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Math is All Around Us: Representing and Solving One-Step Addition and Subtraction Word Problems within 20

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Introduction

Students in the U.S. continue to have difficulty in solving word problems. This is a huge concern because the ability to apply math concepts in the real world context is our ultimate goal of mathematics education and we are failing our students. I chose to explore the use of manipulatives based instruction following the concrete-pictorial-abstract approach because these methods are research-based practices that have been referenced and suggested within the Common Core Progressions. These strategies have been found effective with students with disabilities as well as students experiencing difficulty mastering mathematical concepts at a pace on par with their peers. Students with disabilities as well as language barriers have been found to have success utilizing concrete manipulatives and visual models.

Background

I teach primary autistic support for the Pittsburgh Public School District. PPS is the second largest district in Pennsylvania, comprising 54 schools serving approximately 25,000 students in Kindergarten through Grade 12 and is located in an urban setting.

My classroom is located at Pittsburgh Arlington, a Pre K to 8 school in the southern part of Pittsburgh. The school serves over 500 students: 63% of the student population identify as African American, 23% identify as Caucasian, 11% as multiracial, 1% Hispanic and 1% Asian. The school is a Title 1 school, 83% of the students qualify for free or reduced lunch. Pittsburgh Arlington Pre K-8 is currently designated as a Focus school. Focus Schools are the lowest 10 percent of Title I schools across the state. This is based partly on how students perform on the Pennsylvania System of School Assessment (PSSA) and/or the Keystone Exam scores.

Students and their families are facing poverty related barriers. Many students reside in single-head-of household families, have inadequate housing, face homelessness, have a parent who is incarcerated, a caretaker facing mental health concerns or substance abuse issues and are exposed to chronic trauma and stress associated with urban poverty. These traumas include but are not limited to community violence, sudden or violent loss of a loved one, domestic violence and abuse. Despite these barriers, most days the children come into school with smiles on their faces, eager to learn.

Absenteeism is a concern with 36% of the students attending Pittsburgh Arlington classified as being chronically absent, indicating they missed 10% or more days of school during the past school year. These absences occur for a variety of reasons. Among my students, a third of my class was classified as homeless and missed over 25% of the school days last year due to housing issues as well as related transportation needs. Because of the high likelihood of students missing days of instruction, it is important for my lessons to be flexible and responsive to individual student progress.

My classroom is a regional support program and I provide support to students needing intensive autistic support services. I work with a team of professionals to meet the broad ranging needs of my students. Our team includes: two paraprofessional support staff, a speech therapist, an occupational therapist, a physical therapist and myself. Students attend from beyond the neighborhood feeder, potentially drawing from 34 schools servicing primary students throughout the school district. I teach a multi-grade classroom consisting of 9 students ranging from Kindergarten, 1st and 2nd grade. All students in my classroom have a diagnosis of Autism Spectrum Disorder. This is their primary disability; however, many students have one or more coinciding disorders including: Speech and Language Disorders, Epilepsy, Anxiety Disorder, Tourette Syndrome, Attention Deficit, Emotional Disturbance, Disruptive Behavior Disorder, Intellectual Disability and Obsessive Compulsive Disorder. Additionally some children qualify for Occupational and Physical Therapy due to delays in fine and gross motor development.

Objectives

I aim to develop a unit of study on representing and solving problems involving one-step addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, part-part-whole relationships and comparing, with unknowns in all positions, using objects, drawings, and equations to represent the problem with a concentration on problem construction and teaching practices that effectively promote access, scaffolding, the use of concrete and pictorial models and representation with equations.

Rationale

Study of the fourteen types of one-step addition and subtraction word problems within 20 expands on work started in kindergarten and sets the foundation for problem solving with addition and subtraction in second grade. The comprehension of word problems and the mechanics of addition and subtraction are to a considerable extent separate issues, although understanding what the operations mean, which is supported by work with word problems, is important support for mastery of the mechanics. It is highly desirable that students develop a strong grasp of addition and subtraction word problems in these primary grades to be prepared for future learning. Students will encounter these problem types again in later elementary grades with fractions, and again in middle school with variables. Mastering addition and subtraction situations with whole numbers provides students with foundational knowledge that they can build upon when integrating first fractions, and then variables, into their mathematical repertoires of needed understandings for college readiness.

Since the work contained in this unit will be ongoing. I have decided to utilize multiple themes that correlate with cross-curricular areas of study and community based experiences for my word problems. I will develop 3-4 collections of problems, each based on related scenarios. Basing these themes within a familiar context has been found effective when teaching students with speech and language barriers. Utilizing items that are commonly known and situations that are familiar to students reduces the language demands and need for extensive background knowledge, therefore allowing students to access the mathematical content.

Content

Readiness Skills for Problem Solving

The Committee on Early Childhood Mathematics has concluded that once children demonstrate counting skills including knowledge of number word list and cardinality, they begin to recognize relationships between quantities and develop counting strategies to solve simple word, or story, problems. (Cross, 2009) Despite the challenges that word problems pose for older students, young children actually appear to have greater success with problems posed in context of a story. This is especially true in problems that depict action or in the case of part-part-whole problems, but others (change, start unknown & comparison) are more difficult. At first children will attempt problems that they understand and can act out, either through role-play or by interacting with physical objects. The problems need to include numbers within their counting range. However, as young children have opportunities to apply counting strategies in real world context, they show advancement in their counting skills and develop more advanced strategies and begin to build a range of strategies for computation. (Van de Walle, 2013) Studies have shown children as young as preschool and kindergarten can successfully apply counting strategies to solve word problems involving addition, subtraction, and even multiplication or division. (Cross, 2009)

Problem Types

It is important that students develop an understanding of the operations of addition and subtraction within the wide range of situations where addition and subtraction can be applied. In Grade 1, The Common Core State Standards advocate that students solve problems of 3 types including change problems, part-part-whole problems and comparison problems. Change problems are categorized into two subtypes based on whether the initial quantity increases or decreases. Comparison problems are categorized into two subtypes based upon the difference between the two quantities being described as more or less than the other. These sub-types of problems are further divided according to position of the unknown quantity. This yields 6 types of change problems, and 6 types of comparison problems. There are only two types of part-part-whole problems, according as the whole is unknown, or a part is unknown. These 14 categories make up a taxonomy developed by mathematics educators, consisting in all of fourteen types of one-step addition and subtraction word problems. The complete taxonomy is provided as table I on page 88 of the Common Core State Standards (http://www.corestandards.org/the-standards/mathematics). A more detailed discussion is given below.

Research indicates that some of the 14 types are more challenging than others. I will discuss this issue more thoroughly as I discuss each type. I plan on systematically introducing the problem types, discussing and comparing the structure of the problems, then providing students opportunities to practice newly introduced problem types in a mixed format with problem types previously introduced. Once a problem type is introduced, I intend on practicing the problem type, giving students the opportunity to demonstrate consistent proficiency for minimum of 2 consecutive days in isolation and within mixed practice for minimum of 2 consecutive days before moving on. The pacing will be based upon student performance and individual learning needs. I will initially begin with number sets within 5, then based on student need, I will scaffold the number sets within 10 and within 20. However, I do not intend to slow the introduction of additional problem types until all students are able to work within the larger number sets, as we will continue to work on building these proficiencies outside of the context of word problems. I will discuss later and in more detail how I will make decisions about differentiating the number sets.

Change Problem Types

Change problems are dynamic, involving change over time. The change can be an increase or a decrease. Subtypes consist of change increase (aka add to) and change decrease (aka take from) problem types. These problems should be introduced and intermixed together in order to illustrate the inverse relationship between addition and subtraction. The physical action of joining and separating that occurs in change problems lends to acting out the problem, making these problems easier for students to comprehend and tackle. Change problems have been found to be the simplest problem type; therefore I plan on introducing change problems first.

Change Increase, Result Unknown & Change Decrease, Result Unknown

Change problems with result unknown have been found to be the easiest for children to deal with. The Common Core State Standards advocate that change increase, result unknown and change decrease, result unknown problems be introduced and performed to mastery within 10 during Kindergarten. Since, many of my students enter the year performing below grade level expectations, this will be my starting point. I want to ensure my students have ample experience with this basic problem type.

Change increase, result unknown problem example: 3 + 2 = __

3 children are playing tag at recess. 2 more children come to play. How many children are playing tag now?

Change decrease, result unknown problem example: 5 - 1= __

5 children were playing tag at recess. 1 child went inside. How many children are playing now?

Change Increase, Change Unknown and Change Decrease, Change Unknown

As I see students demonstrating proficiently solving change increase, result unknown and change decrease, result unknown within number sets to 5, I will introduce change increase, change unknown problem types, followed by the introduction of change decrease, change unknown problem types.

Change increase, change unknown problem example: 3 + __= 5

3 children are playing tag at recess. Some more children come to play. There are 5 children playing now. How many children joined the game?

Change decrease, change unknown problem example: 5 - __= 3

5 children were playing tag at recess. Some children quit playing. There are 3 children playing now. How many children quit?

Change Increase, Start Unknown & Change Decrease, Start Unknown

Change increase, start unknown & change decrease, start unknown are the most complex problem of the change class. The Common Core State Standards advocate for the introduction of change increase start unknown & change decrease start unknown in the 2nd grade. Consequently, this problem type will be introduced at a later time.

Change increase, start unknown problem example: __ + 2 = 4

Some children were playing tag at recess. 2 more children came to play. There are 4 children playing now. How many children were playing tag to start?

Change decrease, start unknown problem example: __ - 3 = 7

Some children were playing tag at recess. 3 children quit playing. Now there are 7 children playing. How many children were playing tag at first?

Part-Part-Whole Problems

Part-Part-Whole problems, sometimes known as collection problems, are problems in which some quantity or collection of objects is made up of two parts. This class, though similar to change problems, is slightly more difficult for young children. These problems have been found difficult for students because there is no physical action, making them less transparent and harder to model for young children. There are two types of part-part-whole problems, part-part-whole with an unknown whole or total and part-part-whole with an unknown part or addend. I intend first to introduce part-part-whole, total unknown problems, then part-part-whole, part unknown problems.

Part-Part-Whole, Total Unknown

In part-part-whole, total unknown situations, two quantities or parts are put together to make a third quantity.

Part-part-whole, total unknown problem example:

There are 2 girls and 3 boys playing tag. How many children are playing tag?

Part-Part-Whole, Part Unknown

In part-part-whole, part unknown situations, a total quantity is taken apart to make two quantities.

Part-part-whole, part unknown problem examples:

18 children are playing tag. 8 children are from Mrs. Brulia's class. The rest are from Ms. Loftas' class. How many of Ms. Loftas' children are playing?

10 children are playing tag. There are some boys and 2 girls playing. How many boys are playing?

Comparison Problems

Comparison problems are the most challenging of the 3 types of one-step addition and subtraction word problems. Thereare three types of compare problems: difference unknown, larger unknown, and smaller unknown. Compare problems involve relationships between quantities. Children are asked to determine the difference between the smaller and larger set. This third set or value is a non-specific subset of the larger set and tells the difference between the two amounts or how many more/less. Comparison situations typically do not involve a physical action, making it difficult for students to act out or model the problems. Moreover, comparison problems include language that is complex for young children, especially those facing language delays. The following terms are commonly used to describe the relationships present in the problems: fewer, less than, more, bigger, greater, longer, shorter, older, younger, heavier, lighter, etc. The comparison problems can each be posed in two ways, one uses words indicating greater and one uses words indicating lesser. This language can be consistent with the needed operation or inconsistent. See the examples below for more discussion of consistent and inconsistent language.

The Common Core State Standards calls for the introduction of addition and subtraction comparison problems in first grade. However, prior to the introduction of comparison problems students are exposed to comparison situations. These situations require students to visually compare sets to determine which is more than, less than, or equal to based on perception of length and density of the sets. In addition, students utilize match and unitizing count strategies to determine which set is more than, less than, or equal to other sets. The ability to apply comparative language and the ability to identify the relational situations between quantities are necessary components of solving the comparison problems. Many of my students have language delays and this is an area of weakness. I intend to spend time addressing these precursor skills to ensure they have the underlying foundation to promote success with these problem types.

Comparison, difference unknown

When solving compare problems, students demonstrate greater ease navigating comparison, difference unknown problems utilizing the terms more and bigger than. I will introduce these problems, then intermix comparison, difference unknown problems utilizing fewer and less than.

Comparison, difference unknown problem example:

8 students from Mrs. Brulia’s class are playing tag. 14 students are playing from Ms. Loftas’ class. How many more students from Ms. Loftas’ class are playing than students from Mrs. Brulia’s class?

Comparison, bigger unknown

When students are attempting compare with a bigger unknown problems stating the difference with the term more is simpler for students because the relationship between the quantities and operation are consistent.

Comparison, bigger unknown, consistent language (more, greater, larger) problem example:

Students are playing tag at recess. 8 students are from Mrs. Brulia’s class. Ms. Loftas has 6 more students playing from her class than are playing from Mrs. Brulia’s class. How many of Ms. Loftas’ students are playing tag?

Comparison, bigger unknown, inconsistent language (less, fewer) problem example:

Students are playing tag at recess. 8 students are from Mrs. Brulia's class. Mrs. Brulia has 6 fewer students playing from her class than are playing from Ms. Loftas' class. How many of Ms. Loftas' students are playing tag?

Comparison, smaller unknown

When students are attempting compare problems with a smaller unknown problems stating the difference with the term fewer is simpler for students because the relationship between the quantities and operation are consistent.

Comparison, smaller unknown, consistent language (fewer, less) problem example:

8 students from Mrs. Brulia's class are playing tag. Ms. Loftas has 6 fewer students playing tag than Mrs. Brulia. How many students from Ms. Loftas' class are playing tag?

Comparison, smaller unknown, inconsistent language (more, greater, bigger) problem example:

8 students from Mrs. Brulia's class are playing tag. Mrs. Brulia has 6 more students playing tag than Ms. Loftas. How many students from Ms. Loftas' class are playing tag?

Building Conceptual Understanding through the Use of Tiered Number Sets

Doug Clarke, a researcher in mathematics education, indicates children follow a developmental path as they build conceptual understandings and apply these understandings as strategies to solve addition and subtraction problems. He developed a framework that recognizes 6 key growth points or transitions in how students approach solving addition and subtraction problems. (Clarke, 2001) His trajectory indicates children demonstrate increasing conceptual understanding as they apply increasing more complex strategies.

->	->	->	->	->	->
Count all	Count on	Count back Count down to Count up to	Basic strategies: Doubles Commutativity Add 10 Tens facts Known facts	Derived strategies: Near doubles Add 9 Build to the next 10 Fact families	Extending & applying addition and subtraction mentally by using basic, derived and intuitive strategies

I plan to utilize Clarke's growth points to vary the difficulty of problems by altering the numbers used. Sequential in nature, the complexity of the number sets used in the problems need to be tiered to develop flexibility with the understanding of numbers and place value, expecting students to demonstrate fluency of facts to 5, fluency of facts to10, making ten, decomposing a number to create a 10, adding to 10 and decomposing teen numbers, demonstrating an understanding that it is comprised of one ten and varying ones (1-9). I will make use of various strategies including the use of manipulatives, count all, count on, and recompose solution strategies and developing early connections to the measurement model to build conceptual understanding. Each of these strategies is discussed in more detail later in this text.

Initially I will introduce problems including numbers to 5. This allows me to begin working with students on problem solving who may have a limited counting range. I will intentionally choose number sets to assess the students' ability to decompose 5 into 4 and 1, 3 and 2, and 5 and 0. I will also utilize visual representations of number sets to determine if students are able to recognize these by sight or if they need to count out the items to determine the value of a set.

Number sets will be increased to include partner numbers to 10 with students demonstrating flexibility with numbers within 5 and counting ranges to 10 and beyond. I will begin choosing number sets including 5 as a partner number to encourage students to view 5 as a part of the larger number. I will be assessing students' ability to decompose 6 into 5 and 1 more, 7 into 5 and 2 more, 8 into 5 and 3 more, 9 into 5 and 4 more and 10 as 2 sets of 5, noticing when given visual representations, if they rely on subitizing or cardinality. I then focus on utilizing number sets that form ten: (10, 0), (9, 1), (8, 2), (7, 3), (6, 4) and (5, 5).

Once students demonstrate flexibility and fluency with numbers within 10 and a counting range equal or greater than 20, I will increase the number sets to partner numbers to 20. I will utilize numbers to illustrate adding to 10 and decomposing teen numbers, demonstrating an understanding that a teen number is composed of one ten and some ones (1-9). Finally, I will choose number sets to foster decomposition and recomposition of numbers to form 10s within the solution strategy.

Addition/Subtraction Single-Digit Solution Strategies and Correlating Levels

Consistent with Clarke's trajectory, the Common Core Progression Documents recognize 1st grade students typically utilize three levels of single-digit solution strategies as they develop conceptual understanding in problem solving: direct modeling by counting all, counting on and derived facts or decomposition of numbers to convert to an easier problem. Children approach problems differently depending on the level of understanding they have developed.

Level 1. Direct Modeling by Counting All or Taking Away

Children represent situations or mathematical problems with objects, drawings, or fingers. They create a model for the situation by creating groups to represent the addends or by separating a total amount into groups. Then, counting the resulting total or group can give the answer.

Level 2. Counting On

Children at this level do not need to count or model each number in the situation. The child sees a "part", understanding its value. So, the child will start at this number, omitting the count of this addend and beginning with the number word, count up. The child employs some way of keeping track of the count such as fingers, objects, mentally imaged objects, body motions or count words. For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking technique is used to determine the value of an unknown addend.

Level 3. Derived Facts

A child at this level begins to use break-apart strategies to decompose and compose numbers to make an easier problem. Children decompose an addend and compose a part with another addend to convert to the values in the problem to form 10 or utilize embedded place value concepts.

Strategies

Progress Monitoring

Student progress needs to be monitored across instructional targets including counting range, ability to subitize sets and the quantity of those sets, flexibility and fluency with various number sets, success with problem types and level of solution strategies. I will monitor individual progress, and use my knowledge to determine when to move onto a problem type with a more complex structure, when to increase the types of number families utilized in the problems and when students can benefit from explicit strategy instruction.

Teaching to Mastery

Children need ample opportunity to develop mastery of each problem type. The pacing is to be determined by the needs of the students and individual progress monitoring data will guide the instructional path. Student work and pacing will be adapted accordingly to allow for re-teaching as needed.

Problem Analysis

Often incorrect answers to word problems are the result of correct mathematical calculations performed using incorrect problem representations and equations. This indicates students may misunderstand the conveyed meaning or language of the word problem. In response to this, I will work with my students until they learn to examine the problem and identify critical information. I will focus their attention on what the problem is asking, and prepare an estimate of the type of answer they may come up with so they can judge the reasonableness of their approach and solution. Some questions I will ask include: What's happening? What are we trying to find out? Do you think the solution will be a big number or a small number?

Teaching with Manipulatives

I intend to utilize developmental approaches to promote conceptual understanding of the problem solving process through the use of manipulatives including: unifix cubes, ten rods, unit cubes, Cuisenaire rods, rod tracks and ten frames. It has been widely accepted that manipulative based instruction promotes understanding of a variety of mathematics concepts. Use of manipulative materials in teaching mathematics has been documented for hundreds of years. The use of concrete manipulatives to promote conceptual understanding of the problem solving process is supported by the learning theory work of Piaget and Bruner. I will assist my students to explicitly connect the manipulatives with the symbolic work. This is explored further in the discussion below regarding the use of the concrete-pictorial-abstract approach.

Concrete-Pictorial-Abstract Approach

The concrete-pictorial-abstract approach, while proven beneficial to all students, has been noted to be particularly effective with students with academic deficits, speech and language needs, as well as with ESL students. Hands-on manipulatives or real life objects are used to demonstrate the concept, then students create pictorial representations and represent their thinking with a number sentence. This visual, pictorial modeling is often missing from many programs traditionally used in the U.S. It provides a transition from the mathematical process, to words, to the more abstract equation. This process serves both as a learning process and a scaffold because as students build mathematical understanding they can later rely on more efficient abstract algorithms once the conceptual understanding is developed. This is of course closely related with using manipulatives.

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, proposes that there are three steps essential for pupils to develop conceptual understanding of a mathematic concept. Students must be actively engaged in learning and move through 3 stages: enactive, iconic, and symbolic. The enactive stage involves allowing a student to investigate a new skill or idea by acting it out with real objects such as manipulatives, involving themselves in tactile and kinesthetic experiences. In the iconic stage a student demonstrates understanding of the hands-on experiences by relating them to or constructing visual models, drawings and/or pictures of the problem. In the symbolic stage learners are able to abstractly model mathematics through the use of numbers and symbols to represent their process with mathematical equations.

Explicit Strategy Instruction

Explicit instruction will be provided on specific solution strategies as students move through the developmental conceptual levels. Progress monitoring data will be utilized to identify student instructional needs and students will be flexibly grouped accordingly. I will model strategies in a teacher led format, lead students through various strategies step by step, explaining concepts and calling out critical connections to ensure learners are making connections, as new vocabulary is introduced. During these sessions, as the instructor, I will determine the manipulatives used, the format in which a pictorial drawing is laid out, and the sequence of the steps. Students will be walked through the step-by-step process while I or classroom staff explains the rationale behind the strategy.

Think-Alouds

Think-alouds can be used to begin to teach metacognition strategies in children. (Van de Walle, 2013) I will demonstrates the steps needed to solve a problem while verbalizing the thinking process, using strategies that mirror student entry points and conceptual levels. I will model a problem using concrete materials and then build a pictorial model, while talking through the steps and identifying the reasons for each step in my strategy. During the process I will use questioning strategies to begin to engage students and transfer the responsibility to the student until they are discussing aloud their thinking and process.

Early Development of the Measurement Model

Knowledge of addition and subtraction can be constructed through the context of measurement. The introduction of linear measurement has several connections to counting and cardinality. Often students are first introduced to measurement through the use of non-standard tools. For example students are asked to measure items with objects. They are asked to line up cubes or paperclips and indicate how many units long it is. This builds on their knowledge of counting and cardinality. They are tasked with applying 1-to-1 correspondence to count objects representing the attribute of a unit of length. Measurement assigns a number to this less concrete attribute. The introduction of standard measurement tools through the use of objects of standard size assist in the understanding of the space as a unit. This leads to the ability to use standardized measurement tools such as a ruler with understanding and promotes the understanding of number line models and fractions. The measurement model can be developed by work with cubes and rods of various lengths. Students can work with concrete tools modeling addition and subtraction through the use of 1inch cubes, tiles, rulers, base ten blocks, Cuisenaire rods and tracks. Measuring arbitrary objects raises the issue of non-whole numbers. Cuisenaire rods avoid this, but I will need to be careful to deemphasize the colors.

Language Support

Language development has a significant effect on the accessibility of instruction. I will utilize many research-based scaffolds to provide additional support for students facing language barriers that impact student achievement regarding the problem solving process. I will: utilize concrete and visual models, utilize graphic organizers, explicitly teach academic vocabulary, utilize teacher modeling and explanation, utilize multimedia to enhance comprehension, build background, clarify key concepts by using present tense, shorter sentences, fewer clauses; using examples related to school contexts; using graphics and arrows to illustrate points; and using white space and color to accentuate important information, integrate oral and written instruction into content area teaching; incorporate structured opportunities to speak with partner or small group; provide structured opportunities to write, utilize sentence frames and sentence starters. (August, 2014.)

Activities

The majority of students will be entering the classroom for the 1st time, thus I will need to assess students' needs. Introduction of the unit will be based on student readiness. My teaching will need to be adaptive, making it impossible to predict exactly how things will go. Student performance will more specifically influence the implementation of the above-described strategies. However, these lessons are intended to convey a general overview of possible activities.

Sample Lesson #1

The following materials are needed: projector, wifi connection, The Math Curse by Jon Scieszka and Lane Smith video, spray bottle, whiteboard, dry erase markers, manipulatives, student math journals and colored pencils.

I will introduce the lesson stating, "Today we are going to learn about being problem solvers. First, we are going to watch a video about a boy who is a problem solver. Then we are going to practice being problem solvers ourselves." Then, the class will view the video animated read aloud of the book The Math Curse. In order to activate previous knowledge, I will discuss the story with children, ensuring the understanding that we encounter math in our everyday actions. I will have students make connections between math and their daily happenings and allow students to take turns sharing their thinking in response to the story. I will set the goal for the lesson stating, "Before we begin practicing our classroom rituals and routines, I'm going to put a Math Curse on you guys just like the boy in the story! Not to worry though, this a good curse not a scary one! (I will spray a thinking potion over the students, just for fun!) Ah, now you will think of everything through the lens of a problem solver."

Expectations will be modeled for students, during an opening activity I will connect the lesson to instruction of classroom routines stating, "Since, we've been practicing lining up and waiting in our transition area we are going to model some problem solving about lining up." Since our class size is small, to create a transition area, I create a ten frame on the floor with tape. Students line up in designated spots and I lead them out in groups of 5. I will project the problem, "3 children are standing in line ready to go to Art class. 2 more children join them. How many children are ready to transition?" on the screen and have students read it aloud with me. When introducing the 1st problem, I will utilize explicit strategy instruction, calling on student volunteers to act out the problem. As children act out the problem, I will model the count all strategy and represent the problem using pictures and a number sentence. Students will document the problems in their math journals. I will gradually release responsibility to students by completing another problem, choosing students to be the problem solvers and actors.

Problems:

5 children are standing in line. 2 children leave the line because they forgot to put away their materials. How many are in line now?

Children will be divided into small, differentiated groups. Groups will work with varying levels of support based on student needs. Staff will work with the children providing supports as needed. Students will work together to complete change increase result unknown and change decrease result unknown word problems. Manipulatives will be provided and students will represent their solutions visually and with a number sentence in their math journals. In order to assess learning, groups will share out their solutions and discuss how they solved the problems.

Sample Lesson #2

The following materials are needed: photos of students engaged in classroom activities, whiteboard, dry erase markers, manipulatives, student math journals and colored pencils.

I will introduce the lesson stating, “Today we are going to continue to learn about being problem solvers. We are going to create a class book of word problems starring our class.” In order to activate previous knowledge, I will share some of the photos and have students make connections between math and their daily happenings. I will allow students to take turns sharing their thinking. I will set the goal for the lesson stating, “As we have been busy learning, we have been taking photos of the class. Today we are going to work in groups, looking at photos and creating change problems about the things happening in the pictures.”

I will model expectations for students during an opening activity. A picture will be projected on the screen and I will engage students in a think aloud as we create a problem related to the story. Then, children will be divided into small, differentiated groups. Groups will work with varying levels of support based on student needs. Staff will work with the children providing supports as needed. Students will work together to create change increase result unknown and change decrease result unknown word problems. In order to assess learning, groups will share the problems the created.

Sample Lesson #3

The following materials are needed: student generated math problems, whiteboard, dry erase markers, manipulatives, student math journals and colored pencils.

I will introduce the lesson stating, “Today we are going to continue to learn about being problem solvers. We are going to solve the word problems that our friends have created.” In order to activate previous knowledge, groups will choose a problem that they would like to include in the class book. They will take turns sharing a problem with the class. I will set the goal for the lesson stating, “Today we are going to work in groups and solve the change problems that our friends have created.”

I will model expectations for students during an opening activity. The problem that the class created as a group will be projected onto the screen and I will engage students in a think aloud as we solve the problem. Then, children will be divided into small, differentiated groups. Groups will work with varying levels of support based on student needs. Staff will work with the children providing supports as needed. Students will work together to solve the change increase result unknown and change decrease result unknown word problems created by their peers. They will utilize manipulatives, create a visual of their thinking and identify a correlating number sentence. In order to assess learning, students will share their solution strategies to the student generated problems and solutions and respond to questions. The problems and student solutions will be assembled to create a class book.

Appendix

Theme- Classroom Happenings:

The problems shown below are just a selection from a larger collection.

Add to

Add to result unknown

I have 3 books. Then I get 4 more. How many books do I have now?

3 children are working in the math center. 6 more join them. How many children are working in the math center now?

9 students put their coats in the locker. Then 6 more students put their coats in the locker. How many students have put their coats in the locker?

Add to change unknown

I have 3 books. Then I get some more at the library. Now I have 7 books. How many books did I get at the library?

There were 4 folders at my workstation. Mrs. Brulia gave me some more. Now I have 10 folders. How many more folders did Mrs. Brulia give me?

Kameron makes 5 block towers. Jaylen makes some block towers, too. There are 15 towers. How many towers did Jaylen build?

Add to start unknown

I have some books. Then I went to the library to get 4 more. Now I have 11 books. How many more books did I have before?

I have some folders. You have 10 folders. Altogether we have 12 folders. How many folders do I have?

Kameron makes some block towers. Jaylen makes 4 block towers. They made a total of 20 towers. How many towers did Kameron make?

Take from

Take from result unknown

Kameron has 10 books. He gave 3 to Jaylen. How many books does he have left?

Bri'Air buys a bag with 10 crackers in it. He eats 6 of them. How many crackers are left?

6 people are waiting at school for the bus. Then 4 of them go home. How many people are still waiting for their bus?

Take from change unknown

We have 8 library books. We return some of them. Now we have 5 library books. How many books did we return?

Bry'Air buys a bag with 10 crackers in it. He eats some of them. He has 5 left. How many crackers did he eat?

6 people are waiting at school for the bus. Then some of them go home. There are 2 still waiting. How many people already went home?

Take from to start unknown

Jaylen has some crayons. He gave 3 to Kameron. Now Jaylen has 7 crayons left. How many crayons did he start with?

We had some library books. We return 3 of them. Now we have 5 library books. How many books did we have to start?

Bry'Air buys a bag with some crackers in it. He eats 5 of them. He has 5 left. How many crackers were in the bag?

Part-Part-Whole

Part-Part- whole, whole unknown (Put together/ take apart total unknown)

7 boys and 2 girls got in line to go to Music class. How many students are in line?

5 children said they liked the story. 4 children said they didn't like the story. How many children discussed the story?

9 students have a special class 4th period. 5 students have music class. The rest have art class. How many students have art?

Put together/ take apart addend unknown

Jaylen has 13 coins. He has 9 dimes and some nickels in his pocket. How many nickels does he have?

I have 12 counters. I have 9 red counters. How many counters are not red?

9 children listened to the story. 5 children said they liked the story. How many children didn't like the story?

Compare

Compare difference unknown

Bry'Air has 10 crayons. Jaylen has 14 crayons. How many fewer crayons does Bry'Air have than Jaylen?

There are 2 girls in line. There are 7 boys. How many more boys are in line than girls?

There are 2 girls in line. There are 7 boys. How many fewer girls are in line than boys?

Compare bigger unknown

Bry'Air has 8 books. Jaylen has 6 more books than Bry'Air. How many books does Jaylen have?

Bry'Air has 8 books. Bry'Air has 6 fewer books than Jaylen. How many books does Jaylen have?

Bry'Air has 10 crayons. Jaylen has 6 more crayons than Bry'Air. How many crayons does Jaylen have?

Compare smaller unknown

Jaylen has read 14 books. Kameron has read 6 fewer books than Jaylen. How many books has Kameron read?

Jaylen has read 14 books. Jaylen has read 6 more books than Bry'Air. How many books has Bry'Air read?

Jaylen has 18 comic books. Kameron has 9 fewer comic books than Jaylen. How many comic books does Kameron have?

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