Curriculum Units by Fellows of the National Initiative
2013 Volume V: Energy Sciences

Mathematics of Energy Efficiency: Use Less, Save More

Curriculum Unit 13.05.05, published September 2013
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Introduction

In the media and on the political stage, the main topic of discussion is renewable energy and its environmental and economic benefits. There is such a great urgency to switch, and to switch now! So, why doesn't our society make the leap and switch from non-renewable to renewable energy sources? Well, it's not that easy. To fully understand requires a deeper knowledge of and exposure to what are renewables and what is their overall impact to our communities. Thus, it is our job as educators to be informed and to provide awareness of non-renewable and renewable energy options early and often to our students. Gaining awareness of the options gives our students the ability to make conscious decisions about their role in the local/global effort to be environmentally friendly. In this unit, students will make real-world mathematical connections to the origin of our electricity, the impact of our use on the environment, the potential of wind energy and biodiesel, and the evaluation of our energy practices. Through demonstrations and inquiry-based learning opportunities, students will acquire and retain mathematical concepts which include writing, solving and graphing linear equations and inequalities, systems of linear equations and inequalities, and direct and inverse relationships. Students will also analyze data statistically using box-and-whiskers plots, mean absolute and standard deviation, z-scores, and curve of best-fit.

Non-renewables are energy sources that can be used up like gasoline in our cars. Gasoline is made from petroleum, remains of creatures that lived many years ago and are now buried deep in the earth, subjected to high temperatures and pressures which caused physical and chemical changes, and for that reason it cannot be replenished. Examples of non-renewables include petroleum, coal, nuclear and natural gas. Renewables are energy sources that can replenish itself like wood for a fire. Wood comes from trees; new trees grow to replace those harvested, hence, it is renewable. Examples of renewables include biomass, wind, solar, geothermal, and hydropower. Each of these proposed renewable energy sources is expensive to implement and has unique environmental challenges. Combine this fact with no widespread infrastructure to use supposed sources and you arrive at the core reason why we have not made the leap and switch from non-renewables to renewables. Consequently, my students will come to the inescapable conclusion that they are limited to our current source of energy. However, by the end of the unit, perhaps my students will realize that with a few personal adjustments in habit we can use energy wisely and carefully through energy conservation, also known as the 'fifth fuel.' [1] Energy conservation is decreasing energy use, not depriving yourself of life's simple pleasures. However, according to Daniel Yergin, author of *The Quest: Energy, Security, and the*

Remaking of the Modern World, it is "applying greater intelligence to consumption, being more clever about how energy is used—using less for the same or greater effect." [2]

So, who has time for energy conservation? WE ALL DO!! My Algebra I students meet in 90 minute block classes and are mostly comprised of 9 th graders with a few 8 th and 10 th graders who will discover we can conserve with just a few minor adjustments. This unit will be implemented throughout the academic year following the Algebra I pacing chart established by Richmond Public Schools. At the culmination of this unit, my students, newly named Energy Inspectors, will conduct an audit of their home by making a list of items used in their home or on the go that use electricity and evaluate the cost of usage at a particular point in time and over a year's time, taking into account the time of day, year or season in which they are most likely to use these items. Then, they will write a proposal to their caregivers including a top ten list of ways to conserve energy.

Rationale

Currently, I am in my seventh year of teaching secondary mathematics, specifically Algebra I and II, to inner city youth at Franklin Military Academy (FMA), the nation's first public military school founded in 1980. FMA is a 6 th -12 th grade choice-school located in Richmond, Virginia. Students apply to FMA in the 6 th grade and again in the 9 th grade. Middle school students are not guaranteed a slot in the high school, but, are strongly encouraged to apply. Through an extensive application process, we accept children all over the city with various socioeconomic backgrounds and family dynamics. So, our students are neither the cream of the crop nor your child in need of strong discipline. A high percentage of our students are economically disadvantaged and/or are in circumstances in which their caregivers are incapable of helping with homework, researching, preparing for tests, college or the work force. In addition, FMA just concluded our 2 nd year of school improvement with an emphasis on mathematics. We are awaiting the final results of testing this year; however, the preliminary results suggest we continue to struggle in mathematics.

After reflection on my teaching practices, I plan to employ inquiry based learning activities more vigorously to increase engagement, foster critical thinking and, therefore, increase retention of mathematical concepts and skills. I hope to achieve this through interdisciplinary planning under the theme of energy. Some activities in this unit are repeated in a chemistry or physical science class; so, do not feel like you are stealing the science teacher's thunder. Most science teachers will appreciate the math teacher giving students a foundation of scientific concepts and principles in context. Also, to increase achievement across grade levels, my colleagues and I are having meaningful discussions about vertical articulation, early intervention, and professional development opportunities that encourage collaboration amongst mathematics and science teachers.

President Obama intends for energy efficiency to play a major role in cutting carbon pollution while keeping our economy strong. [3] Energy conservation is not free. It requires investment of time and money. The Obama administration's energy platform is focused on energy efficiency investments as a tool to strengthen our economy and environment. [4] As a result, we need individuals who are trained with the necessary knowledge and skills for a career in residential and commercial energy efficiency related occupations. This unit will expose my students to this sector of our economy. This year, I want the industry of energy efficiency to be considered as a possible career pathway.

Background

When I come home, the first thing I do is flip the light switch and my lights are on, like magic. Assuming I pay the electricity bill on time, what process allows the lights to come on upon the flip of a switch? Likewise, I am able to plug in one of my various appliances and I can bake a peach cobbler, wash my clothes, or watch television. I perform these activities without regard for where the electricity comes from or the impact of my use on the environment. For many adults and students, this is their reality. The background information will answer the question "Where does our electricity come from?" The focus is in on Virginia; however, you may use some of the same resources to find information specific to your state. Then, I will provide information about the environmental effects of coal mining and some renewable energy options namely biodiesel and wind. Finally, my research will look at reasons and ways to increase energy conservation practices related to electricity and fuel usage. While I go through the background, perhaps you may get some great ideas for lessons/activities you can do beyond what is presented in the lesson plans.

My seminar, Energy Sciences, helped me to understand the scientific and social factors associated with non-renewables and renewables. Yergin describes the evolution of renewable best when he says "it is one of innovation, entrepreneurial daring, political battles, controversy, disappointment and despair, recovery and luck." [5] While the campaign for renewables has restarted after somewhat of a hiatus, it is important to recognize all primary energy sources have positive and negative impacts on the environment. We can use algebraic reasoning to model, project and verify current concerns with both types of energy sources. From a social aspect, we can consider Dr. David MacKay's comment which cleverly outlines the realistic views of the British on renewables; I imagine the sentiments are similar in the U.S.:

> Wind: Not in my backyard! Wind farms? "No, they're ugly noisy things." Shallow offshore wind: Not near my birds! Offshore wind? "No, I'm more worried about the ugly power lines coming ashore than I was about a Nazi invasion." Biomass: food, biofuel, wood, landfill gas: Not in my countryside! [6]

How do Virginian's obtain their electricity?

First, let's define the terms energy, power and electricity. Energy is the ability to work. There are two types of energy: kinetic, also known as working energy, and potential, also known as stored energy. Energy is in everything and comes in different forms: heat (thermal), light (radiant), motion (kinetic), electrical, chemical, nuclear and gravitational. Power is the measure of energy used over time; power, P, varies directly with energy, E, and inversely with time, t, $P = E/t$. Electricity is the flow of an electrical power or charge through a conductor. It is actually a secondary source of energy or energy carrier because it is the result of the conversion of primary sources of energy such as coal, wind, solar energy, etc.

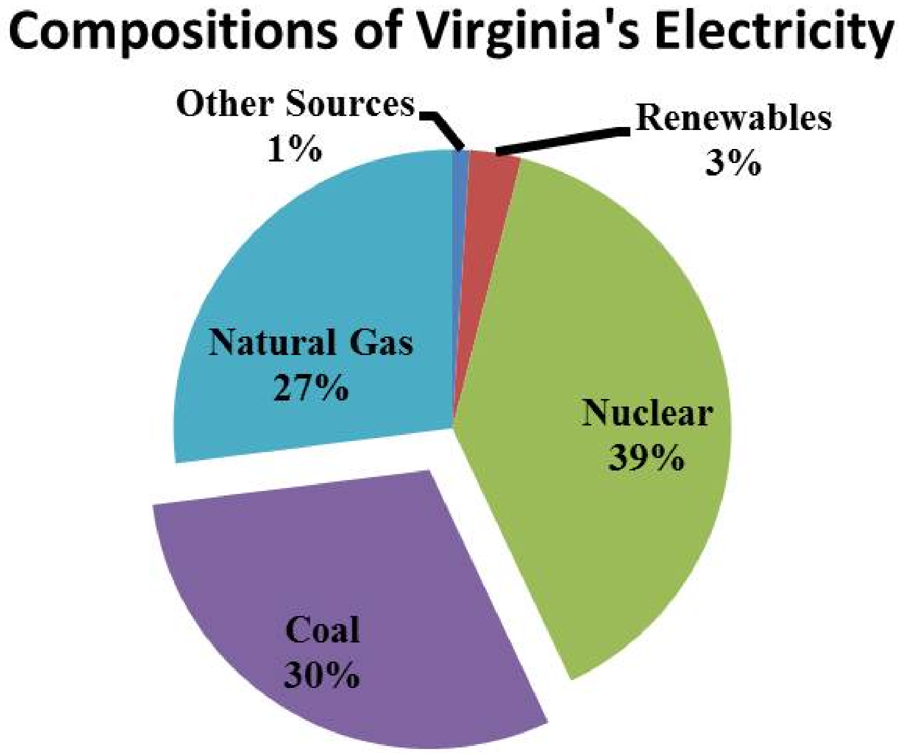


Figure 1. Breakdown of Virginia's energy sources [7]

Coal and natural gas make up 57% of the primary energy sources of Virginia's total electricity production. The rest is nuclear power and renewables. Virginian's pay an average retail price of 8.87 cents per kilowatt-hour and are ranked 23 [rd] lowest cost in the U.S. The unit will specifically look at the production and consumption of coal.

Dominion Virginia Power runs Chesterfield Power Station in Chesterfield, VA, about 15 miles south of Richmond on the James River. It is the largest fossil-fueled power station in the state and generates 1,600 megawatts (MW). The average daily consumption of coal is 8,400 tons. Nevertheless, Virginia imports more electricity than any other state except California and the demand keeps growing. [8]

Coal is plant material buried deep in the earth many years ago, subjected to high temperatures and pressures which caused physical and chemical changes, altering the plant material into peat and then into coal. Coal reserves are discovered through geological explorations which include creating a geological map of the area, conducting geochemical and geophysical surveys, and then, drilling. The area will become a mine if it is large enough and of sufficient quality so the coal can be retrieved at a reasonable cost. Coal is mined in two ways: surface (which includes mountain top mining) and underground mining (which includes strip mining). Once mined, coal is washed of its impurities, treated depending on its intended use and arrives at the power station by truck, train or ship.

So, how is coal, a primary energy source, processed to get electricity, a secondary energy source? It is placed into a hopper where it is crushed into small chunks and the conveyor belt moves the coal to the coal pile which holds a 30-day supply. From the coal pile, the conveyor belt moves the coal to the station and feeds it into a pulverizer that grinds the coal into fine pieces, like baby powder. Coal contains potential chemical energy and is combustible, which means it can catch on fire and burn easily. The powdered coal is blown into the combustion chamber of a boiler where it instantly combusts and releases kinetic thermal energy. The walls of the boiler are lined with tubes which contain purified water. The heat turns the water into high pressure steam which is used to turn the turbine, a massive drum with propeller like blades. This transforms kinetic thermal energy into kinetic motion energy. The turbine turns a magnet wrapped in copper coil to produce electricity in the generator, turning kinetic motion energy into kinetic electrical energy. Electricity is sent to a transformer that increases the voltage to 30,000-400,000 volts (like you increase water pressure). Before the power can be delivered to your homes, the electricity is lowered to safer voltage at an electrical substation where a transformer reduces the voltage to 100-250 volts (like you decrease water pressure). Finally, the electricity moves over distribution lines to your home. [9] Flip the light switch and the lights come on. Surprisingly, while living things are able to store energy from the sun, mankind has yet to find an efficient way to store electricity, so the national grid must balance supply and demand on a minute-by-minute basis.

Coal Mining Experiment

Coal mining companies are often the largest employers in the area which makes their existence essential to the families in the area. On the other hand, their existence means large areas of land are disturbed. This creates environmental challenges including soil erosion, dust, noise and water pollution, removal of all vegetation, and release of greenhouse gases such as methane and carbon dioxide. As a result, the coal industry is charged with rehabilitation, also called reclamation, of coal mine lands during and after use. Power companies are also charged with reducing release of pollutants and greenhouse gases such as such as mercury and sulfur when burning coal. New technologies are constantly developed to reduce the environmental impact of both coal mining and burning. [10]

Students will simulate mountain top coal mining and its effect on the surrounding land. They will use algebraic models to describe the effects of production and consumption of coal on our environment using greenhouse gas emissions data from the Energy Information Administration (EIA). Problems may contain conversions depending on how the data are presented.

Wind Energy: Building windmills

Wind is air in motion created by the unequal heating of the earth's surface by the sun's radiant energy. Think of the power exhibited by the wind speeds of a hurricane or a tornado. Imagine we are able to harness this energy, a renewable source, to power communities. In fact, this idea is not so novel. In the mid-west of the U.S., windmills are used to grind grain, pump water and provide electricity. The wind's velocity, which can be measured with an anemometer, is a factor in calculating the amount of electricity wind turbines can generate. Wind power, P, varies jointly with half of air density,P, area swept by the turbine blades, A, and the cube of the velocity, V, $P = (1/2)PV^3$. Air density is the mass per unit volume of earth's atmospheric gases and measures 1.25 kg/m^3 at sea level and 68 °F. Windmills are tall, and the air density decreases at higher altitudes and higher temperatures. The air density decreases by 3% for every 1000 additional feet in height and 1% for each additional degree in temperature. [11]

On June 25th, President Obama introduced his Climate Action Plan. The document lays out his vision to slow the effects of climate change. Wind energy plays a significant role in this endeavor. [12] The Department of Energy is planning to build a research facility off the Virginia coast near Virginia Beach, Va. The intent of the research facility is to test technologies such as remote sensing designed to determine the potential power of offshore winds. Researchers hope the results of their findings will lead to increased capital from investors by providing valid data, thus, increasing production of wind plants, also called wind farms. [13] Virginia has a strong wind resource off the coast. Preliminary research completed by Virginia Coastal Energy Research Consortium showed almost 10% of our electrical power can come from wind energy. Onshore, Dominion Virginia Power is currently developing the Bluestone River Wind Project in Tazewell County, southwest Virginia, which could produce nearly 80 MW. [14] If Dominion is successful, Bluestone River Wind Project will provide electricity to approximately 80,000 people, that is one megawatt per 1,000 people. [15]

Significant environmental benefits to wind energy include no release of harmful greenhouse gases and it does not interfere with agriculture. Another significant benefit is the ability to produce power in remote locations all over the world. There are generators transforming the wind's kinetic energy into electrical energy for people who otherwise would not have access to electricity. However, like coal production and consumption, there are negative environmental impacts of wind farms. For example, other than the coast, most of the windy areas in Virginia are in national parks, like Blue Ridge Parkway. [16] This is an obvious conflict of interest; after all, national parks were established "to conserve the scenery and the natural and historic objects and the wild life therein and to provide for the enjoyment of the same in such manner and by such means as will leave them unimpaired for the enjoyment of future generations." [17] Other impacts include the consistency of wind power, as it is not always windy which makes it difficult to implement into existing power grids (recall energy supply and demand must be balanced by the minute), the aesthetics to the landscape or ocean view, the harm to birds and bats, and, in some cases, the noise. In addition, there is a large transportation cost because the majority of wind turbines are mass-produced in Europe or in the Midwest. Hence, there is a large need for skilled workers to support the development of a wind energy supply in our region. [18]

Students will build a device that can measure wind speed and a windmill that can do work by picking up an item. Students will then calculate the amount of work and power needed to pick up the weight. Work is the measure of the weight lifted through height; Work, W, varies jointly with mass, m, acceleration of gravity, g, and height, h, $W = mgh$. Students will also calculate the wind energy created by a fan set on different speeds: low, medium and high.

Biodiesel: Alternative Fuel

Have you heard the story about the guy who ran his car on Canola oil? Too good to be true, right? Correct, that guy did a little more to the composition of the Canola oil than just putting it into his fuel tank. The simple chemistry involved using oil, methanol and sodium hydroxide. The product is biodiesel and glycerol which separates well with biodiesel on top and glycerol, a heavier substance, at the bottom of the container. Methanol and sodium hydroxide are dangerous substances. Glycerol is then removed because it will damage engines. Water is used to wash it out of the fuel, leaving biodiesel only. With no engine modifications, biodiesel can be used to power any diesel engine.

Biodiesel is a renewable fuel produced from vegetable oils, animal fats or recycled restaurant grease. In the early 1900's, Dr. Rudolf Diesel demonstrated the diesel engine and ran it with peanut oil. Diesel engines were marketed to farmers with the notion they could grow their own fuel. Indeed, farmers ran their large trucks, tractors and machinery on vegetable oil until the arrival of low cost and convenient petroleum diesel. Since 2004, West Point, VA has housed a biodiesel plant built by Pacific Biodiesel that uses soybeans made in Virginia. [19] Governments back the manufacturing of biofuel due to concerns about the environment and security of oil. Using biofuels instead of gasoline reduces carbon emissions, creates jobs and keeps money within the community. [20] While biodiesel is good for your vehicle because it provides increased fuel lubricity and extends the life of the engine, there are some concerns. Biodiesel gels like petroleum diesel does in cold temperatures. This can be avoided by purchasing biodiesel treated for winter use. There is also a conflict with the food industry because of increasing demand on the industry to supply both food and fuel to the world. The U.S. Department of Agriculture expected world grain use to reach 20 million tons in 2006, 14 million tons for the fuel industry in the U.S. and 6 million tons to supply the world's food necessities. [21] As a result, the price of food is up and it is the highest it has been in 20 years, since almost everything we eat can be used to create biodiesel. [22] The World Bank is concerned about high prices putting an additional 44 million people into poverty. [23] Moreover, most governments do not have measures in place to limit the amount of land used for biofuels as not to compete with the food industry, thus, strengthening hunger in impoverished countries.

Students will make biodiesel from old and new oil then perform titrations to determine if the biodiesel has the correct pH. Students will use compound inequalities to describe the acceptable range of pH, fuel efficiency of biodiesel, crop production and sales, and oil content of crop. Students will evaluate the sustainability of a crop by calculating the energy return on energy investments to ensure that the energy input is significantly less than the energy output. Students will also use systems of equations to model and solve problems related to the production and production cost of biodiesel.

Energy Efficiency: Use Less, Save More

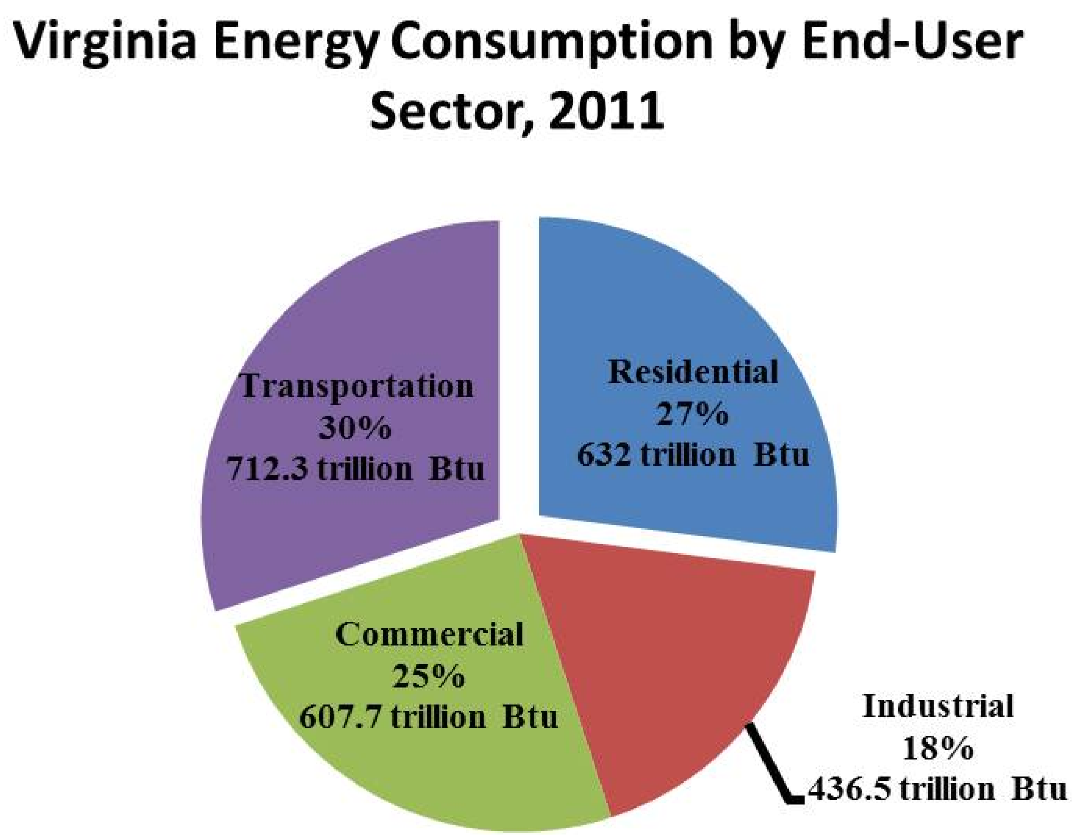


Figure 2. Breakdown of how energy is used in Virginia [24]

Dominik Saner, researcher with the Institute of Environmental Engineering in Switzerland, says residential and private transportation energy use accounts for more than 70% of carbon dioxide emissions. [25] The energy Virginians use in the residential and transportation sector make up 57% of the total energy generated. The future of energy in Virginia requires a comprehensive approach that combines efficiency programs with renewable energy sources. Many of us are not in situations where we can construct a windmill for our individual use. So, we are left to think of ways to make our current non-renewable primary energy sources last longer by decreasing the demand for energy. If we are doing our part at similar commitment levels, the ratio of consumption described could stay the same, but the total amount of consumption is reduced.

There are a variety of ways to conserve on the road that does not include converting your car to a diesel engine and making your own fuel. If you need incentive, gas in New Haven, Connecticut is currently $3.93/gallon. Gas in Virginia is about sixty cents cheaper, temporarily. So, until there are significant changes in the manufacturing of automobiles, we are faced with the high gasoline prices. One way to beat the cost is to drive at slower constant speeds and accelerate/decelerate at a slower rate when merging into traffic on a highway or local road. We waste gasoline driving at higher speeds due to rolling and internal vehicle friction and drag force. Also, turn off your car instead of idling; restarting your car takes the same amount of gas used in 0.2 seconds of idling. Finally, keeping your tires properly inflated will save money and make your gasoline last longer. [26]

Like your car, there are a variety of ways to save in your home. Dominion's website has many practical energy conservation tips customers can implement during the winter and summer months. Some tips are simple and require no investments, while others require an investment which will pay for itself within months, sometimes weeks. Some examples of low to no cost energy conservation tips include: setting your thermostat a few degrees higher in summer and lower in winter, keeping your shades closed when the air conditioner is on and opening your shades during the winter to let the sun's radiant energy heat your home, cleaning/replacing filters, washing dishes or clothes in the early morning or late evening, using cold water to wash dishes and clothes, sealing air leaks at entrances, grilling meals outside, unplugging appliances not in use and turning off lights not in use. Examples of higher cost investments include: using more efficient appliances, installing ceiling fans, using a tankless-water heater and replacing lighting with more efficient bulbs. Local power companies also offer energy calculators on their website to evaluate electrical usage in your home and business facilities; understanding specific energy use by location can reduce consumption and save money. [27] If preferred, the power company will do a standard residential in-home energy audit as well. The power company also offers approved programs that help families conserve energy conveniently and offer an incentive. Some of the teachers in my seminar benefit from programmable thermostats that provide indicators of critical peak times. The most popular, or rather the most visible, program is ENERGY STAR®. In fact, there are mortgage loan incentives for having a home run by appliances under ENERGY STAR® guidelines. [28]

Students will perform an energy audit of their home, write a proposal to their caregiver making suggestions and create a flyer describing 10 things we can do to conserve energy. We will use the formula: $C = WNHDP/1000$, where W is the power in Watts consumed by a device, N is the number of devices in use, H is the number of hours per day the device is in use, D is the number of days in a year when the device is in use, P is the price of electrical service in dollars per kilowatt-hour ($0.0887), and C is the total cost in dollars per year for using the device. We divide by one thousand to convert Watts to kilowatts. This formula accounts for other factors that simpler energy formulas do not take into account.

Teaching Strategies

I need something to connect our students on day one to our class and the content in a positive way. Therefore, I will start the year and unit with a survey. Dr. Judy Willis, author of *Learning to Love Math: Teaching strategies that change student attitudes and gets results*, believes collecting information about your students is an effective way to get what we need to link them to topics through interest or integration of information about them into math problems. [29] My survey will serve a dual purpose of collecting biographical information about my students' prior experience in math class and an assessing their knowledge of energy and energy efficiency habits. The survey may pique students' interest in how the information will be used. Teachers can encourage real-world connections through inductive reasoning by having students make predictions and, as we go through implementation of the unit, provide prizes to those who were accurate or close. Making real world connections integrates creative problem solving, communication, collaboration, and critical analysis, all of which are 21 st century skills we want our students to have. [30]

According to Willis, 50% percent of high school students living in large U.S. cities drop out of school. This creates a phenomenon in which there is a likelihood the parents of our students will have graduated from high school rather than our students graduate. [31] One reason for high dropout rates is boredom and relevance of content to their life. Speakers can help students recognize how the math concepts and skills learned in class relate to careers they might find appealing. [32] Dominion Virginia Power's Speakers Bureau conducts free educational presentations to schools. I will invite them to my class for two visits. During the first visit, the speaker will focus on the basics of electricity (how it is generated, distributed and used) and renewables (how they play a part in the diverse mix of energy sources). The second visit will focus on energy conservation and careers with one of the nation's leading energy companies.

Additional teaching strategies will include demonstrations, hands-on-activities to include inquiry-based learning opportunities, and discrepant events that illustrate scientific principles related to energy. Grabbing the attention of our students at the beginning of the lesson and throughout the lesson through one of these media creates cognitive dissonance. By creating disequilibrium, our students innately feel this vehement need to find a greater understanding of why the outcome did not match their prediction. Students are totally engaged at this point and it is our job to pounce on the opportunity to educate directly or indirectly. Some of the activities I can facilitate/perform and explain while others will be explained by our science teachers.

Lesson 1: Coal Mining Experiment

For the Warm-Up, find out what students know about the origin of their electricity, environmental concerns related to energy, and energy efficiency practices by giving an energy survey. You can use the information to determine groupings. Students will then view a video that connects coal to electricity. [33]

Activity 1: Coal Mining Experiment

For this activity, groups will consist of one person who is fairly knowledgeable about energy and another person who may not be as proficient. Students will simulate mountain top coal mining using a chocolate chip cookie and a toothpick or paperclip. The cookie is the mountain, the chocolate chips are the coal and the toothpick or paperclip is the retrieval tool. Students will estimate the number of chips they can retrieve without picking up the cookie; remind them they cannot pick up a mountain. In the real-world, the area will only become a mine if it is large enough and of sufficient quality so the coal can be retrieved at a reasonable cost. Have students write down the factors they considered in order to arrive at their estimate. Then, students will use the toothpick or paperclip to retrieve the chocolate chips. After they retrieve all the chips they can, have students compare their actual number of chips to their estimates and think about was it difficult to recover the coal from the ground? What impact does mining have on the surrounding land? Is the land suitable for future use? Is there another way to make mining have less of an impact on the land? [34]

Activity 2: Coal Mining Math

According to EIA, the average emission rates in the U.S. from coal-fired generation are: 2,249 lbs/MWh of carbon dioxide, 13 lbs/MWh of sulfur dioxide, and 6 lbs/MWh of nitrogen oxides. [35] For each 0.00053 short tons or 1.07 lbs of coal, one kilowatt-hour (kWh) is generated. [36] Students will use this information to write an algebraic expression to model the relationship between the production/consumption of coal and the average emission rates of a greenhouse gas. Teachers can tier the information by providing the data with no conversions or scaffold it by having students do the conversions one step at a time using dimensional analysis. Important conversion factors are 0.001 MWh = 1 kWh and 2000 lbs = 1 short ton. It is important for teachers to be intentional about students seeing the units cancel. For example, the algebraic expression for the amount of nitrogen oxides emissions in pounds for any amount of short ton of coal produced/consumed is

$$\frac{6\ lbs}{1\ MWh} \times \mathrm{C}\ short\ tons \times \frac{2000\ lbs}{1\ short\ ton} \times \frac{1\ kwh}{1.07\ lbs} \times \frac{0.001\ MWh}{1\ kwh}$$

where *C* is the amount of coal produced/consumed in short tons. Students will then evaluate the expression for given replacement values and look at long term data to compare emissions in various years.

For homework and in preparation for the Dominion Virginia Power speaker, [37] students will view the videos below, which contain different perspectives and give unique information, to construct a verbal or pictorial sequence of where energy comes from. I will split the class into two large groups who will transfer their individual information to a large common paper.

Dominion Power:

http://www.youtube.com/watch?v=Pz2AXbnfSUo&feature=youtu.be

National Grid: How we get electricity and gas using types of energy and an

introduction to renewable energy http://www.youtube.com/watch?v=__zB80SagIk

RCC Power:

http://www.youtube.com/watch?NR=1&feature=endscreen&v=0mjT8ETB128

A possible extension can include students calculating the curve of best fit using the coal production/consumption data to calculate and extrapolate the amount of greenhouse emissions, coal production or coal consumption in the future. Students can also use linear equations to model supply and demand and solve them using systems of equations. In addition, since coal contains potential chemical energy and is combustible, teachers can do a combustion demonstration. For safety reasons (and a chance for the science teacher to remind students about safety), collaborate with the science teacher on this demo. Finally, you can arrange a field trip to the local power plant. I plan to take my class to visit Chesterfield Power Station, the largest fossil-fueled power station in the state. It generates over 1,600 MW and has an average daily consumption of 8,400 tons of coal.

Lesson 2: Wind Energy: Building Windmills

For the Warm-Up, students will construct a device that measures wind speed and then measure the wind speed at different times during the day for a week and record their answers. Students will display measurements in a box-and-whiskers plot and determine if wind is a good source of energy in Churchill, Richmond, Va. [38] In order to use the wind as a source of energy, we must have a steady source of wind.

Activity 1: Wind Energy: Building Windmills

In small groups, students will construct a windmill that can do work by picking up a certain amount of weight. Students will then calculate the amount of work and power needed to pick up the weight. Work is the measure of the weight lifted through height; work, *W*, varies jointly with mass, *m*, acceleration of gravity, *g*, and height, *h*, *W=mgh*. Power is the measure of energy used over time; power, P, varies directly with energy, E, and inversely with time, t, *P = E/t*. The two equations are related since energy is the ability to do work, so E=W. Students will then work in groups to complete a virtual lab [39] on wind energy; there will be a competition of who creates the most efficient and economical wind farm.

Activity 2: How much wind power does a fan generate?

Students will calculate the wind power created by a fan set on different speeds: low, medium and high. Wind power, P, varies jointly with half of air density, P, area swept by the turbine blades, A, and the cube of the velocity, V, $P = (1/2)PV^3$. Air density is the mass per unit volume of earth's atmospheric gases and measures 1.25 kg/m^3 at sea level and 68°F.

For homework, students can write, graph and evaluate linear models for the following individual facts: wind integration costs are approximately $10/MWh, the decrease in air density is 3% for every 1000 additional feet in height and 1% for each additional degree in temperature (air density is 1.25 kg/m^3 at sea level and 68 °F), and the Bluestone River Wind Project in Tazewell County, southwest Virginia, could produce nearly 80 MW (1MW of wind energy will provide electricity for approximately 1,000 people).

A possible extension can include looking at a map of wind power potential along the Virginian coast and having students write a system of linear inequalities to describe the area. Students can also use a system of equations to find the intersections of the boundary lines. Students can write energy equations using the language of direct and inverse relationships. They can also make tables and graph direct and inverse relationships. Have students pay attention to independent and dependent values. In some cases, negative numbers do not exist in the domain or range. Finally, students can calculate a curve of best fit for wind energy related data and extrapolate the wattage of future wind power.

Lesson 3: Biodiesel: Alternative Fuel

For the Warm-Up, students will be presented with a table that has no headings but describes pipeline incidents and related injuries and fatalities. [40] They will predict what the data could represent. Teachers can only answer yes or no questions. This activity encourages students to look at possible trends in data and other details that may be of use.

Activity 1: Biodiesel: Alternative Fuel

Students will prepare for lab by gathering information on biodiesel (history, composition, prevalence, advantages and disadvantages, etc.). Students will make biodiesel from two types of oil (used and new). Encourage students to retrieve a variety of waste oils and label with the location. New oil requires 4 grams of lye per liter of oil, while used oil requires more depending on the quality. Students will determine by chemical analysis how much lye is needed for the old oil.

Activity 2: Biodiesel Math

Biodiesel has to be a specific pH, 8.5, but has a tolerance pH of 0.5. Students will use a compound inequality to describe the allowable range of the pH (between 8 and 9). Compound inequalities will also be used to describe fuel efficiency of cars which use biodiesel, crop production and sales of soybeans, and expected yield of oil content from soybeans. Students will use a quadratic model to maximize fuel efficiency of biodiesel and systems of equations to model and solve problems related to the production and production cost of biodiesel. Finally, students will evaluate the sustainability of a crop by calculating the energy return on energy investments.

A possible extension can include students analyzing yearly oil and biodiesel production statistically using mean absolute deviation, standard deviation and calculating the z-score for a certain amount of gallons. Teachers can use this opportunity to explain the difference between mean absolute and standard deviation.

Lesson 4: Energy Efficiency: Use Less, Save More

For the Warm-Up, each student in the class will view different videos from Dominion Virginia Power in their energy efficiency series. [41,42] Students will write down notes and report out to the class.

Activity 1: Energy Efficiency: Use Less, Save More @ Home

Students will act as Energy Inspectors and conduct an energy audit of their home. First they will make a list of items used in their home or on the go that use electricity. They will gather information about usage such as what time during the day they use this item and how many times during the year is it used (seasonal, monthly, weekly, etc.). They will then use this information to evaluate the cost of usage. We will use the formula: $C = WNHDP/1000$, where W is the power in Watts consumed by a device, N is the number of devices in use, H is the number of hours per day the device is in use, D is the number of days in a year when the device is in use, P is the price of electrical service in dollars per kilowatt-hour ($0.0887), and C is the total cost in dollars per year for using the device. We divide by one thousand to convert Watts to kilowatts. For example, a washing machine has an average rating of 1150 W. Let's assume you only have one washer and you wash clothes every 2 weeks. A wash cycle lasts about 33 minutes and you usually have 3 loads: white, light and dark colors. This means the washer runs for approximately 1.65 hours. How much money does the washer cost in 1 year? The calculations show

$$C = \frac{\text{WNHDP}}{1000} = \frac{1150 \text{ Watts} \times 1 \text{ washer} \times 1.65 \text{ hours} \times 26 \text{ days} \times \$0.0887}{1000 \text{ watts}} \approx \$4.37/\text{yr.}$$

Once students calculate this they can investigate if there are more cost efficient options and calculate how long it would take to get the money back if an investment is made in a more cost efficient option.

Activity 2: Energy Efficiency: Use Less, Save More on the Road

Given a table, students will model the following for 3 types of cars: rolling friction force as a function of velocity, gas used as a function of rolling friction force, constant speed drag force as a function of velocity, gas used as a function of constant speed drag force, and total gas used to overcome forces as a function of velocity. Then, they will calculate how much money can be saved by the reducing rolling friction force and constant speed drag force.

For homework, students will make a top ten list of ways they are able to conserve energy in their household and write a proposal to their caregivers. They will use research from the internet to support their proposal.

A possible extension could include student modeling acceleration/deceleration and use of gasoline; then, calculate how much money can be saved by accelerating/decelerating at a slower rate when merging into traffic on a highway or local road. Students can also calculate the amount of money a taxi driver can save by turning off their car instead of idling. Restarting your car takes the same amount of gas used in 0.2 seconds of idling. [43] Students can research occupations in the field of residential and commercial energy efficiency on the Energy Efficiency and Renewable Energy (EERE) website. [44]

Appendix A: Content Objectives

Virginia DOE did not adopt the Common Core, so teachers are held to the 2009 Virginia Standards of Learning for Algebra I, which are comparable to the Common Core.

A.1: The student will represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values using data related to emissions of greenhouse gases during the consumption and production of coal. The relationship between consumption/production of coal and emissions of greenhouse gases will become apparent through concrete, pictorial, symbolic and verbal representations.

A.4cf/A.5cd: The student will solve real-world quadratic and multistep linear equations and inequalities in one/two variables, and systems of equations and inequalities to determine the allowable range of the pH, fuel efficiency of cars which use biodiesel, crop production and sales of soybeans, and expected yield of oil content from soybeans and production and cost of production for biodiesel. Finally, students will evaluate the sustainability of a crop by calculating the energy return on energy investments. Students will evaluate the reasonableness of a mathematical model and use graphing calculators to solve problems and to verify algebraic solutions.

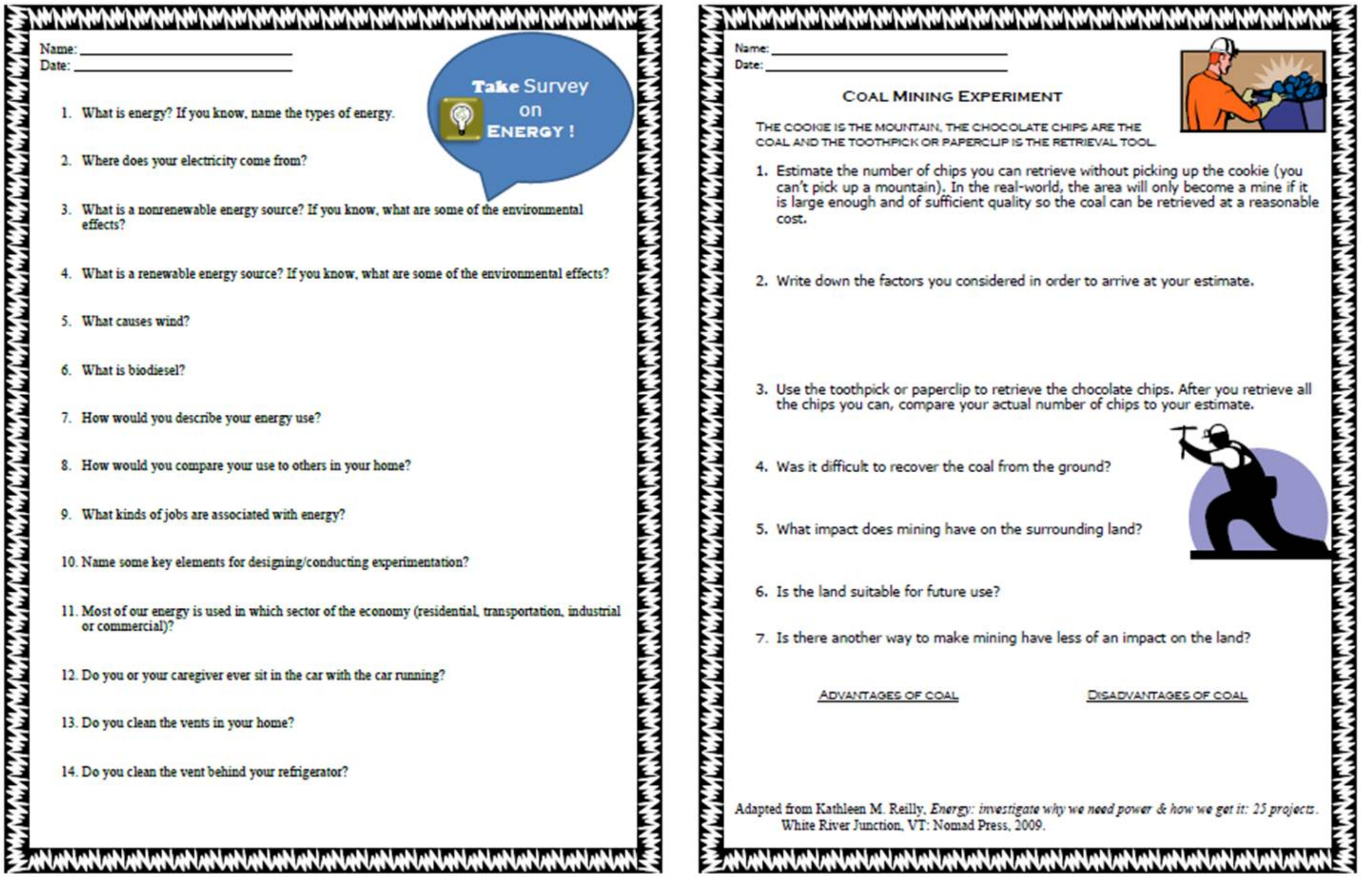
A.8: The student will analyze real-world relations to determine whether a direct or inverse variation exists, and represent a direct or inverse variation algebraically given a table of data related to gas consumption and velocity or energy output over time.

A.9: The student will interpret variation and calculate and interpret mean absolute deviation, standard deviation, and z-scores for data related to oil and biodiesel production and determine the implications from which the data derive. The student will compare and contrast mean absolute deviation and standard deviation.

A.10: The student will compare and contrast multiple univariate data sets, using box-and-whisker plots created from data collected by measuring wind speeds with different tools.

A.11: The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems related to coal production/consumption, greenhouse emissions and wind energy, using mathematical models.

Appendix B: Lesson Handouts



Name: __________________________
Date: __________________________

1. What is energy? If you know, name the types of energy.
2. Where does your electricity come from?
3. What is a nonrenewable energy source? If you know, what are some of the environmental effects?
4. What is a renewable energy source? If you know, what are some of the environmental effects?
5. What causes wind?
6. What is biodiesel?
7. How would you describe your energy use?
8. How would you compare your use to others in your home?
9. What kinds of jobs are associated with energy?
10. Name some key elements for designing/conducting experimentation?
11. Most of our energy is used in which sector of the economy (residential, transportation, industrial or commercial)?
12. Do you or your caregiver ever sit in the car with the car running?
13. Do you clean the vents in your home?
14. Do you clean the vent behind your refrigerator?

Name: __________________________
Date: __________________________

COAL MINING EXPERIMENT

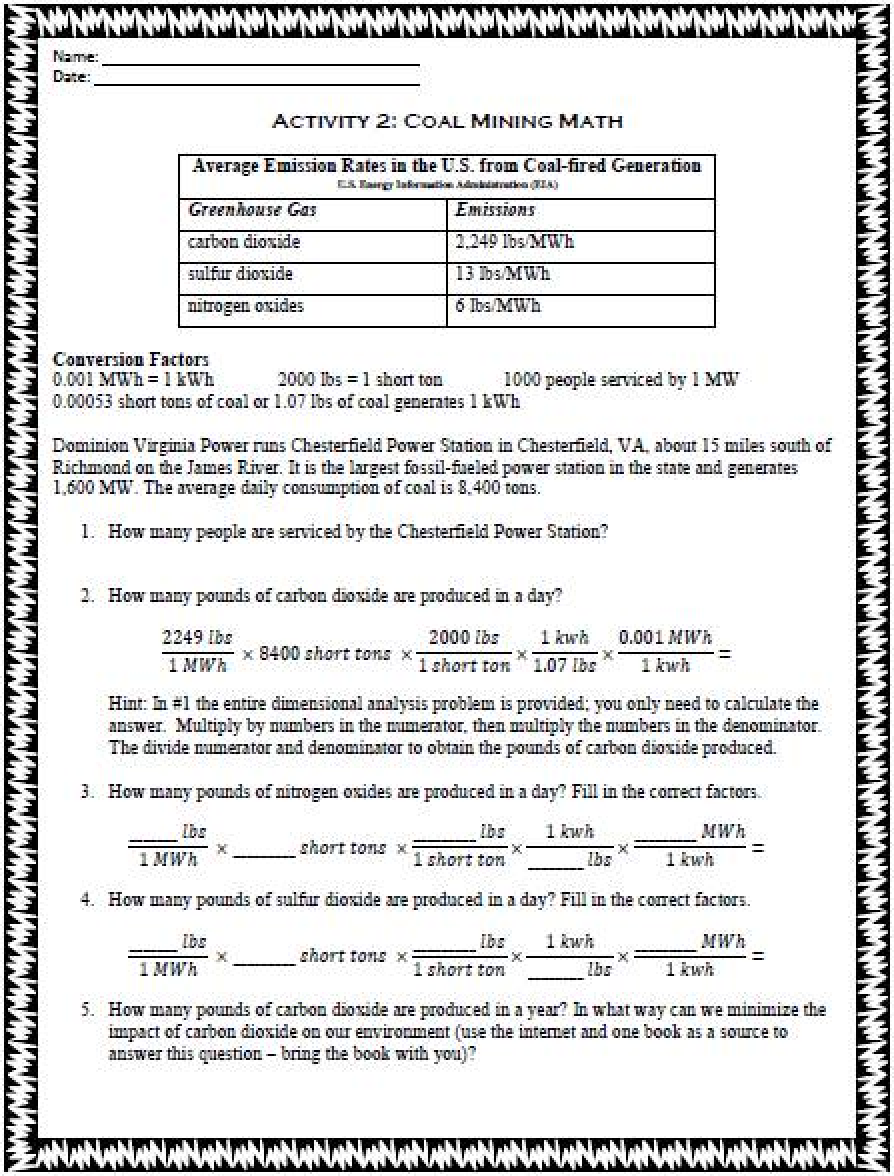
THE COOKIE IS THE MOUNTAIN, THE CHOCOLATE CHIPS ARE THE COAL AND THE TOOTHPICK OR PAPERCLIP IS THE RETRIEVAL TOOL.

1. Estimate the number of chips you can retrieve without picking up the cookie (you can't pick up a mountain). In the real-world, the area will only become a mine if it is large enough and of sufficient quality so the coal can be retrieved at a reasonable cost.
2. Write down the factors you considered in order to arrive at your estimate.
3. Use the toothpick or paperclip to retrieve the chocolate chips. After you retrieve all the chips you can, compare your actual number of chips to your estimate.
4. Was it difficult to recover the coal from the ground?
5. What impact does mining have on the surrounding land?
6. Is the land suitable for future use?
7. Is there another way to make mining have less of an impact on the land?

ADVANTAGES OF COAL | DISADVANTAGES OF COAL

Adapted from Kathleen M. Reilly, *Energy: investigate why we need power & how we get it: 25 projects*. White River Junction, VT: Nomad Press, 2009.

Figure 3. Lesson 1 - Energy Survey and Coal Mining Experiment


Name: ______________________________

Date: ______________________________

Activity 2: Coal Mining Math

Average Emission Rates in the U.S. from Coal-fired Generation U.S. Energy Information Administration (EIA)	
Greenhouse Gas	***Emissions***
carbon dioxide	2,249 lbs/MWh
sulfur dioxide	13 lbs/MWh
nitrogen oxides	6 lbs/MWh

Conversion Factors

0.001 MWh = 1 kWh   2000 lbs = 1 short ton   1000 people serviced by 1 MW

0.00053 short tons of coal or 1.07 lbs of coal generates 1 kWh

Dominion Virginia Power runs Chesterfield Power Station in Chesterfield, VA, about 15 miles south of Richmond on the James River. It is the largest fossil-fueled power station in the state and generates 1,600 MW. The average daily consumption of coal is 8,400 tons.

1. How many people are serviced by the Chesterfield Power Station?

2. How many pounds of carbon dioxide are produced in a day?

$$\frac{2249\ lbs}{1\ MWh} \times 8400\ short\ tons \times \frac{2000\ lbs}{1\ short\ ton} \times \frac{1\ kwh}{1.07\ lbs} \times \frac{0.001\ MWh}{1\ kwh} =$$

Hint: In #1 the entire dimensional analysis problem is provided; you only need to calculate the answer. Multiply by numbers in the numerator, then multiply the numbers in the denominator. The divide numerator and denominator to obtain the pounds of carbon dioxide produced.

3. How many pounds of nitrogen oxides are produced in a day? Fill in the correct factors.

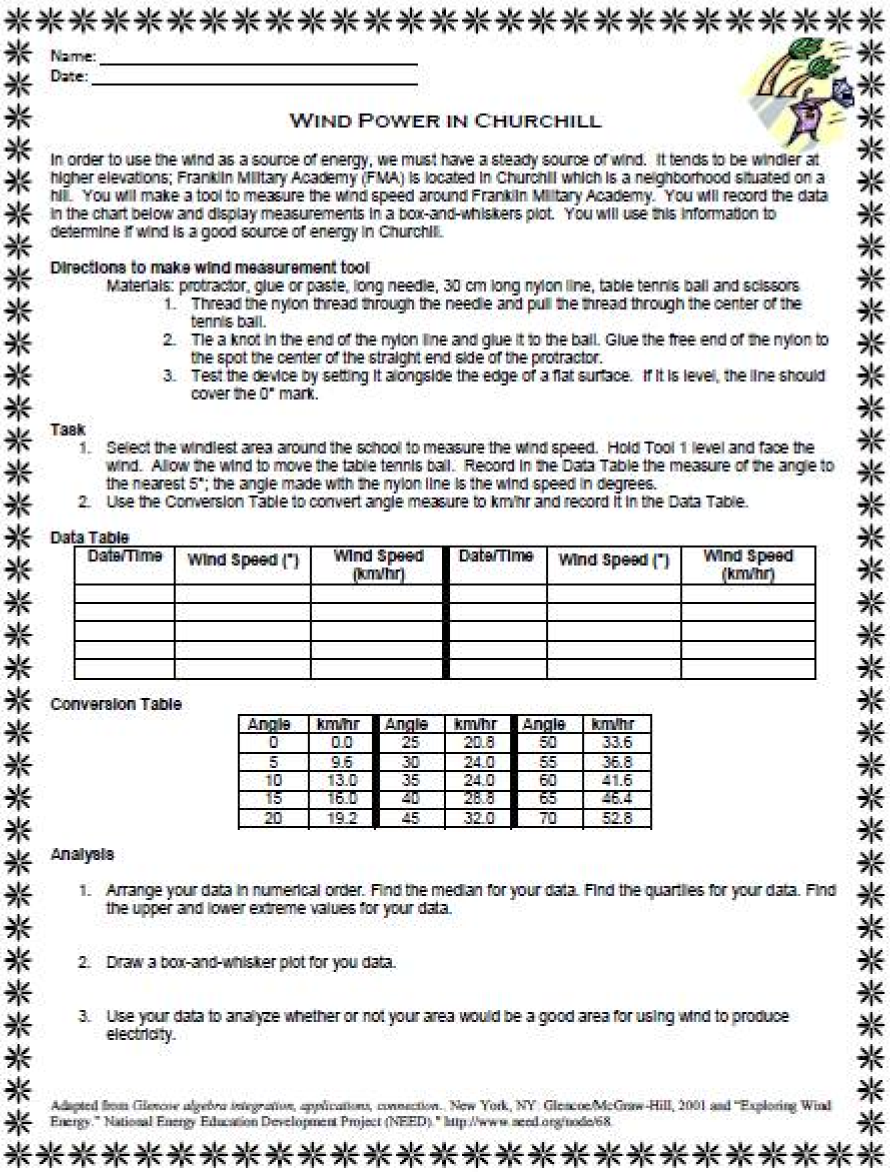
$$\frac{\_\_\_\_\ lbs}{1\ MWh} \times \_\_\_\_\ short\ tons \times \frac{\_\_\_\_\ lbs}{1\ short\ ton} \times \frac{1\ kwh}{\_\_\_\_\ lbs} \times \frac{\_\_\_\_\ MWh}{1\ kwh} =$$

4. How many pounds of sulfur dioxide are produced in a day? Fill in the correct factors.

$$\frac{\_\_\_\_\ lbs}{1\ MWh} \times \_\_\_\_\ short\ tons \times \frac{\_\_\_\_\ lbs}{1\ short\ ton} \times \frac{1\ kwh}{\_\_\_\_\ lbs} \times \frac{\_\_\_\_\ MWh}{1\ kwh} =$$

5. How many pounds of carbon dioxide are produced in a year? In what way can we minimize the impact of carbon dioxide on our environment (use the internet and one book as a source to answer this question – bring the book with you)?

Figure 4. Lesson 1 - Coal Mining Math



Name: ______________________________

Date: ______________________________

WIND POWER IN CHURCHILL

In order to use the wind as a source of energy, we must have a steady source of wind. It tends to be windier at higher elevations; Franklin Military Academy (FMA) is located in Churchill which is a neighborhood situated on a hill. You will make a tool to measure the wind speed around Franklin Military Academy. You will record the data in the chart below and display measurements in a box-and-whiskers plot. You will use this information to determine if wind is a good source of energy in Churchill.

Directions to make wind measurement tool

Materials: protractor, glue or paste, long needle, 30 cm long nylon line, table tennis ball and scissors

1. Thread the nylon thread through the needle and pull the thread through the center of the tennis ball.
2. Tie a knot in the end of the nylon line and glue it to the ball. Glue the free end of the nylon to the spot the center of the straight end side of the protractor.
3. Test the device by setting it alongside the edge of a flat surface. If it is level, the line should cover the 0° mark.

Task

1. Select the windiest area around the school to measure the wind speed. Hold Tool 1 level and face the wind. Allow the wind to move the table tennis ball. Record in the Data Table the measure of the angle to the nearest 5°; the angle made with the nylon line is the wind speed in degrees.
2. Use the Conversion Table to convert angle measure to km/hr and record it in the Data Table.

Data Table

Date/Time	Wind Speed (°)	Wind Speed (km/hr)	Date/Time	Wind Speed (°)	Wind Speed (km/hr)

Conversion Table

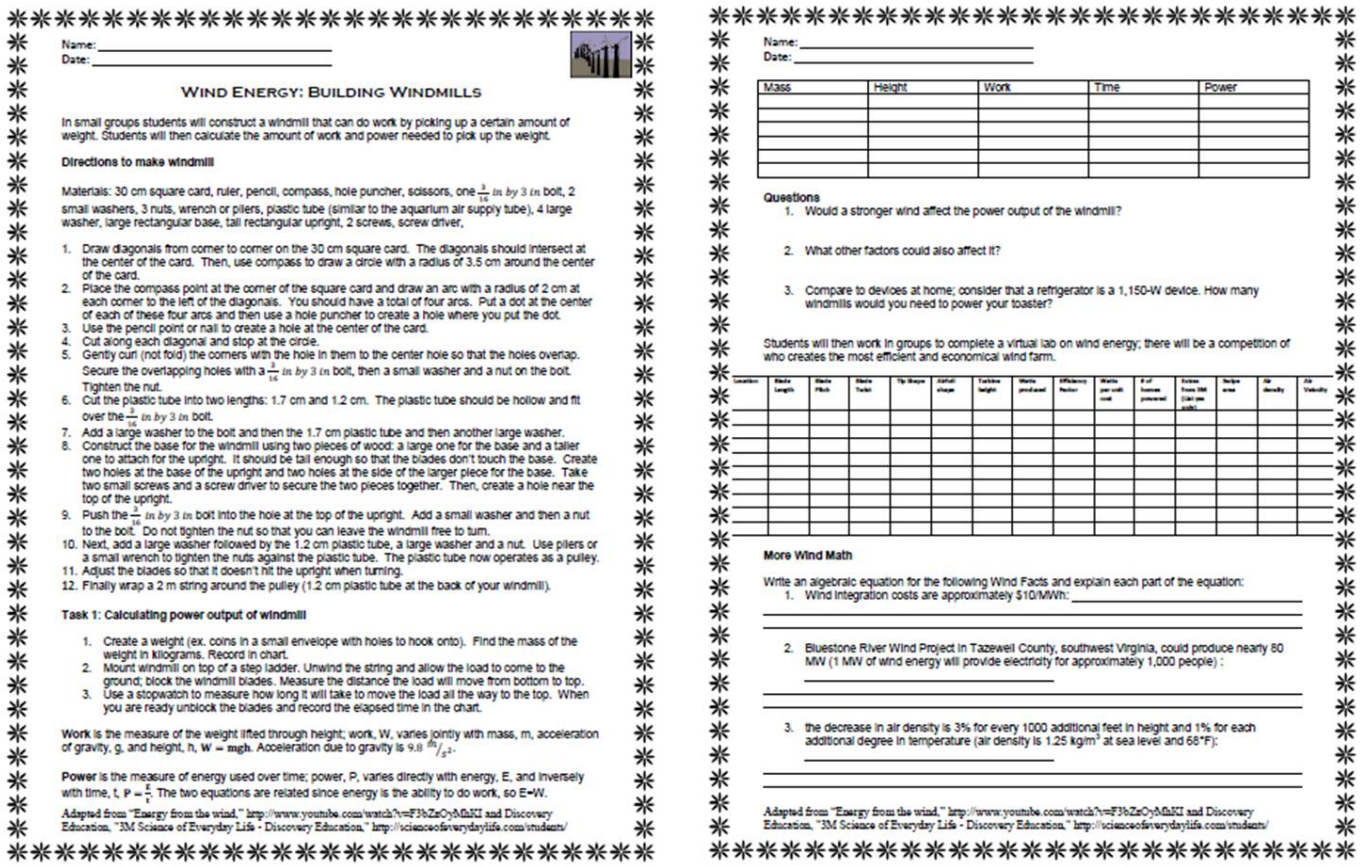
Angle	km/hr	Angle	km/hr	Angle	km/hr
0	0.0	25	20.8	50	33.6
5	9.6	30	24.0	55	36.8
10	13.0	35	24.0	60	41.6
15	16.0	40	28.8	65	46.4
20	19.2	45	32.0	70	52.8

Analysis

1. Arrange your data in numerical order. Find the median for your data. Find the quartiles for your data. Find the upper and lower extreme values for your data.
2. Draw a box-and-whisker plot for you data.
3. Use your data to analyze whether or not your area would be a good area for using wind to produce electricity.

Adapted from Glencoe *algebra integration, applications, connection.*. New York, NY: Glencoe/McGraw-Hill, 2001 and "Exploring Wind Energy." National Energy Education Development Project (NEED)." http://www.need.org/node/68

Figure 5. Lesson 2 - Wind Power in Churchill



Name: ______________________________
Date: ______________________________

Wind Energy: Building Windmills

In small groups students will construct a windmill that can do work by picking up a certain amount of weight. Students will then calculate the amount of work and power needed to pick up the weight.

Directions to make windmill

Materials: 30 cm square card, ruler, pencil, compass, hole puncher, scissors, one $\frac{3}{16}$ $in\ by\ 3\ in$ bolt, 2 small washers, 3 nuts, wrench or pliers, plastic tube (similar to the aquarium air supply tube), 4 large washer, large rectangular base, tall rectangular upright, 2 screws, screw driver,

1. Draw diagonals from corner to corner on the 30 cm square card. The diagonals should intersect at the center of the card. Then, use compass to draw a circle with a radius of 3.5 cm around the center of the card.
2. Place the compass point at the corner of the square card and draw an arc with a radius of 2 cm at each corner to the left of the diagonals. You should have a total of four arcs. Put a dot at the center of each of these four arcs and then use a hole puncher to create a hole where you put the dot.
3. Use the pencil point or nail to create a hole at the center of the card.
4. Cut along each diagonal and stop at the circle.
5. Gently curl (not fold) the corners with the hole in them to the center hole so that the holes overlap. Secure the overlapping holes with a $\frac{3}{16}$ $in\ by\ 3\ in$ bolt, then a small washer and a nut on the bolt. Tighten the nut.
6. Cut the plastic tube into two lengths: 1.7 cm and 1.2 cm. The plastic tube should be hollow and fit over the $\frac{3}{16}$ $in\ by\ 3\ in$ bolt.
7. Add a large washer to the bolt and then the 1.7 cm plastic tube and then another large washer.
8. Construct the base for the windmill using two pieces of wood: a large one for the base and a taller one to attach for the upright. It should be tall enough so that the blades don't touch the base. Create two holes at the base of the upright and two holes at the side of the larger piece for the base. Take two small screws and a screw driver to secure the two pieces together. Then, create a hole near the top of the upright.
9. Push the $\frac{3}{16}$ $in\ by\ 3\ in$ bolt into the hole at the top of the upright. Add a small washer and then a nut to the bolt. Do not tighten the nut so that you can leave the windmill free to turn.
10. Next, add a large washer followed by the 1.2 cm plastic tube, a large washer and a nut. Use pliers or a small wrench to tighten the nuts against the plastic tube. The plastic tube now operates as a pulley.
11. Adjust the blades so that it doesn't hit the upright when turning.
12. Finally wrap a 2 m string around the pulley (1.2 cm plastic tube at the back of your windmill).

Task 1: Calculating power output of windmill

1. Create a weight (ex. coins in a small envelope with holes to hook onto). Find the mass of the weight in kilograms. Record in chart.
2. Mount windmill on top of a step ladder. Unwind the string and allow the load to come to the ground; block the windmill blades. Measure the distance the load will move from bottom to top.
3. Use a stopwatch to measure how long it will take to move the load all the way to the top. When you are ready unblock the blades and record the elapsed time in the chart.

Work is the measure of the weight lifted through height; work, W, varies jointly with mass, m, acceleration of gravity, g, and height, h, $W = mgh$. Acceleration due to gravity is $9.8\ ^{m}/_{s^2}$.

Power is the measure of energy used over time; power, P, varies directly with energy, E, and inversely with time, t, $P = \frac{E}{t}$. The two equations are related since energy is the ability to do work, so E=W.

Adapted from "Energy from the wind," http://www.youtube.com/watch?v=F3bZzOyMhKI and Discovery Education, "3M Science of Everyday Life - Discovery Education," http://scienceofeverydaylife.com/students/

Name: ______________________________
Date: ______________________________

Mass	Height	Work	Time	Power

Questions

1. Would a stronger wind affect the power output of the windmill?
2. What other factors could also affect it?
3. Compare to devices at home; consider that a refrigerator is a 1,150-W device. How many windmills would you need to power your toaster?

Students will then work in groups to complete a virtual lab on wind energy; there will be a competition of who creates the most efficient and economical wind farm.

Location	Blade Length	Blade Pitch	Blade Twist	Tip Shape	Airfoil shape	Turbine height	Watts produced	Efficiency Factor	Watts per unit cost	# of homes powered	Extras from 3M (List per unit)	Swipe area	Air density	Air Velocity

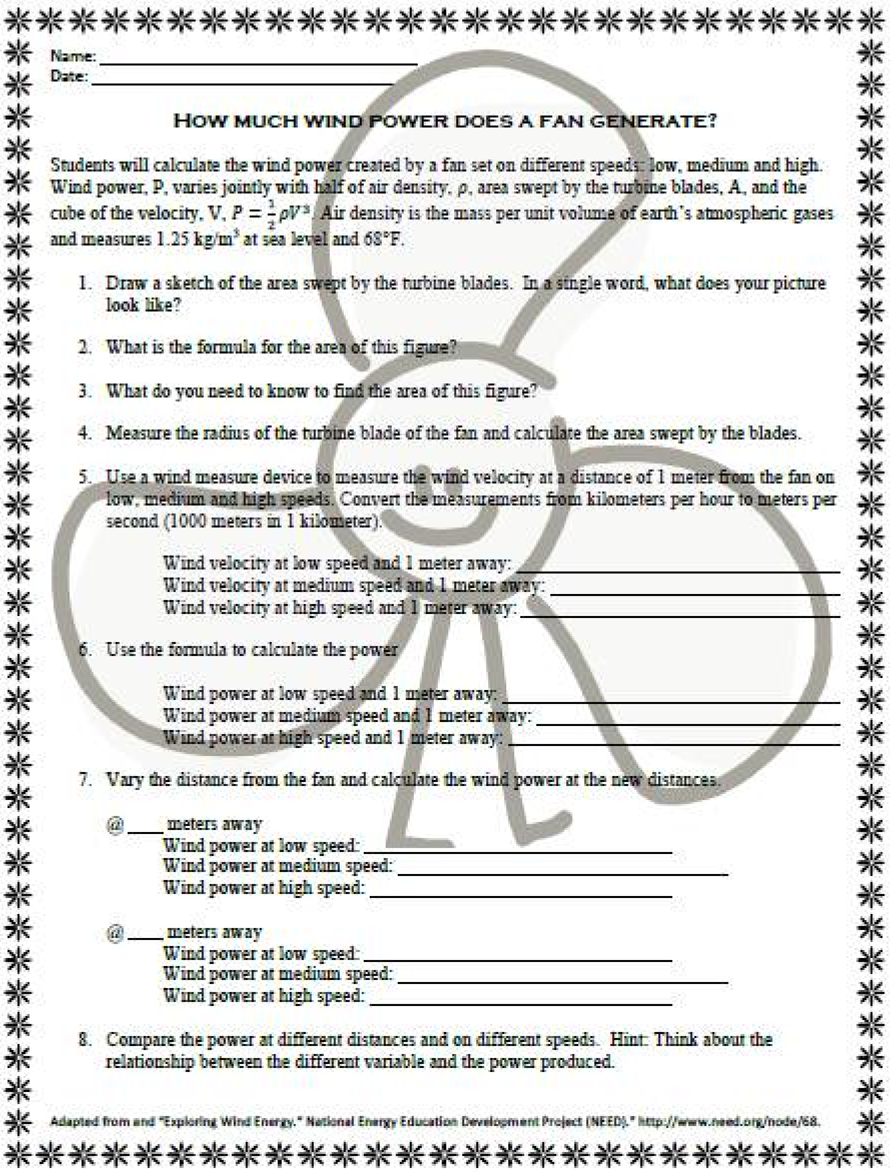
More Wind Math

Write an algebraic equation for the following Wind Facts and explain each part of the equation:

1. Wind integration costs are approximately $10/MWh: ______________________
2. Bluestone River Wind Project in Tazewell County, southwest Virginia, could produce nearly 80 MW (1 MW of wind energy will provide electricity for approximately 1,000 people) : ______________________
3. the decrease in air density is 3% for every 1000 additional feet in height and 1% for each additional degree in temperature (air density is 1.25 kg/m^3 at sea level and 68°F): ______________________

Adapted from "Energy from the wind," http://www.youtube.com/watch?v=F3bZzOyMhKI and Discovery Education, "3M Science of Everyday Life - Discovery Education," http://scienceofeverydaylife.com/students/

Figure 6. Lesson 2 – Building a Windmill and Wind Math



Name: ______________________
Date: ______________________

HOW MUCH WIND POWER DOES A FAN GENERATE?

Students will calculate the wind power created by a fan set on different speeds: low, medium and high. Wind power, P, varies jointly with half of air density, ρ, area swept by the turbine blades, A, and the cube of the velocity, V, $P = \frac{1}{2}\rho V^3$. Air density is the mass per unit volume of earth's atmospheric gases and measures 1.25 kg/m^3 at sea level and 68°F.

1. Draw a sketch of the area swept by the turbine blades. In a single word, what does your picture look like?
2. What is the formula for the area of this figure?
3. What do you need to know to find the area of this figure?
4. Measure the radius of the turbine blade of the fan and calculate the area swept by the blades.
5. Use a wind measure device to measure the wind velocity at a distance of 1 meter from the fan on low, medium and high speeds. Convert the measurements from kilometers per hour to meters per second (1000 meters in 1 kilometer).

   Wind velocity at low speed and 1 meter away: ______________________
   Wind velocity at medium speed and 1 meter away: ______________________
   Wind velocity at high speed and 1 meter away: ______________________

6. Use the formula to calculate the power

   Wind power at low speed and 1 meter away: ______________________
   Wind power at medium speed and 1 meter away: ______________________
   Wind power at high speed and 1 meter away: ______________________

7. Vary the distance from the fan and calculate the wind power at the new distances.

   @ ____ meters away
   Wind power at low speed: ______________________
   Wind power at medium speed: ______________________
   Wind power at high speed: ______________________

   @ ____ meters away
   Wind power at low speed: ______________________
   Wind power at medium speed: ______________________
   Wind power at high speed: ______________________

8. Compare the power at different distances and on different speeds. Hint: Think about the relationship between the different variable and the power produced.

Adapted from and "Exploring Wind Energy." National Energy Education Development Project (NEED). http://www.need.org/node/68.

Figure 7. Lesson 2 – Power of a Fan

1992	389	$70.5	68,810	118	15
1993	445	$67.3	57,559	111	17
1994	467	$160.6	114,002	120	22
1995	349	$53.4	53,113	64	21
1996	381	$114.5	100,949	127	53
1997	346	$79.6	103,129	77	10
1998	389	$126.9	60,791	81	21
1999	339	$130.1	104,487	108	22
2000	380	$191.8	56,953	81	38
2001	341	$63.1	77,456	61	7
2002	644	$102.1	77,953	49	12
2003	673	$139.0	50,889	71	12
2004	673	$271.9	69,003	60	23
2005	721	$1,246.7	46,246	48	14
2006	641	$151.1	53,905	36	21
2007	616	$154.9	68,941	53	15
2008	664	$555.8	69,815	59	9
2009	627	$178.0	32,258	66	13
2010	586	$1,336.4	123,419	109	22
2011	599	$336.3	108,663	65	17
Totals	**10,270**	**$5,530.0**	**1,498,344**	**1,564**	**384**

Pipeline Incidents and Related Injuries and Fatalities (1992-2011)

	Number	Property Damage as Reported* (in millions)	Net Barrels of Liquids Lost	Injuries	Fatalities
1992	389	$70.5	68,810	118	15
1993	445	$67.3	57,559	111	17
1994	467	$160.6	114,002	120	22
1995	349	$53.4	53,113	64	21
1996	381	$114.5	100,949	127	53
1997	346	$79.6	103,129	77	10
1998	389	$126.9	60,791	81	21
1999	339	$130.1	104,487	108	22
2000	380	$191.8	56,953	81	38
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2002	644	$102.1	77,953	49	12
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2008	664	$555.8	69,815	59	9
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2011	599	$336.3	108,663	65	17
Totals	**10,270**	**$5,530.0**	**1,498,344**	**1,564**	**384**

Source. Manhattan Institute for Policy Research, "Issue Brief 23 | Pipelines Are Safest For Transportation Of Oil And Gas," http://www.manhattan-institute.org/html/ib_23.htm.

Figure 8. Lesson 3 - Unnamed table

Name: ______________________

Date ______________________

Making Biodiesel

History

Write a brief statement about the history of biodiesel.

Composition of Biodiesel

Write a brief statement about the composition of biodiesel.

Accessibility

Write a brief statement on the accessibility of biodiesel in Virginia?

Advantages/Disadvantages

Describe the advantages and disadvantages of biodiesel.

Caution

1. Biodiesel fuel made at home or in a school lab is experimental. So, burning it in a diesel engine is at your own risk. While well-made fuel will not harm a diesel engine, if you are interested in using biodiesel, you should read further on the subject.

2. Methanol is a powerful alcohol and sodium hydroxide is a corrosive lye. The teacher will handle both substances only.

New oil

Materials List: Vegetable oil (50 ml), 100 mL Graduated cylinder, Sodium Hydroxide (0.2 g), Hot plate with stir bar, Methanol (11.0 mL), Celsius thermometer, 25 mL Graduated cylinder, 125 mL Erlenmeyer flask (with stopper), 50 mL Erlenmeyer flask (with stopper)

1. Measure out 50 ml of new vegetable oil and pour it into a 125 mL flask.
2. Heat the vegetable oil to 50°C on a hotplate using a stirrer. One person in your group should watch the temperature closely so the oil does not overheat.
3. **Perform step under a chemical hood:** Measure 11 ml of methanol in a graduated cylinder and pour into your 50 mL flask. Cap the methanol bottle tightly.
4. **Perform step under a chemical hood:** Weigh out 0.2 grams of sodium hydroxide and add it to the methanol in your flask. Stopper the bottle and swirl gently for a few minutes until all of the NaOH dissolves. You now have sodium methoxide in your bottle, a strong base. Be careful!
5. **Caution:** Be certain that the oil is not over 60°C, or the methanol may boil. When the NaOH is dissolved and the oil is up to 50°C, add the methoxide to the warm oil and stir vigorously for 10 minutes.
6. Stopper the flask and put it in a safe place. Over the next 30-60 minutes, you should see a darker layer (glycerol) forming on the bottom of the bottle, with a lighter layer (biodiesel) floating on top. Complete settling of the reaction will require several hours to overnight.

Adapted from Adapted from: The Biofuel Project: Creating Biodiesel, U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy

Name: ______________________

Date ______________________

Biodiesel Math

1. If the base rate for Sodium Hydroxide is 4.0 g per liter of oil, why did you only use 0.2 g for this batch? How much NaOH would be used to convert 50 liters of new oil?

2. For a given quantity of new oil, what variables could be changed to effect the reaction?

Old oil

All ingredients will remain constant with the exception of the NaOH. Used oil is degraded into various byproducts such as free fatty acids in the oil. These acids will act to neutralize some of the lye used in the biodiesel reaction. New oil requires 4 grams of NaOH per liter of oil, so for used oil we will need to add extra lye to make up for that neutralized by the free fatty acids. We will conduct a chemical analysis called titration to determine the exact amount of extra lye required.

Materials: sample of lightly used oil and heavily used oil labeled accordingly, pipette, beaker, 90-99% isopropyl alcohol

1. Using a pipette measure 1.0 ml of oil, from one sample into a small mixing beaker.
2. Measure 10 ml of isopropyl alcohol using a graduated cylinder, add this to the oil, and swirl to mix.
3. Add 5 drops of phenol red to the beaker containing 10 ml of isopropyl alcohol and 1 ml of oil to be tested. The solution will appear yellow at an acid pH, and will turn pink when the pH is between 8 and 9.
4. Add lye-water in 0.5 ml increments, counting as you go, until the oil alcohol solution turns pink or purple and stays that way for 30 seconds or more. The number of milliliters of lye-water it took to turn the solution pink is "X", where X + 4.0 grams = L, the total number of grams of lye needed to make biodiesel from 1liter of this particular oil.
5. Repeat all the steps under New Oil substituting the correct amount of NaOH.

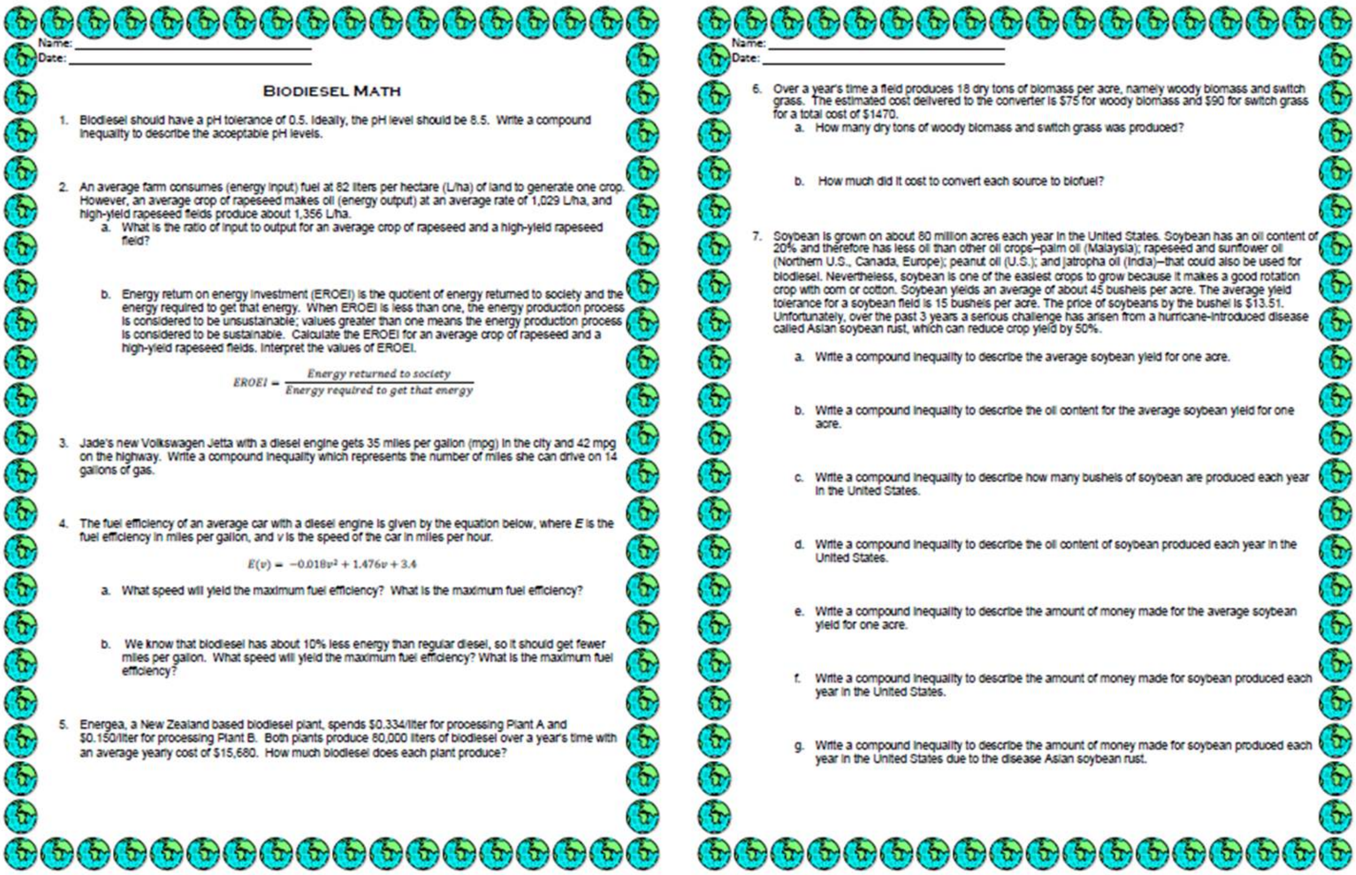
Washing the Bio-Diesel

Materials List: Biodiesel which has settled overnight, Transfer pipettes, 200 mL Distilled water, 50 mL Graduated cylinder, 3 g $MgSO_4$ (anhydrous), Separatory funnel

1. VERY CAREFULLY, decant the biodiesel into the separatory funnel. When only a few mL remains, stop decanting and use a transfer pipette to remove the remaining liquid. Be sure not to transfer any of the glycerol or soap layers.
2. Gently add about 50 mL of warm distilled water to the biodiesel. Stopper the funnel and rock it end-over-end until the water starts to take on a little bit of soapiness, which may take a few minutes. Regularly invert the bottle and open the stopcock to relieve any pressure. Do not shake the funnel! You will want to bring the water and biodiesel into contact without mixing it too vigorously. The biodiesel contains soap and if you shake too hard, the soap, bio-diesel, and water will make a stable emulsion that won't separate.
3. Put the funnel on a ring stand, crack the cap, and drain away the soapy water.
4. Add more warm water and keep repeating the sloshing and draining process. Each time there will be less soap and you can mix a little more vigorously. If you go too far and get a pale-colored emulsion layer between the bio-diesel and white, soapy water, don't drain it away; it's mostly bio-diesel. Just keep washing and diluting until the water becomes clear and separates out quickly. It takes a lot of water. But if the emulsification layer persists, try applying heat, adding salt, and adding vinegar, in that order.
5. After draining the last wash water away, add 3 g $MgSO_4$ and swirl until it's perfectly clear. The anhydrous $MgSO_4$ absorbs any remaining water in the biodiesel.

Adapted from Adapted from: The Biofuel Project: Creating Biodiesel, U.S. Department of Energy, Office of Energy Efficiency and Renewable Energy

Figure 9. Lesson 3 – Making Biodiesel



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Date: ____________________

BIODIESEL MATH

1. Biodiesel should have a pH tolerance of 0.5. Ideally, the pH level should be 8.5. Write a compound inequality to describe the acceptable pH levels.

2. An average farm consumes (energy input) fuel at 82 liters per hectare (L/ha) of land to generate one crop. However, an average crop of rapeseed makes oil (energy output) at an average rate of 1,029 L/ha, and high-yield rapeseed fields produce about 1,356 L/ha.
   a. What is the ratio of input to output for an average crop of rapeseed and a high-yield rapeseed field?
   b. Energy return on energy investment (EROEI) is the quotient of energy returned to society and the energy required to get that energy. When EROEI is less than one, the energy production process is considered to be unsustainable; values greater than one means the energy production process is considered to be sustainable. Calculate the EROEI for an average crop of rapeseed and a high-yield rapeseed fields. Interpret the values of EROEI.

$$EROEI = \frac{Energy\ returned\ to\ society}{Energy\ required\ to\ get\ that\ energy}$$

3. Jade's new Volkswagen Jetta with a diesel engine gets 35 miles per gallon (mpg) in the city and 42 mpg on the highway. Write a compound inequality which represents the number of miles she can drive on 14 gallons of gas.

4. The fuel efficiency of an average car with a diesel engine is given by the equation below, where *E* is the fuel efficiency in miles per gallon, and *v* is the speed of the car in miles per hour.

$$E(v) = -0.018v^2 + 1.476v + 3.4$$

   a. What speed will yield the maximum fuel efficiency? What is the maximum fuel efficiency?
   b. We know that biodiesel has about 10% less energy than regular diesel, so it should get fewer miles per gallon. What speed will yield the maximum fuel efficiency? What is the maximum fuel efficiency?

5. Energea, a New Zealand based biodiesel plant, spends $0.334/liter for processing Plant A and $0.150/liter for processing Plant B. Both plants produce 80,000 liters of biodiesel over a year's time with an average yearly cost of $15,680. How much biodiesel does each plant produce?

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6. Over a year's time a field produces 18 dry tons of biomass per acre, namely woody biomass and switch grass. The estimated cost delivered to the converter is $75 for woody biomass and $90 for switch grass for a total cost of $1470.
   a. How many dry tons of woody biomass and switch grass was produced?
   b. How much did it cost to convert each source to biofuel?

7. Soybean is grown on about 80 million acres each year in the United States. Soybean has an oil content of 20% and therefore has less oil than other oil crops–palm oil (Malaysia); rapeseed and sunflower oil (Northern U.S., Canada, Europe); peanut oil (U.S.); and jatropha oil (India)–that could also be used for biodiesel. Nevertheless, soybean is one of the easiest crops to grow because it makes a good rotation crop with corn or cotton. Soybean yields an average of about 45 bushels per acre. The average yield tolerance for a soybean field is 15 bushels per acre. The price of soybeans by the bushel is $13.51. Unfortunately, over the past 3 years a serious challenge has arisen from a hurricane-introduced disease called Asian soybean rust, which can reduce crop yield by 50%.
   a. Write a compound inequality to describe the average soybean yield for one acre.
   b. Write a compound inequality to describe the oil content for the average soybean yield for one acre.
   c. Write a compound inequality to describe how many bushels of soybean are produced each year in the United States.
   d. Write a compound inequality to describe the oil content of soybean produced each year in the United States.
   e. Write a compound inequality to describe the amount of money made for the average soybean yield for one acre.
   f. Write a compound inequality to describe the amount of money made for soybean produced each year in the United States.
   g. Write a compound inequality to describe the amount of money made for soybean produced each year in the United States due to the disease Asian soybean rust.

Figure 10. Lesson 3 – Biodiesel Math

Name: ____________________
Date: ____________________

ENERGY EFFICIENCY: USE LESS, SAVE MORE @ HOME

You are an Energy Inspector. Your job is to conduct an energy audit of your home.

1. Make a list of items used in your home or on the go that use electricity.
2. Search the item or the internet for information about the wattage.
3. Gather information about usage such as what time during the day the item is used and how many times during the year is it used (seasonal, monthly, weekly, etc.).

Use the information to evaluate the cost of usage using this formula:
$C = \frac{WNHDP}{1000}$, where **W** is the power in Watts consumed by a device, **N** is the number of devices in use, **H** is the number of hours per day the device is in use, **D** is the number of days in a year when the device is in use, **P** is the price of electrical service in dollars per kilowatt-hour ($0.0887 cents), and **C** is the total cost in dollars per year for using the device. We divide by one thousand to convert Watts to kilowatts.

For example, a washing machine has an average rating of 1150 W. Let's assume you only have one washer and you wash clothes every 2 weeks. A wash cycle lasts about 33 minutes and you usually have 3 loads: white, light and dark colors. This means the washer runs for approximately 1.65 hours. How much money does the washer cost in 1 year?

$$C = \frac{WNHDP}{1000} = \frac{1150\text{ Watts} \times 1\text{ washer} \times 1.65\text{ hours} \times 26\text{ days} \times .0887\text{ cents}}{1000\text{ watts}} \approx \$4.37/\text{yr.}$$

Item	Wattage & # of items	Time of day	# of hours	# of days	$ per yr	Alternative option (list alternative appliance or practice)	$ Per yr

Source. Veigele, William J. How to save energy and money at home and on the highway: the mathematics and physics of energy conservation and reduction of consumer energy costs. Boca Raton, Fla.: Universal-Publishers, 2009

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PROPOSAL

Make a top ten list of ways to conserve energy in your household. Write a letter of proposal to your caregiver informing them of the results of the audit. Use the internet to research alternatives in appliances, time of use, etc. Keep a list of the websites used to write your proposal and to create your list. In your letter, include calculations of how long it would take to pay for investments with savings.

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
7. ____________________
8. ____________________
9. ____________________
10. ____________________

Websites

1. ____________________
2. ____________________
3. ____________________
4. ____________________
5. ____________________
6. ____________________
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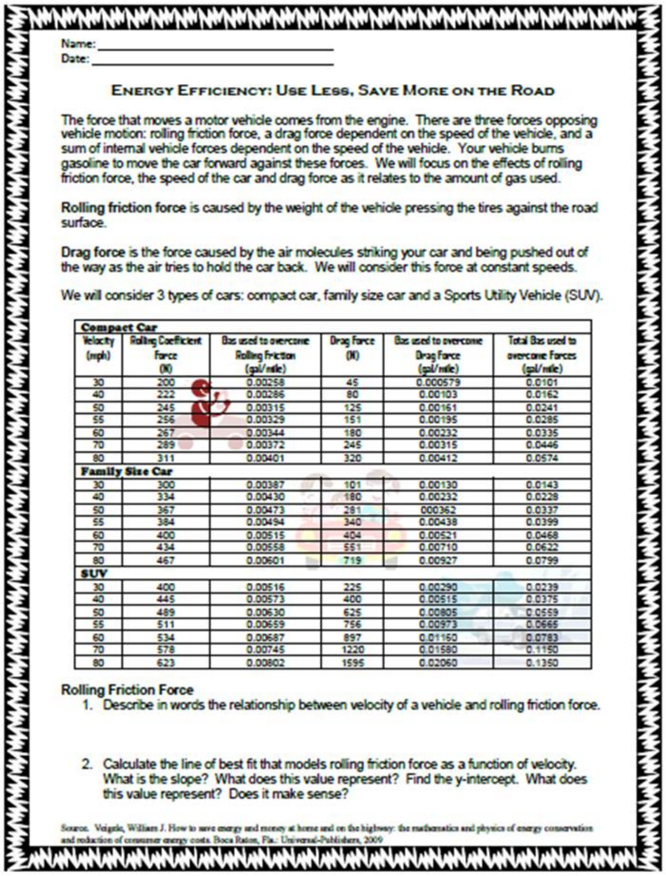
9. ____________________
10. ____________________
11. ____________________
12. ____________________

Figure 11. Lesson 4 – Home Audit: Save Home



Name: ______________________________

Date: ______________________________

Energy Efficiency: Use Less, Save More on the Road

The force that moves a motor vehicle comes from the engine. There are three forces opposing vehicle motion: rolling friction force, a drag force dependent on the speed of the vehicle, and a sum of internal vehicle forces dependent on the speed of the vehicle. Your vehicle burns gasoline to move the car forward against these forces. We will focus on the effects of rolling friction force, the speed of the car and drag force as it relates to the amount of gas used.

Rolling friction force is caused by the weight of the vehicle pressing the tires against the road surface.

Drag force is the force caused by the air molecules striking your car and being pushed out of the way as the air tries to hold the car back. We will consider this force at constant speeds.

We will consider 3 types of cars: compact car, family size car and a Sports Utility Vehicle (SUV).

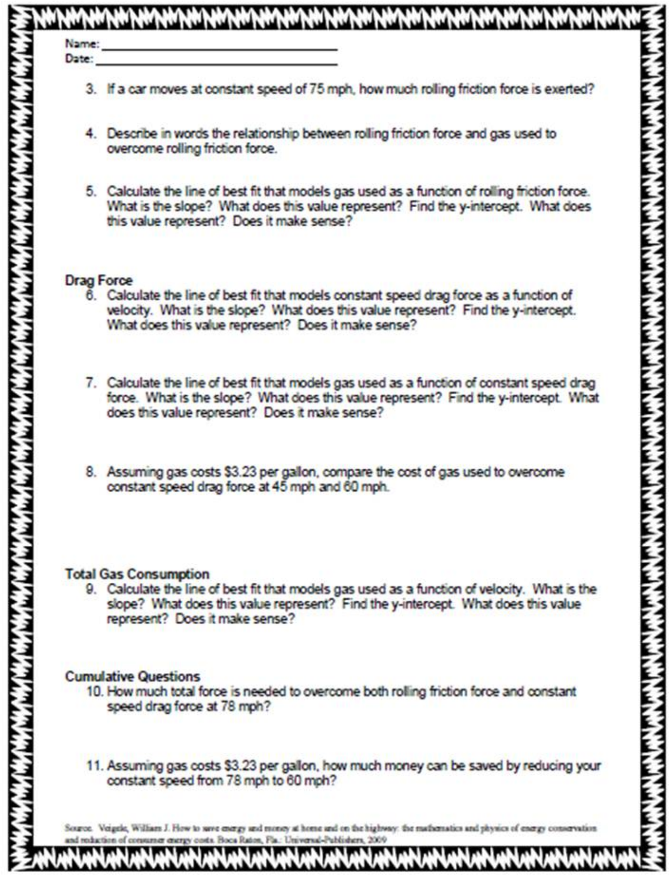
Velocity (mph)	Rolling Coefficient Force (N)	Gas used to overcome Rolling Friction (gal/mile)	Drag Force (N)	Gas used to overcome Drag Force (gal/mile)	Total Gas used to overcome Forces (gal/mile)
Compact Car					
30	200	0.00258	45	0.000579	0.0101
40	222	0.00286	80	0.00103	0.0162
50	245	0.00315	125	0.00161	0.0241
55	256	0.00329	151	0.00195	0.0285
60	267	0.00344	180	0.00232	0.0335
70	289	0.00372	245	0.00315	0.0446
80	311	0.00401	320	0.00412	0.0574
Family Size Car					
30	300	0.00387	101	0.00130	0.0143
40	334	0.00430	180	0.00232	0.0228
50	367	0.00473	281	000362	0.0337
55	384	0.00494	340	0.00438	0.0399
60	400	0.00515	404	0.00521	0.0468
70	434	0.00558	551	0.00710	0.0622
80	467	0.00601	719	0.00927	0.0799
SUV					
30	400	0.00516	225	0.00290	0.0239
40	445	0.00573	400	0.00515	0.0375
50	489	0.00630	625	0.00805	0.0559
55	511	0.00659	756	0.00973	0.0665
60	534	0.00687	897	0.01160	0.0783
70	578	0.00745	1220	0.01580	0.1150
80	623	0.00802	1595	0.02060	0.1350

Rolling Friction Force

1. Describe in words the relationship between velocity of a vehicle and rolling friction force.

2. Calculate the line of best fit that models rolling friction force as a function of velocity. What is the slope? What does this value represent? Find the y-intercept. What does this value represent? Does it make sense?

Source: Veigele, William J. How to save energy and money at home and on the highway: the mathematics and physics of energy conservation and reduction of consumer energy costs. Boca Raton, Fla.: Universal-Publishers, 2009



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Date: ______________________________

3. If a car moves at constant speed of 75 mph, how much rolling friction force is exerted?

4. Describe in words the relationship between rolling friction force and gas used to overcome rolling friction force.

5. Calculate the line of best fit that models gas used as a function of rolling friction force. What is the slope? What does this value represent? Find the y-intercept. What does this value represent? Does it make sense?

Drag Force

6. Calculate the line of best fit that models constant speed drag force as a function of velocity. What is the slope? What does this value represent? Find the y-intercept. What does this value represent? Does it make sense?

7. Calculate the line of best fit that models gas used as a function of constant speed drag force. What is the slope? What does this value represent? Find the y-intercept. What does this value represent? Does it make sense?

8. Assuming gas costs $3.23 per gallon, compare the cost of gas used to overcome constant speed drag force at 45 mph and 60 mph.

Total Gas Consumption

9. Calculate the line of best fit that models gas used as a function of velocity. What is the slope? What does this value represent? Find the y-intercept. What does this value represent? Does it make sense?

Cumulative Questions

10. How much total force is needed to overcome both rolling friction force and constant speed drag force at 78 mph?

11. Assuming gas costs $3.23 per gallon, how much money can be saved by reducing your constant speed from 78 mph to 60 mph?

Source: Veigele, William J. How to save energy and money at home and on the highway: the mathematics and physics of energy conservation and reduction of consumer energy costs. Boca Raton, Fla.: Universal-Publishers, 2009

Figure 12. Lesson 4 - Save on the road

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