_ratio
14. "Patterns in Nature." http://www.ccmr.cornell.edu/education/modules.
15. Ibid.
16. M. Gail Jones, Michael R. Falvo, Amy R. Taylor, and Bethany P. Broadwell. Chapter 1 in *Nanoscale Science.*