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Cracking the Place Value Code

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Introduction

Have you ever worked hard on an addition or subtraction lesson and found out later that your students didn't understand? When this happens to me, I begin to wonder "Where did I go wrong?" I reflect on my lesson and critique the presentation of my lesson. After spending time reflecting, I am still unable to determine where any misconceptions may have occurred in my lesson. The next day I engage in a conversation with my colleagues and find that they have presented the lesson in a similar manner as me, and they weren't successful either. Together we develop new strategies. These strategies are shared with the students and they might not be successful with those strategies either. So, I began to wonder, "How can I help my students to understand how to add and subtract numbers?" I realized I had never asked myself "Could a lack of understanding of place value be the problem?" I have learned through research and experience that completing computation problems is difficult for students. I also discovered that a lack of emphasis on place value is a main cause of this problem, and I will focus on developing the fundamental skills of place value in this unit.

I teach second grade at Southern Elementary School (Colonial School District) in a self contained classroom with 23 students. My school is located in New Castle, Delaware. Southern Elementary opened in August 2001. There are approximately 929 students in grades kindergarten through fifth grade. The ethnicity of the students is diverse: 42% Caucasian, 42% African American, 9% Hispanic, 5 % Asian, and 3% Multi-Racial. Forty-two percent of the population is low income, 11% of the population is special education, and 7% of the students are English Language Learners. This unit is written for second grade students who are struggling to complete basis addition and subtraction computation problems.

The mission for Colonial School District is to improve the academic achievement of all students. Therefore, staff is working toward the common goal of helping each student to have a successful school experience and be a productive addition to their community when they graduate. To aid staff to be successful, they are encouraged to use various methods to help students understand concepts. There are professional development opportunities that help staff to learn new strategies to help students understand concepts. Staff members are encouraged to provide multiple strategies and opportunities for students to be successful. Therefore, the strategies and methods presented in this unit will help me to achieve my district goal as well as help my students be successful with mathematics during their educational career.

The current math curriculum being used in the Colonial School District is the Investigations curriculum. This curriculum has been used in Colonial for at least 13 years if not longer. The Investigation Curriculum was developed in Cambridge, Massachusetts. When it was first introduced, it was referred to as TERC (Technical Education Research Centers) program. The TERC Company has been in existence for at least twenty years. The company works on various entities of school programs ranging from Science, Social Studies, and Reading just to name a few. Within the last few years, the math portion of the program changed their name to Investigations. The Investigation curriculum provides students with opportunities to discover several ways to solve math problems. Students are also working with a partner to explain their thought process to make sure it is accurate and that they understand their math techniques being used. The discussions that students have are an essential part of the curriculum because it allows the students to see if they understand the concepts that they have learned. Students are encouraged to discover new ways to solve problems the discoveries they make becomes theirs. It is a great way for students to feel valued and that they are in control of their learning experience. There are a series of books for each grade level. The Investigation curriculum is a spiral curriculum. A spiral curriculum is a curriculum that repeats the topic later on within the current curriculum or future curriculum. The topic will be reviewed and then carried a little further the next time so that the skill is constantly being built upon. When the Investigation program was introduced in Colonial, teachers attended training sessions for each book throughout the school year. I believe the Investigation program is a great way for students to begin to see and understand how math works. The unit that I have developed here is designed to also enhance the current Investigations program used by my district.

Content

I have had the pleasure of teaching second, third, and fourth grade students. There was something that I noticed about the grade levels that I have taught: students were unable to complete basic computation problems. As I spoke with teachers that taught different grade levels, they also were having the same problem with their students not being able to complete basic computation problems. Many teachers believe that this problem exists because math programs have moved away from the "skill and drill" method. Others believe that students are being exposed to material that is difficult and confusing. I find the perspectives that teachers have shared with me to be quite interesting. Over the years I have felt like I need to crack a special code to help my students be successful with computation problems. After participating in Roger Howe's seminar at the Yale National Initiative, I believe that I have finally cracked the place value code with regard to basic computation problems. The code is to establish a firm foundation with the base-ten system, which needs to be built early on in a child's education. A strong understanding of the base-ten system will allow students to understand place value and be successful with computation problems.

There are two foundation skills that will be discussed several times throughout this unit: base-ten number system (decimal system) and place value. The base-ten number system is a number system that represents numbers. The numbers usually correspond to the number of units in a given set. This number system is composed of the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The reason it is called the base-ten number system is that it is devised of ten numbers. When the numbers bond together, they determine the place value for the given number. Place value is the placement of the number within the decimal system. For example, 9 + 3 = 12, which has a 1 in the tens place, and a 2 in the ones place. Later in this unit we will stress the importance of thinking of the number 12, for example, as a sum of its place value components 10 and 2. Please remember "decimal system" is referring to the base-ten system.

Mathematicians and educators have been studying and learning about math programs being used in the United States, and what they have found is that schools in the United States have been emphasizing the "skill and drill" method. Many people believe skill and drill is the answer to helping students to understand math computation problems, but it isn't. Skill and drill is a quick way to get the information into a student's mind, but it doesn't provide them with logical reasoning and understanding of how the answer is derived. Logical thinking is what prepares students for tougher mathematical concepts later in their educational careers. A reason that this method isn't beneficial is because a student may know a fact one day and be unable to recall the fact the next day. If you build logical reasoning, when they are having difficulty, they will be able to use reasoning skills to produce the answer to the computation problem.

Another interesting discovery that mathematicians and educators have discovered is that Singaporean students are excelling in their math computation skills. [1] The question that this has sparked in me is "Why are Singapore students so successful?" The answer, I discovered, is that Singaporean teachers aren't expected to teach a lot of content units. Instead, Singaporean teachers have identified just a few skills as essential. The teachers work on the identified skills extensively until the students have them mastered. I then began to reflect on the Common Core Standards being adapted here in the United States. [2] This trend of thinking is evident with the Common Core Standards. A set of essential skills have been identified for students to learn at each grade level. I immediately thought back to when I first began teaching thirteen years ago. I had several performance indicators (standards) that I was expected to make sure my students had mastered. Later, the standards were revised and I had fewer standards that I had to make sure my students had mastered. So, as you can see, it seems the United States is moving toward adapting the same philosophy as the Singaporean model. The Singaporean philosophy is to teach less, but make sure that a great deal of instruction occurs with these few concepts and that students master the skills being taught.

I have spent a great deal of time researching the Singapore Math Program. This program is commonly referred to as "Primary Mathematics" in the United States. I chose to look at this program because as mentioned before, they are excelling in mathematics in Singapore, and I wanted to find out what skills they are using. One thing I did notice about the textbooks in the Primary Mathematics series is that three main features are discussed in the beginning of each book. Those main features are: conceptual, pictorial, and abstract (algorithm). Students should be successful at each of the above steps prior to moving on to the next step within the process.

The first step is conceptual. During this step, the students get a firm understanding of what is being asked in the context of the given situation. Students will know why math ideas are important and when to use the skills. During this step of the process, new ideas will be associated with old ideas. Linking new ideas with old ideas helps students to relate to new information better. The second step, pictorial refers to linking pictures with the mathematical concept. Students will use manipulatives or draw pictures to illustrate the concept that they were just taught. Once the students have a firm understanding of the conceptual and pictorial steps, the third step, the algorithm is taught. The algorithm is a step by step process for solving a math problem. Within the United States, we have been accustomed to working with the algorithm and not emphasizing the conceptual or pictorial step. I then began to wonder why many educators are changing things now after we have been doing it the other way for so many years. What I have learned from my research and during my time in the Yale seminar makes sense. Algorithms are very abstract. The symbols used can be as complex as the procedure used in any given computation problem. Given the fact that the algorithm is so abstract, the student focuses on the procedure needed to solve the problem instead of understanding what the symbols mean. The next thing I noticed is that with this new way of presenting information, parents begin to feel helpless with helping their children. I try to reassure them that it is a process with a purpose. The algorithm will be introduced, but not until a bit later. Unfortunately, there are still some parents who feel the need to continue with sharing their philosophy with their child. I try to remind them of Roger Howe's math rule: "Shortcuts are for the experts!" I hope that this will be the motivation that my parents need in order to withhold the algorithm with their child until the appropriate time.

Many math programs in the United State have begun to adapt concept driven math programs. What I have noticed is that with the implementation of these new concept driven programs, some of the previous skills that were taught have been lost. One of those previous skills would be an emphasis on the base-ten system described earlier. Since this is the number system used in the United States, it is important that I expose my students to this primary number system. By spending time with the base-ten system, I aim to build a firm foundation with numbers. This firm foundation will prove to be beneficial later by allowing my student to be flexible with numbers. Because my students are flexible with numbers, they will be able to complete mental math problems with a great deal of proficiency.

All of the above research has made we wonder, "What would be the most appropriate way to teach my students how to complete computation problems?" During my seminars with Roger Howe, he shared four steps that he urged be established early in an elementary mathematics program to lead to success in computation problems. Each of these steps is extremely important. The four steps I am about to share follow the three steps mentioned above: conceptual, pictorial, and abstract. I never move on to the next step if the current step isn't understood. Moving to the next step prematurely will create frustration and will hinder a students ability to be successful with completing future steps. During this unit, I teach skills simultaneously and not separately as they may appear in current and previous math programs. Concepts are being taught simultaneously because I don't want my student to view them as two separate skills. I want my students to see the relationship between the skills. The discovery of this relationship will allow my students to become great with mathematical computations.

In the next section, I will discuss Howe's four steps mentioned above. I personally found the information that was shared to be quite informative. I hope that you will see how cracking the place value code will help your students be successful with understanding place value.

Flexibility with Facts to 10 and Decomposing Tens and Ones

All students need to be flexible with adding and subtracting number facts to 10 and decomposing teen numbers into tens and ones. Students knowing facts to 10 is a first foundational step towards successful addition and subtraction. Students should be taught their facts without flash cards. Working without flash cards allows my students to become flexible with the parts of numbers that can make a specific whole number. A flexible number sense should lead students to think of the many possibilities of how numbers can used to solve problems. Repeated work with numbers will also help students become familiar with their addition facts and recognize them instantaneously. Rearranging numbers to obtain a certain sum or difference is a great way to reinforce the facts without making it a memorization process. If you give the students the number 6, they should be able to give you the following numbers as sums of 6:

0 + 6	6 + 0
1 + 5	5 + 1
2 + 4	4 + 2
3 +3	

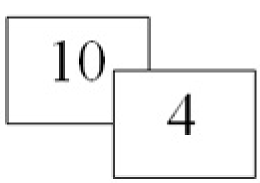
(They do not have to been in the correct order as I have presented them.) I would introduce inverse operations next, which "undo" the previous equation. So, if you have addition, what is the inverse operation? Yes, subtraction is the inverse operation of addition. I introduce subtraction with the following steps: 6 – ___ = 4 (2 goes in the blank). Working with these same numbers, one can form "fact families":

$4 + 2 = 6$	$2 + 4 = 6$
$6 - 2 = 4$	$6 - 4 = 2$

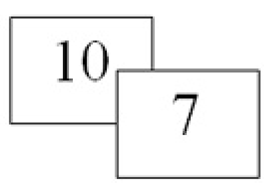
Fact families are a collection of related addition and subtraction equations or inverse operations. However, I am not separating this concept as it is often presented in math programs. Being able to decompose the numbers as explained above will help students to learn how to decompose larger numbers later. Starting with smaller numbers will help you to ensure that your students understand the conceptual part of this process before larger numbers are used.

Using Facts to 10 to Decompose Tens and Ones and Add and Subtract Facts to 20

Once your students have a firm foundation of the facts to ten, spend some time discussing the teen numbers. The teen numbers are 11, 12, 13, 14, 15, 16, 17, 18, and 19. I never considered the teen numbers as being unusual prior to my seminar with Roger Howe. However, in his seminar, he mentioned that teen numbers have funny names: eleven, twelve, thirteen, and fourteen are the most abstract. Fifteen, sixteen, seventeen, eighteen, and nineteen are weird, but a little bit easier to understand than the previous four. Unlike the base-ten number 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, teen numbers do not have a great correlation with their names. Therefore, the number words for teen numbers give an appearance of being abstract for younger children. In Singapore, they have a technique for introducing these numbers to their students, that I feel will help with understanding the teen numbers. They teach the numbers with an emphasis on the base-ten system. When teaching the teen numbers, I use number cards. Eleven would be taught as one ten and one one, twelve as one ten and two ones, thirteen as one ten and three ones, and this technique would continue until all teen numbers were taught in this manner. Next to the numbers I would use a card with ten written on it and a card with the number of ones written on it. They would be overlapped a little to emphasize that the two numbers need to bond together to make eleven.



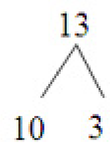
The number 14 with number cards.



The number 17 with number cards.

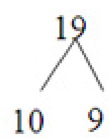
By introducing the numbers this way I allow my students to relate numbers to the base-ten system. The introduction of numbers in this manner also gives students a concrete relationship between the number, word, and the amount the number is representing.

Once student have a firm grasp of this concept, I teach them how to add and subtract by decomposing numbers with the same strategy as I used with the teen numbers. Here is an example of what I am expecting from my students: given the problem of adding 13 + 7, I want my student to recognize that 13 can be decomposed as



so that 13 + 7 = 10 + 3 + 7= ___. Then I would like for them to realize that 3 + 7 = 10. Finally, they would say 10 + 10 = 20. The methods I have described above help students to be flexible when trying to answer computational problems.

Here is an example for subtraction: given the problem of subtracting 19 – 3 = ___, I hope that my students will realize that 19 and be decomposed to



and therefore that they are solving the problem 10 + 9 – 3 = ___. Next, I want them to recognize that nine can be decomposed to 6 + 3. The equation has now become:

10 + 6 + 3 – 3 = ___. Right away my students will ideally recognize that 3 – 3 = 0 due to my work with inverse operations described in the previous step. Finally, they arrive at 10 + 6 = 16. Again decomposing numbers allows for students to be flexible with their addition and subtraction and more fluent with math computations.

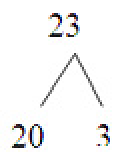
Decomposing a General Two Digit Number into Tens and Ones

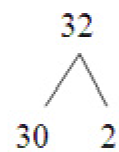
After students successfully carry out addition and subtraction within 20, and gain comfortability with decomposing these numbers into tens and ones, the next step is to carry out these same ideas with general two digit numbers. The numbers that I focus on during this step are the "ty" number. The "ty" numbers are 20, 30, 40, 50, 60, 70, 80, and 90. The goal is to get my students to realize that 20 is composed of two tens, 30 is composed of 3 tens, etc. Once this connect is made, I will look at numbers with tens and ones. An example would be 34. I am expecting my students to tell me it has 3 tens and 4 ones. Once again, number cards are a useful tool to illustrate this. For example, to illustrate the decomposition of the number 34, I would have a 30 written on one card and 4 written on another card. The two cards would be place over one another so that they overlap. Using number cards in this way helps students to understand that the numbers have to bond together to make 34. I practice this with a lot of intensity so that students become fluent with decomposing numbers. The more practice they have will allow the concept to become second nature for them when thinking about two digit numbers. By developing this skill early in my students' educational career, they will increase their abilities and to complete mental math calculations. It is also important to pay attention to how your students talk about numbers. If you ask your student "What numbers make up 32?" 3 and 2 is an incorrect response. Students should respond with either 30 and 2 or 3 tens and 2 ones. This is a great way to emphasize what you just taught them. Decomposing numbers into tens and ones will lend itself nicely to the next step in this process, adding and subtracting number.

Add and Subtract General Two Digit Numbers with Decomposition

By this point in the process a firm foundation of the base-ten system has been established, a great deal of time has been spent with decomposing numbers and the first two steps in the Singaporean approach described earlier. By now we are ready to begin working on general addition and subtraction, and using the algorithm, the third Singaporean steps. The algorithm is a very abstract method, but the foundations previously established in the base-ten number system will aid in students' understanding of the algorithm and what they are being asked to calculate. In the process of learning flexibility with numbers as described above, students will develop their logical thinking skills, which is extremely beneficial in adding tens and ones. As the numbers become larger in their mathematics education, students will reflect on the base-ten model that has been taught as their foundation and they will know that numbers can be decomposed.

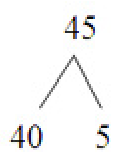
If my students are presented with 23 + 32 = ___ for example, I am expecting them to decompose the numbers as follows:

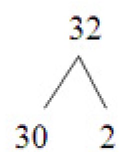




Next, I want them to add 20 + 30 = 50 and 3 + 2 = 5 together. They should arrive at

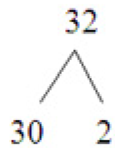
50 + 5 = 55. Prior to this method being shown to them, they may have felt the need to use a calculator because the numbers were so large and overwhelming. I always tell my students "In math, you can move your numbers around to make it comfortable for you." After this seminar I need to say "You can decompose and move numbers around to make it easier for you." Here is an example of what I am expecting in subtraction. 45 – 32 = ___. I would expect the problem to be decomposed as follows:

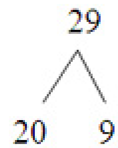




Next the multiples of ten should be subtracted from one another: 40 – 30 = 10 and the ones subtracted from one another 5 – 2 = 3. Therefore, the final answer should be 10 + 3 = 13.

You might be thinking that the previous examples are somewhat special. Indeed, there are two additional very unique situations that can occur in addition and subtraction: "regrouping" and "borrowing". Essentially with regrouping you have enough ones to make a ten and still have some ones, so the extra ones are moved to the tens place as one ten. So, if you have 15 ones, it can be written and one ten and 5 ones. Here is an example of how students think about a regrouping situation. Given the problem of adding 32 + 29 = ___, students who have gained a comfortablility in decomposing numbers into their place value numbers would decompose the numbers:

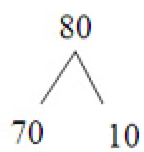


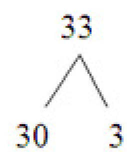


Next, they should add their tens and ones: 30 + 20 = 50 and 2 + 9 = 11. As you see, we have 11 ones. This can be decomposed to 10+1. So, they will take and at the new

10 + 50 = 60. The equation now becomes: 60 + 1 = 61.

The second situation that occurs, borrowing, occurs in subtraction. Essentially, this means you don't have enough to subtract a number from another number, so you move to the next place value to get help. Here is an illustration of borrowing using the base-ten system. Given the problem 80 – 33 = ___, students should now know to decompose 33 as 30 + 3. When they are then faced with the fact that they can not subtract 3 ones from the 0 ones found in the number 80, they should decompose the number 80 as 70 + 10. Thus, they are left with the decompositions:





Then students should subtract 70 – 30 = 40 and 10 – 3 = 7, so 40 + 7 = 47.

I could also begin to develop measurement skills by using the number line. In my research I discovered the saying "We can't subtract a bigger number from a smaller one", (for example the "0 – 3" that students first encounter in the example above), is false! The reason for this being a false statement is that later on, students will learn to subtract a bigger number from a smaller number. If you are telling them this now, they will think that it is impossible to complete their future math computations later. Also, instead of referring to it as borrowing refer to it as decomposing. As you can see, this process has begun to crack the place value code early on and will help my students be successful with their mathematical computations!

Techniques for the Classroom

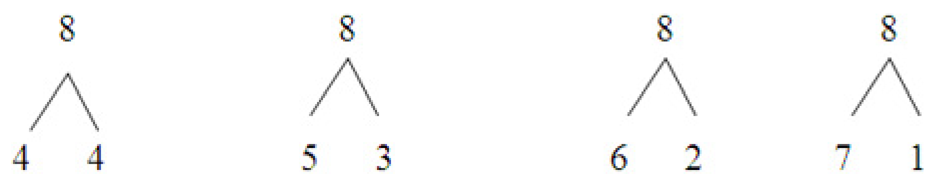
As mentioned early on in this unit, I want my students to see numbers conceptually and pictorially, and also to understand the algorithm, all of which are done in the Singaporean approach to mathematics. There are resources that I use in my classroom that help me to achieve this goal with my students. The order I recommend teaching them in the classroom, depends on whether you're working with a conceptual, pictorial, or abstract example. Thus, one technique below may be more illustrative than another.

Cuisenaire Rods

Cuisenaire rods were created by Georges Cuisenaire in 1952, and are a great way to represent the base-ten number system. I like to use this technique at the beginning stages of the base-ten number system. Cuisenaire rods come in varying colors and each color represents a certain number of units. The rods typically available are: white (1), red (2), light green (3), lavender (4), yellow (5), dark green (6), black (7), brown (8), blue (9), and orange (10). I spend time developing how much each rod is worth and do so by for example, taking a yellow rod and having students figure out how many white units would be needed to create a yellow. I chose white because they are worth one unit, so relevant to this unit's focus on place value and the decomposition of numbers into their base-ten components. It is important to emphasize this point to your students as well once students have an understanding of this, I would present computation problems to them and allow them to use Cuisenaire rods to complete the computation. So, for example a red rod (2) and a black rod (7) together and would add up to 9. Or, and orange rod (10) could be placed above a red and black and students could be asked how many more are needed to make ten. Students should place a white (1) with the red and black. This would help them to see that one more is needed to make ten.

Number Bonds

Numbers bonds were first used in the Singapore math curriculum in the late 1970s. Number bonds are a great activity that can be completed daily to help students build flexibility with adding single digit numbers. They allow students to think of all the numbers that bond together to make a specific number. What I do is present my students with various numbers. For instance I will say "What numbers bond with 8?" They should respond with:



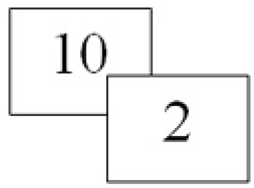
Working with number bonds allows students to become flexible with numbers and realize that there is more than one way to represent a particular number.

Base-Ten Blocks

Base-ten blocks consist of four basic blocks: units (ones), long (tens), flats (hundreds), and blocks (thousands). Many people believe that base-ten blocks and Cuisenaire rods are the same however, they are not. They achieve the same purpose, but they are two different tools. I feel that Cuisenaire rods allow for students to see the decomposition of numbers a little bit better than base-ten blocks. However, base-ten rods allow students to represent the tens and ones in numbers better than Cuisenaire rods. Having said that, "Base-ten blocks should come later in the learning process, once a student has gained a firmer understanding of the base-ten system." This conclusion was gathered from conversations that I have with fellow teachers who have used the base-ten and Cuisenaire rods already. The long pieces allow them to grasp that a ten is composed of ten ones quickly because it is segmented into ten sections that can not be broken apart. I believe that base-ten blocks allow students to add their numbers with ease. Students can also add and subtract with Cuisenaire rods, but it may be more difficult because of all the different combinations. Once your students understand the decomposition of two digit numbers, I encourage you to introduce base-ten blocks.

Number Cards

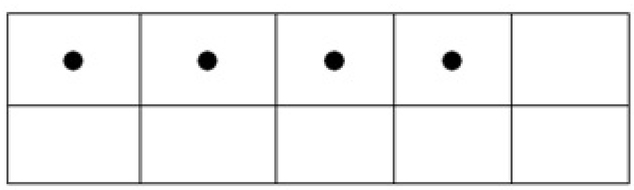
Number Cards are a great way to illustrate the teen numbers. As mentioned previously the teen numbers are weird and this can cause problems with a student's understanding of exactly what each number illustrates. Number cards help to bring some meaning to these weird numbers. For example, in understanding the number 12 students need to understand that it is composed of one ten and two ones. To illustrate this with number cards, I would have a 10 written on one card and a 2 written on another card. I would place the 10 down first and place the 2 overlapping the 10 just a little.



Presenting the numbers in this way allows students to see that twelve is one ten and two ones. The visual that the cards present is phenomenal and for some learners really helps them to understand the teen numbers. It is also a great way to illustrate how numbers bond together. This same process can be completed with the "ty" numbers as well.

Ten Frames

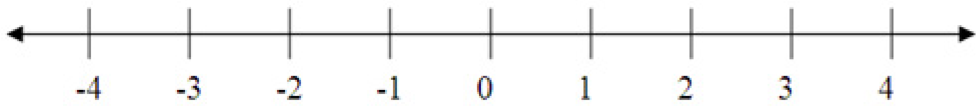
The purpose of ten frames is to get students thinking about numbers in relationship to 10. This tool allows students to see place value easily. Ten frames are arrays made of two rows with five columns.



This illustration represents the number four on the ten frame. For example, I might say "If I have a ten frame with six dots, how many more do I need to make ten?" The response my students should supply is 4. I can also show them a ten frame with one dot and ask "How much less than 10 is my ten frame?" The response should be nine. The different concepts don't have to be taught in isolation, but can be made to play off of one another once students have a firm understanding of the concept that is being presented. Following the activities in this unit, you will find a sample worksheet containing 14 ten frames.

Number Line

The number line is a straight horizontal line with a prescribed origin (labeled "0") and a prescribed unit length (labeled "1"). On the line, the other integers are labeled according to this prescribed unit length, origin, and positive direction. That is, 2 would be marked to the right of 1, and should be the same distance away from 1 as 1 is from 0. This process continues with positive integers continuing to the right of 0, and negative integers to the left.



Students can use this line to help them add numbers. For example, if they have the problem 43 + 12 they would start at 43 and first move ten to the right and land on 53. Then they will move two more spaces to the right and land on 55. If students have a firm understanding of this line, they can also transfer the skills used for adding numbers on this line to measuring objects in the real world.

Introduction to Activities

I will be sharing three activities with you. Each of the activities gives examples of one of the three aspects that were mentioned earlier in my research paper: conceptual, pictorial, and abstract. I would like to reiterate the purpose of each of these steps again. During the conceptual stages, students are trying to understand the concept of tens and ones. After grasping the conceptual stage, the pictorial stage begins. The pictorial stage requires pictures to be drawn to illustrate the tens and ones. The final stage is the abstract, and during this stage, the algorithm is introduced. Each of the activities presented will focus on one of the three stages just mentioned. It may appear the activities are similar, but they are different because they are exposing the students to different stages of the process.

Activity One – Number of Days (Conceptual)

Objective: Students will understand decomposition of numbers.

Materials: Cuisenaire rods, Area to display work created for number of days in school

Procedure:

1. Establish a place to keep track of the number of days you have been in school.

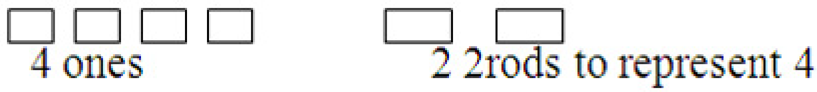
2. Keep a running total of the number of days school has been in session. I like to keep mine on a place value chart similar to the model below:

Hundreds	Tens	Ones
	1	4

3. Make sure you have Cuisenaire rods.

4. Represent the number of days you have been in school with Cuisenaire rods. Emphasize the different ways to create the number of days school has been in session with Cuisenaire rods.

If we have been in school 14 days, I should have a ten rod to represent 10 and then the students could have:



Ask students questions with regard to the number of day you have been in school.

Some examples are:

How many more days do we have to be in school to reach 20 days (Next landmark number)? The answer given our example would be 6 more days. Again have the students represent this with Cuisenaire rods:

Show me this number with Cuisenaire rods. (Make sure all possibilities are shown with the Cuisenaire rods.)

5. Allow students to return to their seats and supply five or more equations that

equal the number of days we have been in school. Some sample equations are:

10 + 4 = 14

6 + 4 +4 = 14

20 – 6 = 14

15 – 1 = 14

14 x 1 = 14

Activity Two – Ten Frames (Pictorial)

Objective: Students will use ten frames to help with adding and subtracting numbers.

Materials: Ten Frames (See Ten Frame Sheet) Make Multiple Copies

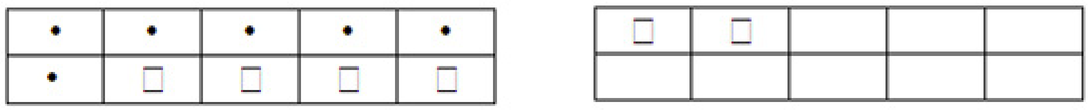
Procedure:

1. Make sure you make copies of the ten frames for your students.

2. Remind your students that the number system we use in the United States is the base-ten number system. The reason it is called base-ten is because it is composed of ten numbers.

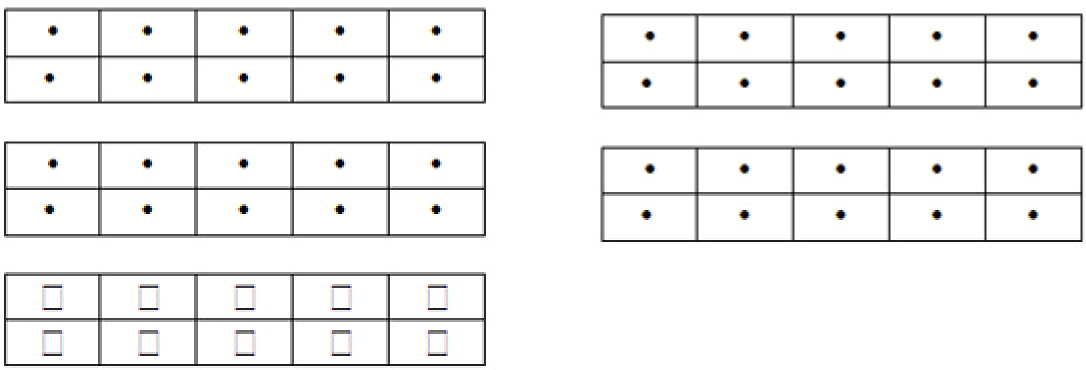
3. Give them a problem. 46 +16. Allow your students to use the ten frames to solve the problem. First, students should decompose the numbers 46 and 16 into tens and ones: 46 = 40 + 6, and 10 + 6. Next, students should add the ones (6 + 6) together.

Here is 6 + 6 on the ten frames:



Your students should realize that 6 + 6 = 12 (one frame is filled completely and the other has two that are filled.

4. Next students should use the ten frames to add the (40 + 10) together as follows:



Students should see from this illustration that 40 + 10 = 50 because you have five ten frames that are full. Looking at the tens and ones yours students should see six ten frames that are full another ten frame containing only two dots, so the total that they are left with is 46 + 12 = 62.

Activity Three – Adding Tens and Ones (Abstract)

Objective: Students will add and subtract numbers with tens and ones.

Materials: Paper

Procedure

1. Make sure students have a firm understanding of decomposing numbers.

2. Present the problem to the students: 25 + 14 =

3. Decompose 25 = 20 + 5, and 14 = 10 + 4

4. Students should add ones 5 + 4 = 9

5. Students should add tens 20 + 10 = 30

6. Add the ones and tens together to get the answer 9 + 30 = 39.

7. If they have the subtraction problem: 36 – 23 =

8. Decompose 36 = 30 +6, and 23 = 20 + 3

9. Students will subtract the ones first 6 – 3 = 3

10. Students will subtract their tens 30 – 20 = 10

11. Students will add the tens and ones together 3 +10 = 13

12. Present the problem to the students 53 – 19 = (This problem requires borrowing.)

13. Decompose 53 = 40 + 13, and 19 = 10 + 9

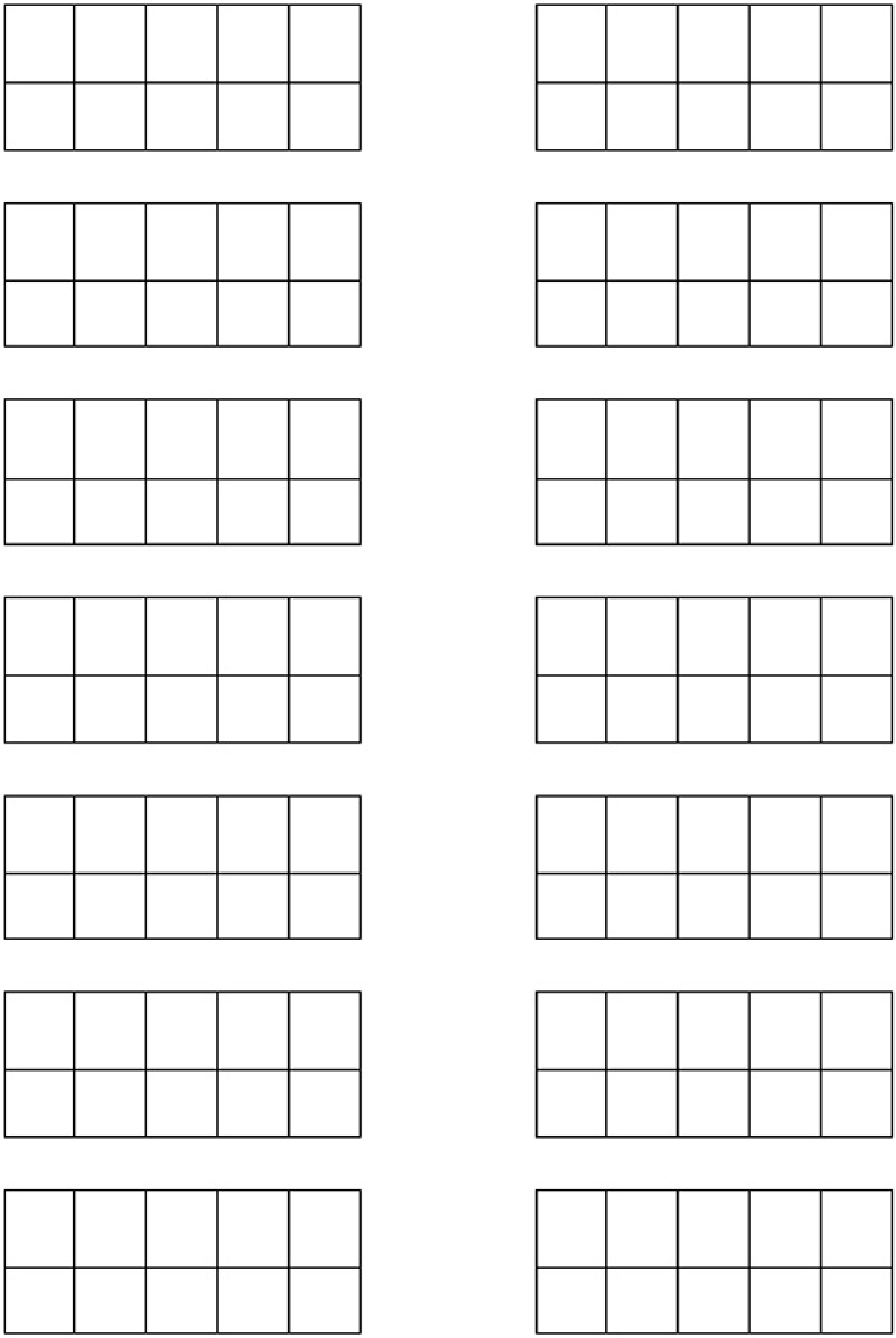
*Notice how I moved a ten into my ones as 10 ones. This get me ready for the next step.

14. Students will subtract the ones 13 – 9 = 4

15. Students will subtract the tens 40 – 10 = 30

16. Students will add the tens and ones together 4 + 30 = 34

Ten Frames



Appendix: Implementing District Standards

Number and Operations in Base Ten (NBT)

9. Explain why addition and subtraction strategies work, using place value and the

properties of operations.

Operations and Algebraic Thinking (OA)

2. Fluently add and subtract within 20 using mental strategies. By end of Grade 2, know

from memory of two one-digit numbers.

4. Use addition to find the total number of objects arranged in rectangular arrays up to 5

rows up to 5 columns; write an equation to express the total as a sum of equal addends.

Measurement & Data (MD)

6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to numbers 0, 1,2 , …,and represent whole-number sums and differences within 100 on a number line diagram.

This unit addresses four of the Common Core Standards Math standards. Students will understand the digits in the hundred, tens, and ones place by understanding and using place value to solve addition and subtraction problems. (# 9 and #4) Students will be able to add and subtract to 20. (# 2) Students will also use the number line to begin to understand how to measure objects. (#6)

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