Curriculum Units by Fellows of the National Initiative
2014 Volume V: Place Value, Fractions, and Algebra: Improving Content Learning through the Practice Standards

Fractions Aren't So Scary! Using the Unit Fraction to Ease the Fear

Curriculum Unit 14.05.02, published September 2014
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Introduction

Fractions are hard, plain and simple! Many of my students fear working with fractions. I hear the moans and groans and see the look of fear when I write on our class agenda that we will be studying fractions. Not only are they fearful, but they have zero memory of what they have learned about fractions in previous years. I hear the "we never learned that" or "I don't remember seeing that". Sound familiar? I KNOW they have been taught about fractions because it is part of our state and district objectives in the lower grade levels. Through reading the research on fractions, I found that this was a common theme. Fractions (rational numbers) are complex; however, they are important for students to understand. Up until now I have focused too much on the procedural way of teaching fractions. My students do not understand why fractions work. They are lacking in the conceptual view of fractions. I have come to realize that I need to focus first on the conceptual view of fractions and then I can proceed to procedural skills. I want my students to leave my class having an understanding of fractions and not just knowing how to work the problem. My goal is to help my students understand fractions are actual numbers and to be able to locate any fraction on a number line as well as being able to perform basic math operations with ease.

I teach fourth grade in a self-contained class, usually around 25 students. Eisenhower International Elementary is a lottery school in the Tulsa Public School district. We are a language immersion school where our students begin learning in a target language of either Spanish or French beginning in Kindergarten. The majority of the students are native English speakers. Teachers instruct students on state and district objectives in either Spanish or French through fifth grade. English Reading and Language Arts are not introduced until the spring semester of second grade. Mathematics instruction is primarily in the target language. In third grade, students are introduced to English vocabulary of mathematics due to state testing.

Our student population is one that is diverse across economic and ethnic backgrounds. However, the majority of the student population is Caucasian and from the middle class. I have on average two to three students yearly in my class that are on an Instructional Educational Plan due to some diagnosed learning disability. Because we only have two third grade classes, two fourth grade classes, and two fifth grade classes, teachers are able to collaborate to determine the needs of our students. We are able to work closely and quickly for intervention. Each class is assigned a teacher paraprofessional. The teacher paraprofessional serves as an extra resource in our classrooms for added instruction in language acquisition, as well as an extra support for remedial and one on one instruction when needed. Teachers, staff, and parents have high expectations for student learning. Eisenhower is known as receiving yearly top state test scores in the state of Oklahoma. Excellence in Education is a priority for our school community. We were designated a National Blue Ribbon School in 2013.

Background

How can I ease that fear of fractions among my students? How can I teach fractions in a way that my students will gain a conceptual understanding of fractions? This unit will be designed to look at the unit fraction as a foundation to build a solid conception of fractions. It will also focus on the idea of a fraction as a rational number. I will use several different models to help students work with rational numbers.

When asked the question, what is a fraction, my students would most likely simply state it is a piece of a whole. I admit that I too have defined fractions as a piece of a whole. From what I have seen in previous grade levels, students are taught the part-whole description of fractions. The part-whole description is primarily used with a set of objects. It is one of four mathematical subconstructs of rational numbers identified by Kieren. [1] One can consider this method of instruction as an introduction for visually seeing a fraction for lower grade levels. However, using the part-whole description could create problems in the upper grade levels when it is required for students to have to use rational numbers in operations. For this reason, Kieren states that it is important for the understanding of all the subconstructs of rational numbers and how they are interrelated. The Rational Number Project has refined Kieren's description of the part-whole subconstruct by adding that it should be related to a specific unit. [2]

The Rational Number Project used Kieren's subconstruct models and created seven different subconstructs of rational numbers. It does not say that one is better than the other and proposes that you may want to use a specific construct for a particular mathematical problem. The first construct, the fractional measure subconstruct, addresses the question of how much there is of a quantity relative to a specified unit of quantity, which is an extension of the part-whole subconstruct developed by Kieren. The second subconstruct, the ratio subconstruct, expresses a relationship between two quantities. The third subconstruct, the rate subconstruct, defines a new quantity as a relationship between two other quantities. The fourth subconstruct, the quotient subconstruct, interprets the rational number a/b as a quotient a divided by b. The fifth subconstruct, the linear coordinate subconstruct, views rational numbers as points on a number line while stressing the importance that they are a subset of real numbers. The sixth subconstruct, the decimal subconstruct, emphasizes properties related to the base-ten system. The final subconstruct, the operator subconstruct, transforms the rational number by using a function concept. [3]

Like Kieren, the Rational Number Project proposes that the subconstructs are interrelated and could facilitate the learning of specific concepts of rational numbers [4] ; For example, using the ratio subconstruct is helpful when renaming fractions or creating equivalent fractions. It seems that these subconstructs can be organized from easiest to hardest and in a progression of learning. If I were to place them in that order it would be: fractional measure subconstruct, linear coordinate subconstruct, quotient subconstruct, ratio subcontract, operator subconstruct, decimal subconstruct, rate subconstruct. After learning about the subconstructs, I realize that I have made the mistake with just sticking to one subconstruct and not paying attention to the others. Along with using the subconstructs to teach rational numbers it is important to incorporate manipulative materials and visual aids. These will be discussed further in the Math Content.

Math Content

Rational Numbers

During the intensive session of the Yale National Initiative, I learned that a fraction is a name for a rational number. A fraction IS an actual number! My students would want to disagree with this idea. They would say that a fraction is made up of two numbers but is not really a number. I want to convince my students that a fraction is an actual number.

In the lower grades, students may have used the part-whole subconstruct to learn about and name fractions (fig. 1a and 1b). This subconstruct may help in just naming simple fractions of sets or groups.

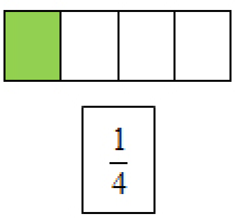


Fig. 1a

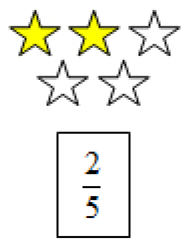


Fig. 1b

However, once we move to more advanced concepts of rational numbers, this subconstruct could cause errors and create misconceptions (fig. 2). [5] One such misconception is that fractions should be added by adding the numerator and adding the denominator.

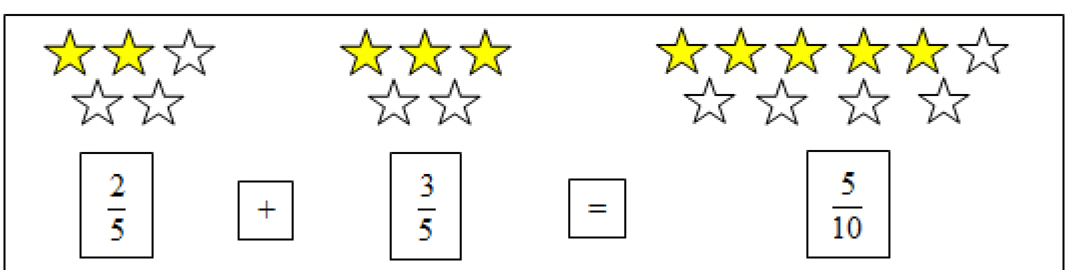


Fig. 2

Of course this equation 2/5 *plus* 3/5 *equals* 5/10 is not correct. Students are using their understanding of basic addition properties to add the fractions. When we add we must be sure to have the same unit. However, in the example in figure 2 the student does not understand that the 2/5 is referring to a group of five stars, and 3/5 is referring to another group of five stars, but that the 5/10 is referring to a group of ten stars. The first two have the same unit, but the total sum is referring to a different unit. This goes back to the misconception of how my students view fractions made up of two numbers. The Part-Whole subconstruct may be ok in the lower grades to initially relate to fractions; however it does not assist with the more advanced concepts of rational numbers. Not only do we need to move away from this subconstruct, but students will also need to reconceptualize the meaning of a fraction. I will teach my students that a fraction is a number that describes a specific unit.

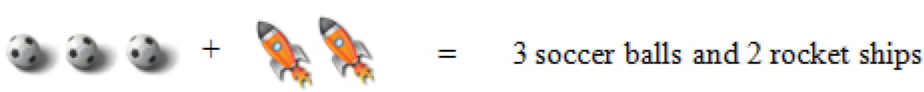
Naming the Unit

It will be essential for students to learn that each number they work with refers to some quantity, often called the "unit" or the "whole". Insisting students to always specify the units for any answer will help to develop "unit consciousness". Developing "unit consciousness" is key for successful fraction learning. For example when you say three, you can mean to say that you have three dogs, or three balls, or three books, or whatever you have three of. Three is describing how many you have of some understood unit. Thus, three is acting as an adjective. Dogs, balls, and books are the objects being named and are nouns that specify the unit. Knowing the unit is essential. It makes a huge difference if you have 3 cents or 3 hundred dollar bills. It makes a huge difference if you have 3 goldfish or 3 whales. This is the idea of the adjective-noun theme created by Herb Gross. [6] He has found that that using this adjective-noun theme of Math as a Second Language can make comprehension of mathematical concepts easier for students. This concept will be especially helpful for the addition of fractions with like and unlike denominators.

This idea emphasizes the importance of having the same unit when doing arithmetic. For example, if you have 3 soccer balls plus 2 soccer balls you will have a total equaling 5 soccer balls (fig. 3). However, if you have 3 soccer balls plus 2 rocket ships, you do not have a total equaling 5 soccer balls or 5 rocket ships. You still have 3 soccer balls and 2 rocket ships (fig. 4). You *could* say that you have 5 things total *if* you don't care about the difference between a soccer ball and a rocket ship. The idea is to be clear in what information you are asking for.

Fig. 3 2 soccer balls + 3 soccer balls = 5 soccer balls

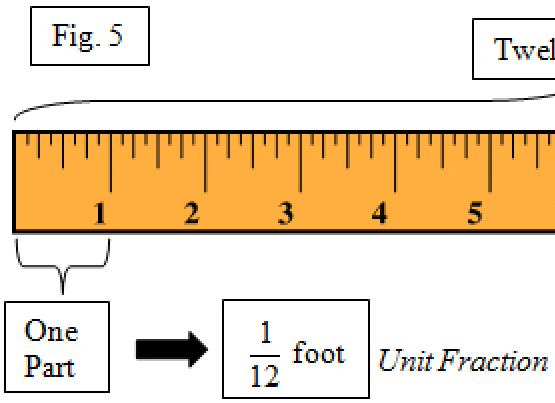
Fig. 4 3 soccer balls + 2 rocket ships ≠ 5 soccer balls or 5 rocket ships



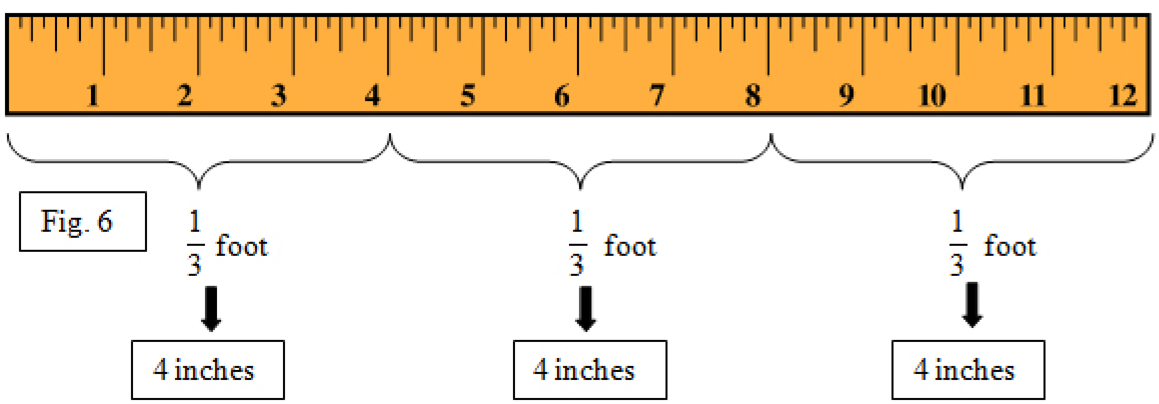
The key is to be precise about the unit. Practice with this is necessary for understanding the number and object relationship.

Unit Fraction

A unit fraction of a given unit can be gotten by dividing that unit into equal parts. If you look at this ruler you know you have one foot. A ruler is easy to use because it is already divided for you. You can see that they already partitioned the ruler in twelve equal parts, 12 inches. This tells us that 1 inch is 1/12 of a foot. The unit fraction is one equal part out of the total number parts to equal the whole unit. In figure 5, foot would be considered your unit.



You can also use the unit fraction to solve for finding 1/3 of a foot. In figure 6, I have partitioned a foot into three equal pieces. Each part is 4 inches long. One-third of one foot is equal to four inches.



You can think of a unit fraction as a base or a starting point. It is important that students know what unit they are working with. Math as a Second Language proposes that when students know what unit they are working with they will have an easier time understanding the concept. [7] In the example above, the unit is foot. You are describing what you have. You have 1/3 (the adjective) of a foot (the noun).

Once Students have an understanding of unit fractions, they can then move on to general fractions. General fractions are fractions such as "two-fifths, three-fourths, four-sixths", etc... The Common Core State Standards for Mathematics (CCSSM) defines a general fraction as several copies of the same unit fraction, that is, as a multiple of a unit fraction. [8] For example, if you have "two-fifths" you would say that it is the same as "one-fifth plus one-fifth" or that it is "two times one-fifth" (fig.7). Looking at copies of the general fraction can clarify the understanding of rational numbers for students.

Fig. 7

$\frac{1}{5} + \frac{1}{5} = \frac{2}{5}$ in other words $2 \times \frac{1}{5} = \frac{2}{5}$

You can then say $\frac{2}{5}$ has two copies of the unit fraction $\frac{1}{5}$.

Models to Use with Fractions

Using math manipulatives has been shown to be very important for understanding mathematical concepts. The Rational Number project, states:

> "There is no single manipulative aid that is "best" for all children and for all rational-number situations. A concrete model that is meaningful for one child in one situation may not be meaningful to another child in the same situation nor to the same child in a different situation. The goal is to identify manipulative activities using concrete materials whose structure fits the structure of the particular rational-number concept being taught." [9]

Students coming into fourth grade have already been exposed to number set and group models. For this unit I will focus on using the Area model and the Linear Model as manipulatives to work with rational numbers. Students must be familiar with both models in my unit. It is always best to have more than one way to solve a problem! Students will be able to show addition, comparison, and multiplication of fractions using both the area model and linear model.

Area Model (bold)

The area model is somewhat similar to an array model in multiplication. It is assumed that students are familiar with array models prior to instruction of fractions using the area model. I will use rectangles to introduce the model. Rectangles are easier than circles to draw freehand for myself and my students and are extendable. Students will show unit fractions using the area models. Usually in an area model, the unit is a rectangle. Students must understand that the rectangle is to be one whole. They then will divide the rectangle to find their unit fractions. In Figure 8, the unit rectangle is partitioned into 4 equal parts. Each sub-rectangle is one-fourth of the whole. It takes four one-fourths to make the whole.

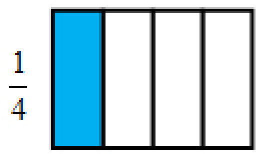
Fig. 8

Equals 1 whole

Equals 4 parts of the same 1 whole

The unit fraction is 1/*d*, with *d* being any number. Here d = 4. In figure 9, one part is shaded. Since there are four equal parts total, students would say ¼ is shaded. One-fourth is the unit fraction.

Fig. 9



Students will need to practice this method naming different unit fractions. At this stage I will introduce the vocabulary of numerator and denominator. The denominator, the bottom digit, is the total parts needed to equal the one whole. The numerator, the top digit, is the number of copies. Once students divide the same whole into different equal parts (*d*), they can compare the different unit fractions and then conclude that the larger the *d*, the smaller *1/d* will be for a fixed whole.

Once students become comfortable with using the area model to show unit fractions, they can progress to showing a general fraction. This will reinforce the idea of a general fraction being copies of a unit fraction. This practice will support the understanding of the unit fraction. To show the fraction "two-thirds" you would begin with one whole. Since the digit in the denominator is a three then you would partition the rectangle in three equal parts. Now we can see that that one part of the whole will be the unit fraction. Since 3 equal sub-rectangles are making the whole, each one is considered to be 1/3. With the area model it is clear that it takes three copies of the unit fraction to equal one whole. Then 2 of the 1/3's makes 2/3: (See Figure 10).

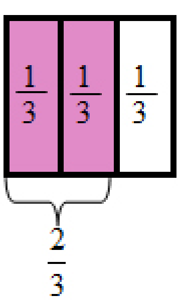


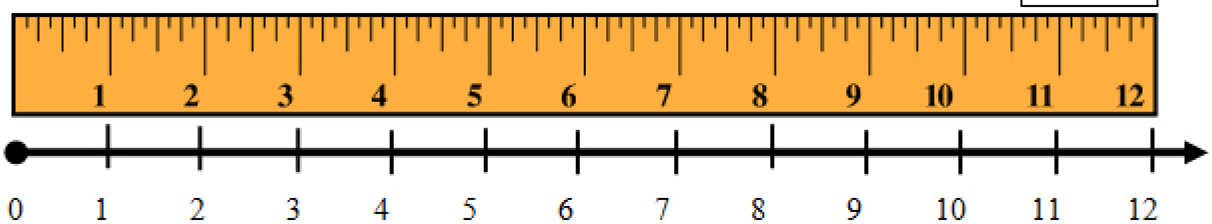
Fig. 10

Linear Model

In addition to the area models the use of the number line can contribute to the understanding of the fractions. [10] It is important that prior to working with the number line that students understand the concept of linear measurement. We are using the number line as a ruler to measure a distance from an origin. Students would also need to have this understanding of a number line. Teachers and students will use the number ray with the origin being 0.

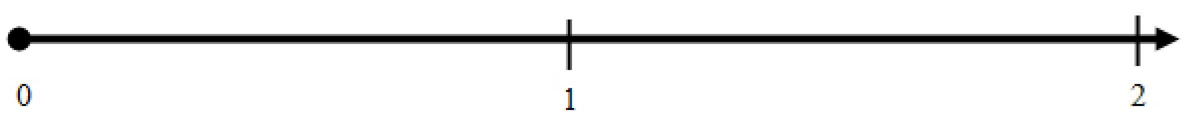
Students can see that a number ray is very similar to a ruler (fig. 11). We assume at this point they have had some experience in using a ruler and we can then have them look at a ruler as if it were a number ray. I use rulers that only show inches in my classroom. Rulers that show centimeters can also work. This could be a good beginning for them to see what a number ray would look like when drawn.

Fig. 11



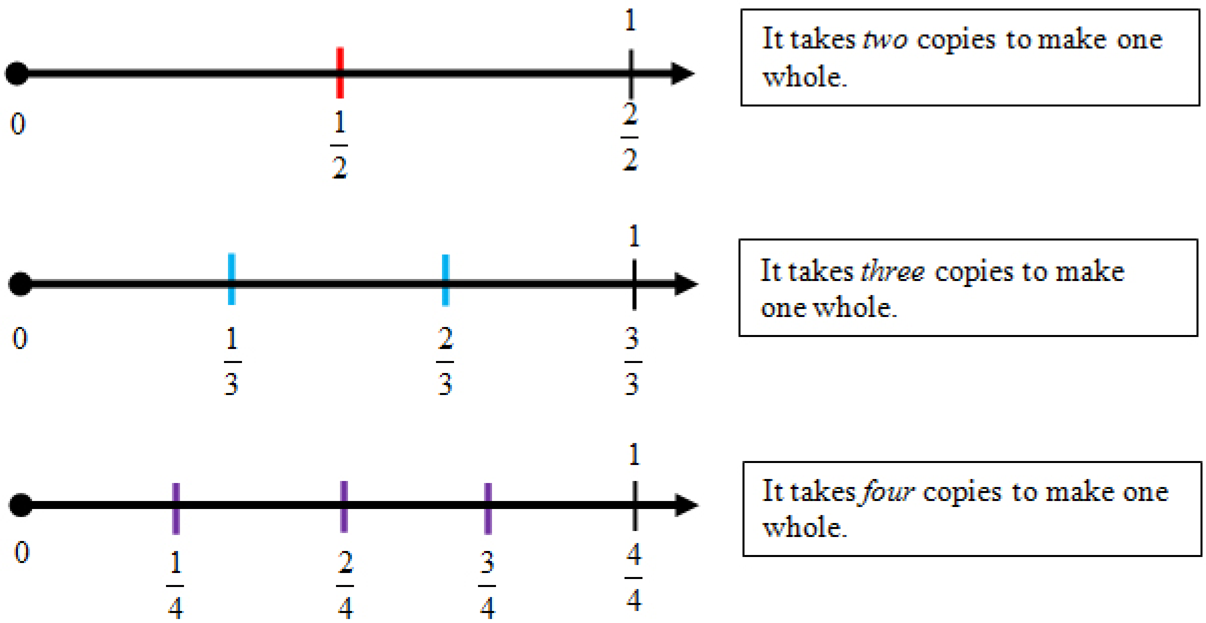
Many of my students are not able to draw and label a number line unless I give them practice. This will allow them to see the spacing between the numbers is also even. Students should be able to relate the above numbers to whole numbers and see that the ruler will have 12 whole numbers and that their number ray will also have 12 whole numbers (fig. 11). Now that they are familiar with drawing a number line, you can enlarge the pieces that you need (fig. 12). To focus on locating the unit fractions, I will enlarge the zero to two piece and use that to find the unit fractions for halves, thirds, fourths, fifths, sixths, sevenths, eighths, ninths, and tenths. The unit fractions will be found between the origin 0 to 1.

Fig. 12



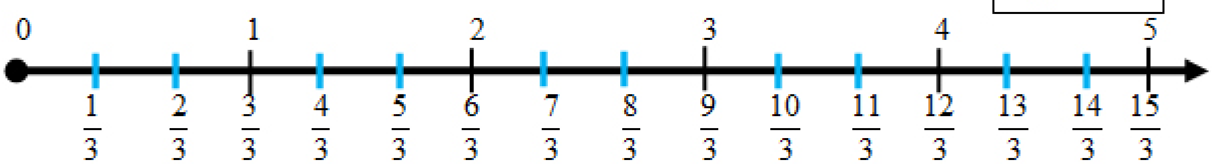
I want them to be able to notice on the number ray, what happens to the actual parts. It is important for students to understand the actual size of each of the unit fractions. Below in figure 13 are examples of halves, thirds, and fourths. The points are labeled according to their distance from the origin. Thus, two-fourths is twice as far from the origin as one-fourth; three-fourths is three times as far as one-fourth. Since it takes four one-fourths to make one, one is equal to four-fourths and is four times as far as one-fourth.

Fig. 13



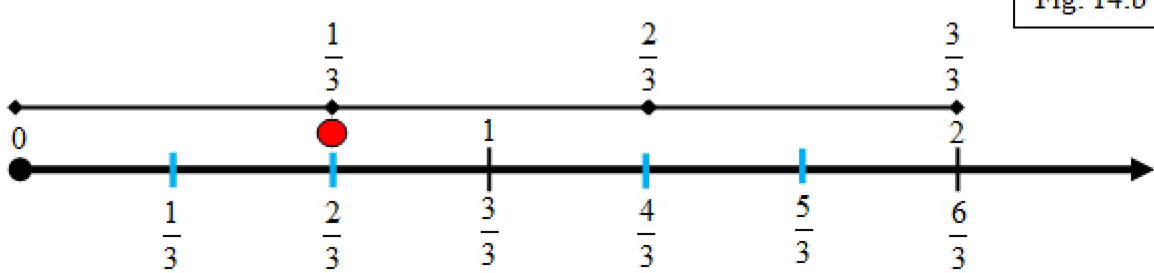
It is also important at this point to note to students they can continue to use the unit fraction to continue past the whole number one to divide the number ray. This will show that they can create any multiple of the unit fraction. They can then see that whole numbers are multiples of a unit fraction as well. Let's use "one-third" to illustrate. The whole number three is nine 1/3's from the origin (figure 14.a).

Fig. 14.a



Once students are comfortable with representing fractions in area models and number line models, they should connect the fraction to division as well. The whole number 2 can be written in fraction form as six-thirds; but we also know that 6 divided by 3 equals

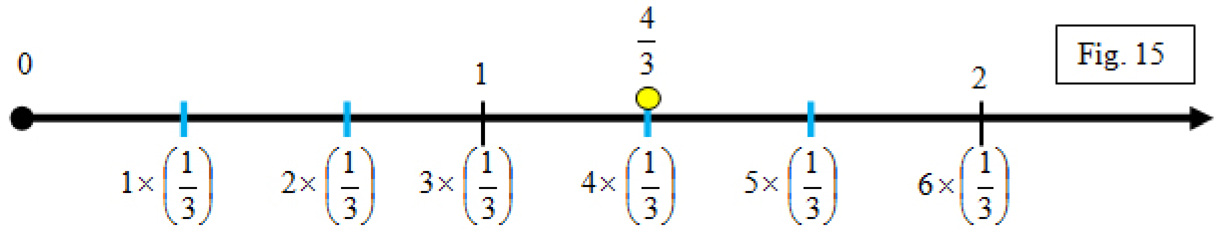
Fig. 14.b



2. The whole number 3 can be written in fraction form as nine-thirds; and 3 is 9 divided by 3. The whole number 5 can be written in fraction form as fifteen-thirds; and 5 is 1/3 of 15, or 15 divided by 3. The number line supports that a fraction is linked to division by showing that it takes 6 copies of one-third to equal two, 9 copies of one-third to equal three, and 15 copies of one-third to equal five (figure 14.a). This extends to

fractions as well. By subdividing the interval from 0 to 2 into 6 copies of 1/3, students can see that 2/3 is also equal to 1/3 of 2, or 2 divided by 3 (figure 14.b)

They can also connect this to multiplication. In the number ray displayed in figure 15, "four-thirds" is four of "one-third". If they multiply "one-third" by four they would have the general fraction "four-thirds".



Once students have an understanding of multiples of unit fractions on a number ray, then they will need to practice finding different general fractions on a number ray. This is key to connecting the number ray with arithmetic of fractions.

Comparing Fractions with Like Denominators

Comparing fractions with like denominators is fairly simple. If the fractions you are comparing have the same unit, or denominator, then you simply have to look at the numerator to compare which fraction is more than or less than. Compare the fractions "three-fifths" and "two-fifths". You would need to first check if they are using the same unit. Yes! Both fractions are using "one-fifths". You then will look to how many "one-fifths" you have in each fraction given. In the first fraction there are three copies of "one-fifth" and in the second fraction there are two copies of "one-fifth". Since three is more than two, "three-fifths" is more than "two-fifths". We can illustrate this by using both the area model and the number ray model. With the area model it is probably best to use separate wholes to illustrate each fraction. These models can be side by side (figure 16) or you could also place them one above the other (figure 17).

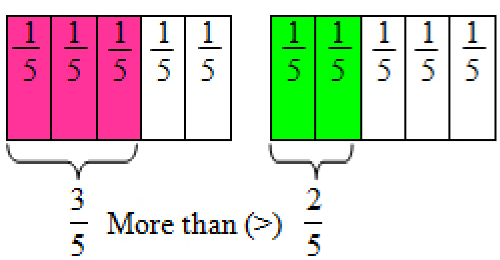


Fig. 16

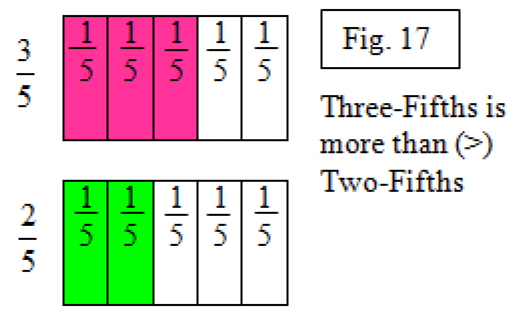
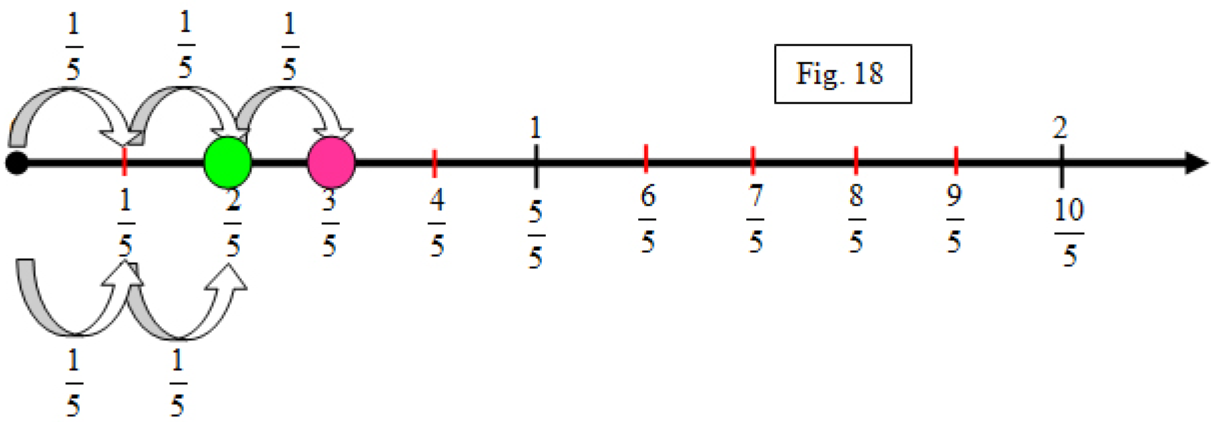


Fig. 17

In the number ray model (figure 18), you are looking for a specific distance from the origin. For the first fraction you are looking for three copies of "one-fifth". Beginning at the origin, you would "travel three "one-fifths" moving in the positive direction. Label the point where you "land". For the second fraction you are looking for two copies of "one-fifth". Again, beginning at the origin you would "travel" two "one-fifths" in the positive direction. Label where you "land". On the number ray you can see that "three-fifths" is further in distance from the origin. Therefore, "three-fifths" is more than "two-fifths". With additional practice comparing fractions with like denominators, students should come to the conclusion that when the denominators are the same, the numerator will tell you which fraction is larger.

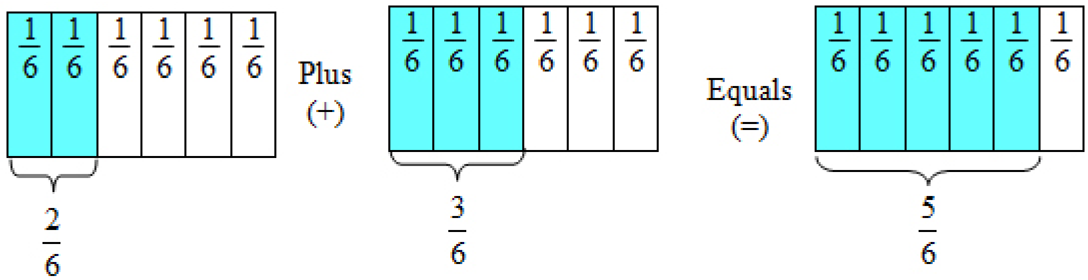


Addition and Subtraction of Fractions with Like Denominators

For illustrating addition and subtraction with the area and linear models, it is best if students are already familiar with doing that with whole numbers. Addition and subtraction of whole numbers and of fractions use the same principles. For the linear model, it is joining (for addition) or comparing (for subtraction) lengths. Then with fractions it is "just the same", except that you are combining multiples of a smaller unit. For the area model, it is pretty much the same as the linear model except that the shapes have some width. When using the area model with the addition of fractions, students must be reminded that each of the fractions to be added should be represented by its own region, disjoint from the other one. It is not so much having separate wholes, as the fact that you want to combine disjoint regions. For subtraction, you are "comparing" regions, which means you fit one inside the other, and find the part of the larger one that is outside the smaller one. Note that the original fractions that you want to add may be larger than 1. It is probably a good idea to do examples where this is the case. You are trying to put the parts together. The sum may be less than a whole or more than one whole.

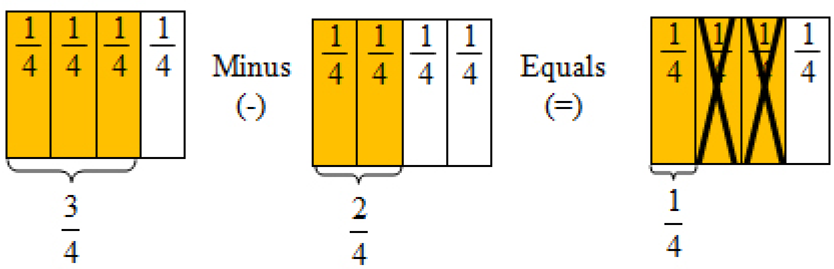
When given the problem to illustrate and solve "two-sixths" plus "three-sixths", student will draw two rectangles (figure 19). They will then look for the unit being described. In this case it is sixths. Reminding students of the adjective-noun relationship would be a good idea to reinforce looking for the same unit. Since the student knows that both rational numbers are describing the same unit, one-sixths, then they can partition both rectangles. Substantial practice with adding rational numbers using the area model with the same unit is needed at this point. Students can see that the numerator is added but the denominator remains the same and can further reason we change the numerator because you are simply putting the pieces together. The unit will remain the same.

Fig. 19



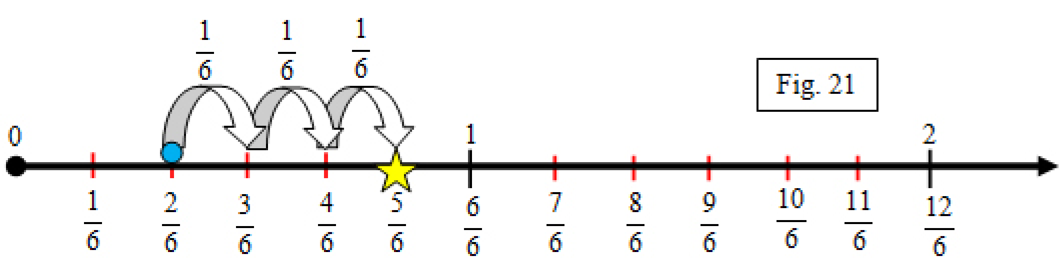
Subtraction of fractions will work the same way using the area model by simply taking away parts of the first area model. Figure 20 shows how to solve for "three-fourths" minus "two-fourths". Again, here we remind students that we are working with two separate wholes.

Fig. 20



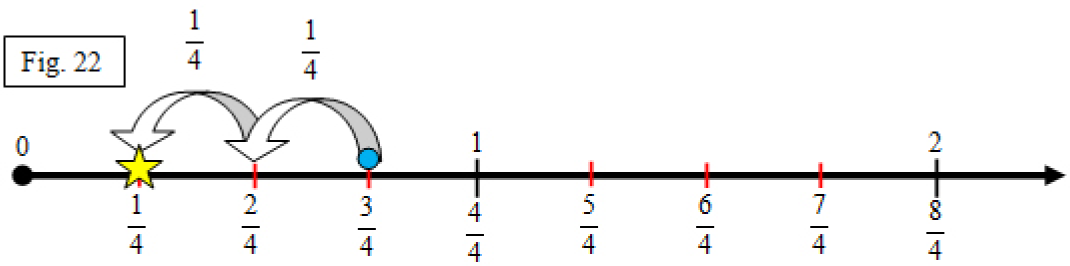
The linear model would further simplify the addition of fractions with like denominators. Just like with whole numbers on a number ray, students can move away a specified distance from the origin 0 and move towards a specified distance to the origin 0. Let's show the example from above on the number ray model. We will illustrate "two-sixths" plus "three-sixths"(figure 21). We know that "one-sixths" is being described, so therefore "one-sixths" is our unit we are working with. We should divide each section between the whole numbers in six parts. Since we are adding we can start with either fraction. When we subtract we will need to start with the "bigger" fraction. "Two-sixths" is two-sixths from the origin. Let's begin there. From "two-sixths" we will then travel three *more* times of the unit fraction "one-sixth". We "land" on "five-sixths".

Fig. 21



We can do the same with the subtraction problem, "three-fourths" minus "two-fourths". We know that "one-fourths" is what is being described and therefore is our unit. We will divide each section between the whole numbers in "one-fourths". Just as subtraction with whole numbers begins with the larger number, so will fractions. It should be said that kids should have understood that if the units or denominators of two fractions are the same, then the numerator will be used to say which fraction is "more than" the other. Students can also locate those fractions on the number line to verify which fraction is "more than". Since "three-fourths" and "two-fourths" share the same unit, "one-fourth", we can look to how many "one-fourths" we have. We can say that we have three "one-fourths" and we have two "one-fourths". Since three is more than two, we can conclude that "three-fourths" is *more than* "two-fourths". We will need to locate "three-fourths" on our number ray first (figure 22).

Fig. 22



"Three-fourths" is three "one-fourths" from the origin. Since we are subtracting we will need to go backward to the point of origin, 0. We will travel two "one-fourths" back to the origin. We "land" on "one-fourth". For proficiency, students will need to continue practice of addition and subtraction of fractions with like denominators.

Renaming Fractions

Many fractions can name the same number. For example, one-half names the same number as two-fourths, three-sixths, four-eighths, and even fifty- hundredths! Understanding repeated subdivision and reconstitution is a "key prerequisite" for understanding addition of fractions with different denominators. [11] Let's take a look at using the area model and repeated subdivision (figure 23). We know that "one-half" can stand for all those other fraction names, but how can we prove it? Subdivision basically says that you can partition an area model more finely to create a same name fraction. You can think back to a multiplication array. But also keep in mind that when you use subdivision, you are creating a new unit. In figure 23 the first area model displays 1 row and two columns. I now have two one-halves. The unit fraction is one-half. I will then divide the area model again in another half. The second area model is divided to show two columns and two rows. The area model displays four parts of the original whole, and students should be able to see that all the parts are equal. Therefore, I now have fourths; each small square is one-fourth of the whole. The one-half of the first area model is the same as two one-fourths of the second area model. In the third area model, I divide the same whole with four rows. This will give me eight equal parts and therefore giving me a new unit of one-eighths. The same one-half fraction is the same as four-eighths. Students should be able to notice that this idea of subdivision resembles an array of multiplication.

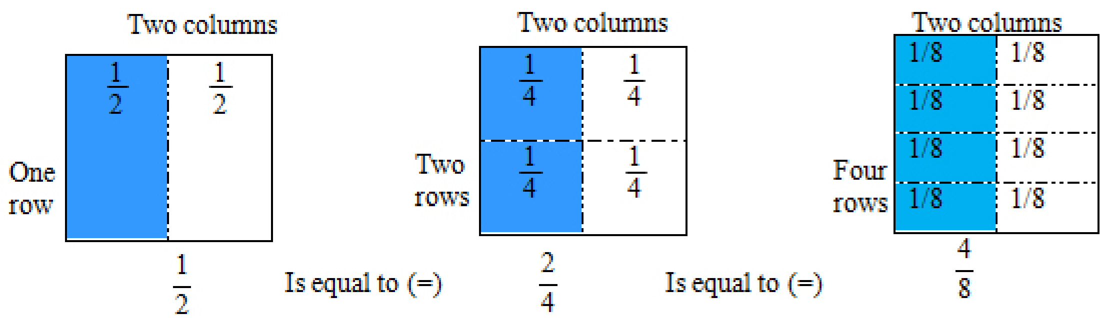


Fig. 23

In understanding these rules, students then can rename any two fractions with unlike denominators to have the same denominator or unit. Using the area model can further ease renaming fractions with unlike denominators. Understanding of this strategy is key to adding and subtracting fractions with unlike denominators. For example, when given 1/3 and 1/2 we can use the area model to rename both fractions. We would first illustrate both fractions with its' own area model. Each model must be of equal size (figure 24.a).

Fig. 24.a

We use the denominator of the other fraction to subdivide the area model. We would subdivide 1/3 in two parts because two is the number in the denominator of 1/2. We would subdivide 1/2 into three parts because three is the denominator of 1/3. It is also important for students to know that they would subdivide in the "opposite" direction. In the example we originally showed both fractions with division of the area model vertically. We would then need to subdivide horizontally to rename the fraction. In figure 24.b, subdividing horizontally is the "opposite" direction. Students can show their original area model divided either horizontally or vertically, but must be sure to subdivide in the "opposite" direction.

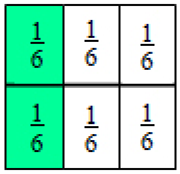
Fig. 24.b

Subdivided in two horizontally

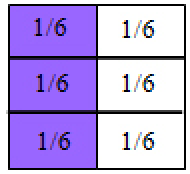
Subdivided in three horizontally

Students will see that each area model now has six parts. Using their knowledge of naming unit fractions, students will be able to recognize that a new unit of 1/6 was created (figure 24.c). They can then see how many multiples of 1/6 is equal to 1/3 and 1/2. We have renamed the fractions to share the same unit. Students should then reason that they can simply multiply the denominators to rename fractions.

Fig. 24.c



2/6 equals 1/3

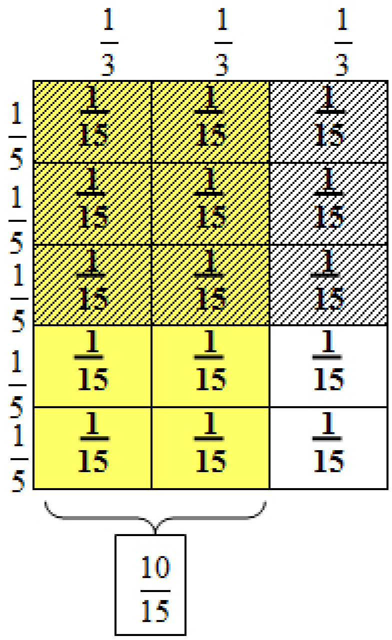


3/6 equals 1/2

Comparing Fractions with Unlike Denominators

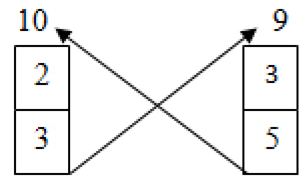
Comparing fractions with *unlike* denominators can be difficult for some students. I suggest reading the section on equivalent fractions before attempting the comparison of fractions with unlike denominators. However if students are confident with the area model then they can master comparing fractions with unlike denominators. The number ray can also be used; however, for this unit I will only use the area model for comparison of fractions with unlike denominators. The technique that will be illustrated can be used to derive the cross multiplication algorithm, and provides a justification for that algorithm. Many people know the "trick" to cross multiply to compare fractions with unlike denominators. However it is not just a trick! There is reasoning behind why it works! Let's compare the fractions "two-thirds" and "three-fifths". They do not have the same unit, denominator, but we can still compare them. We know that we need to illustrate thirds and fifths. The area model will function like a multiplication array (figure 25). We will create an area model with thirds first. Shade the fraction that is being compared. In this case we will shade "two-thirds" in yellow. Using the same area model, we will then divide to show fifths. The equal horizontal strips are each 1/5 of the whole. We are again using repeated subdivision. [12] Essentially, it is creating an equivalent fraction. Once you have the area model divided in "one-fifths", shade the fraction being compared. I have shaded three-fifths in stripes.

Fig. 25



$\frac{2}{3}$ is equal to $\frac{10}{15}$ and $\frac{3}{5}$ is equal to $\frac{9}{15}$, therefore "two-thirds is greater than "three-fifths".

Cross Multiplication Algorithm:
The numerators of the equivalent fraction are the product of the numerator of one fraction and the denominator of the other fraction.



Ten is greater than nine, showing that "two-thirds" is greater than "three-fifths".

You have created an area model with fifteen equal parts. Each rectangle now represents "one-fifteenth" of the whole. Students can connect three multiplied by five equals fifteen with "one-third" multiplied by "one-fifth" equals "one-fifteenth". You can see that "ten-fifteenths" is equal to "two-thirds" and that "nine-fifteenths" is equal to "three-fifths". This area model in itself is the illustration of the cross multiplication algorithm we are so familiar with. The new numerators of the fractions are the products of the numerator of one fraction and the denominator of the other fraction. Now we can compare easily.

Strategies

This curriculum unit will be taught in the beginning of the spring semester. By then, students should have a good grasp of basic number operations and measurement. I will first ask my students for their definition of a fraction. Cooperative learning will be one classroom strategy I use. Students will have a couple of minutes to think and write in their math journal, then share their ideas with their group or partner. Students will ask each other for further explanation during their group chat. Some examples of possible questions are: How did you come up with your explanation? What support do you have for your explanation? Do we have similar or different explanations? Can we both be correct? Group chats will be conducted throughout the unit and will support critical thinking and reasoning. The ideas discussed in this unit can be used for all the operations of arithmetic and fractions.

Introduction of the number ray and area model will be used to locate and recognize a unit fraction. The adjective-noun relationship of numbers and units will be reviewed. The adjective-noun relationship should be introduced with instruction of basic number operations earlier in the year. Students will have many hands on activities to familiarize themselves with both the number ray and area model. Rulers, Hershey's bars, graham crackers, paper folding, and a class yarn number line are some ideas for exploration of fractions.

Teaching strategies vary daily in my classroom. I begin with a question to have my students begin thinking about the topic. I continue with a lecture style addressing student's ideas that were mentioned in our initial group share. Then I will give them a hands-on activities or strategies to aid in problem solving. Students also summarize key points of lessons and create problems in their math journals. The main fraction strategy I will use will be the use of the unit fraction together with the number ray and area model to compare, locate, add, subtract and rename fractions.

Activities

Below is an outline of the lesson progression I plan for this curriculum unit. The unit should take approximately four weeks to complete. Following the lesson progression are examples of some activities to be used in classroom.

Lesson 1: Concept of the Unit Fraction

- Discussion of how students view fractions

- Units-Discussion of using the unit as a noun

- A Unit fraction as *1/d*, d representing the number of pieces of the new unit that it takes to make the original unit

Lesson 2: Unit Fractions using Models

- Using the Linear Model: Emphasis should be placed on distance from the origin

- Using the Area Model

- Using the number line/ray to place *1/d*

- Using area models to represent *1/d*

Lesson 3: General Fractions

- Students will write various general fractions *n/d* as a sum of the unit fraction

- Students will show the general fraction *n/d* as a multiple of a unit fraction *n* x *1/d*

- Using the linear model, students will show the location of a general fraction *n/d* as a multiple of the unit fraction *1/d*

- Using the area model, students will show a general fraction n/d as a multiple of the unit of fraction *1/d*

Lesson 4: Add and subtract general fractions with like denominator using both the linear and area model.

- Students will use their knowledge of the unit fraction in relation to the general fraction to add general fractions on a number line and an area model.

- Students will also do the same to subtract fractions on a number line and with the area model.

- Addition and subtraction should have the same unit (denominator) and relate to separate wholes.

Lesson 5: Comparing Fractions with like denominators

- Students will compare fractions with like denominators using the area model and the linear model

- Students will notice that if the denominators are the same, the numerator will be the deciding factor of which fraction is larger.

Lesson 6: Renaming fractions using the area model (equivalent fractions)

- Students will use the area model to create general fractions to be equal to another general fraction.

- They will understand that when a unit fraction *1/d* is used to further divide the area model, it creates a new unit.

- Students will be able to create a common unit between two fractions with different denominators.

Lesson 7: Comparing Fractions with unlike denominators

- Students will use the area model to compare fractions with unlike denominators.

- Students then will be able to use the cross multiplication algorithm.

Activity Sample #1 Unit Fractions through Paper Folding.

Students will create their own fraction strips to create a visual of the "size" of each unit fraction. I recommend using different colors of construction paper equal in size to create the unit fractions. Focus will be in creating fraction strips for 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/10, and 1/12. The goal for this activity is for students to see that when the paper is folded, several parts are created to make one whole, thus creating the unit fraction. In addition students should be able to compare the different unit fractions and be able to reason that fractions with a larger denominator are actually less than fractions with a smaller number in the denominator. The fraction strips can also be used to show that a sum of a number of unit fractions can equal general fractions.

Activity Sample #2 - What is a fraction? See Appendix 1

Students will be asked to define/explain fractions in their math journal. They can write and/or draw a picture. They will then work in small groups or partners to chat about how they explain fractions. Then they will choose a presenter to present to the whole class their findings. I will record the findings on chart paper or on the board. This will give me an idea of my students' understanding of fractions.

I will then review with my class what we know about the noun adjective theme in relation to mathematics. The focus will be to remind students that units are important when working with numbers. In regards to fractions, it needs to be pointed out that it is just as important to pay attention to the unit. The Naming the Unit section further explains this concept. I will then read The Hershey's Milk Chocolate Bar Fractions Book and introduction to the Unit Fraction.

Students will be given a Hershey Bar and a sheet of Graham Crackers. These are already segmented therefore making it easier to partition. Pass out a paper plate, or napkin, and a Ziploc bag to each student. You can use actual Hershey's Bar and Graham Crackers or you can create copies and laminate to pass out. Students will also have a sheet to record information requested.

Activity Sample #3 Using the Area Model to write a Fraction-See Appendix 2

The goal of this activity is for students to become familiar with the area model. Students will be given a square piece of construction paper. It is important that the paper is not too small and not too big. They will then follow a set of instructions and questions given by the instructor to guide their understanding and writing of a fraction. They will be assigned partners for this activity. After this activity, students can be given graph paper to continue practice drawing and dividing the area model. It is important to stress the importance of EQUAL parts. Teachers can extend with students to do this with other fractions as well.

Extension: Addition can be used with the students' area models they created.

Activity Sample #4 Renaming Fractions with a Partner: I divide, You divide, We Rename See Appendix 3 This is an interactive partner activity to help with the practice of renaming fractions. You can have a print out or have white pieces of construction paper of various sizes of squares and rectangles.

Appendix 1

Naming the Unit Fraction: Hershey's Bars and Graham Crackers

1. Draw what each item looks like in the boxes below. Pay close attention to how they are divided.

2.

Hershey's Bar	Graham Cracker

3. How many pieces are in one whole?

Hershey's Bar:

Graham Cracker:

4. Now that you know how many pieces are in one whole, what is considered to be the noun for each? The noun is known as your unit. (Hint: the total number of parts to equal one whole)

Hershey's Bar:

Graham Cracker:

Use the drawings in number 1. Color in one piece of each of the examples.

Now we write the fraction.

The part you colored will be the number written above the line, the numerator. The total parts will be the number written below the line, the denominator.

Example: *Colored Part (numerator)*

Total Parts (denominator)

5. Write the fraction that you colored in problem 1 for each.

Hershey's Bar:

Graham Cracker:

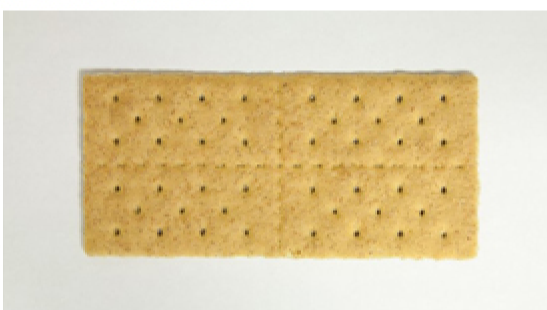
Congratulations! You just named a unit fraction!

6. Describe what you notice about each of the pieces in relation to its own whole. Do not compare the pieces to the other whole. Think about why we wouldn't compare the Hershey Bar to the Graham Cracker sheet.

7. Label each piece of the Hershey's chocolate bar with a fraction.

How many parts will give you one-half of the Hershey's bar? ________________
Write the fraction: ______________________

8. Let's do the same for the Graham crackers. Label each piece of the graham cracker with a fraction.



How many parts will give you one-half of the graham cracker sheet? __________

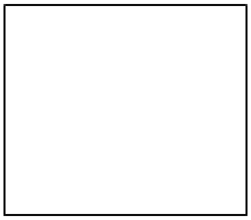
Write the fraction. _________________________________

Appendix 2

Using the Area Model to write a Fraction

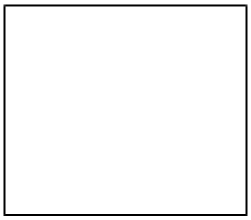
(Students will be given a square sized piece of construction paper)

Here is one whole:



1. What does this equal? ______________________________

Divide the following whole in two *equal* parts:



2. How many equal parts do you have to equal one whole?

   ____

3. Color in one part of the whole green.
4. Color the other part blue.
5. How would you describe the part that you colored green?

   ____

6. How would you describe the part that you colored blue?

   ____

DISCUSS: With your partner share your information. Does your whole look the same? Discuss why they are the same or different.

READ: To write a fraction, you need a denominator and a numerator. The denominator names the *total* number of parts it takes to equal one whole. The numerator can name *one* part or *multiple* parts of one whole. Both the denominator and the numerator are always one number.

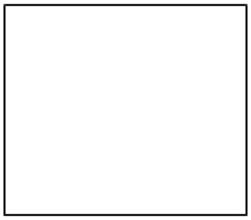
In the example above:

7. What is considered the denominator? ____
8. What is considered the numerator? _____________________________________

Now you can write the fraction: $\frac{\text{numerator}}{\text{denominator}}$

9. Write the fraction for the green shaded part: ________________________

Divide your square again to have four equal parts:



10. How many equal parts do you have to equal one whole?
____

11. Color one part of the whole red.
12. Color the other three parts of the whole yellow.

13. Now write the fractions for each shaded area:

a. Red: ____________________

b. Yellow: _________________

DISCUSS: Check with your partner. Do you have the same fraction? Explain to each other what each number means.

14. How many parts did it take to write the fraction for the yellow shaded part? _______
15. This number is considered to be your _____________________________.

Appendix 3

I divide, You divide, We Rename

Materials needed:

Ruler

Colored Pencils

Two people (can be three)

Instructions:

1. First person will divide and will state the unit fraction they created. You can divide into how many you want.

2. First person will also shade a part of the area model.

3. Second person will state the general fraction that the first person shaded.

4. Second person will subdivide and name the *new* unit fraction.

5. First person will then restate the fraction of the shaded part using the new unit.

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Notes

1. Behr,Lesh,Post, Silver, 92-98

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3. Behr,Lesh,Post, Silver, 98-100

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9. Behr,Lesh,Post, Siver, 102

10. Roger Howe, Three Pillars of Math and Beyond

11. Roger Howe, Thinking of Division III

12. Roger Howe, Thinking of Division III