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Using Place Value to Teach Addition and Subtraction, Let's Count the Ways

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Content Objectives

My goal for this unit is to create lessons that make the transition from simple addition and subtraction into regrouping with 2 digit numbers smoother for students, thus leaving them with a successful mastery of skills they will need to progress in mathematics, including improved automaticity with basic addition and subtraction facts. In a typical progression of a spiraling elementary curriculum, addition and subtraction are revisited using a variety of contexts throughout the year. Throughout my years of teaching experience I have noticed several of my students have lost ground in their understanding of the basic addition and subtraction concepts, making regrouping that much more complicated to understand. It is as if I need to re-teach addition and subtraction each time we revisit it, demonstrating a lack of deep understanding and automaticity with basic facts that are necessary for regrouping in addition and subtraction.

I teach 2 nd grade in a large urban school district with a mobility rate of 15%, low socio-economic population with 98% of students on free or reduced lunch. Over the last several years I have also averaged 3- 4 students per classroom who are part of our Special Educations program, and have an Individual Education Plan, and 5-7 students per classroom who are identified as having some form of ADD or ADHD, some on medication and some not. Within this population, the range of ability varies greatly, and I often find myself struggling to adequately meet all my students' needs. The unit I have developed here will also assist in meeting the diverse mathematical learning needs of students.

My objectives for this unit combine ideas from the Singapore approach to mathematics with the National Core Curriculum for Mathematics [1] , the latter of which my home state has been selected to teach in a pilot study. The main objectives for this curriculum unit that are developed here are the following: 1. Representing and solving problems involving addition and subtraction. 2. Addition and subtraction within 100 to mastery, while gaining a deeper understand of place value. 3. Using place value understanding and properties of operations to add and subtract both without and with regrouping. 4. Relating addition and subtraction to length.

Rationale

What exactly is a "deep understanding of mathematics" and the idea of "learning deeply"? If I was to liken this concept to reading I would say it is the difference between knowing that the letters t-r-e-e spell tree verses a deeper realization that the word tree also relates to the big green thing out in the yard, and how to use of word tree by creating a sentence or story with it. It is where students move past simply knowing words in a sight list to understanding what all the words in a story truly represent. In mathematics, this can mean, for example, that a student does not memorize the fact that 2+2 is 4 but I understands that by taking two separate groups of 2 and combining them, they will always obtain 4, or that 4 things can be broken into two equal groups of 2. A deeper learning of mathematics involves comprehending how numbers work without memorizing "facts" rooted in our base 10 system.

This deep understanding what this unit strives to accomplish. In standard curricula, regrouping is taught as two different chapters, one on regrouping in addition and another on regrouping in subtraction. With in this unit I will be combining concepts I would have previously taught separately. By this I mean to teach addition and subtraction together, allowing students to see how addition and subtraction work together within the base 10 system. While it is very natural to think of a letter as part of the alphabet, it is not so common to incorporate the understanding of numbers with place value within this base 10 system, i.e. thinking of 13 as 10+3, or 57 as 50+7.

By simultaneously teaching addition and subtractions with place value, the student can begin to learn the language that mathematics offers us. This can be seen in the ideals of teaching two digit numbers as some tens and some ones or teen numbers as one ten and some ones (as in the examples above). Place value should not be taught simply as a vocabulary lesson, but used as a tool to teach addition and subtraction. The place value system and addition and subtraction complement each other and work together. This means rather than spending four weeks on chapter one in addition, four weeks on chapter two for subtraction and four weeks on chapter five for place value for example they can be blended together to spend 12 weeks digging deeply into what it means to add and subtract within our base 10 number system. It should be noted that with the deep learning of these concepts, speed with facts may not begin to appear early on. However, with the depth of understanding students will experience, they will successfully be able to move on to more complicated mathematical concepts later. Initially speed might be sacrificed for a deep and lasting understanding.

Background

The following areas provide background information on regrouping in addition and subtraction. This section includes information on place value and the models used to present information, as well as the foundational steps that should be mastered prior to regrouping with two digit numbers. Without a mastery of these first two foundational steps (which can be addressed throughout the entire school year), students may lack the understanding of basic math concepts necessary to regroup. These steps are also included to demonstrate the connections addition and subtraction have to place value. Many curriculums are now structured as spiraling, meaning if students don't understand a concept now they will understand it sometime later. Although this can

be good for review of skills it doesn't lend well to a deep lasting understanding of skills and concepts. The structure of these steps is more of a ladder: it is necessary to fully master one step before progressing to the next.

Place Value

Place value is the method for writing any whole number as a sum of other numbers in terms of its base 10 expansion. For example, the number 1234 denotes 1 thousand + 2 hundreds + 3 tens + 4 ones. The number 0 is used as a place holder to denote that there is not an amount represented in the given place, i.e. in the number 1034 there is nothing in the hundreds place. Each of the other single digit numbers gives an amount that is represented in each place while taking into account that each place in itself is an amount, as in the example above. Each number and each place combine to create their own place value.

Number Line

The number line is a horizontal line, with a designated "0" point, and a designated "1". The one is placed to the right of the 0. The distance between the 0 and 1 becomes the unit length, and dictates the placement of every other integer on the number line. For example, 2 is marked at one unit distance to the right of 1, 3 is marked at one unit distance to the right of 2, and so on. It is important to maintain equal units when working with the number line and when working with number problems to give an answer in number and word form to indentify the unit.

Concrete – pictorial – symbolic

One fundamental ideal within this unit taken from Singapore math [2] is the flow of concrete, pictorial, and symbolic models. By starting with concrete objects or manipulatives students regardless of reading level or even language skills are able to manipulate and discover patterns in numbers. This can include the use of counting bears, base ten blocks or counting tiles, and will allow students who are reading below grade level or may be English language learners to participate in the lesson and begin to build an understanding of the math concept.

After working with concrete models, the next step is to work with pictorial models. It is best to use pictures that relate back to the concrete activity. This makes this step even smother and continues to easily involve struggling readers. The second step begins the process of students transferring hands on learning to paper. While students may see some numbers, at this stage they will not work with mathematical symbols (+, -, =, for example). As students work with pictures and numbers they are exploring how numbers can be taken apart and exploring the part-part-whole aspects of numbers (i.e.13 and 4 are parts of 17 the whole) and students are continuing to deepen their knowledge and understanding of how numbers combine together to build other numbers, and how they may be taken apart.

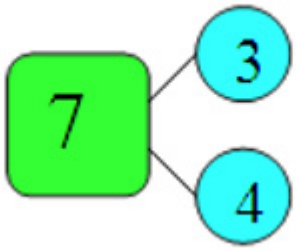
The final model in this Singapore progression is the symbolic. At this stage, students will refine the skills from the two previous steps and begin the transition into paper and pencil work, and then onto mental math. For example students will begin using manipulatives to explore how amounts work together such as combining 4 bears with 3 bears to make 7 bears. Then students will use pictures and or number cards to continue building and breaking apart numbers, and finally use symbols with the numbers to solve problems using algorithms. See the lesson activities for an example related to each of the three models. Teaching just one or two lessons in each area is likely not enough to allow for the mastery of a concept like concrete addition or subtraction. This is essential before moving onto the next stage and integral for a deep and lasting understanding of

mathematical concepts. Each of these three models will be reoccurring throughout this unit.

Addition and Subtraction Facts to 10

Addition and subtraction using the digits up to 10 is a first foundational step towards addition and subtraction of more general (larger) numbers. Prior to beginning addition and subtraction time should be given to concretely and pictorially discuss how to break apart numbers, and then build them back up. For example a student could be given 8 bears and asked to make two groups. Students should become familiar with and practice writing down all the different amounts that can make up the parts or groups of a given number or quantity. Once students have a strong understanding of this, they should be prepared to begin working with basic addition and subtractions. Along with ensuring student have a solid and deep foundation to numbers and how they work together this time will also allow for the opportunity to introduce the concrete – pictorial – symbolic model. This will allow all students to become familiar with a process we will be incorporating throughout the year and allow students who do not yet have a comfort with basic addition to develop their skills and mastery of the concept. To ensure that students who do understand their basic concepts are not distracted I will also intertwine word problem activities.

During this step students will be working with numbers for both addition and subtraction while exploring all they ways they are connected. We will be using fact families where all related numbers are explored. (i.e. 2+3=5, 3+2=5, 5-2=3, 5-3=2), as well as missing addends (i.e. 3+?=5). The concrete model will be explored through the discussion of word problems and with use of manipulatives students can physically act out the problem. An example can be seen in classroom activity 1. Moving into the pictorial model I will utilize the idea from Singapore [3] of "number bonds" which illustrate addition and subtraction without using symbols (+,-,=) It is a part-part-whole way of thinking about numbers (for example 3 and 4 are parts of 7):



Once students have mastered all the ways to take apart and build numbers within 10 we will then move into the symbolic portion using the signs (+,-,=) and working with pencil and paper. Students will revisit all of these skills as they progress, but for a deep and lasting understanding, a good deal of time should be give to allow student to master each step. The suggested time allotment for this step is 3 to 5 weeks, with flexibility to allow for mastery.

Teen Numbers As One Ten and Some Ones

The way teen numbers are named in most languages can be confusing. As our numbers progress it becomes easier to see that twenty-three is 2 tens and 3 ones, but what is eleven? In this second step place value will be introduced and then used as a means to add and subtract. Students will be given an expended amount of time

to gain a deep understanding of what at teen number is and not simply a vocabulary lesson. We will spend time discussing the names of our teen numbers and how we can "rename" them or think of them in a different way. Using concrete models with manipulatives and base 10 blocks paired with a discussion of how eleven is 1 ten and 1 one students will begin to gain an understanding of our base 10 number system and place value.

As students master these concepts and stages they will also begin to develop automaticity for the basic facts, however it is important for later steps that they continue to write out problems. This step lends itself to comparing numbers and working with greater than and less than. The integration of word problems will assist in blending all of these concept areas. The suggested time allotment for this step is 3 to 5 weeks.

Higher Addition and Subtraction Facts Within 20

After steps one and two above, students should be prepared to progress to adding and subtracting within 20 (i.e. using the numbers 1-20), using the same concepts. For example, just as we used base 10 blocks to illustrate adding within 10, 6+8 for example, we will continue to discuss word problems within 20 with base 10 blocks. As we move between the pictorial and symbolic portion of this step students will be directed on how to write out their work using the expanded form. As an example, I might ask students to solve problems such as 14 + 5 = ? , 10 + 4 + 5 = ? From here I will direct student to use the knowledge they have from the first step and their understanding of number bonds to combine 4 + 5 to make 9, then writing 10 + 9 = ? At this point, they should easily see the answer they are working toward is 19. "The Break Apart to Make a Ten" method will also be used in concrete then pictorial and finally symbolic. Within the following example of the symbolic model the Associative Property (i.e. that multiple addends can be regrouped in any ways) is also demonstrated:

$6 + 8 = (4 + 2) + 8 = 4 + (2 + 8) = 4 + 10 = 14.$

Due to the many layers of this step, the suggested time allotment is a flexible 4 weeks.

Two Digit Numbers As Some Tens and Some Ones

In this step we will explore how the –ty numbers (the twenties, thirties, etc) are made up of some 10's. For example, twenty is made of two tens, sixty is six tens, and just as students were taught that 13 is make of one ten and three ones, they will learn that 23 is make of two tens and three ones. These activities will again move through the concrete – pictorial – symbolic model. It is important that this step, as with the step with teen numbers, not be downgraded to a vocabulary lesson. Students will be working with larger numbers and will need to use the skills from previous steps to build or take apart these 2 digit numbers. The skills they learn now will be mirrored and built upon as they work through larger numbers and more difficult concepts. We will continue to work with the base 10 blocks to give student the concert and visual connections for arranging numbers and rearranging them. I will also emphasize writing the numbers in a variety of ways (23, twenty three, 20 + 3) and using them in more complicated word problems.

I will also introduce the use of the number line in this step allowing students to become accustomed to laying out their base 10 blocks on, drawing and writing with the number line or ray. I will make a point to discuss that we are only using part of the line and not all of it so when they are introduced later to negative numbers on the line they will not be confused by the idea that a number line must start with 0 or 1. We will also look at how numbers are related to each other on the 100's chart by mapping out patterns of 10's and 1's. I will allow for at least 4 weeks for this step.

Foundational Step of Regrouping - In Addition and Subtraction, Work Separately With Tens and Ones: Except When Regrouping.

We will now take all the separate strategies of the previous steps and combine them to move into our next step of adding and subtracting 2 digit numbers. Within this step, however, are five smaller steps we will be taking to arrive at the larger goal of successfully regrouping in addition and subtraction problems. As with the previous foundational steps within each of these sub-steps students will be allowed the time needed for mastery of the concept and we will continue to use the concrete – pictorial – symbolic models. The work we have been doing with word problems, number lines, base 10 blocks, place value and place value cards, problem solving skills and the depth of knowledge students have achieved is all utilized in this step.

Each of these sub-steps are simple to understand from an instructional point of view and can easily be rushed through and addressed in only a few lessons rather than taught to mastery. Student will build their mathematical foundation deeply and solidly as they progress assisting them in mastery of regrouping skills, and preparing them to move onto more difficult mathematical concepts. Suggested time allotment will be minimum of a week and a half per sub-step.

Addition and Subtraction using multiples of 10.

Within this sub-step students will only be working with the tens digit. In the first foundational step students were given time and models to develop a deep understanding of the ones place and how numbers work within the ones place. Now they are given that same opportunity to do so with multiples of 10. This is an important step and should not be overlooked. Students are continuing to work with word problems, place value and problem solving skills will have the opportunity to fully explore multiples of 10 before moving on to multiples of 100. This will also show students the steps necessary to break down any number within its place value for faster and effective problem solving. Students will work with a variety of problems in all three of the model areas. For example, with the concrete model students can use the base 10 rods to manipulate and work through word problems, then transition to the use of place value cards and finally, in the symbolic model students should practice solving problems of the following type:

20 + 30 =___, 50 – 30 =___, 50 + ___ = 70, and 50 - ___ = 80.

Addition and Subtraction using a 2 digit number and ones.

In this sub-step students will work with problems that have a digit in both the tens and the ones place. This is a subtle difference that can easily throw student off. It is important here to remember that this sub-step does not involve regrouping. For comparison's sake, a problem that would involve regrouping that we will at this stage avoid would be 12+9= (10+2) +9 = 10 + (2+9) = 10+11 = 10+(10+1) = (10+10)+1 = 20+1 = 21. To avoid confusing problems like this it will be necessary to keep the digits in the ones column in agreement with this idea, which means the sum can not exceed 9. For example, problems modeled like the following should be used:

25+4, 17+2, or 41+5, 49-3, 99-9, or 33+? =38,

and so on. Once again this allows students to work with one place value at a time even through one of the numbers is a two digit number and one is a single digit number. For example with 25 + 4, 25 would be broken into two tens and five ones, next the five ones and the four ones from the original problem will be combined to

make nine, then the two tens and the nine ones will be combined to a sum of twenty-nine. The Associative Property (the ability to regroup in addition is any way desired) is further developed in this stage as well.

25 + 4 = (20 + 5) + 4 = 20 + (5 + 4) = 20 + 9 = 29

Addition and Subtraction 2 digit numbers and multiples of tens.

In this sub-step, students will move on to add and subtract a general 2-digit number with a multiple of ten, chosen so that regrouping is not required. This means limiting problems to those involving numbers whose tens values will not exceed a sum of 9. For example, 23+90 would involve regrouping within the tens place and should be avoided. I will continue to use the base 10 blocks and word problems as the first concrete model of this sub-step, where students will see and develop a deeper understanding of how the tens place works separately from the ones. In the earlier sub-step students only manipulated numbers with in the ones, while now they will build up or take apart the tens while also keeping track of the ones. Again this is a subtle but important step. This is a balancing act of sorts where student are one piece at a time learning to balance the whole of what will grow into a more complicated problem. Once they are able to master these small steps they will have the capabilities to move on to adding and subtracting the more difficult 3, 4 and multi-digit numbers as well as multiplication and division. The Associative Property and Commutative Property (the ability to add numbers in any order) are further developed in this sub-step as illustrated in the following example:

47 + 30 = (40 + 7) + 30 = (40 + 30) + 7 = 70 + 7 = 77

Addition and Subtraction using general 2 digit numbers with no regrouping.

In this fourth sub-step students will now work with both the tens and ones places, however they will still work with only one place at a time. Again, regrouping should be avoided in problems such as 98 + 25 (which requires regrouping in both the tens and ones place). Students will continue to gain a deep and lasting understanding of how the tens and ones are separate places and work independently. As students learn to add or subtract using columns, the understanding of each place value being different is important. Consider the following examples, which illustrate two ways students might add multi-digit numbers.

$$\begin{array}{r} 34 \\ +\ 21 \\ \hline 55 \end{array} \qquad\qquad \begin{array}{r} 34 \\ +\ 21 \\ \hline 361 \end{array}$$

The differences in the above problems are subtle but of course one method (namely the right-most) is fundamentally incorrect. With a strong sense of place value this mistake can be addressed (as well as in later problems containing decimals involving money, for example). While this might seem like a long and drawn out way to teach, one should remember the benefits of long lasting mathematical foundations. After all, we spend weeks working on a given letter and its sound in Kindergarten and weeks on digraphs and diphthongs in 2nd grade. We need to do the same for numbers and place value. The following is an example that demonstrates how utilizing place value, the Associative Property, and the Commutative Property work together allowing students to correctly combine like terms and find the correct solution:

36 + 42 = (30 + 6) + (40 + 2) = (30 + 40) + (6 +2) = 70 + 8 = 78

Addition and Subtraction using 2 digit numbers with regrouping.

Finally at this stage we will address addition and subtraction that involves regrouping. Although it has taken some time to get here the journey we have taken will pay off in the end. Normally I begin working with my students in the 2nd nine weeks of the year on regrouping. They learn the algorithm or short cut so to speak to do this but often never really understand why or how it works. With the journey we have just taken students should now know the why and the how of addition and subtraction. This last sub-step of regrouping is now just a small hurdle to overcome. I also want to point out that by this stage student will have worked with their basic facts long enough and deep enough that they have them memorized and may be beginning, if they have not already, to use various mental math strategies on their own. This is just another benefit of students having a deep lasting understanding of the concept and not just knowing how to compute using short cuts. In this step student will continue to utilize the Associative and Distributive Properties as well as the "Break Apart to Make a Ten" method. Once again we are bringing all the skills we have previously mastered and are using them together. I should note that this is not the stage to regroup up to 100. That will come in the next step and just as before should be built up to. Here is an example of an addition problem with two 2-digit numbers that involves regrouping. Note the repeated use of the Associative Property and the Commutative Property.

46 + 37 = (40 + 6) + (30 + 7) = (40 + 30) + (6 + 7) = (40 + 30) + (6 + 4 +3) =

(40 + 30) + (10 + 3) = (40 + 30 + 10) + 3 = 80 + 3 = 83.

I will have students write out their work at each sub-step to ensure accuracy and the correct understanding of each step of the problem. Short cuts are the luxury of the expert. Once student have mastered a given sub-step they will be allowed to simply show the answer. However, as we progress to the next model they will be required to show all their work until they can show mastery. When they miss a problem I will also require them to go back and redo their work using the expanded form.

Extend Addition and Subtraction Within 1000

Moving on to extend addition and subtraction to numbers up to 1000, students will continue the use of word problems, however at this point the base 10 blocks will be fazed out and replaced with the place value cards. Students will use the hundreds rods discussed in the teaching strategies for some concrete models but this will get to be too cumbersome and difficult when we add multiples of hundreds. Using the place value cards will continue to give students an object to hold and manipulate for the concrete and pictorial models. They will lay out the cards and physically rearrange them before they transition back into symbolic.

Because students have mastered the previous 2 digit addition and subtractions skills and have been given adequate time to work within each sub-step I will not repeat those sub-steps. However, we will work with adding just in the hundreds column for a time and then adding work with hundreds, tens and ones independently for a time (no regrouping) before we add the step of regrouping. When we do get to regrouping we will begin with the ones and tens only before moving onto regrouping within all three. This can be extended into 3rd grade for continued master of hundreds and then into thousands. It continues to be extremely important that students be allowed to thoroughly explore deeply as they move into greater positions of place value. If one small step is missed it leaves room for a week foundation in their

understanding.

Teaching Strategies

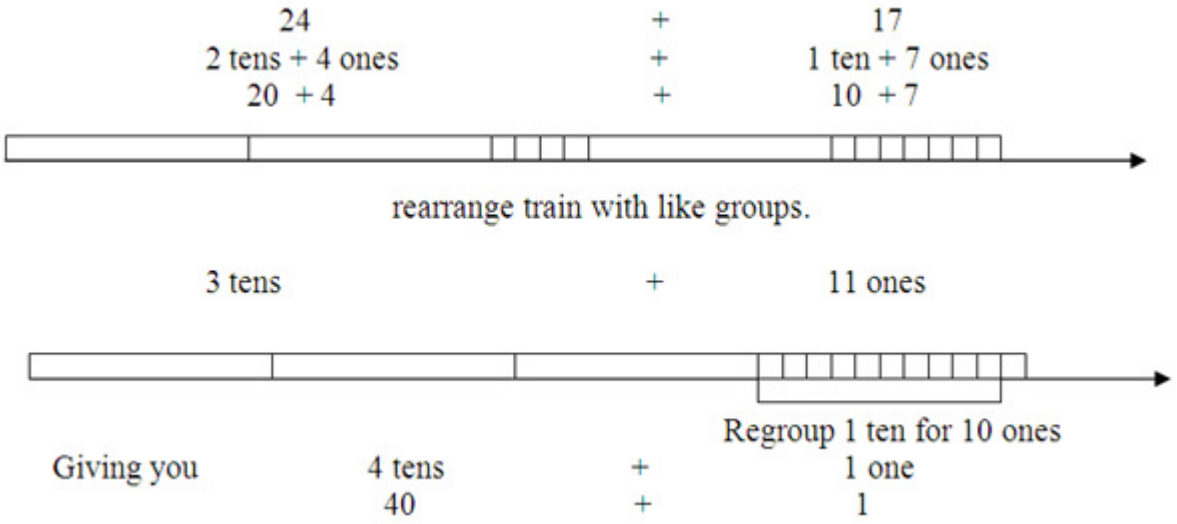
I feel it is important to state once more that each of the following strategies or materials can be incorporated and used together. This incorporation is what allows student to gain a stronger understanding of each concept and in essence provides more instruction time.

Manipulative Materials

The first model used in each step above is the concrete, so I will use a variety of hands on manipulatives for students to hold and move on their own. This will connect with tactile and kinesthetic learners as well as visual students giving even the lowest reader the ability to participate in the lesson, and allowing all students to begin to develop their own understanding of the mathematical concepts. Although I will be using a variety of manipulatives (bears, dinosaurs, tile pieces), base 10 blocks will be my main hands on item. This set consists of one centimeter cubes to represent the ones, a "tens rod", which is a flat rod with makings to denote the ten cubes that make up the rod, and a "hundreds flat" that looks like a hundred cubes connected into a square. Base 10 blocks allow student to easily see the transition into the expanded form of a given number. Each number can be represented using the pieces. To show 243, students would lay our 2 hundreds flats, 4 tens rods and 3 ones cubes. In addition to the common way of grouping base 10 blocks to form squares of 100 units, I will also lay them out long ways like a train to further demonstrate blending of place value, counting and length.

The Number Line to Assist in Computations and Place Value.

By utilizing the number line within my unit I am able to once more teach multiple concepts at once while not overwhelming students. I will begin by using the number line in our addition and subtraction by having students place their base 10 blocks on a pre-printed number line that matches block size. They will be asked to then rearrange their pieces into like trains in order to find the answer to a given problem. For example, to find the answer to the addition problem 23 + 15, students will have 2-tens rods and 3 ones cubes to make 23, and 1 tens rod and 5 ones cubes to make 15. They will lay these out on the number line (in the order above), and will be asked to match "like" values, that is, they will then identify that they have 3 tens rods and 8 ones cubes. At this point student should be able to identify that the answer to this problem is 38. Rods may also be used to illustrate regrouping by simply regrouping 10 ones cubes into 1 tens rod or taking one tens rod and breaking it into ten ones cubes. These types of activities allow for the concrete illustration of a concept. The number line may also be used to pictorially illustrate addition in a similar manner. Students can simply draw the tens rod and ones cubes onto paper, beginning a symbolic process by labeling the rods and cubes and the groups they make. This is illustrated in the figure below:



Place Value Cards

Place value cards are simply cards with place value numbers written on them. For example, the ones cards will be numbered 0 to 9, tens cards numbered 10 to 90, hundreds cards numbered 100 to 900 and so on. These cards will be used throughout the year beginning with the ones to allow once more for a deeper understanding of what each number means and to gain a comfort in using this tool. As we progress through the numbers students will simply lay out the cards in the concrete model to demonstrate a given number. If we are working with 52 bananas for example students can lay out the 50 card and the 2 card. This will lay the groundwork for student to see the connection between a number and its expanded form, as well as the written form they will be working with later. As we attempt to work through to the hundreds and the thousands these cards will become part of our concrete model. For example, rather than having 389 dinosaurs out we can work with the 300 card, the 80 card and the 9 card. Closer to the end of the year, students will create their own set of cards to work with as we delve into hundreds that they can take home at the end of the year to work with over the summer.

Word Problems

With the implementation of the new Core Curriculum Standards students will be required to explain and/or demonstrate their thinking and understanding of math concepts. The use of word problems has the ability to give students the link between reading, writing and math. This encourages students to not only give a numeral for an answer but words as well. By using this combination a teacher can model "thinking" out loud or on paper and teach beyond symbolic "fact" statements like 2+2=4. For example, I have the ability to voice how taking two apples from the tree and putting them with the two apples in my basket now means that I have 4 apples in my basket. This also bridges the various reading levels or gaps students may have. I plan to use the chart of 14 different forms of word problems provided in the Core Curriculum Standards [4] to help develop first one, then two step problems (examples of two step problems and how they can be created can be found in the units referenced below) for my students and teach them to create their own, thus making their knowledge personal and applicable to their everyday lives.

As 2 nd graders my students are still in the early stages of reading and struggle with word problems. Many times I notice long word problems with new or unfamiliar words. This causes a problem for young struggling readers. I will have students work on a daily word problem as we begin our math time each day. They will be posted for students to read each day on a flip chart so we can come back and review problems or look back to

work we have already done. Some days students will have the option to work in pairs and some days they will work independently. Students will create word problems based on current classroom vocabulary to assist in recognition and understanding of new vocabulary. As we begin the unit I will begin with simple one step problems that are not focused on regrouping, such as "Timmy has 2 trucks. His dad gives him 2 more trucks. How many trucks does Timmy have?" Word problem types will range from addition, subtraction, comparison, and utilize the missing addend (i.e. solving 2+__= 10). If you would like more information on Word problems in 2007 the Yale National Institute [5] offered a seminar Keeping the Memory in Mathematics: The Craft of Word Problems led by Professor Roger Howe. Some of the units developed for elementary were: Dr. Word Problem – Solving Word Problems With The Four Operations Using Singapore Bar Models, by Valerie Schwarz; Teaching Addition and Subtraction Word Problems to Children, by Tonya M. Shannon; and Crafting Word Problems Even A Child Can Do, by Huwerl Thornton Jr.

Problem solving skills based George Polya's model [6] .

I will focus on his step one which is "understand the problem", and step four which is "reflection of work and understanding". These two main steps will teach students to focus on asking questions throughout the problem, discussing their different ideas, and building the foundation that will be critical for them to move onto more difficult concepts and activities. My students come from mostly low-income homes and are lacking in exposure a broad vocabulary and to higher order thinking skills. I will use these skills throughout the year but as we begin to progress to regrouping and multiple step word problems in this unit it will become important for students to have confidence in discussing with their peers what they understand the problem to be and how they solved it. Through the use of class discussions students will improve their ability to understand and reflect on problems as well as how their peers "think".

Classroom Activities

In this section, I present three lesson samples used to teach place value in addition and subtraction. Each lesson is designed to emphasize one of the 3 models discussed above, "concrete", "pictorial", and "symbolic". Each also demonstrates a different sub-step of 2 digit addition and subtraction. I have also provided a list of word problems used throughout the year.

Activity 1

This activity demonstrates the concrete model, and will be used as a math starter activity for our daily math time. We will be using daily word problems for this activity. Students work in pairs using base 10 blocks to concretely workout the problem. This activity would supplement the sub-step above, on addition and subtraction with a single digit number and a 2 digit number, and covers the following Common Core Standards; 2.1OA, 2.2OA, 2.1NBT, 2.3NBT, 2.5NBT, 2.6NBT, 2.7NBT, 2.9NBT.

This activity will begin with the following word problem on the board. "There were 35 cubes in the tub. Timmy was cleaning up and found 4 more cubes and put them in the tub. How many cubes are in the tub now?" Students will gather their pre-assigned tub (during this unit of study each tub will contain 99 cubes, number cards 1 to 9 and 10 to 90, white board and white board markers) from the shelf and recreate this problem. They will need to be able to explain to the rest of the class each step they took to re-create the problem and

share their reasoning. Each group who would like to share will have that opportunity. Even if each group took the exact same steps they are continuing to model for peers who may still be struggling with this concept while gaining the deeper more automatic understanding of how to problem solve.

Lesson Extension - Students will then be asked to use these same amounts of cubes 35, 4, and 39 and create their own subtraction story. Each group will have the chance to share their story with the class. This sharing of ideas will allow students to hear from other students the ideas we have been exploring and inspire them to think combining these numbers in multiple ways. Students are by this time in the year comfortable with multiple forms of the 14 possible word problems as outlined in the Common Core Standards and a variety of stories should develop.

Activity 2

This activity demonstrates the pictorial model, and will give students the opportunity for independent practice. Students will create word problems using place value cards to solve the problem, and will be given time to present their problem solving skills to the class. This activity demonstrates the sub-step above involving addition/subtraction of 2 digit numbers and multiples of 10 and covers the following Common Core Standards; 2.1OA, 2.2OA, 2.1NBT, 2.3NBT, 2.5NBT, 2.6NBT, 2.7NBT, 2.9NBT.

Students have access to their personal place value cards 10 – 90, and half a sheet of poster paper. Using the poster paper students will write one of the word problems they have created using the place value cards. This lesson will be independent practice; student will be assessed for understanding of concept. Their work will be displayed on our math wall. Students will be asked to use their tens place value cards to help them create a word problem. (Because they only have one of each and no 100 card the problem will not require regrouping) Student will be asked to show me the number cards they are using so I can assess weather or not they used correct number bonds. Students will then create an addition or subtraction story using their numbers. However, I will remind them they need to leave one of the numbers out of their story so that other students will have the opportunity to discover the missing number for themselves.

Lesson Extensions - These problems can be used during our daily word problem activity or as independent or pair working group activities during center time.

Activity 3

This activity demonstrates the symbolic model, and is used in a whole group setting as a guided practice. Students will use the expanded form and correct mathematical symbols to find the answer to problems. This activity also demonstrates sub-step 5 Regrouping with 2 digit numbers and covers the following common core standards; 2.1OA, 2.2OA, 2.1NBT, 2.3NBT, 2.5NBT, 2.6NBT, 2.7NBT, 2.9NBT.

I will use this as one of the first symbolic model activities within this particular sub-step. As a class we will work the following problem out on the smart board: 35+27 = ? I will write this problem on the board and ask various students to help write out the next step. The first step will be to write it out in expanded form 30 + 5 + 20 + 7. I will ask them what to do next. The next step will be to group like numbers (30 + 20) + (5 + 7). At this point some of the students may want to combine the 10's so we are left with 50 + (5 + 7). It is now necessary to break apart a number in order to add the remaining 5+7. This could be breaking the 5 into 2 and 3, then combining 3 with 7 to make 10, or it could be breaking the 7 into 2 and 5, then combining the 5's to make 10. Both ways are correct and we will write them both on the board and discuss how in the end they will both work to give us the right answer. These methods are illustrated by the following:

50 + 5 + 7 = 50 + (2 + 3) + 7 = 50 + 2 + (3 + 7) = 50 + 2 + 10 = 60 + 2 = 62

50 + 5 + 7 = 50 + 5 + (5 + 2) = 50 + (5 +5) + 2 = 50 + 10 + 2 = 60 + 2 = 62.

This is the format that we have been and will continue to work with until students can show mastery.

Word Problems

The following word problems are based on the "add to" and "take from" categories in the Common Core Standards.

Addition and Subtraction using multiples of 10.

1) There are 30 marbles in a jar. 40 more marbles are poured in. How many marbles are in the jar now?

2) There were 60 marbles in a jar. Some more marbles were poured in. Then there were 80 marbles in the jar. How many marbles were poured in with the 60?

3) There were some marbles in a jar. 50 marbles were poured in. Then there were 90 marbles. How many marbles were in the jar to start?

4) Fifty marbles were on the table. I lost twenty. How many marbles are on the table now?

5) Forty marbles were on the table. I lost some marbles. Then there were ten marbles. How many marbles did I lose?

6) Some marbles were on the table. I lost thirty. Then there were sixty. How many marbles were on the table before?

Addition and Subtraction using a 2 digit number and ones.

1) There are 23 pennies in my wallet. I find 6 more pennies. How many pennies are there now?

2) There are 42 pennies in a tray. Some more pennies are added. Then there were 48 pennies in the tray. How many pennies were added to the tray?

3) There were some pennies on a desk. 8 pennies were added. Then there were 69 pennies. How many pennies were there before?

4) 18 pennies were in a bag. 5 pennies fell out. How many pennies are left?

5) 67 pennies were in a bowl. Some were taken out. Then there were 61 pennies left. How many pennies were taken?

6) Some pennies were on a desk. 5 pennies were taken off the desk. There were 33 pennies left. How many pennies were on the desk before?

Addition and Subtraction 2 digit numbers and multiples of tens.

1) There were 25 cookies on a tray. 40 more cookies were added. How many cookies are there now?

2) There were 13 cookies on a tray. Some cookies were added. Now there are 63 cookies. How many cookies were added?

3) There were some cookies on a tray. 40 cookies were added. Now there are 79 cookies. How many cookies were there before?

4) 79 cookies are on a tray. 50 cookies were eaten. How many cookies are left?

5) There were 57 cookies on a tray. Some cookies were eaten. Now there are 27 cookies. How many were eaten?

6) There were some cookies on a tray. 40 cookies were eaten. Now there are 84 cookies. How many cookies were there before?

Addition and Subtraction using general 2 digit numbers with no regrouping.

1) 24 dogs were at the park. 53 more dogs came to the park. How many dogs are at the park now?

2) 42 dogs were at the park. Some more dogs came to play. Now there are 59 dogs at the park. How many more dogs came to play at the park?

3) There were some dogs at the park. 31 more dogs came to play. Now there are 64 dogs. How many dogs were there before?

4) 37 dogs were at the park. 14 dogs went home. How many dogs are at the park now?

5) 46 dogs were at that park. Some dogs left. Now there are 21 dogs at the park. How many dogs left?

6) There were some dogs at the park. 32 dogs left. Now there are 10 dogs at the park. How many dogs were there before?

Addition and Subtraction using 2 digit numbers with regrouping.

1) 14 ducks were in the pond. 18 ducks came to swim in the pond. How many are in the pond now?

2) 38 ducks were in the pond. Some more ducks came to the pond. Now there are 53 ducks in the pond. How many ducks came to the pond?

3) There were some ducks in the pond. 36 more ducks came to the pond. Now there are 53 ducks in the pond. How many ducks were in the pond before?

4) 61 ducks were in the pond. 28 ducks flew away. How many ducks were in the pond now?

5) 53 ducks were in the pond. Some ducks flew away. Now there are 17 ducks in the pond. How many ducks flew away?

6) There were some ducks in a pond. 37 ducks flew away. Now there are 18 ducks are left. How many ducks

were in the pond before?

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End Notes

1. http://www.corestandards.org/the-standards/mathematics
2. Math in Focus: The Singapore Approach, Marshall Cavendish Int. Teachers Guides 1A, 1B, 2A & 2B
3. Math in Focus
4. Table 1 in the glossary of the Mathematics section of the Common Core Curriculum.
5. http://teachers.yale.edu/units/index.php?&skin=h
6. A Handbook for Primary Mathematics Teachers, Marshall Cavendish Int.