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Curriculum Units by Fellows of the National Initiative
2014 Volume V: Place Value, Fractions, and Algebra: Improving Content Learning through the Practice Standards

# Introduction

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The seminar on "Place Value, Fractions, and Algebra: Improving Content Learning through the Practice Standards" took its inspiration from the Practice Standards of the Common Core State Standards for Mathematics (CCSSM), which were developed in 2009 – 11, and have been adopted by the large majority of states. The Practice Standards are eight in number:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The seminar took the point of view that these standards are not to be taught directly. Rather, they are habits that develop over time when students experience good teaching of challenging material. Accordingly, the Fellows of the seminar concentrated on writing units that give coherent and connected views of important topics of the mathematics curriculum, and that are built around collections of challenging problems.

Fractions were one of the topics especially mentioned in the somewhat unwieldy seminar title, and for good reasons. One is fractions have been one of the topics with which U.S. mathematics instruction has been least successful. A second is that the Common Core has adopted a new and potentially more promising approach to fractions, by starting with the idea of a unit fraction. In particular, this approach gives students a better chance of revising their understanding of the idea of number, so that they can come to think of fractions and whole numbers as being on an equal footing. Unit fractions are fractions with 1 in the numerator: ½, 1/3, ¼, 1/5, etc. A unit fraction, say ¼, of some quantity, is another quantity, of which it takes 4 copies to make the original quantity. So, ¼ of a foot would be 3 inches; ¼ of an hour would be 15 minutes; ¼ of a quart would be one cup. This approach gets students thinking of fractions as numbers that express quantity relationships, rather than "so many out of so many," an approach that leaves many not even thinking of fractions as numbers, but as two juxtaposed numbers.

Three of the units deal with fractions. All use the approach through unit fractions, and make substantial use of two models for fractions that were discussed in the seminar: number line models and area (aka "cornbread" or

"brownie pan") models. Josephine Carreno develops her unit for fourth graders who are still getting used to fractions, and presents them with many different models. Patricia Lee's unit is for fifth graders who will need to study fractions intensively to stay in the hunt with mathematics. She emphasizes the number line because it is a comprehensive model that is especially good at conveying the idea that fractions belong to one overall system, and that any two of them can be compared for size. Rajendra Jaini's unit is for his chemistry students who need to increase their facility with fractions in order to deal with stoichiometry and other aspects of chemistry.

The topic of place value is represented by Torrieann Kennedy's unit for her second grade class. She emphasizes role of the base ten units (1, 10, 100) in expressing 2- and 3-digit numbers and in addition and subtraction. Ann Agostinelli and Marissa Brown have produced units dealing with algebra. Ann's unit focuses on introducing symbolic expressions in ways that help students make sense out of the new language known as algebra. Marissa's unit is built around understanding linear functions from multiple points of view – from real-world contexts or scenarios, from tables, from graphs, and from symbolic equations. Key to connecting tables with graphs is the idea of *rate of change*, and its geometric analog, *slope*. A major take-away for her students is that an algebraic relationship is linear if and only if the associated function has constant rate of change if and only if the associated graph is a straight line.

Finally, Nancy Rudolph's unit stretches the boundaries of the seminar. She has written a unit to help her pre-calculus students better grasp the idea of inverse function. In so doing, she must sharpen their understanding of the formal structure of functions.

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