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Fearless Problem Solvers Can "Express" Themselves Mathematically

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Overview and Rationale

"Shortcuts are the privilege of experts." This has been a recurring phrase in Roger Howe's seminar entitled "Place Value, Fractions, and Algebra: Improving Content Learning through the Practice Standards," and it rings especially true to me as I navigate guiding my students through the Common Core State Standards for Mathematics (CCSS-M). Beginning the CCSS-M as eighth graders puts my students at an immediate disadvantage, as they will have gaps in learning some key content that has now been relocated from eighth grade in our former state standards to lower grades in the CCSS-M. For students who have not felt much success in math, this can dampen their enthusiasm for learning. While it can be tempting to teach them "tricks" to quickly fill these gaps in order to get to the grade level content, the goal of this unit is to help build background knowledge that will deepen their understanding of foundational ideas in order to internalize it and apply it to new situations.

Developing algebraic thinking in students whose foundations in mathematics are limited presents a unique challenge in the middle school algebra I classroom. Students express frustration at learning this "new language" and grappling with representations of numbers they have never seen before; nor do they see the relevance of it in their lives. My goal in this unit is to build a strong foundation for students to explore and understand expressions and equations in a coherent progression based on their prior knowledge. It is also my goal for them to apply their understandings to investigate the mathematics behind topics that matter to them. The core of the unit will be devoted to developing strong, fearless problem solvers who become experts who have earned the privilege of applying shortcuts with understanding to real world problems that matter to them.

Through focused problem sets designed to gradually extend subtopics of expressions, students will construct meaning about the properties of numbers and variables as we discover how these things work together in algebra. By reordering topics to present a logical progression of ideas, I will scaffold my students' learning to prevent burnout and promote the Standards for Mathematical Practice in the CCSS-M. Sense-making, tinkering, and developing problem solving strategies and skills will provide the core of our class discussions. Students will work to define algebra within their own contexts to apply it in a culminating project where they investigate issues that are important to them and apply their algebraic reasoning and skills to come up with viable solutions to share with our school community.

Context

Teaching 7 th and 8 th grade mathematics in an urban elementary school whose population is extremely diverse poses substantial challenges, but also offers substantial opportunities. Our school is a public, neighborhood school that serves about 850 students from pre-kindergarten through eighth grades who live within our attendance boundaries. With over 30 languages spoken among the families who attend my school, many cultures and backgrounds are represented in our building. We also serve students from a nearby boys' group home for displaced youth, so social-emotional needs are prevalent and require special attention in our planning.

The diversity represented in our student body is also evident in our staff. Our vision of shared leadership is truly realized, with all staff taking on managing roles to provide diverse programs to our students in partnership with local agencies. Entering our third year with a new administration, our school has seen some turnover with staff. Stagnant, low standardized math test scores indicate a need for change in the way mathematics is taught. With students whose second language is English making up over half of our population, our reading scores are notably higher than our math scores. To address this disparity, we are beginning a team teaching model to fully include and reach the potential of all of our students. This rebuilding phase is exciting and is allowing us to experiment with various techniques to increase our effectiveness as a school, but growing pains are inevitable.

Part of the vision of our administration requires high school algebra I for all 8th grade students, regardless of their math backgrounds leading up to this benchmark year. This creates an extremely difficult teaching context in which I attempt to balance addressing deep gaps in prior knowledge with the rigorous pace required to successfully meet the algebra standards. With completely heterogeneous classes, the range in readiness is challenging to accommodate effectively. This unit sets out to rethink the first unit of our year to better engage students in problem solving and mathematical reasoning within contexts that matter to them and to transform algebra from something dreaded to something seen as useful in their daily lives.

Teaching Strategies

One of the strongest features of the CCSS-M is the Standards for Mathematical Practice (See Appendix). Addressing a clear need for students to develop problem solving skills, these practice standards are identical from kindergarten through twelfth grades and describe the habits of strong mathematicians. According to the *TIMSS Videotape Classroom Study* (Stigler, Gonzales, Kawanka, Knoll, & Serrano, 1999), teachers often "design lessons that remove obstacles and minimize confusion [where] procedures for solving problems would be clearly demonstrated so students would not flounder or struggle." Lessons that are planned from this perspective limit students' abilities to deeply understand content. I want to do the opposite, but not with abandon. As explained below, I have come to believe through my research that successful problem solving experiences depend not only on rich tasks, but also on the careful planning and execution of lessons that include time for reflection and consolidation. In order to promote rich mathematical exploration and discussion, I will use the Five Practices Model (Smith and Stein, 2011) to design and execute thoughtful lessons.

The planning process requires first, specifically setting a goal and selecting a set of high quality problems that will help students reach it. While this seems like an obvious and perhaps simple step in preparing a lesson, much thought is required to align the task with the mathematical and problem solving goals of the lesson, as it serves as the foundation for all other parts of the work. Consideration of the difficulty and interest levels it involves will be of primary concern in this unit, as well as making relevant connections to students' lives.

Anticipating, the first of the Five Practices, requires me to engage with the task prior to administering it with students to anticipate possible solution strategies students may come up with during the lesson. At this point I also try to anticipate misconceptions that may surface during their work so that I am prepared with questions to redirect them. While it will be unlikely that I will anticipate all of the ways students will solve the problem, taking the time to think it through in different ways will better prepare me to quickly analyze them during class while guiding student groups.

As students work, I will be *monitoring* and *selecting* their responses, paying attention to their strategies and asking individuals and groups about their thinking while they progress collaboratively through the task. During this time, I will be able to identify students whose thinking should be shared with the class during our whole class discussion and summary. Having a monitoring tool with which to keep track of students' strategies is a quick way to stay organized and remember what I see at each group. I will take note of general approaches such as pictorial, verbal, algebraic, etc., so that each type of strategy is represented in the whole class culminating discussion of the day's work. I will also make a checklist of strategies I anticipate, with room for unanticipated strategies that may come up, so that it is easy to organize their sharing out in a way that logically develops the key mathematics from the task. I plan to be careful to share many strategies so that every learner has something to connect to.

Sequencing and *connecting* occur as part of the whole class discussion after student groups have had time to work together on the task. Sequencing student responses is directly related to achieving the goal of the lesson, as students who share their strategies help inform their peers' understanding as well as their own. A typical sequence might begin with a concrete strategy such as a drawing and move toward more abstract, algebraic strategies. This approach allows everyone an entry point into the discussion, but leaves room for extension as appropriate. While students present their strategies, the teacher helps connect the responses and tie them back to the goal of the lesson through highlighting patterns among and the level of efficiency of each shared idea. In recent years there has been a paradigm shift toward conceptual teaching of mathematics. I favor this, but also believe that, unless carefully executed, there is a danger of leaving too many loose ends for students to truly synthesize their learning. This step of connecting offers an opportunity for everyone to get on the same page with what was learned that day and to walk away with common vocabulary and knowledge.

The CCSS-M Standards for Mathematical Practice cannot be taught in isolation; they occur spontaneously during effective problem solving activities. In my research a description that resonated with me was, "These eight Practices call attention to what it means to be mathematically proficient, to go beyond simply memorizing facts and formulas. When our students experience mathematics through these Practices, they have repeated opportunities to make sense of the ideas and to build a deeper understanding of skills and concepts," (Connell, 2013).

Unfortunately, most students do not intrinsically value the struggle that understanding mathematics presents. They often have preconceived notions about who is "good" or "bad" at math, based largely on answer-getting and not strategic thinking, which prevent them from persevering in problem solving (Mathematical Practice Standard 1). In my students' cases, they have not had much, if any, experience grappling with problems they did not already know the algorithm for solving. Therefore, I plan to draw attention to evidence of these practices as they surface during student work time in class through verbal praise, and through student self-reflection in writing on a weekly basis where they will assess themselves on the practices they exemplified that week, providing evidence to support their claims. These will be featured on a classroom display.

Focusing on problem solving has proven benefits. However, many of my students lack a lot of basic number sense, which hinders their progress and can lead to frustration and abandonment of rich problems. To address this deficit in a meaningful way, I will create strings of number talk prompts with interesting patterns that illustrate the properties of arithmetic and elicit a variety of strategies from students. This strategy is based on the work of Sherry Parrish who explains, "Classroom number talks, five- to fifteen-minute conversations around purposefully crafted computation problems, are a productive tool that can be incorporated into classroom instruction to combine the essential processes and habits of mind of doing math" (Parrish, 2010).

Establishing a clear protocol for number talks is important, and I plan to follow Parrish's fairly closely as we put this routine into practice 2-3 times per week. Students will gather on carpet in a corner of my room for the talks so that they use mental math to solve rather than paper, pencil, or calculator, and to make this routine a clear part of class with a designated space. The problem will be presented and written on the board. Students solve mentally and indicate by placing their thumbs on their chests when they have arrived at a solution using one strategy. This personal signal to the teacher eliminates the raising of hands, which often discourages others from continuing on their solution paths because they feel that someone already knows the answer and therefore the process is over. Students are encouraged to find multiple ways to arrive at their solutions, and they add fingers up to indicate each new strategy they find. This keeps everyone engaged during the wait time needed to give everyone an opportunity to think through their processes.

Once nearly all or all students have at least a thumb up, I ask for answers to get them out of the way. Often there will be more than one answer, especially at the beginning of a new type of problem string. I list all of them at the top of the paper and circle the one we are able to prove at the end. Next, I ask for volunteers to defend one of the answers on the list. If I know a student had a misconception and she or he volunteers to defend the answer I tend to call on that student first to illuminate the misguided thinking and diversify the thinking that is shared; we often learn more from incorrect answers than from correct. This, of course, depends on a strong classroom culture where mistakes are valued and safety has been established. This process continues until we have exhausted the contributions. Many misconceptions flesh themselves out along the way, but the process certainly takes longer when we first begin and students are learning the nuances of number talks. Over time, the five- to fifteen-minute range becomes much more realistic.

All of these strategies share the common thread of student understanding guiding lessons and determining the direction of the unit. An essential component of this unit, and arguably of effective teaching, is one that is counterintuitive to the traditional view of mathematics teaching and learning: flexibility. We want our students to become flexible in their approach to and understanding of mathematics, and are now realizing the importance of modeling this in our teaching. This requires depth and breadth in our own competency of what we teach, which is something that was significantly lacking in my own experiences as a student and something I have worked hard to develop throughout my post-high school education that continues today.

The process of filling our own content gaps as teachers is easier stated as a need than accomplished. My certification was for kindergarten through grade 9 all content areas, but after two years of teaching sixth grade all subjects, I knew that mathematics was my passion. I participated in a teacher education program at the University of Chicago called SESAME where I took classes from university professors to earn my endorsement in middle school mathematics. Since then, I have attended hundreds of professional development events, but had never encountered the depth of content learned in these courses until engaging in this summer's seminar. While the level of content I learned was beyond what I teach, my students have benefited greatly from my greater depth of knowledge that allows me to connect what I am teaching to their future learning; this perspective enables me to understand the importance of the content I teach, and to communicate it better to my students. My participation in this summer's seminar reinforced the benefits of content-based learning.

Mathematics Background

In researching the development of understanding expressions in the CCSS-M, I have sectioned the progression into six big ideas with the skills needed to grasp each of those big ideas: writing numerical expressions; representing unknown quantities using variables; representing word problems as expressions; evaluating expressions for given values; transforming expressions, motivated by number tricks; and equivalent expressions,. I have included descriptions, too, of the types of activities that will be used to elicit these skills and ideas while building on the prior knowledge students bring to class. Many of the topics overlap, but the subtle shifts along the progression are essential for students to understand, which is why the divisions are made as they are.

Throughout these chunks of the units, challenging word problems will be woven so that students develop an understanding of the relationships between the contexts of problems and their expressions. The results of a teacher-researcher study revealed what are often considered algebraic problems may lack algebraic thinking. They stated, "...implementation can impact how algebraic a lesson really is. For example, the lesson could have easily turned into a problem of 'adding on by twos' when students completed a function table, potentially rendering the lesson an arithmetic task instead of an algebraic one" (Ernest and Balti, 2008). As a result of the study, three instructional strategies that "helped keep the algebra in the foreground of the lesson" were identified.

The first strategy Ernest and Balti established is *using unexecuted expressions*. By leaving a sequence uncomputed, (for example, 3 x 2 + 2), students are able to connect each term and operation of the expression to its original context. This encourages sense-making about the problem, which leads to a grasp of why the expression works after generalizing it. Often, students are adept at "answer-getting" but have little understanding of what their numerical results represent or why they make sense in the context of the problem. While they initially formulate rules to represent situations, it is helpful to practice using unexecuted expressions to gain insight into students' reasoning and to address any misconceptions they have before simplifying them.

The second strategy is *using large numbers*. When students investigate a relationship, they often solve for new inputs using the previous inputs. For example, if they see consecutive outputs of 5, 8, 11, and 14 for the inputs 0-4, they may state the rule as "add 3 each time the input increases by 1." While this is a true statement, it requires recursivity, or knowing the previous term to find the next. By asking students to determine the output when the input is 100, or 1000, or any arbitrary large number, they are forced to realize the inefficiency of using their current rule to determine it. This shift in seeing the "horizontal" relationship between variables in a table is often difficult for students, but posing a problem that makes it the only feasible way to solve gives them a purpose for using it.

All of the work outlined in the previous two paragraphs still does not guarantee algebraic thinking and reasoning, especially in the absence of *using representational context*. Often, students are able to generate expressions without making clear connections to the contexts to which they relate. Again, this shift away from being satisfied with answer-getting makes the learning important beyond the given task; students need to be able to make connections across various representations in order to develop strategic thinking that will enable them to attack diverse problems effectively. This can be promoted through presenting a variety of contexts for problems, including visual representations, and consistently asking what each term in an expression stands for. This will be discussed further in the section on variables, where students' misconceptions become very evident if they are not in the habit of explicitly defining units in their calculations and solutions.

Writing Numerical Expressions

Students have been writing numerical expressions throughout their careers working with operations, such as

$2 + 3$ $\qquad$ $5 + 4 + 6$ $\qquad$ $2 \times (3 \times 5)$ $\qquad$ $8 \times 2 + 8 \times 10.$

They bring important prior knowledge to the table in the form of experience working with the properties of operations informally in the context of operations of whole numbers. Most of the experiences they have had with expressions have been limited to whole numbers, and while occasional fractions and variables have been included in isolated problems, the understanding of an expression as showing a relationship between or among quantities is not yet evident. In order to give meaning to expressions, students must become more flexible in their definitions of them and formalize their understanding of the basic properties for arithmetic, especially the Commutative, Associative, Identity, Inverse Rules for Addition and for Multiplication, and the Distributive Rule. This will be accomplished through diverse experiences with problem sets delivered through Number Talk strings over several days that require them to build on what they know to incorporate numbers in different forms and a wide selection of expressions.

Students will start to see what powerful tools expressions are, especially in that they can be used flexibly to accomplish specific purposes students will need to address in solving problems. In essence, expressions are "recipes" for computations that will become increasingly complex as students progress through algebra. Reaching this realization will require them to see expressions as objects in their own right and to connect their meaning to the contexts in which they are developed. By building solid understanding of numerical expressions, they will be better able to transition into using variables with meaning in expressions and eventually apply this knowledge to equations.

Representing Unknown Quantities using Variables

The introduction of variables is typically done without fanfare; it is simply presented as replacing numbers with letters. However, this provides little sense of their importance and utility. Variables need to be understood as standing for any quantity within an expression, whose meaning is determined by the surrounding context. It is the meaning that is often lost when students begin using variables. Interviews with students revealed that "the ability to verbally describe a method does not necessarily entail the ability to symbolize that method mathematically," (Nesher and Kilpatrick, 1990). This suggests that there are stages in the development of understanding notation.

So how do students approach problems in algebra? "Many of the cognitive studies in the learning and teaching of algebra have focused on students' approaches to equation solving. These approaches have been classified into three types: intuitive, trial-and-error substitution, and formal," (Nesher and Kilpatrick, 1990). Often, especially in algebra, we rush into formal approaches by teaching students established algorithms without connecting to or building on their intuitive approaches. I believe that this is especially detrimental when introducing variables because it assumes too much prior understanding by the student and, if left unaddressed, can lead to a weak foundation that will eventually crumble.

Since working with expressions is a crucial step toward proficiency with equations, it is important to meet students where they are, honoring their prior knowledge and intuition, and building toward formalization. To do this, students need many experiences with problems that logically progress toward a point where their intuitive and trial-and-error methods (my students refer to this as "guess-and-check") are no longer effective or efficient. Eventually, they will see the need for variables, too, to express generalizations in a formal language. A sample problem set might progress like this:

1. Each group of desks in our classroom seats four students. How many students will fit in 2 groups? 4 groups? 7 groups? How did you find that?
2. Each row in the movie theater seats 12 people. How many rows would a group of 24 people need? 48 people? 156 people? How did you find that?
3. Mr. Jimenez rented square tables (only one person could fit at each side of the table) for the 8th grade luncheon. He pushed the tables together to form one long row. How many guests could be seated with a row of 10 tables? 20 tables? How did you solve? Can you find a rule to tell how many people could be seated in a row of *n* tables? Explain how your rule relates to the situation.

When students calculate the solutions to each set of problems, the questions repeatedly ask, "Why do you think that?" or "How did you figure that out?" This mantra will make the transition into writing expressions easier for students to see. When different inputs are given, asking what they are doing each time develops structure. Then the question becomes, "Can we represent that relationship somehow so that it works with any number?" Students may naturally insert a variable, depending on their previous exposure, or they may verbalize that there is a step at which any number can be inserted into the operation. Either way, their variables now have meaning and students are ready to evaluate expressions knowing what their answers mean.

Beyond understanding how variables work, students will need to consistently label their work with units to connect the computations to the contexts of problems. This will build from our prior work with fractions and will serve as a focal point of discussion. I have come to believe that understanding units is key to students making sense of the problems they encounter; it is another often overlooked part of problem solving that seems obvious to us as experienced mathematicians, but can create gaps in understanding and reasoning for our students who are new to working with unknowns in these contexts.

Evaluating Expressions for Given Values

The purpose of doing all the work needed to construct expressions to represent situations is to make our lives easier when we need to solve for a variety of inputs that range in value from very small to very large numbers. Still working from word problems, students will construct expressions to solve and then will practice evaluating for given values. I will integrate negative and fraction inputs to increase their comfort and capacity dealing with non-whole numbers.

This point in the unit leads to more formalization of the thinking students have been using, so I will model organizing their inputs and outputs into tables to help forge the path into equations and graphing functions that will come later in the year. Students will see the power of having generalized expressions to use in solving problems by the ease in which they are able to find outputs. They will begin to notice patterns in tables and make conjectures about extending single expressions into equations that state two or more expressions are equal.

Number Tricks: Formalizing Notation to Transform and Simplify Expressions

Putting a positive spin on the mystery of mathematics is something that number tricks can accomplish in helping students navigate the road to formal notation which will enable them to transform expressions. Number tricks are series of instructions given to students that lead to some predictable answer. For example, if students choose a number, add 5, subtract 2, multiply by 3 and then subtract 3, they will end up with 3 times their starting number plus 6. I will play the game with students having worked out the simplified form of the expression so that they can give me any input and I will be able to quickly tell them their output. I can also ask them for the output and add four to get their input. ??

After working with some examples of number tricks where there is a simple rule like x + 1to get from the starting number to the ending number, I will expose students to some where the variable cancels and there is a constant answer. This type of number trick will lead us nicely into simplifying expressions and the effects of applying the basic rules of arithmetic to a series of steps.

Repeated rounds of number tricks will be played with students to pique their interest and then the mystery will be revealed. We will represent each step in symbolic form, building upon the previous steps. Finally, we will apply the properties of arithmetic to simplify the expressions as far as they can be to reveal the trick of how I was able to "guess" their numbers so quickly. We will formalize the process of simplifying through other examples, and students will craft their own number tricks to represent in words and symbolically to practice these skills.

Equivalent Expressions

Now that students have gathered many tools for working with expressions, they will apply them to compare expressions and test for equivalence. Since students are now comfortable with the fact that numbers can be represented in various forms, they will formalize some of the conventions that allow this to occur. We will explore two methods for determining equivalence, both of which apply concepts students have been exploring in this unit.

One way to test for equivalence in one-variable expressions is to evaluate the expressions for the same input value. If the resulting output is always the same, for any input, the expressions are equivalent. (In fact, for the expressions we will construct, which will all be linear in the variable, it will be enough to check for 2 values; but I will ignore this point in class, because later they will probably encounter more complex expressions for which many comparisons may be needed.) An example of this would be comparing 2(x + 6) and 2x + 12. If I evaluate each for x = 2, I will get an output of 16. Connecting this to number tricks, I could give the recipe of "Choose a number, add six, then multiply the sum by 2," or, "Choose a number, multiply it by 2, then add 12 to the product," and I would get the same result if my chosen number was 2. Students will practice this skill given various expressions with whole numbers, fractions, and exponents.

The other method to test for equivalence is simplifying to check for representations of the expressions in the same form. Using the example above of 2(x + 6) and 2x + 12, students can apply the Distributive property to the first expression to make it represented in the same form as the second, proving they are equivalent. This will be a point to introduce the idea of using the Distributive Rule to combine like terms so that when they see expressions like 2x + 4 - 5x + 6 and -3x + 10, they will be able to simplify to show that they always yield the same result. This concept of equivalence will lead into our next unit where we will explore equations and the basic moves for solving.

Representing Word Problems as Algebraic Expressions

While students encounter word problems throughout the progression of the unit, it is a topic that deserves special attention, as it is frequently feared and avoided. At this point in their learning, students are ready to start generalizing the patterns they have been noting in how expressions are formed and what they mean. At this point, I will introduce a famous problem, "Diophantus's Age," which poses a math problem in verse:

His childhood lasted one sixth of his life; he grew a beard after one twelfth more; one seventh later, he was married. Five years later, he had a son. The son lived exactly half as long as his father. Four years later, Diophantus died. How old was he?

Initially daunting, this problem can be presented in a way that caters to students love affair with "guess and check" and extends it to a method of "Guess-Check-Generalize" (Cuoco, 2008). To solve this problem, students make a guess at Diophantus's age when he died. Sometimes interesting discussion will arise about lower life expectancies in earlier times, but most students guess somewhere between 60-100 years.

Next, students apply the verses to their guesses. As they come to solutions, they adjust their guesses up or down based on their results. They will follow this process with a total of 3 guesses until we can generalize. I will have three different students share their guesses and processes, writing the guesses in a different color than the rest of the steps. Students will see repetition in what we're doing, and we will replace the guesses with variables to generalize the expression and solve as an equation.

Ending this unit with this problem will lead us into working with equations. The transition at this point will provide an empowering view for students to take, as they will feel accomplished in solving what had appeared to be a difficult problem using a method that is transferable to any word problem requiring an equation to solve. It will also set the stage for our culminating unit project where they will apply their knowledge about expressions to represent an issue they care about mathematically.

Culminating Project

Recently, *ChicagoMagazine* published a two-part story about the gross misrepresentation of crime statistics in Chicago. The details are horrifying and treat human life as a disposable commodity, so the graphic content is not appropriate for my sensitive young learners. However, it launched an urgency in me to share the power of its approach: investigating numbers that people in power would like us to simply accept and believe to be true representations of what is going on.

Helping students see the relevance of mathematics, particularly algebra, to their lives is one of the most difficult parts of my job. Students often tell me that no one in their families understands it either, so why should they have to? This is an important question. I understand that the skills developed through algebraic thinking make students more versatile and critical consumers of information, but the reality is that their experiences have not required them to apply these skills in the situations where problems typically apply them, such as buying cars or comparing cost of t-shirt companies.

Contrary to their beliefs, students do use this thinking in their daily lives for things they care deeply about such as computer and video games, music and other media, calculating their grades, and navigating the selective enrollment process for high school. This project aims to help them see the advantages mathematical thinking can provide in those contexts, as well as in helping them counteract injustices they face as a result of their race, English proficiency level, and socioeconomic status.

The project will develop over the course of this unit, beginning with weekly lessons during our Response to Intervention block that connect the issues around making informed decisions when choosing family phone plans, pricing pizza, understanding where the money goes when buying scalped concert tickets, and deciding whether or not to get a warranty on electronics. By engaging in analysis of these situations together, students will develop an understanding of how to apply mathematics to answer questions they have about situations they encounter. Throughout this time, they will generate lists of questions they have about issues and situations they care about, as well as gathering articles that speak to them in periodicals as starting points.

At the end of the unit, students will share their ideas in small groups and with the whole class to narrow their focuses. Students will use a graphic organizer to help focus their research to determine what they need to know, how they will find out, and how they will know when they have answered their question. They will have time in class to research, meet with me, and develop arguments that are mathematically sound to address the issues they choose. Communicating their findings will take place via posters, multimedia presentations, or videos that they will create and share at our first quarter report card pick up with their families who attend. A secondary but equally important goal of the culminating project is to set the tone for a year of mathematics learning with positive support and interest from the homes of my newly empowered students.

Appendix 1

Implementing District Standards

This unit has been written in accordance to the Common Core State Standards (commonly referred to as the CCSS), which have been adopted by the Chicago Public Schools. Over the course of this unit, students will explore the structure and conventions of expressions and apply them to situations and contexts that are important to them. They will also develop the eight habits of effective mathematicians outlined in the CCSS, called the Standards for Mathematical Practice. Students will synthesize their learning and habits throughout the unit by engaging in carefully sequenced activities to develop content and skills required for their culminating project of researching, answering mathematically, and presenting a question related to a context they care about.

Specifically, this unit will develop the following content and practice standards:

Content Standards

HSA.N.Q.1 - Reason quantitatively and use units to solve problems.

HSA.SSE.A.1 - Interpret expressions that represent a quantity in terms of its context.

HSA.SSE.A.1A - Interpret parts of an expression such as terms, factors, and coefficients.

HSA.SSE.B3 - Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

HSA.SSE.A2 - Use the structure of an expression to identify ways to rewrite it.

HSA.SSE.A.1B - Interpret complicated expressions by viewing one or more of their parts as a single entity.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Appendix 2 – Sample Problem Sets

Writing Numerical Expressions

1. Write a sum that is negative with one positive and one negative addend.
2. Write a sum that is positive with one positive and one negative addend.
3. Write an addition sentence with a negative outcome.
4. Write a subtraction sentence with a positive outcome.
5. Write a subtraction sentence with a negative outcome.

Representing Unknown Quantities using Variables

1. How many fives are in 345?
2. How many tens are in 560?
3. How many halves are in 31?
4. How many sixths are in 45/2?
5. How many eighths are in 72/4?

Evaluating Expressions for Given Values

1. Evaluate the expression (3x + 5x – 6x + 2x)/x for each of the following values of x.
2. 6

3. 22
4. -13
5. 4

Number Tricks: Formalizing Notation to Transform and Simplify Expressions

1. ite each of the following as algebraic expressions and simplify completely. 1. Choose a number. Multiply by -2. Subtract 8. Subtract 12.
2. Choose a number. Divide by 2. Add 3. Subtract 6. Multiply by 4.
3. Choose a number. Add 6. Divide by 2. Subtract 3. Multiply by 2.
4. Choose a number. Subtract 2. Multiply by 5. Add 6. Divide by 3.

Equivalent Expressions

Match each of the pairs of equivalent expressions below.

1. 3(x + 2) – 4a. 1 + 4x
2. -4(2 - x) + 9b. –x + 6y
3. 12x – 3x + 4 – 5 + 2xc. 3x + 2
4. 2x + 5y – 3x + yd. 11x – 1

Representing Word Problems as Algebraic Expressions

Write an expression to represent each of the situations below.

1. A rectangle has length 2x and width 4x – 3. Write an expression to represent the area of the rectangle, then write an expression for the perimeter.
2. A garden is five feet longer than twice its width. Write an expression to represent the area of the garden, then write an expression for its perimeter.

Resources

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