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Exponential Explosion: Analyzing Scientific Notation and Its Application to Astronomy and Order of Magnitude

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by Troy Julian Holiday

Overview

Math and Science have been connected ever since their origin. While it is known that mathematics originated before Science there is no doubt that the two share a relationship. In fact, one of my most challenging obstacles is getting students to recognize the relationship between math and science. No disciplines exist in isolation and yet my students do not realize that mathematics is a critical support for science. For that reason, I have chosen to implement this seminar into my Science classes at Wagner Middle School in Philadelphia, Pennsylvania. In particular, I intend to connect the math concepts of scientific notation with Astronomy and other real life situations.

This unit is designed for 6 th grade students, however they can be adjusted to meet the needs of any classroom that discusses scientific notation. The duration of the curriculum is also something that is tentative. However, most of the information should take a month to a month and a half to complete.

There are many objectives that I hope to accomplish over course of this unit.

The primary objective for this unit is to enable students to be able to use scientific notation proficiently and understand its implications to the real world. To accomplish this several other supplementary objectives will need to be addressed. The following objectives should be addressed in conjunction with the primary objective:

A) Understanding the meaning of 'order of magnitude' (size/place value) and how it is expressed in scientific notation

B) Understanding how the leading digits in scientific notation give good approximations to the number

C) Applying scientific notation to real-life scenarios and understanding its relation to Astronomic concepts.

Accordingly, the unit focuses on accuracy and order of magnitude, the two major concepts that make up scientific notation. The literature I have read about scientific notation focuses on real-life scenarios that use the applications of the scientific notation to express very big and very small numbers in relevant terms. The unit also aligns with Pennsylvania State Core Standards for sixth grade science, which can be found in an appendix of this unit. Processing the wealth of information made available to me on this subject has led me to believe that mathematics is the language of science, helping to explain the unexplainable. The better students understand that the better they will understand scientific law and theory.

In Astronomy, there is a major emphasis on order of magnitude and how it is used to express overarching concepts. These concepts can be neatly and compactly expressed using the technique of Scientific Notation. By understanding scientific notation, students will master these ideas and gain a deeper understanding of Astronomy. They will also realize that these concepts are simplifying life through the use of the notation and rounding. The object will be to get the students to see that each succeeding step provides more simplification of the concepts by utilizing the ideas of scientific notation. Presently, when my students attempt to solve Scientific Notation problems, they become frustrated and frequently give up. This could be due to the missing connection to relevant examples in their life. For that reason, many of the examples or problem sets will pertain to some aspects of their life. These connections should alleviate much of the frustration that make it impossible for them to progress towards proficiency, and hopefully also help them relate the ideas to familiar contexts. Eliminating such struggles and replacing that with the confidence they gain from experiencing early success is invaluable. It is reasonable to hope that their success in the more basic ideas can then be transferred to more complex concepts. Their familiarity with the examples will then share a bond with the concepts being taught in the lessons because they will connect one idea with another. This should result in better test results and increased confidence.

Rationale

Following the completion of the unit, students will have achieved a proficiency in concepts fundamental to mathematics and Science. This is of critical importance since statistics show that US students are falling significantly behind in math and science. The latest PISA (Program for International Student Assessment) results reported that Americans were receiving lower scores than schools in over half of the developed countries in the world. This implies that US students are not achieving a robust understanding of key principles of mathematics. Thus, many questions have to be answered to find a solution to this systemic problem. The immediate question of this unit, is how can students overcome the challenge of understanding scientific notation. The unit provides a solution to the question by providing strategies that reinforce the concepts of accuracy and order of magnitude. Essentially, this is the foundation of scientific notation. Understanding these concepts could help students deal with more complex, abstract concepts, such as Black Holes and The Big Bang Theory.

For my students in particular, much of the emphasis will be placed on understanding what scientific notation represents. My students struggle with this concept equally if not more so than other concepts in astronomy. Therefore, we will spend a significant amount of time on the topic.

The students in my class dress the same, talk the same and also come from similar backgrounds. They consist of kids from urban areas whose families generally do an adequate job of providing for their children. It is understandable to see how these characteristics can make my class seems as though it lacks diversity. On the surface it may appear that way, but when analyzing intellectual levels, the students in my class can range from a elementary to high school levels in math and reading. This requires diverse strategies when attempting to attain the objectives with the overall class. Thus, by focusing on the fundamentals and less complex concepts, I will give the students a higher chance for success. Successes, whether small or large, have proved to perform wonders for my students, giving them the confidence they need to excel in my class over the course of the year.

Astronomy is both fascinating and challenging to my students,. This is, in part, due to aforementioned relationship shared between math and science. Specifically, students seem to face most of their challenges when applying problem-solving skills. This could be due to their inexperience with the practice. According to some theorist, individuals construct their own knowledge bases, which is to say that human learning is largely a constructive process [2] . Therefore, without much relevant information to draw from students can become discouraged, inhibiting their growth as a learner. The constructive learning model implies that learners do not simply add new information to their "store of knowledge", but connect the new information to already established knowledge structures and construct new relationships among these structures [2] . These same theorists also attribute the recognition of patterns as an indicator of the ability to understand content. According to their research, it is likely that pattern recognition skill may involve episodic or imagistic memory representation built up through hours or years of extensive practices in the task domain [3] . Because of this, I will implement the strategy of establishing procedures, which the students can use when problem solving with scientific notation. By consistently reviewing these procedures the students will cement concepts in their memory until it becomes second nature and they are able to apply the skills whenever necessary. This is similar to the model used in Japan where teachers are known to spend entire class period or more actively exploring and discussing the variety of approaches that might be taken to solve a particular problem. This supports the argument that depth in contrast to breadth, serves as a stronger method for instruction in the classroom.

As mentioned before my intentions for the unit are to enable students to break down barriers blocking their understanding of the fundamentals of scientific notation. A byproduct of this will be the student's ability to solve real-life problems by applying scientific notation concepts. Currently, my student's struggles seem to stem from their misunderstanding of the fundamentals of Mathematics. In addition, they become frustrated when the problem appears to be a challenge, i.e. word problems or algebraic type problems involving variables. I see these struggles accentuated when the students attempt to process scientific notation concepts in regard to Astronomy. Frustration is also evident when the problem resembles the type of standardized testing question they often see. Their struggles appear to stem from a lack of understanding that has manifested itself during their progression through each grade level. The type of thinking necessary to overcome these obstacles has been lost in time due to teacher's inability to connect fundamental ideas.

In particular, many of the strategies utilized in this unit are drawn from things the students experience in their everyday life. Temperature and currency are two examples that are sure to relate to the interest of the students. In fact, my students give more credence to currency then almost any other representation of success. Therefore, intertwining currency with scientific notation almost assuredly will allow the students to achieve some type of success. It also shouldn't be much of a stretch to expect the same results with distance, time, and size and its relation to light speed since these concepts are also a major part of a student's life. As the unit approaches it's conclusion the idea of estimation will be explicitly discussed so that the students may identify its practical uses and understand its application to scientific notation and accuracy. I also intend to compare aspects of computer science with scientific notation. This will intentionally be done towards the end of unit.

Background

The goal of the unit is to have my students be competent and confident in working with numbers of all sizes, using the tool of scientific notation. To accomplish this, I will have them work with sets of numbers drawn from various topics of interest, with an emphasis on astronomy, since that is an important topic in our syllabus, but also from other areas that should interest them. For example, an early lesson might discuss the volumes, masses, and densities of the planets and the Sun. Other topics I plan to deal with include:

- Currency Values
- Wealth and Income
- Time
- Distance
- Size
- Information and memory in computers.
- HR Diagram

One of the high points will be discussing size, temperature and luminosity of stars,

as encoded in the Hertsprung-Russell diagram.

Students should understand that base 10 notation is a concise method for writing numbers, in which the position of each digit determines it value. They should easily be able to identify the place value of numbers up to three digits, including the one, tens, and hundreds place. This also applies to either side of the decimal point, where the three digits to the right represent tenths, hundredths and thousandths. Next students need to understand how place value operates. They should recognize that an increase of each place value represents an increase by a factor of 10. The same is true for a decrease in place value with the place value decreasing by a factor of 10. This makes the next place value 10 times bigger or smaller depending on which direction the place is moved.

To help express this idea, students will practice decomposing numbers like 752 into 700 + 50 + 2 showing the value of each digit in the number. We will call the individual terms like 700 and 50 and 2, *single place numbers,* since each one has only one non-zero digit. It should be explicitly discussed that these numbers could be expressed using the power of 10 just as our base 10 number system describes. This can easily be done by writing each 'single place number' as a-digit-times-a-power-of-10. Students will perform that task by explaining how 752 can be expressed as 7 x 100 + 5 x 10 + 2 x 1. This notion becomes even more clear when it is expressed in exponent notation as follows: $7 \times 10^{2} + 5\times10^{1} + 5 \times 10^{0}$. As mentioned before this pattern of notation can be used on both sides of the decimal place. To continue with this idea, students will write out how 58.33 can be expressed using scientific notation. Many students may come up with the following as an answer:

$$5 \times 10^{1} + 8\times10^{0} + 3 \times 10^{-1} + 3 \times 10^{-2}$$

If done correctly this should prepare the students for how notation also explains how the Law of exponents works, giving the reciprocal of the number when it is represented by a negative exponent. Simply put, changing the sign of the exponent turns a power into its reciprocal (i.e. $10^{-1} = 1/10$, $10^{-2} = 1/10^{2}$ or 1/100). A lesson on fractions is not included in this unit, however it can be explored if needed.

The conclusion of the previous activity will lead us to compare the sizes of the single place numbers. The idea here is to get the students to understand the significance of the single place numbers and eventually how rounding makes numbers easier to understand. Therefore, students will compare the sizes of all single place numbers identifying which numbers are greater than others. They should conclude that the largest single place number is larger than all others combined. For example, in the number 752, the 700 is larger (in this case, much larger) than the 50 + 2. Further, the 700 + 50 already represents almost all the number. The missing 2 is negligible for many purposes.

Similar reasoning applies to numbers of any size. If we take

7,524 = 7000 + 500 + 20 + 4,

then the 7000 is (again, much) larger than the 524, and the 7000 + 500 = 7500 is much much larger than the 24 represented by the last two digits. It is always like this.

This is the motivation for scientific notation. If we have a number like

$752 = 7x10^{2} + 5x10^{1} + 2x10^{0}$,

then the largest base ten unit involved in it is $100 = 10^{2}$. The idea of scientific notation is to express the number relative to this unit. So we write

$700 = 7x\ 10^{2}$, and $50 = 5x10^{1} = 5x10^{-1+2} = (2x10^{-1})x10^{2}$, and $2 = 2x10^{0} = (2x10^{-2})x10^{2}$.

Putting these together gives

$752 = 7x10^{2} + (5x10^{-1})x10^{2} + (2x10^{-2})x10^{2} = (7 + 5x10^{-1} + 2x10^{-2})x10^{2}$

$= (7 +.5 +.02)x10^{2} = 7.52x10^{2}$.

Thus, the whole number is expressed as a multiple of its largest base ten unit. This is the basic idea of scientific notation.

Further inspection of this idea will lead my students to understand the role of accuracy in scientific notation. For example, they will compare the first two single place numbers to each other and identify that the first two are ten times larger than all others combined. Taking it one step further the students will be asked to take the first three single places and come to find that they are at least 100 times larger than all the others. This will lead to their understanding of how significant figures are determined by observing how stating a number to its first three places gives you very good approximation of the original number. A student then may ask for an example of such a scenario. Comparisons will be made to objects give you accurate estimations by rounding to the first three place, like the radius of the Earth, which is approximately 4000 miles (More accurately, 3960 miles, or about 1% less). Here they will see that the error made in this estimation is unimportant for the most part. They will find that two or often, even one digit gives you a good approximation. This will be made clear in each step by rounding the numbers to the first one, two, and three places.

It is essential to reiterate the realization that scientific notation makes computation of large and small numbers quite simple. Students might believe that numbers of these magnitudes are too difficult to compute because of their size. However, if the students identify what scientific notation does to a number, they can begin to accept the reality of how simple calculating these numbers can be. To capture these ideas, students will be asked to use zeros to represent single place numbers. This will be followed with instructions to attach exponents to the zeros in the single's place and write out the proper notation. They should respond with answers that look like the following:

$10^3 = 1000$, $10^5 = 100,000$, $10^7 = 10,000,000$.

Hopefully, by performing this task students will begin to see how the process of factoring out the largest power of ten allows us to obtain scientific notation. It also saves us the time of writing all of those zeros. This will help to highlight the value of using scientific notation.

For students to completely understand scientific notation they must become more familiar with how to write out the notation. To help with that, students will be given a set of numbers and asked to convert those numbers into scientific notation. For instance, they will be given the number of seconds in a year to be 31,560,000.0 and then asked to convert to the most appropriate form of scientific notation. Another problem might give them the mass of the sun to be 1,989,000,000,000,000,000,000,000,000,000,000 grams in which they would have to convert the number to the most appropriate form of scientific notation. They could also be asked to give the closest approximation to each number as they can, using rounding techniques. This should get them to realize how rounding a number is connected to the notation of the number.

In addition, they will again be asked to round their answers to a specified number of significant figures, i.e. 1^{st} digit, two leftmost digits, etc. The reinforcement of this idea is critical for obtaining the objectives and will remain a common theme throughout much of the unit. They will follow that by comparing the rounded numbers to the actual value, and expressing each difference as a percent error. This will help them to see that the main information is found in the leading digits. To add rigor to the lesson, I will ask my students to express the numbers in different units. For example, they could convert minutes to years, or kilograms to milligrams. This purpose of this activity is to get my students to understand what numbers are significant and how their magnitudes are affected when the units are changed. This will also lead into a discussion on accuracy, which will be discussed more thoroughly later in the unit. More problems to practice their writing of scientific notation can be found in the appendix.

Another example that could be used to employ the order of magnitudes is money and how its applied to everyday life. The examples will take a specific value, like the value of money in Zimbabwe (1 Zimbabwe dollar = $\$10^{-17}$ USdollars), and compare it the value of money in the United States. When they compare the numbers they should keep in my mind the theory of a base ten number system and recognize how significant the difference is. They will be able to do so by subtracting values from one another writing out the value in scientific notation.

In addition, they could be asked how much a pile of million dollar bills in Zimbabwe currency weighs, if each Zimbabwe bill were the same size as an American dollar, and the total pile was equal in value to one American dollar. (The weight of a dollar bill can obtained via web browsers on the Internet) Then to ensure their understanding they will be asked discuss the differences in the two problems. By computing these numbers students recognize the way notations of a number can make concepts more clear. Several more problems like the ones above will be used to reinforce the idea that scientific notation can represent magnitudes of very big and very small numbers. These problems can be found in the appendix. It should also be noted that many of the problems given would require access to additional information via the Internet or any other database of information.

Then students could then be asked what the ratios imply and if any connection could be made about earth and moon behaviors because of the ratio between the mass of the earth and moon. Understanding ratios will further lead the students on a path of connecting accuracy to scientific notation. Additionally, comparing very large numbers and finding normal sized numbers to replace these very large numbers can get students to see that these large numbers can be handled quite easily and can produce information they can comprehend. Additional problems on comparing magnitudes and comparing their ratio can be found in the appendix. What follows will be examples of activities that are meant to emphasize the relationship that accuracy shares with scientific notation.

When evaluating different sets of data, scientists depend on the accuracy of results. Accurate data provides scientist with the reliability they need when declaring theories and laws of science. This is found to be true in many other activities as well, including the government and pharmaceutical industries, where the accuracy of numbers can have dramatic effect on profitability. The complexities of science sometimes determine the accuracy of results. Often, the more precise the science, the more accurate the results need to be. The inaccuracy of results in science and the industries mentioned above could be costly and fatal depending on the circumstance. Frequently these results are given as estimations. Estimations help when calculating very big or very small numbers by eliminating many insignificant numbers and focusing on the numbers that matter. These numbers tend to have the largest magnitude and are emphasized because their inaccuracy could cause the most change in the result. My students need to understand that basic principle to progress through the unit. Therefore, they will analyze the difference in ratios of many sets of problems and speculate on what they can infer from the differences It is important to note that estimations can often be misleading even from the most reliable of sources. My students will be given then following example to highlight this point. NASA's original claim was that the probability of a space shuttle crash was about 1 in 100,000. My students will be asked to examine this further to check its validity. After taking a closer look into NASA's claim, they will see it suggest otherwise. If the claim is true and we assume that a space shuttle takes off everyday, then we can calculate that NASA should expect to have a crash once every 300 years. This can easily be calculated by the following equation:

$$\frac{1 \text{ crash}}{100{,}000 \text{ flights}} \times \frac{1 \text{ flight}}{1 \text{ day}} \times \frac{365 \text{ days}}{1 \text{ year}} \approx \frac{1 \text{ crash}}{300 \text{ years}}$$

Obviously, NASA has not been fortunate enough to experience that type of probability for crashing, proving how misleading estimates can sometimes be.

Accurately expressing numbers in scientific notation, involves two main ideas. First, significant figures must be established and consistent throughout the problem. Second, relative error of estimations helps make clear the importance of the leftmost digits by comparing the ratio of the estimation to its original value. Both of the ideas allow a person to obtain a better handle on how accuracy is expressed and emphasized in scientific notation. Significant figures refers to the number of digits we are sure are correct when giving the result of a measurement. The number of digits that are usually included for significant figures rarely goes past 4 digits. Claiming any level of accuracy past 4 digits can be deceptive because it insinuates that we know more about the number than we actually do. We are limited by our technology; This leads us to only rely on numbers we can be absolutely certain about. The leftmost higher magnitude digits found in the result represent these numbers. If the significant figures are not identified at the start of the problem, my students could become confused. Students will reflect on that by revisiting the mass of the Sun. As an introductory question, I will ask the class to compute problems using the mass of the Sun. Half the class will use scientific notation and the other half will use normal base ten notation. Following the exercise my students should recognize that it was significantly easier to complete the problem using scientific notation. Accordingly, they should see that this makes identification of appropriate significant figures critically important when computing scientific notation problems.

Students will begin to understand this even more by participating in an activity that concentrates on the accuracy of the leftmost digits. They will be asked to create 10 numbers at random. The criteria for the numbers selected include: three of the digits should be over 3 digits; and 3 more of the numbers should be just 3 digits. Next, they will compute what percentage of a whole number the first digit, the first 2 digits, and the first 3 digits of a number accounts for. Furthermore, problems of this type will be reinforced through many more problems given in the appendix, leading my students to comprehend the significance of the leftmost digits. Some of the problems involve the collection of data from the Internet, making this an appropriate exercise for this developing multiple skills. They will take the problems and round each piece of data to one and two places and compute the ratios of the following: I) the numbers they find on the internet to answer basic data questions II) the same number rounded to the first digits III) the same number rounded to first two digits. Following that they could round the answers they get in I) to the first and second digit and then compare the numbers they get to II) and III). The repeated use of rounding and computing ratios will assist my students retention of the concepts and illuminate the overarching objective of the unit. They will find that they always get the same answer, or close to it (although having only one digit accuracy sometimes might lead to significant discrepancy). This will help to convince the students that they don't need very many digits before they have a pretty good handle on the number. This should be one the most valuable lessons they can learn from the unit and could prove to be quite useful throughout their life.

It is extremely important that scientists and mathematicians have a method to check that their estimations are accurate. Accordingly, an expression was devised to identify the accuracy of estimations. The following expression is referred to as Absolute Relative Difference. It also can be referred to as Relative Error or Relative Change. The expression demonstrates how accuracy can be calculated as well as the importance of the leftmost digits. It is properly utilized when we have some number/quantity represented by Q. It then follow that we approximate the number Q by Q'. Students will be given an example of that case represented by the length of the calendar year. They will read how the original calendar year was represented by 365 days, however, over time, it became clear that 365 was not an accurate account of the amount of days in a year. Thus, a new Julian calendar was submitted giving a more accurate account of the length of the year at 365.25 days. Once again, this was found to be inaccurate. This lead to the development of a new Gregorian calendar year which identifies the length of the year as 365.25 – 1/100 + 1/400. Believe it or not that number was still found to be inaccurate and today we use the very precise number of 365.242199 days in the solar calendar year. In this case we see that understanding the value of the leftmost numbers gives us insight to inaccuracies of the past. The students will then be instructed to use the equation below and take the actual length of the year and have that subtracted by the previous estimates of the length of the year. That number can then be divided by the actual length f the year, giving us the absolute relative difference. This will indicate how accurate the original estimates of the calendar year were. That situation can be represented with subsequent equations:

$$\left\| \frac{Q' - Q}{Q} \right\| \quad \text{or} \quad \left(Q' = (1 + r)\, Q = Q + rQ \right)$$

As expressed earlier in the unit, students learn best when the information being taught to them relates to some aspect of their life. Accordingly, included in this unit will be examples that are applicable in a variety of ways to each of the students I teach. The researched information of each example below should assist the teacher in understanding the relationship that is shared between the examples and the student. Detailed exploration of each application of scientific notation is necessary to obtain a robust understanding. As a result, the background information that follows should also be shared with the students.

A few examples that would be of particular interest to the students would be the fact that the richest man in the world, Warren Buffet, is worth more than most countries with a net worth of nearly 62 x 10^9, or 62,000,000,000 dollars. Also, the students might find it interesting that the cost of all real estate in Manhattan is at 169 x 10^9, or 169,000,000,000 dollars. (This figure is for the land only, not the buildings.) Wrapping your mind that much money can be difficult but clearly it can be seen how scientific notation could make it much easier. We could ask, for example, how many Warren Buffets it would take to buy all the land in Manhattan. After we discuss the relevance of the amount of money people possess, they will use that information to compute information relating the Sun's mass to the Warren Buffet's Wealth.

The next applications talks about the great sizes of objects in space. Much of the information that is included in this section will reinforce ideas gone over in the previous paragraph regarding time as an example for using scientific notation. However, one distinction during this section of the unit will be the focus that is given to the "*Powers of 10*" video that will be shown and its relevance to accuracy. In the video, the narrator discussed the way our planet along with the universe can be broken down into powers of 10. The video is extremely effective at giving the audience accurate visualizations to help comprehend how our universe is broken up into powers of 10. However, how these sizes can be accurately calculated is a mystery to the students, since there isn't a way for people to actually experience these values first hand. At this point students will begin to recognize the importance of scientists' estimation as well as how accurate they often are. In turn, this will help the students to see how accuracy is connected to scientific notation.

This concept will be carried over to the lessons when students attempt to grasp the immense sizes and distances that exist in space. There are many examples that can be used to express those concepts, including planet and star size in conjunction with distance from planet to planet and galaxy to galaxy. For size, the examples are similar in that we will compare the sizes of planets and stars to one another, all using scientific notation to express the values. Included in these examples will be another revisit to the mass of the Sun. The Sun has the greatest mass our solar system, therefore comparing other objects in the solar system to our Sun reiterates the importance and usefulness of the notation and rounding of numbers. When comparing the different values, students will take the opportunity to practice their estimation skills making the values as accurate as possible. Once this section of the unit is complete, the students will have had an adequate amount of real-life examples to apply scientific notation and master the foundations that produce the intricate yet straightforward method of manipulating numbers.

Distance

The idea of a light year is a critical concept for the students to comprehend because the many uses it has in Astronomy. However, the idea of it sometimes perplexes students because they can no wrap their minds around the actual size of the number. To be specific, the light year is used to describe distances in space. It can sometimes be misrepresented by the light speed, which is a derivative of a light year. The fact that most of space is a empty vacuum with celestial bodies sporadically occupying parts of it makes its necessary for scientific notation to exist. It helps to explain the distances of space in between these bodies since the space is so vast.

Because of its vastness the empty space in between can only practically be represented by light years. As a result, light year is represented as 10^{16} meters, 10 trillion kilometers, or 6 trillion miles. Again, the conversion of these units can be good practice for understanding how valuable estimation can be. The introduction of the light year provides an excellent opportunity to compare the orders of magnitudes, which can be seen in the different ways a light ear represented above. Also, this will help to emphasize how size can be represented in various ways using scientific notation. This would also be a good time to point out how using only one digit provides a very good approximation of the distance one light year is. In accordance, my students will be given the fact that light speed travels at 299, 792, 458 meters per second. Then they will be given the distance from the Sun to the Earth and asked to calculate the time its takes the light to reach earth based on this information. They will come to find that it only takes 8 minutes for light leaving the Sun to reach Earth, even though it has to cover a great distance. This intends to emphasize the great distances and great speed of objects in space and how they can be managed fairly easily by using the correct notation.

Further questions that can be used to address those concepts might go as follows: How big is the solar system in light years? Or express distance of closest star Proxima Centauri in scientific notation. In addition, another tangible situation that can be used to emphasize the light year concepts considers the distance the amount of cars driven in the US travels in a year and comparing that to distance in space. The question could ask: What is the distance 150 million cars in the US and they travel in one year, if the average car travels 12, 000 miles per year? When students are done completing this problem they should share it with the class for discussion purposes. The answer should resemble something like the following: 150 million = 1.5×10^{8} , and 12,000 = 1.2×10^{4} , so the product is 1.8×10^{12} miles, which is about 1/3 of a light year! To follow up this question they could be asked how long would it take us to drive to the star Proxima Centauri at that rate? The students can then compare that answer to the different distances that specific planets are from Earth. That type of problem can be adjusted in various ways to meet the needs of students. However, yet again it is important to underscore the use of scientific notation to make these events easier for people to process. These questions are pertinent to assessing whether students are obtaining enough understanding to apply the concepts in other context. During these exercises students will experience the practice of identifying appropriate size and accuracy leading to an absolute understanding of scientific notation. Additional problems to supplement these ideas can be found in the appendix.

Luminosity, Temperature, Size

The example I will use to illustrate the application of scientific notation to temperature will involve the Hertzsprung-Russell diagram (HR Diagram). The HR Diagram compares luminosity (absolute magnitude), temperature, color and size of a star. It brilliantly demonstrates the relationship all of those characteristics share with each other. The values expressed on the diagram are very large numbers and thus using scientific notation is necessary to understand the comparisons. In fact, the luminosity scale on the HR diagram is not linear, but exponential. A change of magnitude by 1 signifies a tenfold change in brightness. I will take care to explain this carefully, and help students to read the HR diagram correctly. Once students understand what the chart is trying to express along with the way it tries to expresses it, they can then attempt to answer a problem like the following:

The Sun is 6,000 ^{o} K, how much hotter is the sun than our solar system's next closest star, Proxima Centauri? The students will also be asked to express all values in scientific notation and then interpret the meaning of the result. They can also be asked to compare ratios of the different luminosities of stars since the luminosity is represented in base logarithmic fashion using base 10 exponents to express their value (10^{-3}, 10^{-2}, 10^{-1}, 10^{1}, 10^{2}, 10^{3}). The complexities of the HR diagram can be difficult to understand, however after much practice and experience with scientific notation the students should be prepared to handle the intricacies of the diagram. Their experience will help them to identify the explosion in numbers that takes place as they move around the HR diagram.

When comparing the magnitudes to one another the students can identify the differences along with how those differences correlate with other qualities expressed on the chart. Recognizing the correlation will provide students with an understanding of how movement up or down one order of magnitude (place value) can dramatically affect the qualities of stars.

Computer Science

Bits can be defined as any form of information that is converted into binary digits each having a value of 1 or 0. This may seem too simple at first, however bits can be organized in an infinite amount of various strings of bits that relay information to our computer. When the numbers start to become too big or small for practical use, they are given prefixes like kilo, mega, and giga to clarify the amount of memory being utilized.

Here, students will gain more practice with the notation and relate it to another relevant aspect of their life. This provides yet another chance for students to identify how the notation makes abstract ideas in their lives more clear. By this time, they should start to realize that the use scientific notation offers clarity on many ideas that once may have seemed too complex. Through a more thorough examination of computer science the students will identify that eight million bits is equal to one megabyte, which is the equivalent of one short novel on stored on a computer (8,000,000 bits= 1 megabyte= 1 short novel). To simplify the information even further, students will be asked to convert that those numbers and many more to scientific notation, i.e. 8 million bits can be also looked at as 8 x $10^{\wedge 6}$ bits. When finished with this application of scientific notation students should understand that 8 million bits= 1 megabyte= 8 x 10^{6} bits.

Using order or magnitude to discuss computer science presents students with ideas that are tangible and relevant since computers are a major part of their lives. Identifying the connection to scientific notation allows students differentiate between memory size and the abilities that more memory size offers computers. This directly relates to order of magnitude, in that students will be able to clearly recognize that the higher the order or magnitude the more capabilities computers have. Comparing these orders of magnitude to computer memory and then to devices they use every day makes this concept evident to the students.

Strategies

In each lesson, I will ask my students to write and read numbers, to do calculations, to make comparisons, to do approximations and to make meaningful conclusions. As we work with these numbers, I will visit and revisit the basic facts of base ten notation, and the meaning of scientific notation, and with issues of approximation until my students are comfortable with them, and tell me that I don't need to explain any more.

It would be important to start with a short review the basic structure of base ten notation,

for example that

732 = 700 + 30 + 2

so that 732 is really shorthand for the sum of certain special numbers, like 700, and 30

and 2. Each of these numbers is a digit, meaning 1,2,3, 4,5,6,7,8,9 or 0, times a base ten unit. The base ten units are 1, or 10, or 100 = 10x10, or 1000 = 10x10x10, and so forth.

Each one is the product of a certain number of 10s. The number of zeros in the unit

tell how many tens have been multiplied together to make it. I will call a digit times a base ten unit a *single place number*. [3]

Writing a number explicitly as a sum of single place numbers is called *expanded form*.

If a number has zero as a digit, then we don't have to explicitly write the term corresponding to the zero. For example

4093 = 4000 + 90 + 2.

No confusion is caused by leaving out 000, because each single place number in this sum tells us how large it is. But when we recompress the number, we need to put in the zero in the hundreds place, or the 4 won't correctly signal that it is representing thousands.

After a brief review like this, I will write a fairly large number on the board, for example

1,988,929,000,000,000,000,000,000,000,000,

which tells the mass of the Sun in kilograms. I will ask my students to write this number in expanded form. I hope they think it is a lot of work! When they complain, I will tell them about a shorter way – exponential notation.

In exponential notation, instead of writing out all the zeros in a base ten unit, you just count how many there are, and put that number at the upper left of 10. For example,

1000 = 10x10x10 = 10^3 , and 1,000,000 = 10x10x10x10x10x10x10 = 10^6 and so on.

After this discussion, I will ask them to write the first number I gave them using exponential notation for the base ten units. For the example above, this would be

1x10^{30} + 9x10^{29} + 8x10^{28} + 8 x10^{27} + 9x10^{26} +2x10^{25} +9x10^{24}.

This is still some work, but a lot less than writing all the zeros! After doing this, we would work with some other similar numbers. For this particular lesson, I would also give them the mass of Earth and the Moon in kilograms, and their volumes in cubic kilometers, and perhaps the comparable figures for some of the planets.

All lessons will begin with an introductory questions to prepare the students for what they learn that day. This will warm up their brains and get them in the right train of thought. Accordingly, the following questions will be used to introduce two of the lessons: "How long would it take you to count to 1 million? or "How long is a million seconds? 1 billion? 1 trillion?" or "How much would 10 American dollars be worth in Zimbabwe (3.33 x 10^{-17} = 1 Zimbabwean cent)". These questions and ones like them, encourage students to think about size and how it can be measured with scientific notation. Additionally, they will recognize how significant order of magnitude is when comparing numbers. This will be most evident when the students observe the extreme differences in the answers they got for the introductory question. The teacher will discuss these concepts aloud with the class following a group discussion at their assigned table. The time needed to successfully complete these questions range from 7-10 minutes. Time to expound on the concepts will be given during the lesson through built in supplementary task. In the time given, students should have prepared themselves to achieve the objectives of the day. This should be accomplished by successfully completing each of the given tasks.

In general, all lessons included in this unit will contain a specific theme or objective that guides the entire lesson. As seen above, there are particular themes to be covered in each lesson given to my students. Specifically, I plan to include, order of magnitude, accuracy, rounding, and the notation of numbers as common themes interweaving the lessons together. Overall, my students will witness how these ideas make their lives a whole lot easier when attempting to get a handle on various concepts. These intentions will be made clear in the introductory lesson. They will be asked to write out huge numbers such as the mass of the Sun in kilograms and a light year measured in kilometers. These two numbers will be used because they allow the students to see how these numbers can easily be easily understood through the use of proper notation. This will be a reoccurring theme that will persist throughout the entire unit. It will help to reinforce the importance of scientific notation and how it simplifies their life. These concepts can also serve as a springboard into other topics like accuracy and absolute relative error, enhancing their understanding.

As mentioned before, I would sell each idea on the basis that it simplifies things. To start this discussion, the introductory lesson will tell them that there is an even simpler way to write the mass of the Sun. This will lead to an introduction on the idea of the law of exponents, and negative exponents, where we would factor out 10^{30} , and then write the quotient in standard base ten notation, and there we would have scientific notation. Next I could say that, although scientific notation is really wonderful compared with the regular base ten notation, we would still would like not to have to write out all those digits, so we would approximate our number with a simpler one – like 2x10^{30} . A discussion on the relative sizes of the places will follow, making sure they understand that the leftmost place is largest, and worth more than all the others combined. My students can then begin their introduction into relative error, by replacing the number above by 2x10^{30} , and observing how much error it makes. After computing it, they will be asked if it a big number – they will probably say yes. Then I would say, but we should compare the error to the number we are approximating – is it big compared with the total mass? When they divide the error by 2x10^{30} , they will find that, relatively speaking, the error is quite small. Many problems like this one will be given to the students before they actually receive a formal definition for relative error.

Once they become more familiar with writing these types of numbers, they should have the skills to compare the ratio of the mass of the Sun to the mass of Earth, and the mass of the Moon to the mass of Earth. Next, they would do the same calculations with the approximations they have found. Then, they should verify that the results are good approximations to the computed ratios. Once they have computed the ratios of volume and of masses, they should compare these two, again by division, and conclude that Earth is a lot denser than the Sun, and the Earth is denser than the moon. Then we could have a discussion of why this might be. The eventual understanding of these ideas should lead my students to provide the explanations instead of myself.

Eventually my students should be able to understand relative error since it a highlights a major concept of scientific notation. When using scientific notation numbers are usually expressed within using 1 or more of the first 3 digits. Those digits are then multiplied by a power of 10. To understand this better my students will learn how we determine the leftmost digits to be so accurate. They will start with a number like, 2×10^{4} . In the case of the example above, knowing the 1 st digit of the base 10 number always tells you at least 50% accuracy of the actual value. Therefore, using 1000 to represent a number like 1,484 puts you at least 50% accuracy of knowing the actual value of 1,484. Students should see that by computing the subsequent expression 1000 1,484 2,000 and so 1000/1,484 > 1000/2000 = Â½ = .5 x 100= 50%. It ought to be clear that as the first digit of the base 10 number increases so does the accuracy. This will be done multiple comparisons of numbers. This could be followed, by asking the students what happens if more digits are added to the estimation. They should see the more digits added the more dramatic the increase of accuracy. Using the 1 st first two digits of a base 10 number will always put you within at least 90% accuracy of the value (3.4×10^{5}). Following that same pattern, using the 1 st 3 digits of a base 10 number will always tell you 99% of the number. This is illustrated in the following expression:

149,000 149,623 150,000 so 149,000/149,623 > 149,000/150,000 = 149/150 > 99/100 = 99%.

All of this is possible because of the expression (x/x+1) with x representing the digits used to get the estimation. Mastering this concept can become very powerful in proving how practical applying scientific notation can be.

The next technique implemented will provide the students with an introductory question connecting scientific notation to temperature, luminosity, and size in astronomical terms. This will be done with introductory questions that stimulate their curiosity. The type of question asked will encourage them to think about the significance of big and small numbers. For example, the question used to introduce this lesson on temperature, luminosity, and size will have them simply identify the difference in temperature between our sun and another familiar star. This should prepare the students by leading them to think about how size is related to other qualities of a star. Because the qualities of a star can be measured with such huge numbers scientific notation can be used to express these values. This will assist the students with achieving the objectives of the unit.

Once they become comfortable with the concepts they will begin with the application of scientific notation to temperature and its correlation to star color. Students will be provided a Hertzprung-Russell Diagram. A Hertzprung-Russell or HR Diagram is a chart that displays the absolute magnitude (luminosity/brightness) of a star and its connection to its temperature and color. The chart does an excellent job of illustrating the connection that exists between all three characteristics of a star. It also brilliantly demonstrates the vast differences amongst the stars by using scientific notation to represent numerical value. The HR diagram itself will help the students to better understand scientific notation by comparing the different magnitudes to each other. The difference in magnitudes can also be compared to other qualities of star represented by the diagram. Therefore, one example might allow the students to observe how the greater magnitude of a star's luminosity strongly correlates with the size of star. This helps to prove how scientific notation can be used to compare several qualities of a star illuminating another purpose of this specific representation. An example of the HR Diagram can found in the appendices to supplement classroom activities.

Although all the strategies mentioned can serve the purpose of realizing the objectives, the strategy my students will rely on most involves repetition and practice of many problem sets. Provided in the appendices is an example of the types of problems I will give my students. Ultimately, I feel what best serves the students is the consistent practice of the theories discussed throughout this unit. As with any thing in life, the more you work with concepts, the more comfortable they will become to you. Consequently, the students will spend more time practicing the ideas as opposed to the more time spent listening to lectures. In effect, the students will "learn by doing" gaining knowledge from their experiences with success and failure in the unit. After all, practice makes perfect.

Activities

All Lessons Are Based Off a 75 min period. Time spent for each part of the lesson is tentative and should be based off student understanding.

Lesson 1 Introduction

Objectives

Students will identify and understand number facts/relationships for numbers 1-10. They will be expected to apply these number relationships to scientific notation in order to become more familiar with the concept. Finally, students will utilize scientific notation to understand basic astronomy concepts. They will also be the time that the mass of the Sun along with the distance of a light year are introduced to serve as a example of how using Scientific Notation make our lives more simple.

Warm up (10 min)

Students will enter the classroom and answer the following question: How long would it take you to count to 1 million? 1 billion? 1 trillion? Once the students have answered the questions in their notebook, the teacher will ask them share out their answers. The answers shared with the class will stimulate a discussion about place value and order of magnitude. During this time the teacher will explain the meaning of place value along with order of magnitude and its application to Astronomy.

Direct Instruction (15 min)

A discussion will be held following the warm up question to explain the different strategies used to find their solution. The students will share their answers with the class by demonstrating their strategies on the board. Following that, the teacher will break down the concept by elaborating on the estimation that is involved in answering this type of question. They will understand how accurately estimating is dependent on the amount of leading digits used. Next the teacher will provide the examples of the mass of the Sun and distance of a light year to emphasize the importance of estimation. It will then be explained to the students how learning this idea will be useful to them in the future.

Guided Practice (20 min)

At this point students will be introduced to the commonly known number facts for numbers 1-10. The students will then identify various number relationships as the teacher demonstrates the processes on the board. Students will then be asked to create their own number facts with the groups at their table. The students will then be asked to reflect the same facts using Cuisenaire Rods. To build on their understanding they will also be asked to use the rods to create new number facts and relationships on their own. The students will then share out the examples with the class and on the board. One student from each group will come to board and produce the fact/relationships they created with their groups. Included in their explanation will be a description of how they created their relationships with and without the rods. This section of the lesson will conclude with the students discussing the implications of the relationships with their groups. (20 min)

Independent Practice (25 min)

Students will then begin their independent work by identifying something a million seconds old, billion, and trillion seconds old. They will also be asked to convert all results to days and years. This section will conclude with the answers being explained to students to check that they are understanding the concepts.

Closing (5 min)

Students will be asked during the conclusion of the lesson to think about more examples that exemplify the importance of using scientific notation and how efficient it is at expressing numbers neatly. An example of this question may be to have the students identify anything else that uses trillions to describe its value. It should also be reiterated that mastering number relationships will make it easier for the students to grasp scientific notation as they progress through the unit.

Homework

See Appendix for scientific notation worksheet to accompany this lesson.

Lesson 2 Luminosity, Temperature, Size

Objective

During this lesson students will be expected to apply the scientific notation to the luminosity, temperature, and size of a star. Additionally, students will compare and contrast orders of magnitude and understand its implications when describing the qualities of stars.

Warm Up (10 min)

Students will enter the classroom and begin their introductory question for the day's lesson. At the point students should be more comfortable with scientific notation due to extensive practice they will have had prior to this lesson. Accordingly, students should handle the following question rather easily: If the star Proxima Centauri's temperature is 3,000 K, how much cooler is it than the star Bellatrix with a temperature of 22,000 K. Express values using scientific notation and explain what we can hypothesize about the star's qualities based on the values given.

Direct Instruction (25 min)

Students will spend this part of the lesson sharing and discussing their answers from the introductory question. Then, the teacher will explain the answer by describing the relationship that exists between the qualities of a star and its temperature. Two particular qualities affected by a star's temperature include a star's luminosity and size. The students should recognize how prominent the relationship between a star's luminosity, temperature, and size is and discuss the implications with their groups. During their discussion the teacher can facilitate by clarifying misconceptions. For example, a student may wonder how scientists know these answers so accurately. This would provide a perfect opportunity to elaborate on the estimation techniques used in science and particularly astronomy. Then, the teacher would explain how scientific notation enhances the accuracy of a value by focusing on a few leading digits. Thus, making the estimation almost as accurate as it could possibly be. With time permitting the teacher could expound on the concept by explaining to the students the efficiency of using 1-3 leading digits in Scientific notation. Again, understanding how the leading digits determine accuracy should enlighten the students. The 1 st leading digit puts you within 10% accuracy which is considered to be moderately good accuracy. Using 2 leading digits puts you within 1% accuracy, which is considered quite good accuracy. Finally, three leading digits puts you within .1 % accuracy which is considered to be very good accuracy. (This concept may take an extended amount of time to complete, therefore is to the teacher's discretion on much of the information to include in the lesson) Following that the teacher will reiterate the importance of scientific notation and connect the all the ideas learned to the HR Diagram.

Guided Practice (10 min)

Students will receive a copy of their own HR diagram to look over as the teacher models how to read and interpret the diagram. They will then be asked to compare luminosities to each other using scientific notation as their values. They will analyze these values with their groups and indentify the implications of increased and decreased luminosity in regards to temperature and size.

Independent Practice (25 min)

After the students have completed their group work they will share their results with the teacher and class. The correct results will be explained in detail leading the their complete understanding. Accordingly, students will then perform independent work by answering several word problems given by the teacher. The questions associated with this part of the lesson can be found in the appendix.

Closing (5 min)

The teacher will provide answers to the students while the students explain how they obtained their answers.

Homework

Students will complete HR Diagram worksheet given to them to be discussed the next day in class.

Lesson 3 Currency

Objectives

The students will expected to compare and contrast orders of magnitude with currency in practical and sometimes impractical terms. They will also be expected to apply scientific notation when representing currency values.

Warm Up (10 min)

This lesson can be expected to be completed towards the end of the unit and thus the students should have a strong understanding of scientific notation. Students will enter the classroom and begin the lesson with this introductory question: If the country, Zimbabwe in Africa, has a currency value of 10^{-17} when compared to the American dollar, how much money would you have if you went there with 10 dollars?

Direct Instruction (15 min)

The teacher will then explain the introductory question to the students as they break down the significance of the answer. To answer the problem they will understand that all that needs to be done is to move the decimal over 17 times to the right. This profound number will have the students in awe bringing them to understand deeper concepts about the answer. Understanding the implications behind the number might lead the students to ask more questions. The teacher can capture that moment use it as a 'teachable moment' and discuss the national deficit in conjunction with other huge numbers all along tying it back to scientific notation.

Guided Practice (20 min)

Students will then be given currency order of magnitude charts. The teacher will model how to read the chart and interpret its meaning. It will be particularly interesting to witness how our consumer interest compare with the different orders of magnitude of currency (i.e. cars, home, planes, etc.). Following that discussion students will identify several significant values and compare them to each other.

Independent Practice (25 min)

The students will group together and discuss the gaps and differences in currency magnitude. Then they will move on to other "money issues" that concern orders of magnitude and scientific notation. Students will be given problems asking them to compare Gross National Product (GNP) of countries with GNP per capita. Another 'money issue' considers income distribution and calculating the total range. The students will solve these problems and others with their groups.

Closing (5 min)

Students will attempt to capture concepts of the day by discussing its relevance to their lives. It should be evident how scientific notation makes it easier to understand and calculate numbers such as the ones used in the lesson.

Homework

The students will receive the following 3 questions to be completed by themselves at home. The results of the homework will be discussed the next day in class:

1) If everyone earned the median wage, what would be the total payroll of the country? How does that compare with this compare with the total earned income? Explain the difference in value.

2) Compare the national debt to GNP. How long would it take us to pay off the debt if we put all our income towards it?

3) How many Billionaires would it take to buy Philadelphia if the total value of all real estate in Philadelphia was $115 x 10 ^9^ ?

Notes

1. Edward A Silver and Sandra P Marshall, "Mathematical and Scientific Problem Solving: Findings, Issues, and Instructional Implications," in *Dimensions of Thinking and Cognitive Instructions*, ed. Beau Fly Jones et al. (Hillside, New Jersey: Lawrence Erlbaum Associates, Inc., Publishers 1990), 266.
2. >2) Silver, "Mathematical and Scientific Problem Solving", 270.
3. >3) Roger Howe and Susana S Epp, "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra," Essay, Yale University 2008, 39.

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1) Edward A Silver and Sandra P Marshall, "Mathematical and Scientific Problem Solving: Findings, Issues, and Instructional Implications," in *Dimensions of* http://www.facebook.com/profile.php?id=54202258*Thinking and Cognitive Instructions*, ed. Beau Fly Jones et al. (Hillside, New Jersey: Lawrence Erlbaum Associates, Inc., Publishers 1990)

2) Lloyd Motz and Jefferson Hane Weaver, *The Story of Mathematics* (New York, New York: Plenum Press, 1993)

3) Lawrence Weinstein and John A. Adams, *Guesstimation: Solving The World's Problems on The Back of a Cocktail Napkin* (Princeton, New Jersey: Princeton University Press, 2008)

4) Roger Howe and Susana S Epp, "Taking Place Value Seriously: Arithmetic, Estimation, and Algebra," Essay, Yale University 2008

5) Sam Dillon. "Top Test Scores from Shanghai Stun Educators," New York Times, December 7, 2010.

Standards for Astronomy in the State of Pennsylvania

3.1D: Explain scale as a way of relating concepts and ideas to one another by some measure

3.2A: Explain and apply scientific process and scientific knowledge

3.4B: Relate energy sources and transfers to heat and temperature

3.7B: Use appropriate instruments and apparatus to study materials

Appendix

Problem sets for scientific notation

Order of magnitude/ expanded form

4) What place value is the 3 in the following number 12.35?

5) What is the highest order of magnitude for the number 12,357?

6) Express the following numbers in expanded form:

1. 15, 237
2. 170, 452
3. 17.95
4. .0347
5. 1,453, 976, 421

7) Convert the following expanded form numbers to their normal decimal notation:

a. $5 \times 10^{3} + 2 \times 10^{2} + 4 \times 10^{1} + 6 \times 10^{0}$
b. $7 \times 10^{5} + 4 \times 10^{4} + 0 \times 10^{3} + 1 \times 10^{2} + 8 \times 10^{1} + 3 \times 10^{0}$
c. $8 \times 10^{2} + 7 \times 10^{1} + 3 \times 10^{0} + 9 \times 10^{-1} + 4 \times 10^{-2}$
d. $1 \times 10^{0} + 3 \times 10^{-1} + 5 \times 10^{-2} + 4 \times 10^{-3} + 7 \times 10^{-4}$

Money

What is the median household income of the US?

Identify the highest minority household income in the US? Compare the number in question 1 to question 2 and explain what that means

How many people have cell phones in the US?

How many families are on welfare? Compare this number to the number in question 4. Is there a connection? Explain why or why not

Identify the profits Comcast made last year.

Identify the profit (deficit) of the Philadelphia School District last year.

What is the ratio of question 6 to question 7? Explain what might be the cause of this terrific gap in profits.

What is the mean wage of a student that graduates high school? College? Graduate School?

Compare the ratios of mean wages for each level of school mentioned in question 9. Explain what the difference could mean.

Applications of Scientific Notation

Name

Scientific Notation is an important way to represent very big, and very small, numbers. Here is a sample of astronomical problems to test your skill using this number representation.

Problem 1: The Sun produces 3.9×10^{33} ergs per second of radiant energy. How much energy does it produce in one year?(Hint: a year is about 3.1×10^{7} seconds long.)

Problem 2: One gram of matter converted into energy produces 3.0×10^{20} watts of energy. How many tons of matter in the Sun are annihilated every second to produce its luminosity of 3.9×10^{33} ergs per second.

Problem 3: The mass of the sun is 1.98×10^{33} grams. If a single proton has a mass of 1.6×10^{-24} grams, how many protons are in the Sun?

Problem 4: The approximate volume of the visible universe (a sphere with a radius of about 14 billion light years) is 1.1×10^{31} light years. If a light year is 9.2×10^{17} centimeters, how many cubic centimeters does the visible universe occupy?

Problem 5: The NASA data archive at Goddard Flight Center contains 25 terabytes of data from over 1000 science missions and investigations.(1 terabyte= 10^{15} bytes) How many CD-roms does this equal if the capacity of a CD-rom is 6×10^{8} bytes? How long would this take, in years, to transfer this data by a dial up modem operating at 56,000 bits/second? (Note: 1 byte = 8 bits)

Scientific Notation - Estimation

Money

Currently, the US is in the midst of the greatest recession since the 1920s. A recession is a general slow down in economic activity. In plain English, most people don't have as much money as they used to. This does not include companies like Apple, Pfizer, General Electric, and CBS who currently have a combined total value of 1.2 trillion dollars. That doesn't include their property value or income, just cash or cash equivalent they could spend tomorrow. Based on this information, estimate what these companies can buy if they released their money.

a. How many years could the companies pay the rent if the average for all US residents renting is $\$26.5 \times 10^{9}$ /month?

How many US home could they buy if the mean price of a US home of is $185, 000?

How many incomes could they provide if they the average household income was $50,221?

How many per capita (per person) incomes could they provide if the average per capita income was $27,041?

Compare a to b and b to c to find the ratios. What does the difference mean?

What is the relative error if you use $50,000 for the average household income, and 27,00 for the per capita?

a. If the average value of foreclosed homes in 2011 was $165,000 and there were 40,460 foreclosures in US during that year, how many of the foreclosures could the company buy?

6. Compare this answer to the answers in question 1. How are they different? What does this mean?

a. How many years could they pay gas bill if the average cost to fill up the tank each month was $250?

8) What is the average cost to fill up one tank of gas?

9) What variables could causes the estimates to be inaccurate?

10) The average income per capita is $27,041. What percentage of a person's income is spent on gas per month, per year?

a. How much money could they give each US family, if there were approximately, 1.3×10^8 families in the US?

a) Describe the best way a family could spend this money.

b) How much richer would Philadelphians with the combined additional income provided by the companies? (Note: The population of Philadelphia is 1.5×10^6 people)

Problem Suggestions

Population

1. What is the population of the world?

2. What is the population of the 10 largest countries (in terms of population)?

3. What percent of the world population is in the 10 most populous countries?

4. What is the ratio of the most populous country to the 10th most populous?

5. What percent of the world population is in China? In India?

Land Area

1. What is the total land area in the world?

2. What is the area of the 10 largest countries (in terms of area)? Also find: the area of the US not counting Alaska.

3.What is the percent of the total land area in the 10 largest (in terms of area) countries?

4. What is the ratio of the area of the largest country to the smallest?

5. What percent of the world's land area is in Russia? In Canada?

Population Density

1. What is the average population density of the world?

2. Suppose you don't count Antarctica as land area. What is the population density of the rest of the world?

3. What is the population density of China?

4. What is the population density of India?

5. What is the population density of the rest of the world, not counting India and China?

6. a) What is the population density of the US?

b) What is the area of Alaska? What is the population of Alaska? What is the population density of Alaska?

c) What is the area of the US not counting Alaska? What percentage is the area of Alaska, of the area of the rest of the US?

d) What is the population of the US not counting Alaska? What percentage is the population of Alaska, of the population of the rest of the US?

e) What is the population density of the US, not including Alaska?

f) What figure is most relevant to you, the answer to a), or the answer to e)?

g) What would it be like if the US had the same population density as India?

7. What is the population of Pennsylvania? What is the area of Pennsylvania? What is the population density of Pennsylvania?

Automobiles

1. How many automobiles are there in the US?

2. How many people per automobile are there? How many automobiles per person are there?

3. How many miles does an average car get driven in a year?

4. How many miles do all the cars in the US go in a year?

5. What fraction of a light year is the answer to 4? How long would it take us to drive the distance to the nearest star?

6. How many times the distance to Pluto is that?

7. How many gallons of gas do all the cars in the US use in a year?

8. How many pounds of CO_2 do all the cars in the US produce in a year? (Figure 25 pounds per gallon.)

9. How many miles of roads are there in the US?

10. If all the cars in the US were evenly spread out on all the roads, how far apart would they be?

Solar system

1. What is the radius of Earth?

2. What are the radii of all the planets? (Debate whether to include Pluto.)

3. What is the ratio of the largest to the smallest? What is the ratio of Earth to the largest? To the smallest?

4. What are the distances of the planets to the Sun? (These are not constant. Take the average of the largest and the smallest.)

5. What is the radius of the Moon? What is the distance from the Moon to Earth? (Again, take an average.)

6. What is the radius of the Sun?

7. What are the volumes of all the planets and the Sun?

8. What is the ratio of the volume of the Sun to the volume of all the planets put together? Suppose that Earth were the size of a pea, and all other distances were in the same proportions as they actually are.

9. What would be the distance from the Moon to Earth?

10. What would be the radius of the Sun?

11. What would be the distance of Earth from the Sun? Would the Sun and Earth fit inside the classroom?

12. What would be the distance from the Sun to Jupiter? Would the Sun and Jupiter fit inside the school grounds?

13. Same question as 12, but for Neptune or Pluto.

14. What is the ratio of the radius of the Moon to the radius of the Sun? What is the ratio of the distance from Earth to the Moon, to the distance from Earth to the Sun? For this compute two different ratios first using the smallest distance from the Earth to the Moon, and then using the largest distance.

15. How do the ratios, of radii, and of distances, compare?

16. What does the answer to 15 have t o do with eclipses of the Sun? Does it have anything to do with eclipses of the Moon? What are the relevant ratios for eclipses of the Moon?

Cities

1. What are the 10 largest cities in the world, in terms of population?

2. What is the ratio of the largest to the smallest?

3. What percentage of the world's population lives in these 10 cities?

4. What are the 10 largest cities in the US, in terms of population?

5. What is the ratio of the largest to the smallest.

6. What percentage of the population of the US lives in these 10 cities?

7. Choose any country from the following list: England, France, Germany, China, Mexico, Australia, Canada, Brazil, Japan, India. Answer the same three questions for this country.

Money

1. What is the median wage in the US?

2. What is the median household income in the US?

3. What is the reason for the difference between 1 and 2?

4. How many households are there in the US? How many workers are there?

5. What is the ratio of the second number in 4 to the first? What does it mean?

6. If every worker made the median wage, how much money would that be?

7. If every household had the median income, how much money would that be?

8. What is the ratio of 6 to 7? Why might they be different?

9. Suppose that Philadelphia had households of average size, and they all made the median income. How much money would that be?

10. What is the total Gross National Product (GNP) of the US?

11. What is the ratio of 9 to 6? What could be reasons for the difference?

12. Who are the 10 richest people in the US and what are the net worths? What is the ratio of the smallest to the largest?

13. What is the total net worth of these 10 people?

14. What is the ratio 13 to 6?

15. What is the ratio 13 to 9? What does this mean?

16. What is the total value of real estate (all land and buildings) in Philadelphia?

17. What is the ratio of 16 to 13? Could the richest 10 people buy all of Philadelphia? How much could they buy, or how much would they have left over?

18. If your parents have a credit card, find out the annual percentage interest rate. If they don't, find the annual percentage rate on the credit card of someone you know. How much is that per month?

19. If your parents have a savings account, find out the annual interest rate. If they don't have a savings account, go to a bank and find out the interest rate on their standard savings accounts.

20. Which is larger, the annual interest on a savings account, or the monthly interest on a credit card?

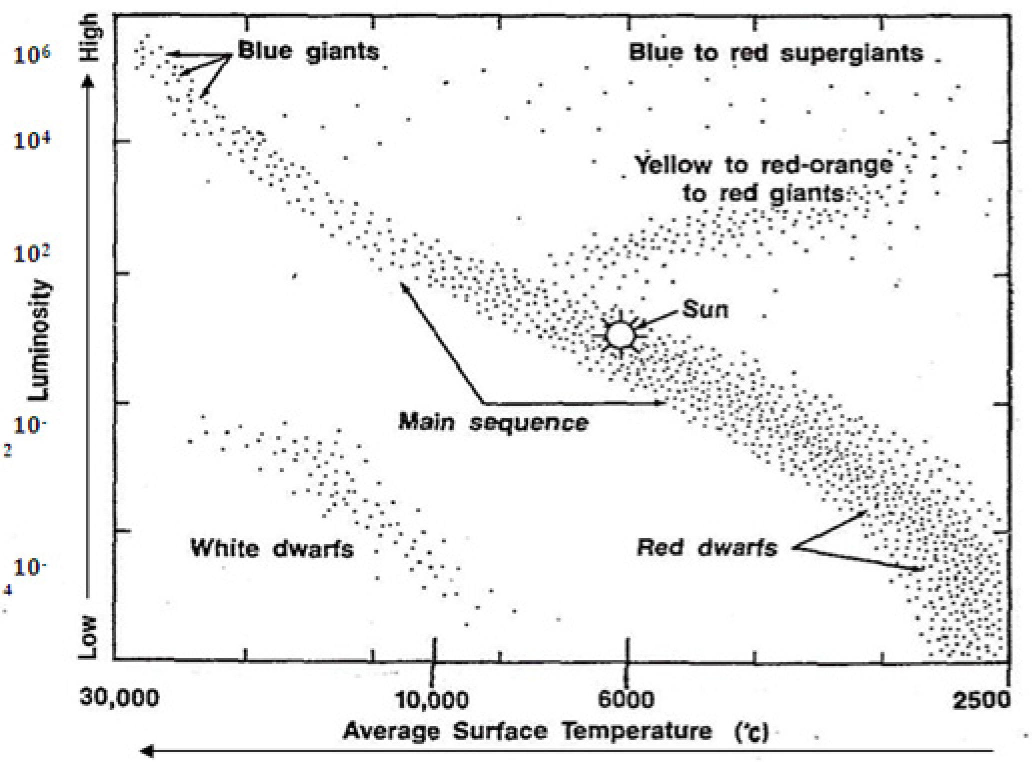
The Hertzsprung-Russell Diagram Practice Worksheet

Name ________________________________ Period _____ Date ____________

The Hertzsprung-Russell diagram is actually a graph that illustrates the relationship that exists between the average surface temperature of stars and their absolute magnitude, which is how bright they would appear to be if they were all the same distance away. Rather than speak of the brightness of stars, the term "luminosity" is often used. Luminosity is a measure of how much energy leaves a star in a certain period of time. Generally, for stars that are at equal distances from the Earth, the more luminous a star, the brighter it is.

The luminosity of stars is affected not only by temperature but also by size. The most luminous stars would be those that are large and hot. Those that are the least luminous would be small and cool. The color of a star is determined by its surface temperature, which is illustrated on the Hertzsprung-Russell diagram.

The Hertzsprung-Russell Diagram



Answer the questions on the back of this page using the above HR Diagram

THE H-R DIAGRAM

1. What is the approximate surface temperature of the sun? ___________________

2. Would the surface temperature of white dwarf stars be **higher** or **lower** than red supergiants? (Circle one of the bold words)

3. What is the color of the stars with the highest surface temperature? ___________

What is the color of the stars with the lowest surface temperature? ____________

4. Compare the Luminosity of a red supergiant star to that of a white dwarf? Identify the magnitudes and explain what you can interpret from the difference between them.

____

5. The luminosity of each star is measured by using 10 as its base to compare to the Sun. Explain what it means when a star has a luminosity of 10^{-5} .

____

6. How is it possible for white dwarf stars to have lower luminosity than the sun even though the sun is cooler than white dwarfs?

____

____

____

____

7. Plot the following stars.

Star A = 4,000 ^{0} C and low/medium brightness

Star B = 6,000 ^{0} C and high brightness

Star C = 20,000 ^{0} C and low/medium brightness

Star D = 6,000 ^{0} C and medium brightness

Identify the type/color for each star:

Star:	Color:	Type:
Star A		
Star B		
Star C		
Star D		

Orders of Magnitude

(money expressed in United States dollars)

Factor ($)	Long scale	Short scale	Money	Item
10^{-17}	one Zimbabwean cent		$3.33 * 10^{-17}	Exchange rate on February 2 of 2009
10^{-3}	one mill		$0.001	smallest unit of currency, used in pricing gasoline and computing taxes
10^{-2}	one cent		$0.01	used chiefly for making change
10^{-1}	one dime		$0.10	highest common price per page for self-service monochrome photocopying
10^{0}	one dollar		$1	double cheeseburger at McDonald's
			$4	typical drink of gourmet coffee
10^{1}	ten dollars		$10	wristwatch with quartz circuit, 20 lb. sack of rice
10^{2}	one hundred dollars		$100	2 or 3 video games
			$400	approximate annual GDP per capita (PPP) for East Timor (2004, CIA World Factbook)
10^{3}	one thousand dollars		$1,000	used car (15 years old, runs)
			$1,000	midrange personal computer
			$1,000	a nice digital camera, approximate GDP per capita (PPP) for Nigeria (2004)
			$9,117	approximate world GDP per capita (PPP) (2008)
10^{4}	ten thousand dollars		$10,000	cheap new car
			$10,000	approximate GDP per capita (PPP) for Russia (2004)
			$20,000	(Israel, Greece)–$40,000 (Jersey, Norway, United States) – approximate GDP per capita (PPP) in most first world nations (2004)
			$26,000	cost of an average new car
			$30,000	cost of an Engineering degree from an average university
			$35,060	annual income (GNI) per capita (PPP) for employed citizens of the United States, as of 2002
10^{5}	one hundred thousand dollars		$100,000 - $999,999	In the United States, a "six figure salary" is sometimes seen as a milestone of significant wealth, and indicator of higher social class.
			$100,000	small house far from cities
			$100,000	cost of a Law degree from a prestigious

				university
			$101,000	median value of a home in the U.S. in 1990
			$120,000	median value of a home in the U.S. in 2000
10^6	one million dollars		$1,000,000	huge house in suburbs, nice condo downtown in large city
10^7	ten million dollars		$10,000,000	a small hospital
10^8	one hundred million dollars		$100,000,000	large city office building
			$264,000,000	estimated price of an Airbus A380 airplane
10^9	one milliard dollars	one billion dollars	$\$2.5\times10^9$	estimated cost of a B-2 Spirit stealth bomber
10^{10}	ten milliard dollars	ten billion dollars	$\$15.83\times10^9$	Gross Domestic Product of Iceland
			$\$45\times10^9$	cost of the high-speed train from San Francisco to Los Angeles, the route for which is to be constructed by the California High-Speed Rail Authority [1]
			$\$55\times10^9$	cost of a manned mission to Mars with a crew of four astronauts (cost would be spread out over ten years) using Robert Zubrin's Mars Direct plan [2]
			$\$62\times10^9$	fortune of Warren Buffett, world's richest man, as of 2008 [2]
10^{11}	one hundred milliard dollars	one hundred billion dollars	$\$100\times10^9$	budget for reconstruction of Iraq
			$\$100\times10^9$	total cost of the International Space Station [3]
			$\$169\times10^9$	total value of all real estate in Manhattan [4]
			$\$236\times10^9$	Gross Domestic Product of Greece (CIA World Factbook)
			$\$420\times10^9$	approximate United States budget deficit
			$\$972\times10^9$	total cost so far (as of March 2010) of the Iraq War and the War in Afghanistan ($712 billion for the Iraq War and $260 billion for the War in Afghanistan) [5]
10^{12}	one billion dollars	one trillion dollars	$\$1.26\times10^{12}$	total value of all real estate in Florida [6]
			$\$2.5\times10^{12}$	approximate United States annual federal budget as of 2005
			$\$9.06\times10^{12}$	United States national debt as of October 2007 [3]
10^{13}	ten billion dollars	ten trillion dollars	$\$12.39\times10^{12}$	United States GDP (PPP) as of 2005 [4]
			$\$42.7\times10^{12}$	total wealth of all 10.9 million rich people (defined as those with $1 million or more of investable assets) in

				the world (27% of which are women) as of 2010[7]
			$\$53.5 \times 10^{12}$	total of all private household net worth in the United States as of Sep. 2009[8]
			$\$55 \times 10^{12}$	global GDP (PPP)
			$\$62 \times 10^{12}$	value of all real estate in the developed countries (includes $48 trillion residential real estate and $14 trillion commercial real estate) as of 2002 [9]
10^{14}	one hundred billion dollars	one hundred trillion dollars	$\$140 \times 10^{12}$	total value of all world financial assets [10]
			$\$510 \times 10^{12}$	total world derivative contracts as of June 2007 [5]