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Practice Standards = Perfect Problem Solvers

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Mathematics is one of my favorite subjects to teach, and it is one of the most challenging. When I was in elementary school I was taught to memorize facts and formulas without really understanding what they meant or how to apply them to the problems that needed to be solved. I love that the Common Core Standards for Mathematics have problem solving at the core of math instruction, and I love investing time to figure out how to support the standards by teaching my students how to be better problem solvers.

A trend that is negatively impacting education today is that children expect things in an instant, whether it is the answer to a question or the speed in which their video-game or technology works. Students need to be reminded and frequently encouraged to slow down and take their time; to not give up, to persevere through something they don't know; and to check their work and think about whether or not the process or the product makes sense. That is the central message and foundation for the Practice Standards that are embedded within the Common Core Standards for Mathematics. The Practice Standards are a distinctive and essential part of the Common Core Standards. Unlike the grade level standards, which prescribe specific learning goals year by year, they apply uniformly from PreK through 12 th grade. The Practice Standards are:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. [1]

While the Practice Standards should be embedded across the mathematics curriculum, I believe my students especially need to be taught how to apply them in their everyday routines and procedures when engaging in learning. Students need repeated opportunities to apply the Practice Standards to what they're doing in mathematics and they should be taught to recognize how what they're doing is a Practice Standard that will make them a more successful scholar. I am designing this curriculum unit for second grade students, but it could easily be used in other elementary grades.

I am a second grade teacher in a self-contained classroom in a PreK-5 school in Charlotte Mecklenburg, a large

urban public school district. Teachers on my grade level work together and create thematic units with integrated connections across the subjects of math, science, social studies, and language arts. We develop and use a relevant and rigorous curriculum. As we implement the national Common Core standards for Language Arts and Math, and state-created Essential Standards for Science and Social Studies, prior knowledge of students, along with observations and assessments, inform instruction in the classroom. Student assessments include district mandated tests in the subjects of reading, math, science, and social studies. We gather information about our students through self-reflection, portfolios, grade level pre- and post-unit assessments, formative and summative assessments, and classroom observations. In the classroom and at the school, students have access to numerous technologies including computers with internet and instructional software, calculators, overhead and data projectors, TV, VCR/DVD player, CD player, cassette players, and iPads. Some students have access to technology at home. My school's population is about 850 students. There is an approximate one to twenty-two teacher to student ratio. The demographics of my school include 66% African American, 14% Caucasian, 10% Hispanic, 3% Asian, and 4% other. Approximately 2/3 of our students live in poverty.

The seminar I participated in and this curriculum unit will help me to teach my students to set goals to engage in using the Practice Standards in every problem situation they encounter.

At the beginning of the school year 2 nd grade students start a Problem Solving with Place Value unit. I want to include the process of applying Practice Standards to the process of learning place value and problem solving strategies. I envision my students setting goals for using the Standards in order to take pride in the math work they do. I will model for students and provide an example of what each Practice Standard looks like in the context of solving a problem. We will create a class visual to remind us of each Practice Standard and what it implies, and then when students are solving problems they can reference the poster and make a mental note as to which Standard they used to solve the problem and check and prove their work.

As a result of this curriculum unit, I want a paradigm shift to occur in my classroom. Currently students rush to solve problems without attending to what they're doing, or they give up before even attempting to figure out what the problem is asking of them. Through our strategies and activities related various one-step problem types, thinking about the structure of problems, and thinking in terms of the base ten components of numbers, my classroom climate will show evidence of learners who actively engage in all of the practice standards and solve problems with more perseverance, modeling, precision, structure, and reasoning.

Content Objectives

Prior Knowledge: Second graders need to have some basic number sense. They should be able to represent numbers in standard form (i.e. 4). They should be able to count correctly in sequence and understand one-to-one correspondence. Students should know how to add digits together and if they do not know answers immediately, they should at least know how to access tools to use to represent the ones and can count on or count all. I have several assessments that I give my students at the start of the year, to figure out what they know and understand, to find out who is ready to receive and build new knowledge and information, and who may need additional review and practice.

The Lesson Sequence described within this section describes how I will introduce my class to the two major

goals of this curriculum unit: understanding the important parts and meaning of story problems (word problems) and understanding how to use place value to solve the problems. These lessons describe a sequence and not necessarily each day in the unit. Each lesson could take multiple days or even weeks for students to master the objective and be able to attend to precision.

Weaved throughout each lesson I will be building my students confidence with working with solving problems. No one ever says, "I can't read," but so many adults are quick to acknowledge to their child that they "Can't do math" or "Can't do science." As an educator I take great pride in spending time telling my students (and their parents) "Yes you can!" In the book My Kids Can there is a story about a teacher who spends time observing that some students do not attempt problems and she makes it her goal to build up their confidence related to math and to problem solving. The teacher scaffolds instruction for her students, meets individually with those who aren't participating, and practices strategies beginning with what each student already knows. This allows her students to feel more confident when solving problems independently. Even though this sounds like best practices for anything, it is also something we might take for granted. I'm going to identify students who do not feel confident in the subject of math and spend some time building their confidence. I hope they will be more willing and motivated to do mathematical work. [2]

Lesson 1: Flexibly Understand Numbers

Elementary students need to understand numbers through real world experiences. A numeral such as "3" is just a symbol. As students begin to learn 3 it is like a letter. Just as young scholars learn the sounds a letter can make, and that some letters make multiple sounds, they also need to learn the meaning behind the numeral, and that numbers can appear in many different situations. Three can represent three apples, three friends, three cubes, etc. It could also represent one book and two more books, or two toys and one more toy. Three can be expressed by having four stickers and giving one away, or ten pennies and spending seven. There are many more ways to model and help young learners understand something that on the surface may look and seem so simple. (And that's just for one number – there are infinitely many numbers students need to learn, know, and experience!)

Lesson 2: Fluently and Flexibly Add and Subtract within Ten

Second graders need to be able to "Fluently add and subtract within 20 using mental strategies and by the end of 2 nd grade should know from memory all sums of two one-digit numbers." [3] When I was learning my facts I practiced using flashcards and memorized what two symbols equaled. It wasn't until I started teaching math and spending time understanding numbers that I began to understand what it meant to fluently know addition and subtraction facts. There are so many ways for students to access and understand this information. Students need repeated opportunities and experiences with practicing place value and number sense. I'm going to start with numbers to ten and have my students develop number sense using concrete examples. This provides a foundation for what they will later do abstractly. If students are learning 3+4, the 3 represents 3 of something and the 4 represents 4 of the same thing. I will have students count those "things" (or math tools that can represent those things). If students are adding 3+4 they can line up side-by-side next to a number line 3 cubes next to 4 cubes and count the total length of the cubes. Students can also develop the skills of being flexible when adding by shifting the cubes around to realize that 3+4=4+3. (This is called the commutative property of addition.) Furthermore, students can recognize that addition and subtraction are related. If 3+4=7 and 4+3=7 then 7-4=3 and 7-3=4. When students are taught that numbers are related in this way they can begin to see patterns when solving problems with other numbers. I will make sure that my students need to both understand and memorize their facts to ten. This is the foundation that adding and

subtracting all other numbers will build on. As students develop fluency with adding numbers 0-10, they can begin to work on identifying ways to make ten. By having students line up the cubes end-to-end, they are building a foundation for two things: recognizing the size of a number, and also measurement, as it relates to the length of a train of a certain number of cubes. A manipulative model can be turned into a pictorial model on the number line showing a start from 0 jumping (or drawing a half oval) to the number 3 and then counting on 4 more times to end at the number 7.

Lesson 3: Make Ten

Next my students need to be able to build numbers to ten. To first introduce this I'm going to give students ten cubes and have them separate the cubes into two groups as many different ways as they can. They also will record the equations that represent how they have grouped the numbers. Students need to be responsible for knowing all the combinations to ten: 0+10, 1+9, 2+8, 3+7, 4+6, 5+5, 6+4, 7+3, 8+2, 9+1,10+0. In the strategies section of this unit I'm including numerous activities to help them practice and become fluent in making ten. By looking for patterns of how to make ten students are building a foundation for algebra. If they have one number and need to figure out what they need to get to ten then their information can be represented as 4+x=10. Students will not be explicitly taught this, but they will be thinking like this. Students will use the thought process of "what do I need to add to four in order to make ten."

Lesson 4: Higher Addition Facts - Fluency to 20

Once students are fluent with making ten using different combinations then we move into the "teen" numbers. Teen numbers are 11, 12, 13, 14, 15, 16, 17, 18, and 19. When students have a sense of "ten" then they can combine numbers using tens and ones. Students need to be fluent in all the facts to 10, and especially, the decompositions of 10. Then they can use that to make 10 to add within 20. If students are adding 6+7 and they know that 6+4=10 then they can decompose the number 7 into 4+3 and find the solution by making ten and adding the ones just like this: 6+7 = 6+4+3 = 10+3 = 13. Students need lots of opportunities to write these equation sentences in order to demonstrate the flexibility of working with numbers. Rearranging counters in 10-frames can help them understand the sequence of symbolic equations involved in adding by "making a ten". This allows students to accurately solve the problem and apply the Practice Standards 1, 6, and 7. [4] After students learn the basic structure for finding tens they can apply this type of problem solving to any problem they encounter. To further extend their knowledge, when students begin working with problems that involve 2-digit or even 3-digit numbers (as second graders do) they can look for patterns that will get them to the number ten or the number 100 and begin to build new numbers using hundreds, tens, and ones.

There are multiple ways for students to approach learning and understanding numbers. When I was a learner in elementary school I remember having flashcards and timed tests to learn my addition, subtraction, multiplication, and division facts. Now students are expected to not just know facts, but understand numbers. "Memorized knowledge is knowledge that can be forgotten. Internalized knowledge can't be forgotten because it is a part of the way we see the world. Children who have internalized a concept or relationship can't forget it; they know it has to be that way because of a whole network of relationships and interrelationships that they have discovered and constructed in their minds." [5] When children are given numerous opportunities and options for working and manipulating numbers they are engaging in Practice Standards 7 and 8. An example of a child who can make use of structure and express regularity in repeated reasoning would be able to defend that 8+7=15 and that student would show numerous examples of how to prove that information: 8 + 7 = 8+2+5=10+5; 8 + 7 = 7 + 8 = 7 + 3 + 5 = 10 + 5 =15. When students can express numbers and equations in numerous ways they have demonstrated internalizing making a ten.

Lesson 5: Problem Solving – Teaching Vocabulary and Meeting a Variety of Problems.

As students are working to build an understanding of numbers they should be manipulating the numbers within the context of word problems, sometimes also called story problems. Numbers represent units of something and if students know what those units are then they can better visualize how to work with the numbers and begin to solve problems. "Mathematics word problems are as important for their potential to improve reading skills and thinking skills as they are for teaching mathematics. In fact, word problems are the glue that holds mathematics to the real world, and studying them from a language arts point of view, as passages that we want to understand, is as important as solving them." [6] When beginning to solve word problems students must first understand the language of the problem and what the problem is asking them to do. To develop this practice with my students I will model how to read a problem to and with students. Math very much integrates language arts and just as when students take notes when reading a passage or story, they can learn to apply this skill to math. I will model underlining the important part(s) and discussing what the problem is asking and how they know. Word problems should match the skills that students are working on in the content of math. They can also reflect the learners in some way. When I use current events in problems or include the names of my students then students are more interested in solving the problems and the problems will have more meaning them. Some examples of word problems students can solve when building an understanding of ten include:

1. Carlos has 6 markers. Ten markers come in a box. How many markers is he missing?
2. Meg has 3 songs on her iPod. She is allowed to download 7 more songs. How many songs will Meg have in all?

As we progress through various steps, the word problems will progress as well. For example, when my students learn how to decompose numbers into tens and ones, then word problems will relate to grouping things into tens and ones. Later in the school year when my students subtract 3-digit numbers with regrouping, word problems will include situations where students are subtracting 3-digit numbers that involve regrouping. My students will have numerous opportunities to solve word problems aligned to each objective. And, I will continue to provide more examples of how to read a word problem and look for the important parts of a problem.

When students are working with word problems they have an awareness of the unit the number represents. This lends itself to building a foundation for when students are solving math problems at a higher level or in other areas of math. 12+12+12=1 doesn't make sense in the context of a purely symbolic equation, but when described as 12inches + 12inches + 12 inches = 1 yard it does. Also, when students are writing the unit, they are practicing language skills and integrating them into math.

There are 14 types [7] of common word problem situations that if students can recognize and solve they will be able to deal with a wide variety of problems. When students have a foundation for flexibly and fluently making ten and figuring out how to solve word problems then they are ready to progress through the types of learning activities that will help them understand larger numbers.

Lesson 6: Expanded Form of 2-digit Numbers

I will teach my students how to learn and understand 2-digit numbers by representing them in expanded form. My students will show 2-digit numbers using base ten blocks which model how many tens and how many ones are in a number. For the number 27 a student would display 2 ten rods and 7 unit blocks. My students will represent the numbers by describing them as tens and ones. 27 would be described as 2 tens and 7 ones. I

will show my students how to represent the numbers using expanded form. In expanded form the number 27 is represented as 20 + 7. (To reach this understanding, students may need to count 10, 20 and 1, 2, 3, 4, 5, 6, 7). I will make sure to give students the opportunity to describe numbers in expanded form in the context of story problems. *Jose has to put his toys away. His toys fit in boxes of ten and leftovers go on the shelf. If Jose has 27 toys how many boxes of ten will he fill and how many leftovers go on the shelf?* My students will have multiple opportunities to practice the skill of expanding numbers within the context of a problem, and I will assess them to make sure the base ten structure is understood.

An important part of adding and subtracting two digit numbers is adding and subtracting the "-ty numbers" - the multiples of ten. I want to make sure that my students understand that when they add two -ty numbers, the result will again be a -ty number, and the tens digit will just be the sum of the digits of the two addends. To ensure they get this, we will spend time adding collections of groups of 10. We will also have a lot of story problems involving just –ty numbers.

Lesson 7: Adding and Subtracting 2-digit Numbers

Once my students understand how to create the expanded form of a number they can begin adding (then subtracting) 2-digit numbers without regrouping. I will give my students problems to solve and require them to use place value. *Germaine had 35 dollars. He received 41 more dollars on his birthday. How much money does he have now?* After students determine the important parts of the word problem they are ready to begin solving it. I will give my students base ten blocks to model the problem situation, and students will also write it symbolically using numbers. First, students will expand both numbers from the problem into tens and ones. Next students will group the tens and group the ones. Then add the tens and add the ones. Finally they will combine the tens and ones. The following sequence of equations gives the symbolic version of these steps.

35 + 41 = 30 + 5 +40 + 1 = 30 + 40 + 5 + 1 = 70 + 6 = 76

Since subtraction is the undoing of addition it is important for students to practice subtraction situations with addition situations. When students can fluently add 2-digit numbers without regrouping they can progress to subtract 2-digit numbers without regrouping. *Germaine has $76 dollars. He had $35 dollars. He received some more for his birthday. How much money did Germaine get for his birthday?* As with addition, students will expand numbers into tens and ones, but instead of adding the pieces (tens and ones) of the second number, they will subtract the pieces. *One thing that needs to be made clear here is that even though students are subtracting, they are decomposing numbers into tens and ones therefore 76=70+6 not 70-6. A foundation for this is for students to be able to compare numbers and remind them to start with the greater number and subtract the smaller number. It is also helpful to use manipulatives to represent this information first and then move to the pictorial representation and then the abstract. Once the numbers have been expanded into tens and ones I will model how to group the numbers into the tens and ones. Next, students will subtract the tens and subtract the ones. Last, they will add the tens and ones.

76–35 = 70+6–30–5 = 70–30+6–5 = 40+1 = 41

Lesson 8: Adding and Subtracting 2-digit Numbers with Regrouping

Once my students have mastered the art of adding and subtracting 2-digit numbers that do not involve regrouping, they are ready to learn how to regroup numbers. When learners are adding 2-digit numbers that involve a regrouping situation in the ones place, the process includes an additional step from what was described for adding 2-digit numbers without regrouping. *Donna's dad works at the Apple store. In March he*

sold 37 iPhones and 45 iPads. How many devices did he sell altogether? I will begin by modeling for my students writing the numbers in expanded form. Then we will group the tens and group the ones. Next we will add the tens and add the ones. Since the "ones" comes out to a two-digit number, we will make it into expanded form; we will make it into a ten and ones to add the tens to the other tens. I will help my students realize that this process is just reproducing one of the higher addition facts. As they become fluent with adding tens and ones, they may be able to do the recombining process mentally, but when we are just starting to study this process, I will teach them to break numbers apart into their pieces and then put the pieces back together. The final step involves combining the tens and ones. Here is the sequence:

37+45 = 30+7+40+5 = 70+12 = 70+10+2 = 80+2 = 82

I will start teaching how to subtract 2-digit numbers with regrouping by reviewing the ways to represent a number. For example the number 34 = 3 tens and 4 ones (30+4), and it also equals 2 tens and 14 ones (20+14), 1 ten and 24 ones (10+24), and 34 ones (34). This is a good place in the learning sequence to pause and make sure my students are able to be flexible with how they construct numbers. *Abraham had 54 minutes to get all his chores done. He wasted 29 minutes playing a video game. How many minutes does he have left?* Similar to addition and to subtraction without regrouping, I will model for my students how to expand numbers into their pieces of tens and ones. Then I will instruct my students to group the tens and group the ones. (For the first few examples I will use manipulatives to represent this information and then move to the abstract or pictorial representation). After we have grouped numbers into tens and ones we can subtract the tens and ones. (It really doesn't matter what order students subtract in, because regrouping can occur at any time). Regroup 30 into 20+10 so students can subtract 9 ones from 10 ones. Finally, subtract and add it to the equation. Then add the tens and ones. A lot happens during subtraction with regrouping. It is important for students to keep track of the numbers they have used to make sure they've applied the operation where they need to. I model keeping track by underlining numbers that I've already used. My students can still see the number, but have a signal to know it's been used. When assessing student work I can see what numbers my students are using so if a student gets it wrong I can see where s/he messed up or forgot a step.

54-29 = 50+4-20-9 = 50-20+4-9 = 30+4-9 = 20+10+4-9 = 20+14-9 = 20+5 = 25

I plan to give my students pairs of problems, one involving addition and one involving a subtraction in the same "fact family", so that they can see that the regrouping process in the subtraction problem just reverses the regrouping that we did when adding.

Lesson 9: Expanded Form of 3-digit Numbers

As my students become experts at expanding, adding, and subtracting two-digit numbers they are then ready for the challenge of working with three-digit numbers. I will start by introducing how to write and model three-digit numbers in expanded form. My students will model 3-digit numbers using base ten blocks showing how many hundreds, tens and ones. Then we will engage in a discussion that 10=10 ones and that 100=10 tens. For the number 638 my students would display 6 flats, 3 ten rods and 8 unit blocks. They will represent the numbers by describing them as hundreds, tens and ones. 638 would be described as 6 hundreds, 3 tens, and 8 ones. Students represent the numbers using expanded form. In expanded form the number 638=600+30+8. (To reach this understanding, students may need to count 100, 200, 300, 400, 500, 600, then 10, 20, 30, then 1, 2, 3, 4, 5, 6, 7, 8). My students will have multiple opportunities to practice and manipulate numbers to show mastery of breaking numbers apart into the pieces of hundreds, tens, and ones. They will also need to work with numbers within the context of story problems. *Carla was working at a store. She had to organize Legos into boxes of hundreds, tens, and ones. At the end of the day she had 638 Legos.*

How many boxes of hundreds, tens, and ones did she have? What are some other different combinations she could make when putting away the Legos? The second part of this question gives students the opportunity to expand their thinking and prepare for when they subtract three-digit numbers with regrouping. 638 can be organized into 6 hundreds, 3 tens, and 8 ones, but it can also be organized into 5 hundreds, 13 tens, and 8 ones; 6 hundreds, 2 tens, and 18 ones; 1 hundred, 51 tens, and 28 ones; and so many more possibilities. Students will have the opportunity and encouragement to flexibly represent numbers.

Lesson 10: Adding and Subtracting 3-digit numbers without Regrouping

Students will begin to add three-digit numbers using the expanded form of numbers. So first, students will decompose both numbers into hundreds, tens and ones. Just as I gave them practice with adding multiples of 10, I will give them problems that just ask for adding multiples of 100. We will see that the digits behave as if we were adding ones –and in fact, we are adding single-digit multiples of rather large units!

Then we will move and add the hundreds, tens and add the ones. Then they will combine the hundreds, tens and ones. *Marcus and his family went on a road trip. They traveled 274 miles in the first day and 314 miles the second day. How many miles did they travel in the two days?*

274+314 = 200+70+4+300+10+4 = 200+300+70+10+4+4 = 500+80+8 = 588

We will progress from adding 3-digit numbers without regrouping to subtracting 3-digit numbers without regrouping. Students start by decomposing both three digit numbers into their pieces of hundreds, tens, and ones. After students have decomposed numbers into hundreds, tens and ones, they will then group the numbers to subtract the hundreds, tens and ones. We will observe that, when working with units of a given size, the digits behave exactly as when we learned the addition and subtraction facts. That is why it was worthwhile spending time to learn those facts! (Again it is helpful with the larger numbers to underline the numbers used from the original equation so as a way to make sure all the pieces are included in the equation. Then add the hundreds, tens and ones. *Marcus and his family went on a road trip. The total trip was 588 miles. They traveled 274 miles on the first day. How many miles did they travel on the next day?*

588-274 = 500+80+8-200-70-4 = 500-200+80-70+8-4 = 300+10+4 = 314

Once my students have lots of experiences with adding and subtracting 3-digit numbers without regrouping, they can begin to practice adding 3-digit numbers with regrouping.

Lesson 11: Adding and Subtracting 3-digit Numbers with Regrouping

Adding 3-digit numbers is more involved than adding 2-digit numbers, and when regrouping is involved, it increases the challenge. Regrouping can occur between the ones place and the tens place, between the tens place and the hundreds place, or in both situations. It is important for students to keep track of the steps they do. I will teach my students to add 3-digit numbers using the expanded form of numbers. So first, students will expand both numbers into hundreds, tens and ones. Then students will group and add the hundreds, tens and ones. (As numbers get larger and students are working with more pieces, I will have my students underline the pieces they've used to make sure they represent it in the new equation). Then students will further expand numbers to show hundreds, tens, and ones and then add groups of hundreds, tens, and ones. If students have mastered fluency of adding numbers they may be able to skip this step, but I would make sure they have multiple experiences where they are continuing to expand numbers before putting them back together. This increases their number sense and allows them to build the capacity of being an expert at

problem solving and place value. Lastly, students will combine the hundreds, tens and ones. All of this should occur within the context of story problems too. *The elementary school was collecting recycled cans. Tyrone collected 384 cans. Susan collected 269 cans. How many cans did they collect in all?* I will make sure to give my students opportunities to practice all types: regrouping occurring between the ones and tens place, regrouping occurring between the tens and hundreds place, and regrouping occurring in both places.

> 384+269 = 300+80+4+200+60+9 = 300+200+80+60+4+9 = 500+140+13 = 500+100+40+10+3 = 600+50+3 = 653.

Subtracting 3-digit numbers with regrouping can also occur in more than one position. Regrouping can happen between the hundreds and the tens place, between the tens and the ones place, and between both places. First, we will start by reviewing ways to expand numbers into their pieces. For example the number 134 = 1 hundred, 3 tens, and 4 ones (100+30+4), and it also equals 13 tens and 4 ones (130+4), 120 tens and 14 ones (120+14), etc. *Tyrone and Susan were collecting cans for school. They collected 653 cans in all. Tyrone collected 384. How many cans did Susan collect?* Students will expand numbers into their pieces of hundreds, tens and ones. Then they will subtract the hundreds, subtract the tens and subtract the ones. I will use manipulatives to represent this information first and then move to the pictorial representation and then the abstract. Once numbers are expanded we will group numbers. The next step asks for students to subtract the tens which could equal a negative number, (and if students learn to write equations like this they are getting a foundation for working with negative numbers which they will be required to do when they get into higher level math and science, but second graders work in positive numbers). Since we cannot take 80 away from 50, students need to expand the 100. There are so many ways to manipulate this. Once we model how to regroup to subtract the tens the remaining step is to subtract the ones digits. Again students will need to regroup. The following is one out of many possible ways teachers can model solving this type of subtraction situation using place value.

> 653-384 = 600+50+3-300-80-4 = 600-300+50-80+3-4 = 300+50-80+3-4 = 200+150-80+3-4 = 200+70+3-4 = 200+60+13-4 = 200+60+9 = 269

Adding and subtracting 3-digit numbers could also involve a "rollover" situation where carrying in the ones place contributes to carrying in the tens place like in the equation 347+256=603; or borrowing is required in both places like in the equation 603-347=256. I will give students opportunities to practice examples like these within word problems and allow them to define what is happening in order to become proficient in solving problems involving these types of situations. I will make sure my students have mastered regrouping in one place first before introducing problems with "rollover" situations.

Lesson 12: Multistep and Higher Level Problems

Once students have a foundation for solving problems, and they are attuned to really applying the Practice Standards, they need to be given multiple problem series and higher level problems to challenge their thinking and the application of their skills. These problems include similar situations or similar numbers asking for different information. Types of problems like these encompass everything they have learned and challenge how to think about problems and numbers. Multistep problems and open ended problems encourage students to think deeper about the math content they know and apply it to various situations. Students also need to have the opportunity and encouragement to write their own story problems. Student-created word problems reveal whether students understand the math in an equation situation (if you give them an equation to write a problem about), and it allows students to practice their writing skills in the context of math. If students are creating their own word problems correctly then they are showing proficiency of the Practice Standard 1.

Multistep problems can look different for students. This is a really good way to see how proficient students are at reading and understanding a problem, and to scaffold instruction for students who are not. I notice that when students begin to solve multistep problems they tend to only concentrate on one piece of the problem and need to be reminded there are more. For students who persistently only solve one part of a problem, the problem can be framed in a way to let students identify the different pieces individually and not as one whole problem, but rather separate problems put together. Here are two alternate formulations of the same multi-step problem.

> Martha earns $5 dollars a day for feeding and walking her neighbor's dog. She has to walk the dog for 9 days while her neighbor is away. How much money will Martha make after taking care of the dog for 9 days? Her neighbor just left 3 days ago so how much money has Martha made so far? How many more days does Martha have to keep taking care of her neighbor's dog? How much more money will she make?
>
> OR
>
> Martha earns $5 dollars a day for feeding and walking her neighbor's dog. She has to walk the dog for 9 days while her neighbor is away.

1. How much money will Martha make after taking care of the dog for 9 days?
2. Her neighbor left 3 days ago so how much money has Martha made so far?
3. How many more days does Martha have to keep taking care of her neighbor's dog?
4. How much more money will Martha make?

In the Practice Standard 7 sometimes the structure occurs in the numbers and how they are framed and put together, but sometimes the structure occurs in the process students engage in to solve the problem and make sense of how they create their solution or solutions as the case could be with multistep problems. I can also create problems for students where they are adding more than two numbers together.

There are many things for students to learn and a sequence for second grade students to progress through to support their learning in a logical manner where one idea can build upon the foundation of another. As my students become more proficient in displaying and writing numbers in a linear fashion and when I know they understand that the 3 in 134 is really 3 tens, I will then introduce how to line numbers up vertically so students can add and subtract that way as well.

In the next section of this curriculum unit I will illustrate several practical strategies that I will implement in the classroom to support student learning and understanding of place value and problem solving.

Teaching Strategies

Integrate language arts skills into mathematical instruction

Today most of the problems students solve are in the context of word problems (or an even friendlier term we use in second grade is "story" problems). It helps students see equations in real world situations which make the process of solving problems more meaningful to students, and it adds an element of students not just

solving an equation, but engaging in reading and skills related to reading - understanding the main idea and details of the problem. Students have to analyze what is important information and what is not. By implicitly teaching students and acknowledging for students they are integrating reading into their math curriculum then they can begin to see connections between the two subjects and hopefully apply the skills they use to answer questions in reading to answering questions in math. Story problems allow students to "attach meaning to operations" and "extend their understanding of situations involving operations." [8] "Mathematics occurs in forms much more like word problems. It is by doing word problems that students realize the importance and applicability of mathematics. Word problems are also a very effective way to reinforce both concepts and calculational skills." [9] Word problems can be written by teachers to get students to start thinking about numbers and operations in context. They can also be written by students. When I challenge my students to write their own problems to match equations, they are thinking at a higher level to demonstrate they know what is being asked of a situation. I make sure to read word problems to or with students so that students who are struggling readers do not get hung up on the words, specifically the words they do not know or understand, and can spend time focusing on understanding the math behind the words or situations.

Another language arts and math integration is to teach students the vocabulary associated with the different structures that problems can be identified with having and have students categorize what type of problem they are solving. The structure vocabulary includes: Change plus or minus with initial amount unknown, change unknown, or result unknown; Comparison greater or fewer with greater amount unknown, smaller amount unknown, or difference unknown; and, part-part whole with one part unknown or whole unknown. This helps students understand the relationship between the number(s) they know in a problem and the number(s) that are unknown in order to understand how to solve the problem. Examples of all these types of problems for different grade levels can be found with the Common Core support documents. [10]

Language arts is also integrated with math when I teach my students to use labels or units to describe numbers. Sometimes problems ask for students to add or subtract common objects (red balls and red balls) and sometimes they do not (pennies and nickels). When students are solving problems they need to label parts of the problem and especially the solution so others can know what each number is referring to (i.e. the units of the number). Students use this skill so they know when they have common units or to recognize when they need to find common units. This will later be applied when working in measurement, geometry, fractions, algebra and other topics in math. If my students can develop a habit of recognizing and labeling numbers as beginning learners, then they will apply it when working in more challenging math situations as older learners.

Use models to illustrate addition and subtraction and the relationship of how many more.

I try to scaffold instruction from concrete to pictorial to abstract. Some of my students will need to use manipulatives longer than other students and that's ok. Manipulatives help students reason and work with numbers in a concrete hands-on way. Numbers can be represented with any material. The number four can be represented with 4 crayons, 4 cheerios, 4 paper clips, 4 snap/pop/unifix cubes, 4 teddy bear counters, 4 base-ten-block units, or many, many other materials. In my classroom I call manipulatives "math tools" and they are referenced in the Practice Standards 4 and 5. When students engage in the use of manipulatives they are representing numbers in a very concrete way and supporting their reasoning for the operations they are conducting in equations. I always have students represent their models using pictorial representation on paper. When students are working with numbers to 10 or 20 then it's ok for them to use manipulatives to show one-on-one correspondence, however, for numbers past 20 I encourage students to start building number sense by making numbers represented as tens and ones and hundreds tens and ones so for students using math tools in the classroom it is a good time to transition to using Base-Ten-Blocks.

To prepare students for measurement and integrate the idea of measurement of length, and help them understand the relationship of size, students can line up cubes in a train to have a visual of the difference of sizes as a number increases or as a number is compared to another number. This is especially important in helping students understand the difference between one and ten, ten and one-hundred, and even one and one-hundred. Size models can look different for different problems. By lining up the trains on a number line I can reinforce the idea that length combination amounts to addition. (See Appendix II for Length Model).

Number Line

A number line is a perfect tool to use when modeling addition and subtraction situations. Number lines are used to build an understanding of size and distance – foundations for measurement of length. Before even introducing the number line I have students practice modeling equations using snap cubes or tiles which align to using an inch as the unit length for the number line or unit cubes which align to using a centimeter the unit length. My students line up the cubes and place them with the zero edge of a ruler (either inch or centimeter ruler or side depending on what math tool your students are using). My students line up their cubes for an addition problem and they can see the magnitude of the numbers they're working with. When they line them up along a ruler they are attending to the Practice Standards 4, 5, and 6. After students practice pairing using cubes along a ruler they can begin to conceptualize how to create a visual of a number line on their paper and iterate or evenly segment numbers across the number line and model the equation using the number line. *Chris got three books. Then he received five more books. How many books does Chris have now?* (See Appendix II for Number Line model).

Games

Students love to play games. Games are a great way to help them both gain an understanding of what they are learning and also repeatedly practice new knowledge and information. There are three games that I know and use to help students manipulate equations to build ten. It is helpful to play the two card games with number cards. I use cards that have both the number written in standard form and also include a pictorial representation. I always make sure students record the equations they create when playing math game.

"Tens Go Fish" is played like "Go Fish," but instead of asking a partner for a pair, a student asks for a number that would help him/her make ten. Each player gets five cards and places down any two cards that when added together make ten. Then students take turns asking each other for a number that when added to a number in their hand will equal ten. Students replace each card they place down with a card from the "Go Fish" pile until all the cards are used up. Students also record the equations while they get them in order to practice all the different ways to represent ten. [11]

Another game is called "Make Ten." During this game students place number cards in an array of 5 x 4 and take turns finding combinations of ten. (A more challenging version is placing the cards face down and playing it like a memory game trying to find tens while remembering locations of cards). With both of these games students will record the equations they make with their hands in order to help them better commit to memory what they're doing and learning. [12]

"Cover up" is a third game that can be played with tiles or counters. Students count out ten counters and place them on a surface. Students take turns covering some of them. (You can use a book or piece of construction paper to cover them). Then they write an equation using a square in place of the missing number, make a prediction, guess, or reason how many counters are covered and record it in the square. Then students should uncover the counters to check and see if they are correct. I always tell my students to

validate themselves if they got the right answer with a check, star, or smiley face on their paper next to a correct equation.

$4+\square=10$ $4+\boxed{6}=10$

Class Discussions

One of the Practice Standards requires students to "Construct viable arguments and critique the reasoning of others." The best way for students to apply in this practice is to engage in class discussions. Students need to share the work they are doing with others and defend both the process they use and the product they achieve. Class discussions can look like whole group, small group or partner. Sometimes it is hard to engage all students in a discussion if it is whole group, but class discussion is not only a great way for a teacher to assess student understanding, but also for students to share their strategies and thinking with others. At the beginning of the school year I establish the routine/procedure for Class Discussions. We brainstorm why class discussions in math are important and students tell me the rules for discussion time. I record the rules and make them visible as a review for every discussion. In addition to brainstorming rules and expectations for discussions, we discuss the role of the listener. Once my students have developed ownership on why class discussion is important to a mathematical practice I can begin facilitating discussion conversations. This is a great and safe time in the day for my students to ask questions, offer and illustrate different strategies and ways they think about and solve problems, share feelings, review and use vocabulary, explain their process and steps for solving in their own words, and much more. I can anticipate student participation or lack thereof by meeting and having a small group discussion with students who may be struggling and give them the opportunity to practice both the problem solving that they're doing, and their thinking and discussion about what they're doing. As an adult learner I value the time I have to discuss ideas and concepts with my colleagues to learn new strategies and work through struggles I'm having as an educator. Usually a light bulb goes off when I hear how another teacher solves something or figures out how to get his/her students to understand something. Students can benefit from classroom discussions as well.

Ten Frames

Ten Frames are a great way for beginning learners to see ten in a very visual way. As my students progress through their understanding of ten they can be used to see groups of tens and singles. Ten Frames are an array of two rows and five columns that illustrate a quantity. Students use what they can see to figure out how many they need to make a set of ten. An example of a Ten Frame is below.

Ten Frames help students visualize addition or subtraction. Students can see this Ten Frame as 5+2+x=10, 7+x=10, 10-x=7, or 10-7=x. Second grade students can use the variable x or some other symbol to represent the missing information, or they will probably express it in a sentence saying or writing 10 minus something equals 7. After students become proficient with understanding one Ten Frames they can be given multiple Ten Frames to figure out a quantity and/or missing amount. (See Appendix II).

Fact family houses

Operations have relationships. Subtraction is the undoing of addition just like division is the undoing of multiplication. Students can understand numbers better when they begin to see numbers in terms of their relationships, and when students understand their relationships they can choose which way of solving problems works best for them. Some students are more comfortable with solving subtraction problems through counting back while others may choose to count up. When students understand the relationships between numbers they can engage in making these choices. As students are working toward knowing and understanding their math facts and mental strategies it is sometimes helpful for them to visualize the relationship. One of the ways I help students to do this is to teach them how to build number houses. Visualize the house having a rectangular base and triangular roof. In each corner of the triangle students record three numbers that have a relationship. In each corner of the square base students record an equation that illustrates the relationship they have. Fact family houses can be built for any three numbers that have a relationship. Fact family houses can also be used to help students understand that numbers in a word problem have a relationship and students just need to figure out what the relationship is – what part of the family the missing number belongs in. *Jan used 65GB of data on her phone. Her plan only allows her to use 100GB of data each month. How much more data can Jan use this month?* (See Appendix II)

Classroom Activities

This section of my curriculum unit will include mini lessons where I describe teaching students about different types of problems they are going to be solving and model for them how to apply the practice standards while solving them. While working through problem solving I will scaffold both the problems and levels of support I will provide students as they continue to practice problems and become proficient

mathematicians.

Activity 1 - Expanded form with story problems and assessment

Objective: Students will decide how many hundreds, tens, and ones are in a number by completing a table and representing the number in expanded form.

Whole Group Mini-Lesson: I will model writing a table on the board (or creating one digitally with a SMARTboard). In the table we will organize numbers into their pieces of hundreds, tens, and ones, and write the expanded form of the number. As my students progress through the lesson, some numbers they will see the standard form of, some numbers will include the expanded form, and others will include the word form. Here's a sample table that I'll use in my classroom and ask students to help me complete using the given information. The first line models how to complete the table. I will intentionally model writing the units too so students will represent the units in their table.

Standard Form	Hundreds	Tens	Ones	Expanded Form
362	*3 hundreds*	*6 tens*	*2 ones*	*300+60+2*
925				
	4 hundreds	*3 tens*	*0 ones*	
				80+4

Workshop: My students will work independently, with a partner or in a small group to complete a table like the one modeled above. Students will represent numbers by expanding them into their pieces. I will work with a small group and provide additional support by modeling numbers using base-ten-blocks. (They are also available for all students to access if they want them).

Discussion: Students will compare their tables with a partner and discuss any differences. I will give students a story problem that requires them to expand numbers and partners will work together to solve it. *Megan can only fit 10 shirts in a drawer. If she has 37 shirts how many drawers will she fill? How many shirts will be leftover?*

Assessment: I will give students a table like the model and students will complete it as an exit ticket.

Activity 2 – Diagram model or teen number lesson

Objective: Students will be able to add and subtract "teen" numbers by using manipulatives and looking for patterns to make ten.

Whole Group Mini-Lesson: I will model for students how to find groups of ten and ones. I will present my students with a problem. *Jared had to empty the dishwasher. There were 8 plates and 6 cups. How many things did Jared have to put away?* Then I ask students questions to get them to help me identify that if I have 8, how many do I need to make ten? Then if I take that amount away from the 6, what is left. I will model using cubes, ten frames, and symbolic notation: 8+6=8+2+4=10+4=14. I could also model and explore this same problem like this: 6+8=6+4+4=10+4=14.

Workshop: My students will be given multiple problems similar to the one described in the mini-lesson. They can work independently, with a partner or in a small group to solve the problem. Students will receive scaffold levels of support as needed and have access to math tools.

Discussion: Students will discuss with a partner the different ways they created ten. They will also work together to check their work by solving one or two problems a second way. Students will discuss how they know they have the correct answers.

Assessment: I will give my students an exit ticket where they have to show two ways to solve a problem by using this strategy of making ten and adding on the ones.

Activity 3 – Multi-level questions

Objective: Students will solve word problems by identifying the important parts of the problem, drawing a diagram to model the problem, and using place value to find a solution.

Whole Group Mini-Lesson: I will model how to read problems: give students a paper with the story problems. (I

usually leave space on a paper so students can solve the problems right underneath where the problem is written – this helps second graders be organized).

1. Chorally we will read the story problem and have students discuss what the important parts of the problem are. Students will underline the numbers and units in the problem and the question the problem is asking.
2. I model for students how to write an equation to represent the problem. I will represent the problem with a diagram or manipulatives.
3. We will show the numbers using tens and ones (or hundreds, tens, and ones).
4. We will group the numbers so the tens are together and ones are together.
5. We will add the tens and add the ones. (I model for regrouping if necessary).
6. We will repeat the steps for each problem. As I practice with students I scaffold the amount of time I spend questioning students and guiding them through the process described above.

Here are some problems I will use for this lesson:

Students in Ms. Dooley's class are expected to read 35 pages every night during the week.

1. How many pages do students have to read in one week?
2. John reads 32 pages on Sunday and 38 pages on Monday.
3. How many pages has he read in all?
4. How many pages does he have left to read for this week?
5. Maria has read 35 pages every night for 4 nights.
6. How many pages has Maria read?
7. How many pages does Maria have left to read?
8. If Maria wants to take one day off from reading during the week how many pages would you recommend that she read on the two days left? (There is more than one correct answer!)
9. Sarah read 42 pages on Tuesday. She read 10 more than Ricco. She read 15 more than Jillian.
10. How many pages did Ricco read?
11. How many pages did Jillian read?
12. How many pages have the read altogether?

Workshop: I will create problems similar in style using different numbers and allow students to work through solving them independently, with a partner, or in a small group with me.

Discussion: Students will choose one problem and share with a partner how they solved it and what their answer was. Students will compare answers.

Assessment: I will score one of the problems students complete independently and evaluate both their work and their answer.

Appendix I - Implementing Common Core Standards

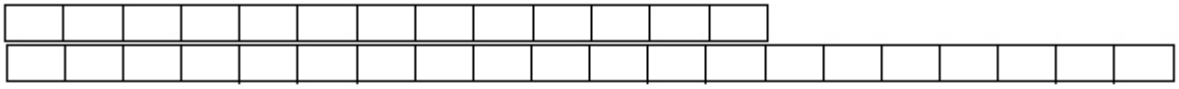
2.OA Represent and solve problems involving addition and subtraction. Second grade students will be given numerous experiences throughout this curriculum unit to apply the practice standards while they are meeting this common core standard. The standard includes students understanding the different types of problem situations and how to solve them, operational fluency within the number 20, recognizing numbers as even or odd and defending reasoning for identification, and using arrays to model repeated addition.

2.NBT Understand place value. Mathematically proficient second graders need to be able to decompose numbers into hundreds, tens and ones, understand the value of each digit, know how to read, write, and represent numbers in multiple ways, fluently count and skip count by 5s, 10s, and 100s, and compare numbers using symbols. Then they need to apply their understanding to the operations of addition and subtraction.

Appendix II – Models

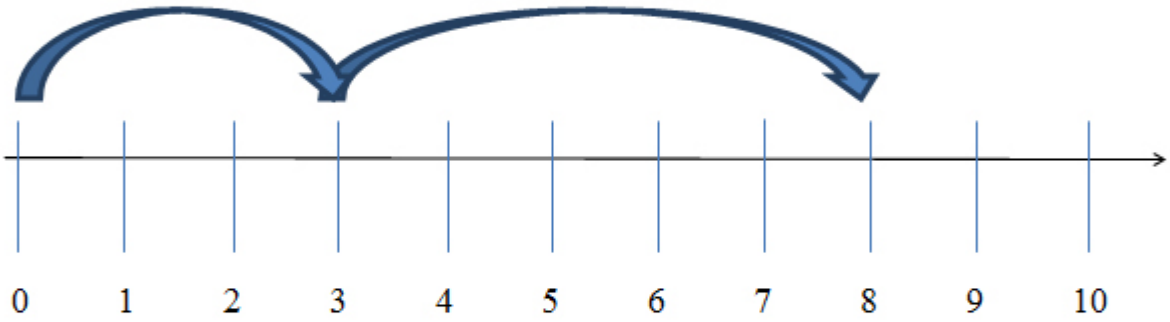
Length

Sam has 13 pretzels. His friend gives him some more and now he has 20. How many pretzels did Sam's friend give him?

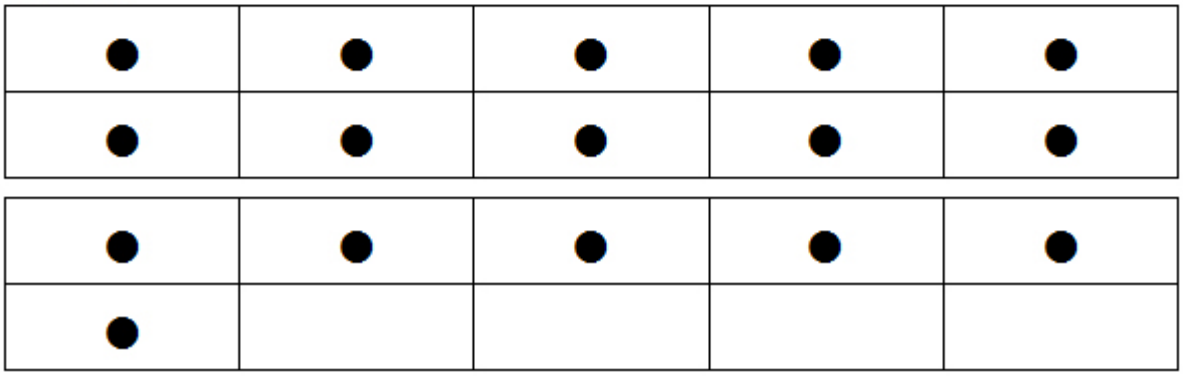


Susan had 14 dollars. Then she found 3 more dollars. How many dollars does Susan have now?

Number Line

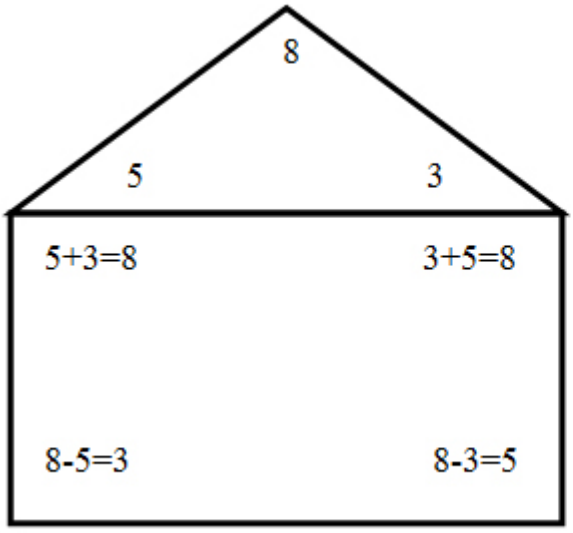


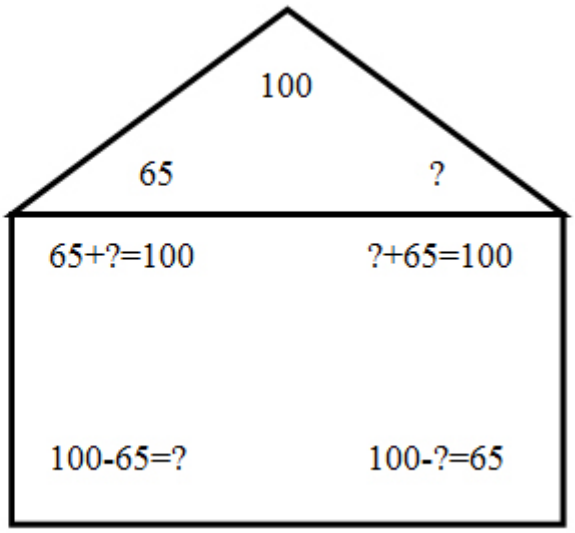
Ten Frames



These Ten Frames can be expressed as 10+5+1, 5+5+5+1, 10+6, or 20-4

Fact Family





Appendix III – Assessment Samples

The "Assessment" word is frequently used in my school and district. "How do you know that students know what you want them to know?" I'm including here some quick assessment items that can be used as a pretest, check in, or post-test for the different types of problems students will be solving. (I usually copy and paste them into a format and font that is appropriate for my your learners).

Making Ten

10 = ____ + 4	8 + _____ = 10	5 + 5= _____	_____ + 3 = 10

Higher Addition Facts

5 + 6 = 10 + _____	7 + 6 = 10 + _____	8 + 9 = 10 + _____	9 + 5 = 10 + _____

Expanded Form (2-digit & 3-digit numbers)

63 = _____ + _____
406 = _____ + _____ + _____
728 = _____ + _____ + _____

Adding and Subtracting 2-digit numbers without regrouping

- Carl had 14 toy orange cars. He also had 13 toy blue cars. How many toy cars did Carl have altogether?
- Janet completed 35 problems. She got 11 wrong. How many problems did Janet get correct?

Adding and Subtracting 2-digit numbers with regrouping

- Lisa and Marcus went to the store. Lisa spent $49 and Marcus spent $36. How much money did they spend altogether?
- Martin and Michael collect baseball cards. Martin has 62 cards. Michael has 48 cards. How many more cards does Martin have than Michael?

Adding and Subtracting 3-digit numbers without regrouping

- Darren is a really good reader. He read 320 pages in January. He read 469 pages in February. How many pages did he read altogether?
- Marissa has to work 655 minutes before she can get a raise. She has already worked 233 minutes. How many more minutes does she need to work before she can get a raise?

Adding and Subtracting 3-digit numbers with regrouping

- Joanna rode her bike 623 miles in the summertime. Her brother Joe rode his bike 389 miles. How many miles did they ride altogether?
- Joanna rode her bike 623 miles in the summertime. Her brother Joe rode his bie 389 miles. How many more miles did Joanna ride than Joe?

Resources

Chapin, Suzanne H., and Art Johnson. *Math matters understanding the math you teach, grades K-8*. 2nd ed. Sausalito, CA: Math Solutions Publications, 2006. This book is an excellent resource for teachers to understand all of the K-8 math instruction.

Gross, Herb . "Taking the Fear out of Math." Math As a Second Language. http://www.adjectivenounmath.com/id71.html (accessed July 10, 2014). This website illustrates ways math is more than just a set of rules.

Howe, Roger. "Taking Place Value Seriously: Arithmetic, Estimation and Algebra." PMET: Preparing Mathematicians to Educate Teachers. http://comp.uark.edu/~ttrzeci/pmet/resources/PlaceValue_Final.pdf (accessed July 10, 2014). This essay shows the progression of place value understanding.

Howe, Roger. "Three Pillars of First Grade Mathematics." Reading, May 2014 from Roger Howe, New Haven , July 7, 2014. This article describes the main "ingredients" for beginning mathematicians.

Howe, Roger. "Briefs on the Rules of Arithmetic," "Briefs on the Number Line," and "Briefs on Place Value." Reading, May 2014 from Roger Howe, New Haven, July 7, 2014. These briefs provided a foundation for this curriculum unit I wrote.

"Illustrative Mathematics." Illustrative Mathematics. https://www.illustrativemathematics.org/ (accessed July 10, 2014).

This website offers sample problems with models, diagrams & pictures related to each of the Common Core

State Standards across all the grade levels.

Investigations in number, data, and space. 2nd ed. Glenview, Ill.: Pearson Scott Foresman, 2008. This is the curriculum that my school district uses. Teacher manuals provide a lot of support, background knowledge, and lesson plans.

"One Step Addition and Subtraction Word Problems." Achieve the Core. http://achievethecore.org/content/upload/2.OA.A.1%20One-Step%20Addition%20and%20Subtraction%20Word%20Problems%20FINAL%20BRANDED.pdf (accessed July 10, 2014). This pdf file has sample questions and model assessments that show the different types of word problems in Common Core.

Parker, Thomas H. , and Scott Baldridge. *Elementary Mathematics for Teachers*. Okemos: Sefton-Ash Publishing, 2003. This resource describes for an elementary math teacher how to sequence information, and struggles to anticipate.

"Progressions Documents for the Common Core Math Standards." The University of Arizona. http://ime.math.arizona.edu/progressions/? (accessed July 12, 2014).

This pdf illustrate the progression of standards within each domain.

Richardson, Kathy. *Developing number concepts using unifix cubes*. Menlo Park, Calif.: Addison-Wesley, 1984. This teacher resource shows ways to support elementary students learning through using unifix cubes.

Richardson, Kathy. *Developing number concepts*. White Plains, NY: Dale Seymour Publications, 1999. This resource helps model moving from concrete to abstract when teaching addition and subtraction. It includes examples and activities.

Rotz, Leyani, and Marilyn Burns. *Lessons for algebraic thinking*. Sausalito, CA: Math Solutions Publications, 2002. This teacher book includes a progression of lessons for how to teach young learners (K-2) algebraic thinking.

"Standards for Mathematical Practice." Common Core State Standards Initiative. http://www.corestandards.org/Math/Practice/ (accessed June 21, 2014).

This website lists and describes the eight Practice Standards for Mathematics

aligned to the Common Core Standards.

Storeygard, Judith. *My Kids Can: Making Math Accessible to All Learners, K-5*. Portsmouth, NH: Heinemann, 2009. The premise of this book is that all kids can learn math and it offers strategies, suggestions, and ideas for a range of learners.

The Singapore model method for learning mathematics. Singapore: Ministry of Education, 2009. This resource book illustrates the framework for mathematics that is used in Singapore and provides visual examples for how they use math tools and illustrations to model and solve problems.

Walle, John A., and LouAnn H. Lovin. *Teaching student-centered mathematics*. Boston: Pearson/Allyn and Bacon, 2006. This teacher resource illustrates specific ways to teach students math using various activities. It was written for grades K-3.

"Welcome to the Curtis Center." UCLS Department of Mathematics. http://www.curtiscenter.math.ucla.edu/index.shtml (accessed July 10, 2014).

This website provides a visual for the relationship and progression from

Common Core State Standards in Mathematics among grade levels.

Notes

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1. "Standards for Mathematical Practice," Common Core State Standards Initiative, http://www.corestandards.org/Math/Practice/ (accessed June 21, 2014).

2. J.Storeygard, My Kids Can: Making Math Accessible to All Learners, 149-157.

3. "Common Core State Standards Unpacking," http://www.ncpublicschools.org/docs/acre/standards/common-core-tools/unpacking/math/2nd.pdf, (accessed July, 2014), 9.

4. "Standards for Mathematical Practice," Common Core State Standards Initiative, http://www.corestandards.org/Math/Practice/ (accessed June 21, 2014).

5. K.Richardson, Developing Number Concepts, 43

6. R.Howe, Three Pillars of First Grade Mathematics, 2.

7. "Common Addition and Subtraction Situations," https://www.engageny.org/file/3401/download/common_addition_and_subtraction_situations.pdf?token=H4FC8_TF0g8pL21wQ-DC7GoJHwg2Dpdabvu9OCZOQAY (accessed July 21, 2014).

8. S.Chapin and A. Johnson, Math Matters Understanding the Math You Teach, Grades K-8, 55.

9. T. Parker and S. Baldridge, Elementary Mathematics for Teachers, 47.

10. "Common Addition and Subtraction Situations," https://www.engageny.org/file/3401/download/common_addition_and_subtraction_situations.pdf?token=H4FC8_TF0g8pL21wQ-DC7GoJHwg2Dpdabvu9OCZOQAY (accessed July 21, 2014).

11. Investigations in number, data, and space.

12. Investigations in number, data, and space.