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Fractions: Building a Strong Foundation Based on Conceptual Understanding

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Introduction

In preparation for my work developing this curriculum unit, I spent three days learning about Singapore Math at Worcester State University in Worcester, Massachusetts. My interest and belief in the way math instruction is delivered in Singapore stems from the fact that Singapore is number one in the world in mathematics. A novel idea I discovered at the conference is that there is nothing Singaporean about Singapore math. The instruction is simply based on philosophies and best practices from educators such as Jerome Bruner and Zoltan Dienes. Singapore math uses the CPA Approach, which stands for conceptual, pictorial, and algorithm. The idea is to develop a visual concept of the skill, carefully match the skill to pictorial representation, and finally move to the algorithm. The ideas and strategies taught through Singapore math are designed to develop students' ability to visualize and generalize. Currently, students tend to learn algorithms that are based on memorization. By engaging my students in a series of hands-on and visual lessons, the students should learn fractions in a meaningful way that will make calculating and problem solving with fractions more meaningful and lasting.

Rationale

Fractions are typically a difficult concept for students. Fractions are particularly challenging for fourth graders for several reasons. Fourth graders not only have to conceptualize fractions, but also calculate and solve word problems using them. Working with fractions requires knowledge of multiplication and division. These operations are relatively new skills in fourth grade.

Additionally, solving simple word problems with whole numbers is a weak area for most students. Accuracy plays a role in being successful. Fourth graders' basic skills are still developing, so students tend to make frequent errors. The rigor of problem solving increases when the problems are multistep, involve multiplication or division, or fractions. Sometimes in fourth grade, students are faced with problems involving all of these skills. There are many factors that contribute to great effort needed to become successful with fractions.

Developing the students' confidence and even an affinity for working with fractions is an especially challenging task for the teacher. For many students, fourth grade provides the first experience to compare and compute with fractions. A bad experience can affect students negatively for a long time or possibly for the rest of their lives. Also, an inadequate foundation and understanding of rational numbers can hinder students' ability to understand higher mathematical and scientific concepts. Therefore it is crucial for students to develop a robust understanding of rational numbers at an early age.

My curriculum unit is designed to teach myself, other teachers and most importantly the students ways to approach fractions without relying on memorizing the "rules" that are typically taught in the United States. The curriculum unit is designed to develop deep, conceptual knowledge and lead the students to discover the algorithms themselves. Developing a strong foundation and understanding of rational numbers is an important component of the curriculum unit. Students typically extend their knowledge of whole numbers to rational numbers. They think of multiplication as repeated addition and generalize that multiplication always makes the product larger. Similarly, students think of division as a model of sharing and incorrectly assume that division always leads to a smaller answer. Both of these ideas only hold true for whole numbers. According to Susan J. Lamon, "In the world of rational numbers, both of these models are defective" [1] . I plan to create a curriculum unit that is not limited by a few defective models, but full of a variety of models to build my students' conceptual understanding of rational numbers.

Background

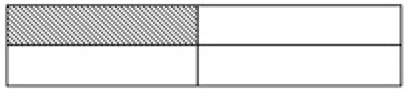
Models of Fractions

Students need to have many opportunities to experience many different interpretations of fractions in order to develop a robust, mature understanding. A deep understanding will provide a solid foundation. There are several different ways to model fractions, and it is important to expose the students to all of them in a variety of ways. Variation is the key to creating a robust understanding and flexible thinking.

Students are typically taught about fractions with part-whole comparisons. The part-whole comparison is just one interpretation of fractions. If students are only exposed to one or two models they will develop a limited understanding and will incorrectly generalize that fractions can only be represented in those one or two ways. The different types of models are listed below.

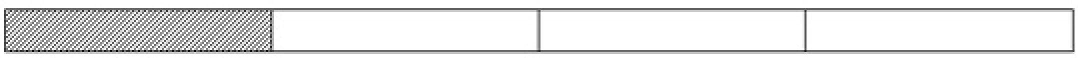
Area Models

Area models can be depicted with squares, rectangles, circles, or liquid volume. Singapore Math recommends using squares and rectangles since they are easier for children to draw. Circles are a little more complicated especially when representing thirds and fifths. The rectangle below is an example of an area model depicting the fraction ¼ by the shaded region.



Linear or Measurement Models

The linear or measurement models include number lines, bars, rulers and scales. Often the fractional parts are not clearly identified on rulers and scales. When using number lines, it is important to emphasize that the unit is the linear distance of the unit and not the point indicating the interval. One example of a linear or measurement model is the bar below, also depicting the fraction 1/4 by the shaded region:



Set Models

Set models are a collection of objects. Fractions can be depicted using set models in many ways. For example, the fraction 1/3 can be shown by beginning with 15 soda bottles, five of which are root beer, so that one third of them are root beer. The collection of shapes below is another example of a set model.

In this collection, 3/8 of the shapes are squares, 1/4 are triangles, and so on.

Constructs of Fractions

Fractions are a difficult concept for children to fully grasp because fractions are multifaceted. In addition to the different types of models there are also many different uses for fractions. Five interrelated interpretations, or constructs have been identified. The five constructs are part-whole, measure, operator, quotient and ratio and rate. [2] Part-whole comparisons are the most dominant construct in education, and unfortunately this construct is limited and leads to misconceptions. Fractions as measures are important and are often illustrated using a number line. When fractions are used as operators, they enlarge or reduce the size of a number or an object. An example of operators would be two times or one half of a whole. Symbolically, fractions represent a quotient. When a fraction is written such as 1/3, the midline means divide. Representing fractions as ratios and rates is another type of construct. A ratio compares two quantities of the same type such as two out of three of the students. A rate compares two unlike quantities as in miles per hour. The many interpretations of fractions demonstrate the underlying complexity that is too often simply represented with a pie.

Division in Fractions

Division is the common thread that runs through all of the constructs. Understanding fractions as division is essential to teaching students to develop a robust understanding of fractions. The operation of division has two distinct models: equal share and partitioning. An example of an equal share is 15 toy cars that need to be put into three equal groups. The partitioning model could also be called the measurement model. A similar problem demonstrating the partitioning or measurement model would be to begin with 15 toy cars and ask students how many packages of five toy cars there are. Both models are equally important and need attention when teaching students how to work with fractions.

Objectives

The objectives of my curriculum unit is to develop the students' conceptual knowledge of the "unit", unit fractions, general fractions, comparing fractions, equivalent fractions, mixed numbers, improper fractions, and adding and subtracting fractions with like, related, and unlike denominators.

The "Unit"

The "unit" is essential in understanding a fraction. Often in an elementary mathematics textbook, a pie or a cookie is used to represent the unit. Students need to understand the unit and that the unit does not have to be one object. In dealing with fractions, it is always important to establish and clearly state what represents one. For example, if the smallest cube in base ten blocks is the unit then a "flat" is one hundred cubes. However, if the flat, or 100 cubes, is the unit, then one cube represents one hundredth.

Another important idea is that a unit changes in each new context. For example, if there are 24 cans of soda in a case and the unit is one can, then four units represents one sixth of the case. However, if there are 24 cans and the unit is two cans then two 2-packs represents one sixth. The above scenario models the need for students to be able to identify the unit in each context.

A real world example of identifying the "unit" occurs all the time in the grocery store. Which is a better deal, 18 ounces of cereal for $3.79 or 12 ounces of the same cereal for $2.88. By calculating the unit price, or the price per ounce, you can determine the best buy. The same problem could be solved by recognizing that 18 ounces is three 6 ounce units and 12 ounces is two 6 ounce units. Then the price could be determined by dividing $3.79 by three and $2.88 by two. This example demonstrates the benefit of thinking flexibly about units. This problem also shows the reasoning skills that will develop if students are provided opportunities to foster their unitizing ability. Providing students with many opportunities to develop their unitizing (chunking) skills will be an essential part of this unit. The curriculum unit will provide opportunities to unitize verbally and visually. By teaching my students to unitize and partition, finding common denominators will become a more innate and natural process for them.

Unit Fractions

The unit fraction is any fraction that has a numerator of one. For example, one third, one half, one fourth, and one eighteenth are examples of unit fractions. Mathematically if a whole is broken into d equal parts, then the unit fraction is $1/d$. Once unit fractions are introduced, I would provide the students with many different types of models of the unit fractions and many opportunities to work with unit fractions of the same unit. For example, I would want my students to contrast 1/2, 1/3, 1/4, 1/5, 1/6 and so on of the same unit so they could determine that as the denominator gets larger the unit is divided into more pieces and the size of the piece gets smaller. The converse is also true. As the denominator gets smaller, the size of the pieces gets larger assuming that the unit fractions are related to the same size unit.

Another important understanding of the unit fraction $1/d$ is that multiplication by $1/d$ equals division by d. Multiplication by a fraction results in a smaller product. This concept tends to be confusing to students because their limited experience with whole numbers leads them to believe that multiplication always makes numbers larger. However, if students understand that multiplication by $1/d$ is really division by d, then the fact that the product gets smaller actually makes sense. In my fourth grade math curriculum I am not required to teach multiplication or division of fractions, however after learning about Singapore math it is very clear that these concepts are an integral part of working with fractions at any level. Multiplication of fractions is a part of naming general fractions. Two thirds is two copies, or two times one third. Five eighths represents five times one eighth. When students convert an improper fraction to a mixed number, they are really dividing by a fraction. If I have two and one third and I want to change it to a mixed number I am really dividing two and one third into as many one third pieces as I can. The answer is 7/3. This is an example of the concept of division being embedded in the work we will do with fractions.

General Fractions

General fractions are related to unit fractions through multiplication. If you have multiple copies (say, n) of a unit fraction (1/d), you have a general fraction n/d. For example, if I have three copies of 1/*d*, and *d* is four, then I would have three fourths, or 3/4. Simply put, a general fraction is multiple copies of a unit fraction. I could also say that I have *c* copies of *1/d* or *c/d.* In this example *c* is called the numerator and *d* is called the denominator. The numerator tells the number of copies and the denominator tells the name of the pieces. You could also think of this in terms of adjectives and nouns. If you had three dogs, the three would be the adjective and the dogs would be the nouns. The adjective–noun model points to the principal that you can add like terms. The adjective-noun model removes some of the mystery and complication students have when dealing with fractions. Students often see a fraction and view it as two separate numbers instead of as one. By applying the adjective-noun technique to the fraction three fourths, the adjective would be three and the noun would be fourths. Thus, making the fraction three fourths seem less abstract. It also helps students to extend the concept of like terms to like denominators.

Equivalence

The next important idea is equivalence. There is more than one way to name a fraction. In fact, the same fraction can be named an infinite number of ways. It is important to understand equivalent fractions because just as with unitizing, sometimes it is helpful to "change the name" of a fraction in order to perform a calculation or to simplify a fraction. This is basically the notion of unitizing in different ways. If I had the fraction, one half, I could also call the fraction two fourths, three sixths, fifty hundredths, and an infinite number of names.

Tools

There are several tools that I plan to use to teach my unit. Paper folding, partitioning and number lines are the tools I have chosen to help my students develop a visual image of fractions in accordance with the Singaporean method.

Paper Folding

Singapore Math uses paper folding as a concrete manipulative to teach fraction concepts. The technique is kinesthetic and visual. Square and rectangular pieces of paper are the preferred shapes. An example is folding a square piece of paper into four equal parts with the same area. One part would represent one fourth. Students could share the different ways that they folded the square into equal parts. Sharing the variations helps students to deepen their knowledge. Then the students can name two parts as two fourths. Paper folding is one tool that will be used to teach the objectives of my unit.

Partitioning

Partitioning is another tool I will incorporate. Partitioning is the process of dividing an object or objects into more parts. The parts should not overlap. All of the object or objects should be contained within the parts, meaning no part should be left over or unused. When working with fractions, these parts must be equal in area. Developing the skill of partitioning will take time to develop in children. Students will begin with simple partitioning activities with concrete objects such as paper folding. The second step in the CPA (concrete, pictorial, algorithm) Approach is for students to partition pictorial representations. For example, students may subdivide each half of a picture into three equal parts. Then the smaller part would represent sixths. Susan Lamon describes the importance of partitioning in this way, "Partitioning is fundamental to the production of quantity, to mathematical concepts, and to reasoning and operations." [3] Over time, more complex partitioning activities will be taught. Two goals of the partitioning activities are 1) to develop a deep understanding of the concept of equivalence and 2) to develop flexible thinking about units. These strategies will build the students' fundamental concepts in a meaningful way, as opposed to memorizing algorithms and rules that are easily forgotten.

Number Lines

I plan to provide my students the opportunity to work with number lines. A number line is built by prescribing an origin, a unit and a positive direction. It is important for the teacher and the students to understand that a number line is a metric entity, visually showing the distance or length from the origin. All of the numbers on the number line refer to a unit and as you move away from the origin you work with multiples of the unit distance. Negative numbers are placed to the left of the origin, although we will restrict ourselves to positive numbers and fractions in this unit. For example, 2/4 represents two copies of the distance 1/4 and it is twice the distance from the origin, zero. Likewise, 3/4 would represent three copies of 1/4 and would be three times the distance from zero.

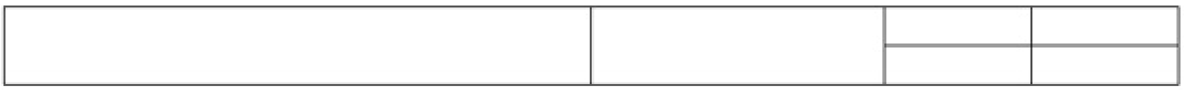
I would begin by having the students "count" by one fourth on the number line. Using the number line above, I would have the students count aloud, "one fourth, two fourths, three fourths..." I would build this skill using different unit fractions such as ½, 1/3, and 1/5. In addition to counting orally, it is important for students to label number lines in the same fractional increments. These exercises help students to understand that the numerator "counts" the multiples of the unit. It is important to have the students go past one whole, so that they develop the conceptual understanding that fractions can be greater than one whole.

Another important use of number lines is to develop a rich understanding of the size of unit fractions and general fractions. It is very important to keep the size of the whole unit constant when students compare the sizes of different fractions. A common problem is that textbooks represent fractions with so many different pictures that it is difficult for students to build a clear concept of the comparable size of fractions. Therefore, the students must develop a meaningful sense of the relative size of the fractions in order to create a strong mathematical foundation.

Strategies

The "Unit"

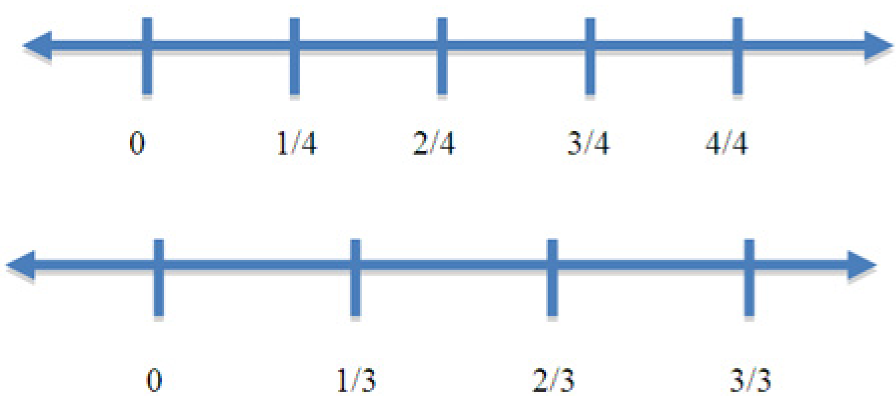
Understanding the unit is also an important part of the process of unitizing or chunking.



Using the image above, I would want my students to be able to identify or "see" in the image one half, one fourth, and one eighth. Developing the students' ability to unitize will help them to be more flexible in their thinking and recognition of the unit. Once the students were skilled in this concept I could provide them with some more challenging unitizing pictures. There are many great examples in Susan Lamon's book that I have referenced in the bibliography. I could also vary this activity by providing the fraction sometimes and other times asking an open-ended question about the different units or chunks that the image could be seen in the image. For example, given a unitizing image, I might ask students "who can see a chunk? Can you show the class the chunk you found and name it? Who sees another chunk? Are there any more chunks?" By providing a more open-ended question, the students have to apply their knowledge of fractions and unitizing more deeply than when a fraction is given.

Unit Fractions

I would model unit fractions with fraction strips, circular fraction pieces, a collection of objects, and a number line. Once students have practiced counting on the number line, I would then move into comparing different size fractions on separate number lines. It is important to keep the size of one whole unit consistent, so students can build their conceptual knowledge of the relative size of fractions. By comparing the number lines below, students can see that one third is larger than one fourth.



The number line is a particularly valuable model since it encourages students to think about the linear representation. Providing my students with different visual representations of fractions will help to develop the flexibility in which they are able to think about fractions.

Improper fractions will be included in all of the different models from the start, so the students understand that fractions can be bigger than one whole. Counting out loud and numbering number lines using different unit fractions is important to help students understand how multiple copies of the unit fraction work.

General Fractions

It is important for students to understand unit fractions before moving on to general fractions. Students need to have a good grasp of the role the denominator plays in understanding the relative size of unit fractions. This understanding is the starting point for conceptualizing the relative size of general fractions.

There are several key concepts and strategies to help students compare the relative size of fractions. The numerator tells the number of copies and the denominator tells the size of the part. The simplest types of fractions to compare are those with the same denominators. In that case, students need only to look at the numerators to compare. For example 4/8 and 6/8, the fact that these two fractions have the same denominators means that the size of the pieces, or the unit, is the same. Thus, they just need to compare four pieces to six pieces, and so can conclude that 6/8 is bigger than 4/8. It is important for the students to be comfortable with this concept.

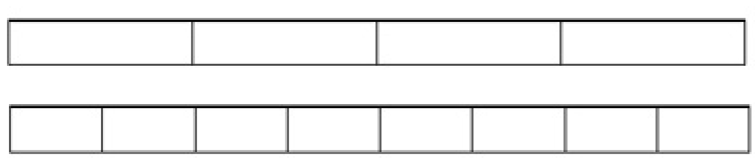
The next simplest types of fractions to compare are those whose numerators are the same. For example with 5/6 and 5/12, both numerators are five. Now the students need to look to the denominator. Remember, the denominator tells the size of the pieces. Therefore, a larger number in the denominator means the size of each piece is smaller. So 5/6 is bigger than 5/12 because I have the same number of pieces and sixths are bigger.

Another key concept is for students to compare a fraction to one half or one. If one fraction is larger than one half and the other fraction is less than one half, then it is very easy to determine which is bigger. Take for example 2/6 and 8/12. Since 2/6 is less than 1/2 and 8/12 is greater than one half, it follows that 2/6 is less than 8/12. However, if both fractions are either smaller than one half or larger than one half, then the students must consider the part size. For example if I am comparing 5/6 and 7/8, both fractions are greater than one half. I notice that both fractions are missing one piece. Now I must look at the part size. I know that sixths are larger than eighths. I can determine that 5/6 is less than 7/8.

As I am working with my students, I know that they must grasp these concepts to really develop a strong understanding of what fractions represent. As I teach I must keep these notions in mind and give my students many hands-on opportunities so they can develop this type of deep understanding.

Equivalence

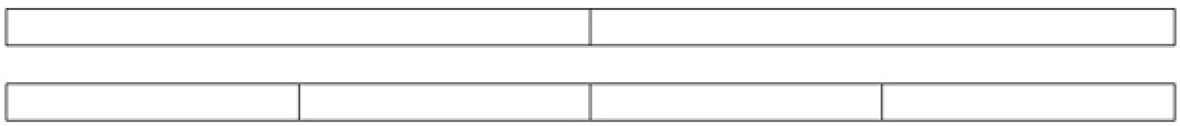
Again I plan to allow the students to explore this concept primarily through paper folding activities based on the Singapore Math model. An example of this strategy would be to give each student a square or rectangular piece of paper. I would ask the students to fold the paper into four equal parts. I would ask them to name the fraction that two of the parts represent. Then I would ask the students to further fold the paper to produce eight equal parts. Then I would ask them how many eighths is equal to two fourths.



I would provide my students with many opportunities for paper folding activities. I would be very careful to scaffold the paper folding gradually moving from easy to more challenging problems. An easier problem would be 2/4 equals 4/8. A more difficult problem would be 6/8 equals 9/12 because the answer is not as obvious. However, the answer can be figured out in the same way by folding paper or by drawing bars.

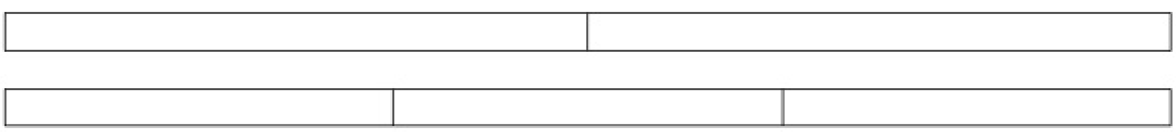
Partitioning strategies will also be used to develop the concept of equivalent fractions. For example, I would present the students with the bars below and ask them to partition them into equal parts.

Example A:



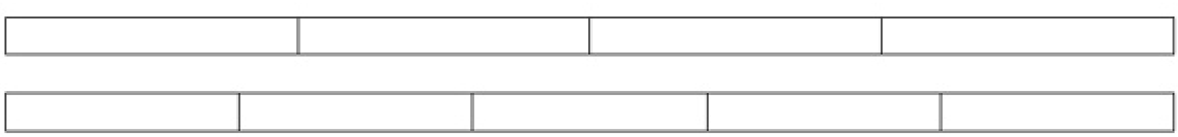
In example A, each segment of the first bar would need to be subdivided into two equal parts. The second bar would remain the same. I would again provide my students with multiple opportunities to partition. Initially the problems would be with concrete objects such as a strip of paper. Then I would move the students to a pictorial representation. I would make sure that the concrete paper strip matched the pictorial representation exactly.

Example B:



In example B, the problem depicts bars divided into halves and thirds. This would be a good introduction to partitioning both bars at the same time. I also would scaffold the example with questions to help the students. I might ask the students, "Can you see where one half would be on the bottom bar? Can you see where one third and two thirds would be on the top bar? Can you finish breaking each bar into equal parts? Into how many parts did you break each bar? Each bar would be partitioned into sixths.

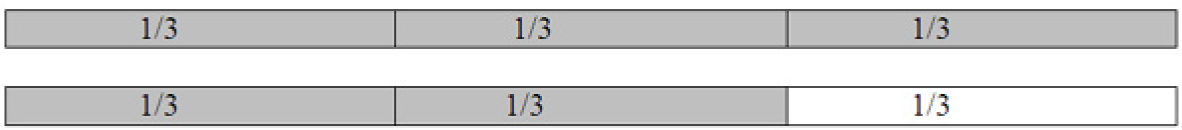
Example C:



In example C, the partitioning strategy is more challenging. Each segment of the first bar would have to be further subdivided into five equal parts. Each part of the second bar would have to be subdivided into four equal parts. Thus both bars will end up divided into twenty equal parts. This type of partitioning will lead to the students' conceptual development of common denominators. The same partitioning strategies are also used when solving word problems using Singapore bar models.

Mixed Numbers and Improper Fractions

The next task would be to teach the students mixed and improper numbers. To develop understanding I will use paper folding, square tiles, pictorial images, number lines, and Cuisenaire® rods. Cuisenaire® rods are wooden or plastic rod-shaped pieces of different lengths and color. The length of the rod ranges from one unit to ten units. Providing a variety of different contexts will enable the students to better generalize the information and apply it. While developing proficiency changing mixed numbers to improper fractions back and forth, students will also hone their skills adding unit fractions with like denominators. Each gray segment below represents one third. So if I add 1/3 +1/3 +1/3 +1/3 +1/3, the result is five thirds (5/3) or one and two thirds (1 2/3), both of which can be pointed out in the rods illustrated below. I would provide several examples that did not require simplifying.



A slightly more difficult problem would be to understand say, 6/4. Following with more difficult problems like this one will help scaffold students' learning and understanding. In this problem, the students would have to transform the improper fraction into a mixed number (1 2/4) and then simplify it (1 1/2). Again, several examples of this type would be required to build understanding.

Adding and Subtracting Fractions

The subsections below provide an outline of how to scaffold instruction for adding and subtracting fractions. The problems are carefully sequenced and should be taught in order. I recommend using paper folding, Cuisenaire® rods or fractions pieces as concrete materials. Rectangles and squares will work well for pictorial representations.

Adding and Subtracting with Like Denominators

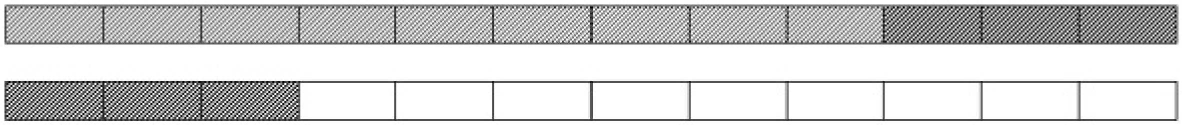
The first concept to develop in fraction addition and subtraction is adding and subtracting fractions with like denominators. Through the paper folding activities previously used with unit fractions, the students should already be adept in adding unit fractions. I would carefully sequence the problems that I used in class and begin with examples where the sum is less than one whole such as three eighths plus two eighths:

$$\frac{3}{8}+\frac{2}{8}=\frac{5}{8}$$

This problem requires straight addition without any simplification. The next type of problems would have a sum of less than one, but would require regrouping as in one sixth plus three sixths:

$$\frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

Finally, I would then move to a problem with a sum greater than one without regrouping such as three fourths plus two fourths. After providing several examples at this level, I would reveal the most difficult type of problem in this sequence. It would require the students to calculate a mixed number sum with regrouping. An example would be nine twelfths plus six twelfths, which can be illustrated using the bars below:



By looking at the shaded bars above, if you add the shaded segments the problem represented would be:

$$\frac{9}{12}+\frac{6}{12}=\frac{15}{12}=1\frac{3}{12}=1\frac{1}{4}$$

I could either work on subtraction problems of similar difficulty interchangeably with the addition, or I could work on addition first, and then scaffold the subtraction problems in the same manner. A sample sequence of subtractions problems would begin with a one step problem such as 5/6 - 4/6 = 1/6. The next problem would be slightly more difficult, requiring simplification, such as 7/8 - 3/8 = 4/8 = 1/2. An example of a challenging problem is 1 6/8 – 4/8 = 1 2/8 = 1 1/4. I would I believe either order would be effective. If addition and subtraction were handled in isolation I would provide some mixed practice for my students once both operations were covered.

Adding and Subtracting with Related Denominators

The next concept to tackle is adding and subtracting fractions with related denominators. In this type of problem while the denominators are different they are related in such a way that the smaller one can be transformed into the larger one with knowledge of equivalent fractions. In particular, the denominator of one fraction is a multiple of the other, such as is the case with 3/5 and 7/10. Once again, paper folding, pictorial representations and Cuisenaire rods will be used to model and depict the problems. Additionally, I would use the strategy of listing out equivalent fractions until a common denominator was discovered. Then the two fractions would have like denominators and could easily be added or subtracted together. I would begin with two unit fractions. In this type of problem, only one unit fraction would need to be transformed into a general fraction. An example is:

$$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$$

I would proceed to a similar problem requiring one of the fractions to be transformed, but that would also include simplifying the answer. Two sixths plus one third would fit nicely sequentially.

$$\frac{2}{6}+\frac{1}{3}=\frac{2}{6}+\frac{2}{6}=\frac{4}{6}=\frac{2}{3}$$

Once the students were successful with this level of problem, I would provide a similar problem, but where the answer was an improper fraction that had to be changed into a mixed number.

$$\frac{5}{8}+\frac{1}{2}=\frac{5}{8}+\frac{4}{8}=\frac{9}{8}=1\frac{1}{8}$$

Remember to begin with concrete modeling such as fraction pieces, followed by pictorial representations such as rectangles that match the fraction pieces. Finally, students should solve the algorithm using paper and pencil.

The last type of problem in the sequence would include an improper fraction that when changed into a mixed number required simplification.

$$\frac{7}{12}+\frac{9}{12}=\frac{16}{12}=1\frac{4}{12}=1\frac{1}{3}$$

Again, the problems would be carefully sequenced to gradually increase the complexity.

Adding and Subtracting Fractions with Unlike Denominators

The students should by now feel pretty comfortable with making like denominators. In the next series of problems, the denominators are not related. Therefore both fractions will have to be renamed with a common denominator. An example is:

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

I would provide the students with several similar examples in which both fractions had to be renamed, but further simplification was not required.

Then I would have the students solve similar problems that also required simplifying the answer. For example:

$$\frac{2}{4}+\frac{1}{6}=\frac{6}{12}+\frac{2}{12}=\frac{8}{12}=\frac{2}{3}$$

Again, it is important to provide several problems of this type. The next step would be to solve a problem where the sum was greater than one, but did not require simplification.

$$\frac{3}{4}+\frac{2}{3}=\frac{9}{12}+\frac{8}{12}=\frac{17}{12}=1\frac{5}{12}$$

The most difficult kind of problem will include changing both fractions to a common denominator, changing the resulting improper fraction to a mixed number and simplifying the answer. An example is:

$$\frac{3}{4}+\frac{3}{6}=\frac{9}{12}+\frac{6}{12}=\frac{15}{12}=1\frac{3}{12}=1\frac{1}{4}$$

Once again, providing your students with several problems of each type is very important for them to conceptual the process. With the many variations of problem types, it is important to develop a deep understanding, so the students can fully understand how to change denominators, when to change an improper fraction to a mixed number and when to simplify.

By carefully scaffolding the problems in a natural progression from easy to more challenging, the students will use their prior knowledge to help them find a solution. The structure of the unit is designed around the philosophy of Singapore Math. It is designed to ensure the success of struggling and average students, while offering opportunities to differentiate the problems for the high achieving students.

Activities

Below is a tentative overview of 21 days of math instruction. Staying on the schedule will depend on the students' abilities to grasp the skills. Certain skills may require more or less time.

I plan to develop fraction concepts by providing the students with unitizing and number line work throughout the year. I use homeroom time as an opportunity to review skills in all subjects. Several times per week I have my students complete a math review. It includes a variety of problems and really helps maintain and sharpen their math skills. I plan to include a unitizing picture and number lines regularly on these review sheets.

Day 1: The Unit

Day 2: Unit Fractions – paper folding

Day 3: Unit Fractions – number lines

Day 4: Unit Fractions – Cuisenaire rods

Day 5: General Fractions – paper folding

Day 6: General Fractions – comparing (same size parts and same number of parts)

Day 7: General Fractions – comparing (comparing to one half and one whole)

Day 8: Test

Day 9: Mixed Numbers and Improper Fractions – concrete square tiles

Day 10: Mixed Numbers and Improper Fractions – paper folding

Day 11: Equivalent Fractions – paper folding

Day 12: Equivalent Fractions – Cuisenaire rods

Day 13: Quiz

Day 14: Adding/Subtracting with like denominators – paper folding

Day 15: Adding/Subtracting with like denominators – pictorial

Day 16: Adding/Subtracting with related denominators – paper folding

Day 17: Adding/Subtracting with related denominators – pictorial

Day 18: Adding/Subtracting with unlike denominators – paper folding

Day 19: Adding/Subtracting with unlike denominators – paper folding

Day 20: Review

Day 21: Test

Activity 1:

The first activity is designed to use fraction strips to compare general fractions. This would be day 5 of the unit outlined above. The activity is designed to help students understand how to compare fractions. The key concepts include when the denominators are the same, when the numerators are the same and how to compare fractions to one half and one.

The students will use fraction strips to complete this activity. I would prepare a set of fractions strips for each student (see appendix 1). (I will include an extra strip of thirds for each student.) For this activity I recommend keeping the unit fractions attached. Students could also fold rectangular strips of paper to make their own representations.

Throughout this activity, I will present students with problems that increase in difficulty according to the scaffolding laid out previously in this curriculum unit. I would begin with some examples involving fractions with the same denominators. For example, I might ask students to compare 3/8 and 5/8. The students would find their strip of eighths and compare three to five. I would have the students share how they found 3/8 and 5/8 and model it on the board with fraction strips. I would ask, "Which is bigger? Do you agree? How do you know?" Then I would give another slightly more difficult problem with like denominators, such as the problem of comparing 4/6 and 2/6. I might ask, "Which strip do I need? Who can help me model 4/6? Who can help me model 2/6? Are these fractions modeled correctly? Which one is smaller?" The correct model would use the sixths strip with four sixths shaded. The second strip would also be sixths, but only two would be shaded. The students should recognize that four sixths (4/6) is greater than two sixths (2/6). You could also include the greater than (>), less than () and equal to (=) signs if you would like to further emphasize the symbolic aspects at play. I would give one more difficult example such as comparing 7/12 and 9/12. Again I would have students model this independently and then have the class review this at the board. I would ask questions such as, "How do I show 7/12? 9/12? Which is bigger? How do you know?"

The next part of this activity involves comparing fractions with like numerators. In a similar way to the exercises described above, I would ask students to work with easier to compare fractions such as 2/3 and 2/5, and ask questions such as "What strips do I need? Can you find 2/3? Can you find 2/5? Which one is smaller?" I would have students model on the board how they depicted each fraction and how they reached their answer. Next, I would move to a more difficult type of problem and ask them to compare 7/10 and 7/8. I might ask, "Which strips did I need? How many tenths do I have? Do I need another strip? Yes. Which one? Eighths. How many eighths do I need? Which one is bigger? 7/8. Is it correct? Yes." Finally, I would move on to an even more advanced type of comparison, asking them to compare 4/3 and 4/6. (You may need to have extra strips of thirds available.) Again, I would ask questions like, "How many thirds do I need? 4. Do I have enough on my strip? No. What should I do? Get another strip of thirds. How many sixths do I need? 4. Is it modeled correctly? Yes. Which is bigger? 4/3." After this, I will have the students complete the worksheet in Appendix 2 at their desks.

Activity 2:

This second activity corresponds to day 6 of the unit described above. The activity continues to explore the different strategies used to compare general fractions. For this activity the students will need a set of fraction strips, as in the previous activity. For the first group of problems the students will compare each fraction to one half. In these problems, two fractions are given to students, chosen so that one fraction is greater than one half, and the other is less than one half. I would have the students find their halves strip and keep it out for the first few problems. For example, I might say, "Let's compare 4/9 and 6/10. How many ninths do I need? 4." I would then model 4/9 on the board using fraction strips and show the students the 1/2 strip. I would ask, "Is 4/9 greater than or less than 1/2?" and then model the comparison of 4/9 to 1/2 on the board. I would continue to ask questions such as, "Do you agree? Yes. How many tenths do I need? Six. Is 6/10 modeled correctly? Yes." Now using the 1/2 strip, I would ask, "Is 6/10 greater than or less than 1/2? Greater than. Do you agree? Yes." I would follow with a slightly more advanced problem by having the students compare 4/10 and 5/6. Students will work with the fraction strips at their desks. I would probe, "Can you see 4/10? Can you see 1/2? Is 4/10 greater than or less than 1/2? Can you see 5/6? How does 5/6 compare to 1/2? So which fraction is greater?"

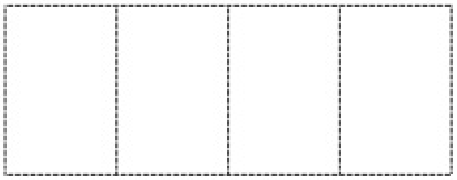
Now I need some help with this next problem because I do not understand it. Compare 4/10 and 2/6. (Continue modeling the problem on the board with fractions strips as students work with the strips at their desks.) I would say, "Is 4/10 greater than or less than 1/2? Less than. Is 2/6 greater than or less than 1/2? Less than. So how can I tell which one is greater? Line up the strips. Does anyone have any other ideas? See which fraction is closer to 1/2. How many tenths equals 1/2? Five. How many more tenths do I need to make 1/2? One. How many sixths do I need to make 1/2? Three. How many more sixths would I need to make 1/2? One. Do you know how I can decide which fraction is bigger now? Compare the part size. Are sixths or tenths smaller? Tenths. Do you agree? Yes. So which fraction is closer to 1/2? 4/10. Do you agree? Yes."

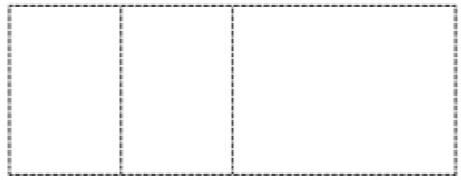
Now compare 2/8 and 4/10. (Continue modeling the problem on the board with fraction strips while students work with their strips at their desks.) Ask the students, "Is 2/8 greater than or less than 1/2? Is 4/10 greater than or less than 1/2? Can I tell which is larger yet? How many eighths equals 1/2? How many more eighths would I need? How many tenths equals 1/2? How many more tenths would I need to make 1/2? Are eighths or tenths smaller? So if I need two more eighths or one more tenth, can you see which one is closer to 1/2?"

Now have the students compare 5/6 and 3/4. (Continue modeling the problem on the board with fraction strips while students work with their strips at their desks.) I would ask, "Is 5/6 greater than or less than 1/2? Is 3/4 greater than or less than 1/2? Do you agree that they are both greater than 1/2? Should I compare them to 1/2 or is there another number to which I can compare them? Do you agree that I can compare them to one whole? How close is 5/6 to one whole? How close is 3/4 to one whole? Do you agree that I need one more unit of each to make one whole? Are sixths or fourths smaller units? So is 5/6 or 3/4 closer to one whole?" Now I would have the students complete Appendix 3 at their desks.

Activity 3:

The third activity is designed to develop the concept of equivalent fractions. It will utilize paper folding as a method of partitioning. Give the students a rectangular piece of paper. Establish that an identical rectangle represents one whole. Ask the students to fold it to make halves. The students may fold the paper different ways as long as the two halves have equal area. Share the different ways the students created their halves by drawing a similar representation on the board. Then ask the students to identify 1/2. Tell the students to now fold the paper again to make fourths. Again share the different ways the students made fourths. Then ask the students how many fourths are equal to one half. They should tell you two. How many fourths would it take to make one whole? They should tell you four. Ask your students to work with their neighbor and show you how many fourths there are in 3/2. They should tell you six, as illustrated with the following paper folds:



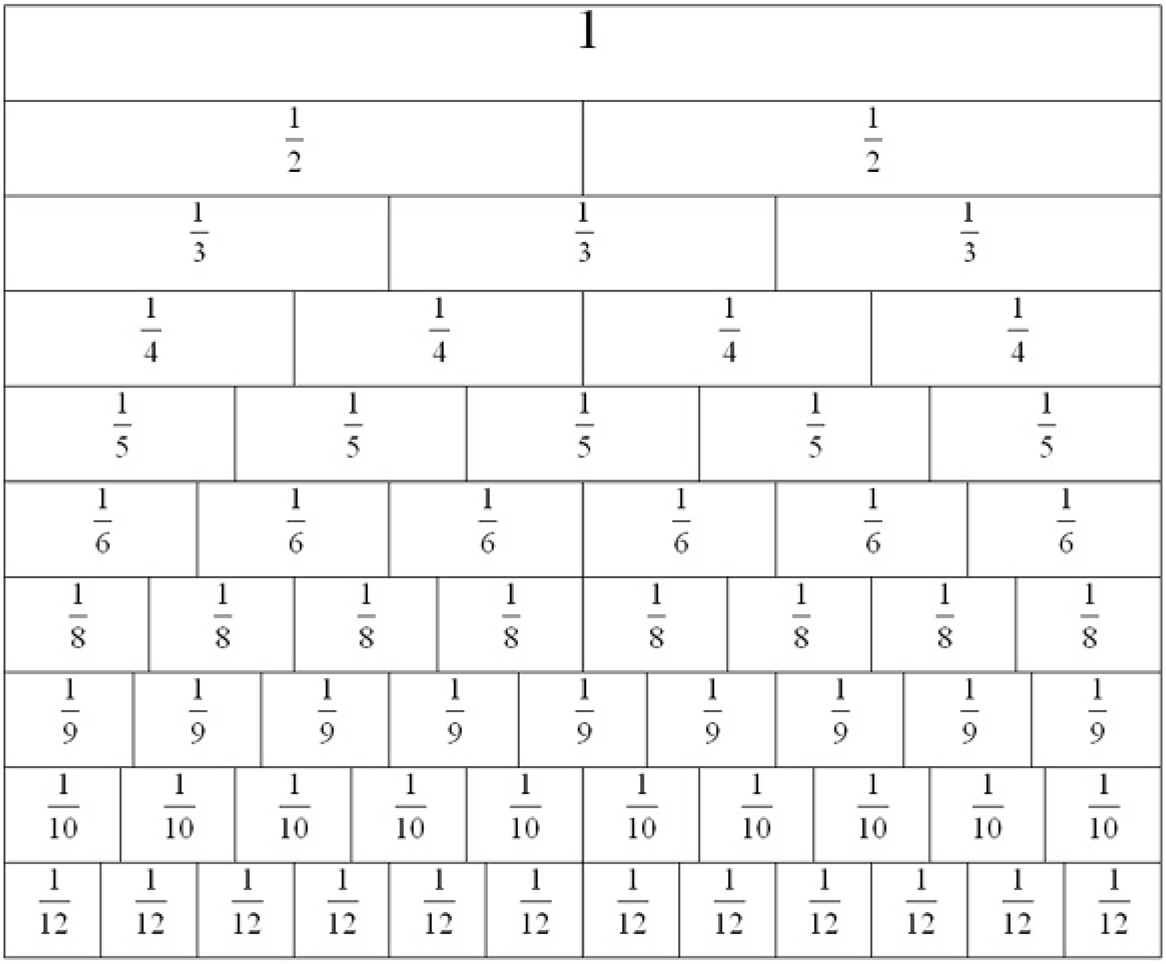


The problem mentioned and modeled above is important to familiarize your students with the notion that fractions can be larger than one whole. A good way to accomplish this goal is to include a problem or two that is greater than one whole as you work through all of the different concepts with fractions.

Next I would ask the students to work independently again with their own rectangle. I would ask them to show me how many eighths are in one fourth. The students would have to fold their rectangle again. They should show you two. I would check for understanding by asking, "How many eighths are in two fourths? How many eighths are in three fourths?" Then I would give the students a copy of the worksheet in Appendix 4. The students should be able to move from the concrete materials to a pictorial representation. Notice how the pictures in the exercise match the rectangular squares used in the lesson. Different materials can be used on different days, but within a lesson be consistent with the models and representations. Notice at the bottom of the worksheet, that the students are challenged to determine the algorithm on their own.

Appendices

Appendix 1: Fraction Strips


1
1/2 1/2
1/3 1/3 1/3
1/4 1/4 1/4 1/4
1/5 1/5 1/5 1/5 1/5
1/6 1/6 1/6 1/6 1/6 1/6
1/8 1/8 1/8 1/8 1/8 1/8 1/8 1/8
1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9 1/9
1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10 1/10
1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12 1/12

Appendix 2:

Name______________________________ Date ______________________

Shade each fraction strip to show the fraction. Then compare each pair of fractions. Circle the bigger fraction.

1) $\frac{5}{6}$

$\frac{2}{6}$

2) $\frac{4}{8}$

$\frac{6}{8}$

3) How you can tell which fraction is bigger when the denominators are the same?

4) $\frac{4}{10}$

$\frac{4}{6}$

5) $\frac{5}{8}$

$\frac{5}{6}$

6) How you can tell which fraction is bigger when the numerators are the same?

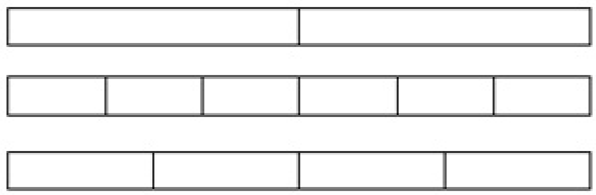
Appendix 3:

Name ______________________________ Date ____________________

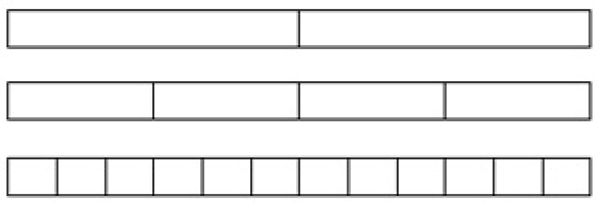
Equivalent Fractions
Activity #2

Compare each fraction to ½ then circle the greater fraction.

1) $\frac{2}{6}$ and $\frac{3}{4}$

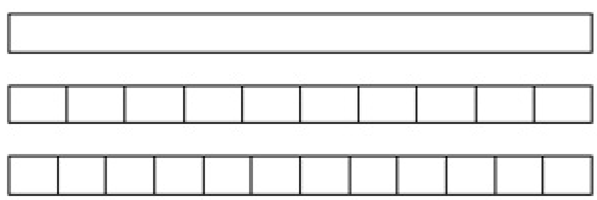


2) $\frac{1}{4}$ and $\frac{5}{12}$



Which fraction is closer to $\frac{1}{2}$? Explain how you know.

3) $\frac{8}{10}$ and $\frac{10}{12}$

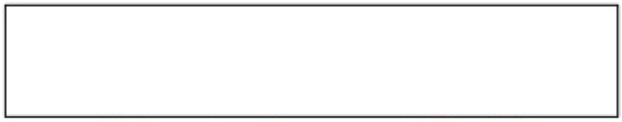


Which fraction is closer to one whole? Explain how you know.

Appendix 4:

Name ______________________________ Date ____________________

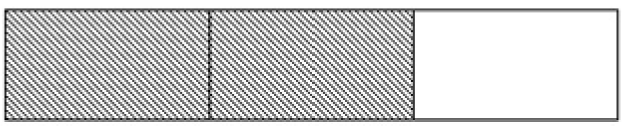
Equivalent Fractions
Activity #3



This rectangle represents 1 whole.

Now look at the rectangle below. Name the shaded part of the rectangle. _______

Look at the rectangle below. Name the shaded rectangle. ________

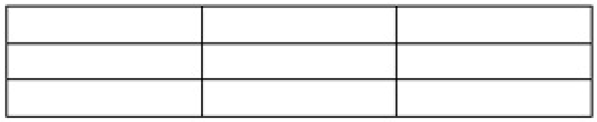


Now using the rectangle above, draw a line to divide the rectangle into six equal parts.

How many sixths equal $\frac{1}{3}$? _______ How many sixths equal $\frac{3}{3}$? _______

How many sixths equal $\frac{2}{3}$? _______ How many sixths equal $\frac{4}{3}$? _______

Now look at the rectangle below that is divided into nine equal parts.



How many ninths would equal $\frac{1}{3}$? ______ How many ninths would equal $\frac{2}{3}$? ________

How many ninths would equal $\frac{3}{3}$? ______ How many ninths would equal $\frac{4}{3}$? ________

Explain how you can figure out how many twelfths equal $\frac{1}{3}$? $\frac{2}{3}$?

____

____

____

____

Appendix 5: Implementing District Standards

Below are the state standards that directly correlate to my curriculum unit. My unit is designed to deeply develop conceptual knowledge of fractions, some of which directly correlate to the state standards from grades 2 and 3. By developing a strong conceptual foundation, the goal is that the students will truly understand fractions in a way that enables them to work with fractions flexibly in many contexts. Below are the standards included in my curriculum unit.

Grade 2 Mathematics Standards

2.3 The student will

a) identify the parts of a set and/or region that represent fractions for halves, thirds, fourths, sixths, eighths, and tenths;

b) write the fractions; and

c) compare the unit fractions for halves, thirds, fourths, sixths, eighths, and tenths.

Grade 3 Mathematics Standards

3.3 The student will

a) name and write fractions (including mixed numbers) represented by a model;

b) model fractions (including mixed numbers) and write the fractions' names; and

c) compare fractions having like and unlike denominators, using words and symbols (>, , or =).

3.7 The student will add and subtract proper fractions having like denominatorsof 12 or less.

Grade 4 Mathematics Standards

4.2 The student will

a) compare and order fractions and mixed numbers;

b) represent equivalent fractions; and

c) identify the division statement that represents a fraction.

4.5 The student will

b) add and subtract fractions having like and unlike denominators that are limited to 2, 3, 4, 5, 6, 8, 10, and 12, and simplify the resulting fractions, using common multiples and factors;

Endnotes

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1. 1. Susan J. Lamon *Teaching fractions and ratios for understanding essential content knowledge and instructional strategies for teachers*. (Mahwah, N.J.: Erlbaum, 1999), 19
2. 2. Charalambos Y. Charalambous, and Demetra Pitta-Pantazi. "Drawing on a theoretical model to study students' understanding of fractions." *Educational Studies in Mathematics* 64, no. 3 (2007), http://www.JSTOR.org/stable/27822662 (accessed July 17, 2011). 295
3. 3. Lamon, 90.

Annotated Bibliography

Beckmann, Sybilla. "Fractions." In *Mathematics for elementary teachers with activity manual* . 3rd ed. Boston: Pearson Addison Wesley, 2011. 39-78.This book is an excellent resource. It provides a very thorough background information.

Charalambous, Charalambos, and Demetra Pitta-Pantazi. "Drawing on a theoretical model to study students' understanding of fractions." *Educational Studies in Mathematics* 64, no. 3 (2007): 293-316. http://www.JSTOR.org/stable/27822662 (accessed July 17, 2011). The information about the subconstructs of fractions was useful. Overall, I find studies to be difficult and not very user friendly.

Edge, Douglas. "Teaching Fractions." In *Teaching Primary School Mathematics*. Singapore: McGraw Hill, 2007. 130-152. This book is extremely valuable especially if you are not familiar with Singapore Math.

Lamon, Susan J.. *Teaching fractions and ratios for understanding essential content knowledge and instructional strategies for teachers*. Mahwah, N.J.: Erlbaum, 1999. The strategies and ideas presented in this book were very helpful and offered insight into teaching young students about ratios and relational thinking.

Parker, Thomas H., and Scott Baldridge. *Elementary mathematics for teachers*. Okemos, Mich.: Sefton-Ash ;, 2004. This book is a very useful resource. The information is presented in a way that is easy to understand.

Seto, Cynthia, and Wee Leng Ng. *Teaching fraction, ratio and percentage effectively*. Singapore: Panpac Education Private Limited, 2010. The background information in the beginning is useful. This book comes with a CD and has a wealth of computer activities for the children. I have not used these yet.