YALE NATIONAL INITIATIVE
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Curriculum Units by Fellows of the National Initiative
2015 Volume V: Problem Solving and the Common Core

Real World Problem Solving in Second Grade Mathematics

Curriculum Unit 15.05.03, published September 2015
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Introduction

Edgewood Magnet School in New Haven, Connecticut is an arts magnet school, integrating the arts across the curriculum. Students in this environment are encouraged to use the strategies of observation, interpretation, and analysis to increase their thinking skills in every subject. With that mission, both teachers and students use unique and exciting approaches to "the basics" and work together to ensure that all learners are included.

For most second graders, the beginning of the year is a time for refreshing knowledge and skills from first grade. The summer away from direct instruction and opportunities for practice and guidance sometimes means a loss of solid understanding of learned concepts in mathematics. This three- to four-week unit is designed to review and build new understanding of one-step word problem solving using addition and subtraction as students develop skills and strategies they will use all year. The students, through a series of mathematical scenarios, will use the problem types identified in Table 1 of the Common Core Mathematics Glossary which covers addition and subtraction.[1]

Objectives

The Common Core concentrates on a clear set of math skills and concepts. Students learn concepts in an organized way during the school year as well as across grades. The standards encourage students to solve real-world problems. [2]

The Common Core calls for greater focus in mathematics. Rather than racing to cover many topics in a mile-wide, inch-deep curriculum, the standards ask math teachers to significantly narrow and deepen the way time and energy are spent in the classroom. This means focusing sharply on the major work of each grade, which for grades Kindergarten through second grade includes concepts, skills, and problem solving related to addition and subtraction.

The New Haven Public School district uses the *Math in Focus Singapore Approach,* a Common Core-based

curriculum for students from Kindergarten to Fifth Grade. The student books and workbooks follow an instructional pathway that includes *learning* concepts and skills through visual lessons and teacher instruction for understanding the how and why; *consolidating* concepts and skills through practice, activities and math journals for deep math understanding, hands-on work in pairs and in small groups; and, *applying* concepts and skills through extensive problem solving practice and challenges to build real world problem solvers. [3]

This approach embeds problem solving throughout each lesson and encourages frequent practice in both computation and problem solving. The word problems appear throughout each chapter and progress from 1-step to 2-step to multi-step. Each chapter concludes with a challenging problem or set of problems that require students to solve some non-routine questions. To solve these problems, the students need to draw on their deep prior knowledge as well as recently acquired concepts and skills, combining problem solving strategies with critical thinking skills, including classifying, comparing, sequencing, identifying parts and whole, identifying patterns and relationships, induction and deduction and spatial visualization.

The second grade text begins with numbers to 1000. Students begin by expressing numbers in standard form (231), expanded form (200 + 30 + 1), and word form (two hundred thirty-one). This is accompanied by concrete representations via base ten blocks, and, for two digit numbers and a few three digit numbers, representation by trains of rods, of lengths 1, 10 and 100. This initial chapter also includes sequencing numbers and comparing using greater than and less than terminology, and then moving right into addition and subtraction of two- and three-digits numbers. Here the take-away should be, if you have more hundreds, the tens and ones don't make any/much difference; and if you have the same number of hundreds, but more tens, then the ones don't make any/much difference. Most of my students (if not all) struggle from the start! They do not seem to have a solid foundation of understanding numbers to 100 or the concept of place value in general. This unit is designed to get ahead of the frustration that the students feel when pushed too quickly before they have a firm understanding of principles of place value and the properties of operations.

This unit launches the school year with 1-step addition and subtraction problems of all types using numbers to 10. The goal is to spend time practicing basic computations with numbers that student can work comfortably with before jumping right into the district curriculum. Once there is a level of understanding with these problem sets (numbers to 10), students will move on to solving 1-step problems using teen numbers and then onto numbers to 100. Most of the curriculum problems at the start of the year require addition and subtraction of 3-digit numbers. Some students will move quickly through the problem sets with numbers to 100 and will be ready to work with the regular curriculum.

For the duration of the unit, the focus will be steadily on solving and later constructing a collection of word problems that provide robust and balanced practice. Problems sets will be based on a scenario which will provide the substance of the story. Each scenario will allow us to extract several problems, changing the numbers and ensuring each set of numbers makes a reasonable problem. This idea looks like the following: *John has 8 crayons in his box. He shares 3 with Sam. How many crayons does John have left in the box? John has some crayons in his box. He shares 3 with Sam. John has 5 crayons left in his box. How many crayons did John start with? John has 5 crayons. Sam has 2 fewer than John. How many crayons does Sam have?John and Sam are sharing crayons. John has 5 and Sam has 3. How many crayons do the friends have together?* The two students participate in several crayon-sharing stories that use the same set of numbers but in slightly different situations. Some situations are more obvious and direct while others take more thinking. It is important to provide opportunities for students to work with and solve the different problem types that can be created from one set of numbers. [4]

Background: Problem Types

The taxonomy of addition and subtraction problem types as identified in the Common Core State Standards of Mathematics Glossary is a framework that sorts one-step problems into three broad classes: *change*, *comparison*, and *part-part-whole*. Each of the three classes is then separated further into a total of 14 problem types sorted out as follows: *change*, in which some quantity is either added to or taken away from another quantity over time; *comparison*, in which one amount is described as more or less than another amount; and *part-part-whole*, in which an amount is made up of two parts. [5]

Within the group of change problems, there are two subgroups: *change-increase*, in which a quantity is added to an initial amount and *change-decrease*, in which a quantity is taken from an initial amount. We might recognize these subgroups more familiarly as "add to" or "take from." Additionally within each of these subgroups, there are three possible unknown quantities. One scenario to show *change-increase*: 2 kittens were playing with some yarn. 3 more kittens join them. Now there are 5 kittens playing with the yarn. Using these quantities, the unknown might be the result (2 + 3 = ?), an unknown quantity of change (2 + ? = 5) or an unknown initial amount (? + 3 = 5). In the *change-decrease* subgroup, there are again three possible unknowns. A scenario for this example: 5 birds are sitting on the branch. 2 fly away. Now there are 3 birds sitting on the branch. Here again the students might solve for the final amount (5 – 2 = ?), the amount of change (5 - ? = 3), or the initial amount (? – 2 = 3). This gives in all six types of change problems.

Similarly *comparison* problems can also be categorized into two subgroups: *comparison-more*, in which one quantity is described as more or greater than another, and *comparison-less,* in which one quantity is described as less or fewer than another. Here again, each of these two subgroups has three possible unknowns, for a total of 6 types. Sam has 6 marbles. James has 8 marbles. James has 2 more marbles than Sam. The unknown quantity may be the lesser amount (? + 2 = 8), an unknown greater amount

(6 + 2 = ?), or the unknown difference (8 – 6 = ?) one quantity that is more and one that is less. Using this same scenario for a set of *comparison-less* problems, the language need to change from "more than" to "less than." Here is a way to present this set with the language adjustment: Sam has 6 marbles. James has 8 marbles. Sam has 2 fewer marbles than James.

Part-part-whole problems are a set of two quantities, the parts that, when put together, make up a whole quantity. This problem type seems very like to *change* category but in this problem type there is no change over time. The two parts play equivalent roles, which allow for only two possible unknown categories: either a part is unknown or the whole is the unknown. There are 4 large dogs and 3 small dogs. There are 7 dogs in all. One of the parts may be unknown (4 + ? = 7 or ? + 3 = 7) or the unknown may be the size of the whole (4 + 3 = ?). Since the parts are interchangeable, there are only 2 types in this class of problems.

The following chart sort these classes and categories into the framework. Located in Appendix A of this unit is a set of example problems illustrating each of these 14 types.

Problem Type		**Unknown Amount**		
Change	Increase Add-to	Initial	Change	Final
	Decrease Subtract from	Initial	Change	Final

Comparison	Greater / More	Smaller	Greater	Difference
	Less / Fewer	Smaller	Greater	Difference
Part-Part-Whole		Part	Whole	

The Scenarios of the Problems

For second graders, life at school is a large part of their world. Most of my students arrived at Edgewood for their Kindergarten year and stayed through First Grade making the year in second grade essentially their third year at the same school. They are comfortable in the building and know many of the other students. They will become the active players in the math stories that I, and we together, will construct. Activities that occur in the classroom, in the cafeteria, on the playground, and on the bus seem to be recognizable situations that will help with the basic understanding of context.

Additionally, there are opportunities for students to incorporate the topics and learning that occur in the other subjects, such as science, social studies, literacy, art, music and, in our school, dance and drama. One example might be to create set of story problems centered on the life cycle of the butterfly, a unit of study each year in second grade. With the common knowledge the students will be obtaining, this content could become the scenarios for word problems. An example might be: *Seven caterpillars climbed up the branch and formed their chrysalises. Later that day, three more caterpillars climbed up the branch and formed their chrysalises. How many chrysalises are hanging from the branch?* Similarly, using the characters in a book read together as a class could provide the characters in a new set of problems. *Curious George had a bunch of bananas. He ate 4 of them. Now he has 3. How many bananas did Curious George begin with?* The use of common or thematic content will not only connect all the thinking and practicing, it will provide tangible and real situations. With an established scenario, students will work with a set of numbers, determining the unknown within each of problems types.

Creating the Problems

A question that is frequently answered with a guess is “What should we do to answer the question to solve the word problem?” The fundamental understanding of what is being asked is not apparent to the students, making the solution inaccessible. Most first graders entering second grade have a basic understanding when the story (problem) is categorized as *final unknown* or *whole unknown*, but most other components of the taxonomy are unfamiliar to them or just too difficult to decode. To begin to help them with their thinking, they will use concrete models, such as themselves (2 children are sitting at the reading table, 4 more join them) acting out scenarios. Many basic materials in the classroom – pencils, notebooks, folders, crayons – can be used to create and design scenarios, with each type of problem represented.

Solving the Problems

Following the overall plan of the Singapore Math program, the students will solve problems using the concrete, pictorial and abstract approach. Because this is a standard approach in our district mathematics instruction throughout the year, the students will begin with this set of strategies to solve problem sets.

Word problems are written as stories and scenarios making language a consideration in crafting the problems for the beginning second graders. Word problems are as much about language and reading as they are about math. If the story is not understandable, how can students begin the know what to do with the numbers

they've been given and the question they've been asked? Thus, words and vocabulary need to be appropriate and useful for the variety of reading levels of the incoming students. The structure of word problems should be understandable and clear, accessible in language as well as numbers. Also, the language, especially the words that express the relationship between the quantities involved, should be discussed to ensure that it is familiar to all students.

This is a clear integration of Language Arts and Mathematics and a method in which students can connect math to the real world, in this case, through the activities they engage in at school. Reading skills and computation skills come together with even the simplest of word problems.

Structure of Problem Collection

The content introduction over the duration of this unit includes a certain sequencing and scaffolding to guide students through the 14 problem types. To begin the unit, students will only be working with numbers to 10. This is an important starting place to ensure that understanding is occurring. Most of my second graders are capable with addition and subtraction to 10, but are not so comfortable with the word problem language. So first, students will be challenged more by the language than the arithmetic. Students will practice figuring out what exactly the problems are asking with problems that they are familiar with before moving on to a new step. Practicing all the problem types will improve and increase strategies and confidence!

With addition and subtraction within 10 mastered, the next phase of the unit moves to numbers to 20. The key is to continue with scenarios that are obvious and repeated as new numbers are introduced. An example of this transition would be these parallel problems:

6 students got on the bus at the first stop. 3 students got on the bus at the second stop. After the second stop, how many students are on the bus? (*change-increase, final unknown)*

Some students got on the bus at the first stop. 3 students got on the bus at the second stop. Now there are 9 students on the bus. How many students got on at the first stop? (*change-increase, initial unknown)*

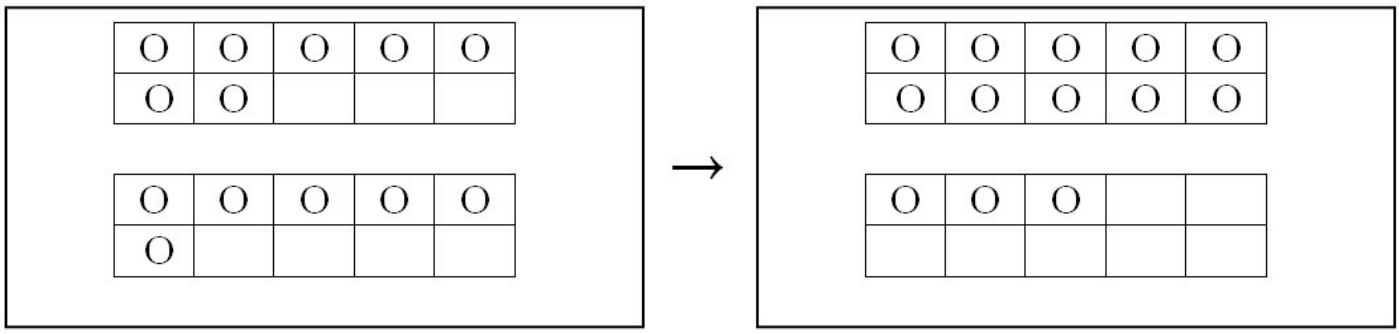
These now become:

11 students got on the bus at the first stop. 7 students got on the bus at the second stop. After the second stop, how many students are on the bus? (*change-increase, final unknown)*

Some students got on the bus at the first stop. 7 students got on the bus at the second stop. Now there are 18 students on the bus. How many students got on at the first stop? (*change-increase, initial unknown)*

When working with numbers to 20, it is essential that students understand that the "teen" numbers (11-19) are really 10 and some ones. Students should work with numbers within 20, creating equations using their knowledge and skill of making a ten first. In the case of 7 + 6, making a new ten looks like this:

7 + 6 = 7 + 3 + 3 = 10 + 3 = 13



Because 7 needs a 3 to make ten, and 6 is composed of 3 + 3, this equation shows the progression of making 10 and some more. Practicing this method using two ten frames demonstrates the process concretely. In the example above, students use the ten-frames to show 7 and 6 separately. To make the new 10, students will move 3 from the 6, which now shows 10 and 3 more or 13.

As mentioned earlier, it is obvious that the most accessible problem types for students entering the second grade are the *change-increase* or *change-decrease, result unknown* and *part-part-whole, whole unknown.* The general go-to strategy for solving a word problem seems to be to just take the two numbers you see and add them together, or maybe subtract, but often the students are just unsure. It seems that these are the most practiced problem types, which leaves students without balanced experience with all 14 types and ultimately without some strategies to employ as they problem-solve. Students need to see a broad range of problems to gain a strong understanding of how addition and subtraction are used and how they are related to each other. The notion of example sufficiency means students should be exposed to a wide array of examples to provide well-rounded practice with the concept. [6]

Teaching Strategies

The approaches for this curriculum unit vary to reflect the learning styles of all students.

The general format is based on the workshop model. The concepts and skills are taught through a series of mini-lessons focused on the objective with the following methods used throughout:

Experiential Learning: Most young students need to begin with hands-on learning. Using concrete models to work out math stories allows students to see the problem and manipulate the pieces as the story progresses. This type of learning is an important first step.

Differentiated Instruction: Lessons and activities will be targeted to maximize learning. The students will use a variety of approaches, working sometimes individually and sometimes in small groups, determined by the complexity of the work. Some students will move more quickly as they master skills and some will need more opportunities for practice.

Cooperative Learning: The students will be given opportunities to work as cooperative groups to create math stories to present to the class. This strategy will allow students to work collaboratively taking on various roles necessary to complete the work, with a focus on success for all.

Classroom Activities

Activity 1: Sequenced Problem Types – Problems to 10

The introduction (and review) portion of the unit covers all problem types but in a sequenced manner. The objective is for students to read and interpret a word problem with guided instruction followed by independent practice. Because of the many problem types, this part will take several days of review and practice before students are comfortable beginning to write their own sets of problems. Based on student need and pace of understanding, I expect this section to be a four- to six-day set of lessons, more if needed.

The sequence is as follows: *part-part-whole*; *change-increase* and *change-decrease*; and finally, *compare-greater* and *compare-fewer*. The following introductory sessions are designed as a whole group activity, with students either at their desks or gathered on the rug close to the board or easel. The whole group portion should be 20 minutes at most. At the close of each session, I will give students between 5 and 10 similar problems to solve. More capable students can begin to generate their own problems during the independent work time.

Beginning with the fundamentals provides a good opportunity to get to know students' skills which is helpful in preparing differentiated work and creating groups,

In this lesson, students will interpret real world problems and with the help of manipulatives and pictures, solve part-part-whole stories using addition and subtraction.

6 girls are playing

3 boys are playing with them.

How many children are playing in all?

Begin the story with the whole unknown as in this example. This type of story is perfect for students to act out right in the classroom. Write the story on the board or chart paper and have students volunteer as actors. Once the students have solved the problem, write the math sentence to show what happened: 6 + 3 = 9 students. Explain that the 2 parts (boys and girls) have made a whole (children). With the students still in acting position, present a new approach to this scenario:

9 students are playing.

6 of them are girls.

How many boys are playing?

With this visual example, students should see right away how many. The important concept to demonstrate is that the parts can be determined when the whole and one part are known, in this case 9 is known as the whole and 6 as one part. Again, write the math sentence to show this calculation: 6 + ? = 9 and include the strategy of starting with the whole to determine the missing part as a subtraction sentence 9 – 6 = 3. Practicing both approaches to the solution will help students connect addition and subtraction and recognize how they are used together.

Since this lesson requires students to read story problems, I will pair fluent readers with those who are less fluent, provide counters for those who want them, and allow partners to work together to solve and problems and share the strategies that they used.

I will use two more examples, like the ones below, to demonstrate, remembering to write the word problem on the board as well as the math sentence. I will also reword the problems to have the part as the unknown.

Hannah has 5 red markers.

She has 3 blue markers

How many markers does Hannah have in all?

7 students are drawing with crayons.

2 students are drawing with colored pencils.

How many students are drawing?

Continuing with this same idea, the next set of problem types includes *change-increase* and *change-decrease*. Although part-part-whole is language that students can adopt and use while discussing their work, the *change-increase* and *change decrease* language is a bit trickier. The use of the word *change* is more appropriate for students to demonstrate that some amount has been either added or subtracted from an initial amount.

Introduce the word problem below which is an example of the unknown result in the change-increase category.

Jason had 8 “caught being good” stickers on his chart at the beginning of the day.

During the school day, he earned 2 more stickers.

How many stickers does Jason have on his chart at the end of the day?

Student can solve the problem as written and, using the same scenario, challenge them to create the *change-unknown* and *initial unknown* story. One example might be:

Jason had some “caught being good” stickers on his chart at the

beginning of the day.

During the school day, he earned 2 more stickers.

At the end of the day, he has 10 stickers.

How many stickers did Jason have at the beginning of the day?

This is an oral activity, with me writing the adjusted version across the board, placing the math sentence underneath. It is important to allow students to work on composing the problem so they can begin the see the relationship between the problems and what the problems are asking.

Result Unknown Version	Change Unknown Version	Initial Unknown Version
8 + 2 = ?	8 + ? = 10	? + 2 = 10

The goal is for students to understand and not just solve. I can informally assess during the discussion of rewriting the text of the word problem, with more formal assessment later in the unit.

The next category to introduce is the *change-decrease* problem types. Following the same format as before, I will introduce the *result unknown, change unknown* and then *initial unknown.*

Crystal collected 7 leaves for her project.

2 leaves blew away in the breeze.

How many leaves does Crystal have left for her project?

Result Unknown	Change Unknown	Initial Unknown
7 - 2 = ?	7 - ? = 5	? - 2 = 5

Again, the goal is for students to understand and not just solve.

The third broad class, *compare,* is more difficult for my 2nd grade students. This requires the text of the word problems to be very straight-forward. Students should not get tangled up when they are learning to take the data from the problem. Remember that using the exact terminology is not the goal, but rather understanding what the problem is asking. Here are three ways I will present a scenario that shows the problem types *comparison-more*, and three ways to show *comparison-less.* Students need to be exposed to and have opportunities to practice all types. Of course, not all of these examples should be used at one time. As I write the problems out on chart paper and post them in the classroom, the students can begin to see and do their own comparing and contrasting as one scenario is explained in different ways. The use of the words “more” and “fewer” should be highlighted and explained as the problem set is introduced and worked on. My role here is to let the students begin to notice the subtle differences in the wording and how it changes the thinking. Simpler is better to start with!

Olivia has 4 more erasers than John. John has 2 erasers. How many erasers does Olivia have? *(comparison-more, greater unknown)* 2 + 4 = ?	John has 4 fewer erasers than Olivia. John has 2 erasers. How many erasers does Olivia have? *(comparison-less, greater unknown)* 4 + 2 = ?
Olivia has 4 more erasers than John. Olivia has 6 erasers. How many erasers does John have? *(comparison-more, smaller unknown)* 6 – 4 = ?	John has 4 fewer erasers than Olivia. Olivia has 6 erasers. How many erasers does John have? *(comparison-less, smaller unknown)* 6 – 4 = ?
Olivia has 6 erasers. John has 2 erasers. How many more erasers does Olivia have than John *(comparison-more, difference unknown* 6 – 2 = ? 2 + ? = 6	John has 2 erasers. Olivia has 6 erasers. How many fewer erasers does John have than Olivia? *(comparison-less, difference unknown)* 6 – 2 = ?

Throughout these introductory sessions, the students and I will brainstorm scenarios that can eventually be used in own word problems. Ideas should generate from school activities and materials, guiding students to think of what students can actually use for manipulatives or, as in the first scenario, be able to act out to solve. By keeping a list of ideas on chart paper as reference material, students won't struggle with vocabulary or appropriate scenarios; they will be on to the task of crafting their problems. This list will prepare the students for the second part of the unit.

Activity 2: Classroom / school scenarios

As stated earlier, words and vocabulary should be accessible to students and not a challenge or hurdle. The goal is to get to the thinking of the stories and plugging in the information that was gathered during the brainstorming session. To begin this portion, review the charts and add more if students have new ideas. It may be helpful for the purposes of composing word problems to have the information in categories, such as these:

Materials We Use

Classes We Attend

Activities at School

Classmate's Names

I will create groups of two or three students to have them write problems of their own to share with the class. Since this lesson requires students to read and write story problems, again, fluent readers and writers with those who are less fluent, provide counters for those who want them, and allow partners to work together to solve and problems and share the strategies that they used.

The goal during this period of time is to challenge students to write the same problem but try it another way, choose a different type as they tell the story. The timing for the student groups to work together will be during arrival time as morning work and during the math workshop portion of math instruction time. This will allow students to work as much as 30 minutes per day with their partners to create some math stories.

I will stress that it is important to keep their collection together as much of their work will become part of the workbook they will create at the end of this unit. Folders and math journals can be helpful, or my collecting the work-in-progress daily is another option.

Activity 3: Science Scenarios

The first unit in 2nd grade science is investigation and research on the life cycle of the butterfly. Students receive caterpillars at the start of the semester and observe and record the changes the caterpillar's life. The work that the students do during their science lessons can become the information and scenarios they can use for crafting word problems.

Using all different problem types, we will write several together as a class. This is an additional opportunity to integrate math very specifically into our science research and work. It is important for students to recognize that, although their learning has been compartmentalized into subject areas, it is essentially impossible to separate it all out into categories. So this portion of the unit uses math, science and reading to help students learn about the life cycle of a butterfly (and other animals as well).

Students will create problem sets that use their daily experiences tracking their caterpillars. Each student has 2-3 caterpillars to observe and record information on, which can become the start of word problems. Examples to start: If Table 3 has 8 caterpillars and 2 caterpillars join that group, how many caterpillars are being observed at Table 3? Here are 28 students in the class. Each student need one cup of caterpillar food. There are 30 cups of caterpillar food. How many more cups of food are there than students?

There are often students who have great interest in other areas of science. This is an area to encourage if students are excited about sharing their knowledge. Some students will be more inclined to use the unit of study going on in class, but throughout the literacy portion of the day, students are exposed to a great deal of non-fiction, or informational text, that could certainly enrich our science word problems.

Throughout the duration of the science unit, students will continue to write word problems of various types to eventually include in our final project, the workbook. These problems can be written during the morning work session, during math workshop, and at the end of science class. By the end of the unit, each student should have two problems to add to the Science chapter of the workbook.

Activity 4: Creation of Workbook / Publishing Celebration

The goal of this portion of the unit is to sort the word problems into “chapters” and create a workbook to share at the Publication Celebration. Chapters will be titled by subject or category, depending on student choice and teacher suggestion. Ideas include Beginning Stories, Classroom Activities, Playground Fun, Science & Math, and Social Studies Connections. Let students be creative with titles!

Students will submit their work which will include at least one word problem for each chapter. They must also submit the solutions to their problems so that they can be included in an answer key. Each chapter will have at least 25 problems, with examples of all types and with varying levels of difficulty. Word problems can either be typed or hand-written for the final workbook, depending on what the students decide as a class. One workbook per student will need to be copied and bound in some manner for the Celebration.

Two weeks before the Publication Celebration, students will create an invitation to give to their family and friends, inviting them to come for a “Celebration of Problem Solving.” Parents and other VIP guests will spend some time working on word problems, moving around the room, visiting many students. The students will share their own specific work with the guests (the word problems they themselves created) and “help” their visitors figure out the answers.

Each student will have a “Comments” sheet for guests to sign and leave comments on their experience working with the student. I will encourage visitors to stop to talk with each student or as many as they can during their visit.

Additionally, this is an opportunity to have some students work as editors and publishers. Creating the workbook will require review and assembly time and these tasks can be delegated and shared by the students who are interested.

Resources

Common Core State Standards for Mathematics,

http://www.corestandards.org/the-standards/mathematics.

Fong, Ho Kheong. *Math in Focus Singapore Math by Marshall Cavendish*. Final ed. Singapore: Marshall Cavendish Education, 2015.

Fuson, Karen C. *Math Expressions*. 2011 ed. Orlando, FL: Houghton Mifflin Harcourt ;, 2011.

Howe, Roger. *Three Pillars of First Grade Mathematics and Beyond*

Howe, Roger, *The Most Important Thing for Your Child to Learn about Arithmetic*

Howe, Roger and Harold Reiter, *The Five Stages of Place Value*

Ma, L. *Knowing and Teaching Elementary Mathematics*, Erlbaum Associates, Mahwah, NJ, 1999.

The Moscow Puzzles: 359 Mathematical Recreations. New York: Dover Publications, 1992.

Polya, George. *How to Solve It: A New Aspect of Mathematical Method*. New Princeton Science Library ed., 2014.

Appendix A: Problem Set

Change Increase / Result Unknown

7 students are in the classroom. 2 more students join them. How many students are in the classroom now?

Change Increase / Change Unknown

7 students are in the classroom. Some more students join them. Then there were 9 students. How many students join the first 7?

Change Increase / Initial Unknown

Some students are in the classroom. 2 more students joined them. Then there were 9 students. How many students were in the classroom at the beginning?

Change Decrease / Result Unknown

9 students were in the classroom. 2 students went home. How many students are in the classroom now?

Change Decrease / Change Unknown

9 students were in the classroom. Some students went home. Now there are 7 students. How many students went home?

Change Decrease / Initial Unknown

Some students were in the classroom. 2 students went home. Then there were 7 students in the classroom. How many students were in the classroom in the beginning?

Compare More / Difference Unknown

Sam has 10 French fries. Emily has 6 French fries. How many more does Sam have than Emily?

Compare More / Greater Unknown

Sam has 4 more French fries than Emily. Emily has 6 French fries. How many French fries does Sam have?

Compare More / Smaller Unknown

Sam has 4 more French fries than Emily. Sam has 10 French fries. How many French fries does Emily have?

Compare Fewer / Difference Unknown

Sam has 10 French fries. Emily has 6 French fries. How many fewer does Emily have than Sam?

Compare Fewer / Smaller Unknown

Emily has 4 fewer French fries than Sam. Sam has 10 French fries. How many French fries doe Emily have?

Compare Fewer / Greater Unknown

Emily has 4 fewer French fries than Sam. Emily has 6 French fries. How many French fries does Sam have?

Part-Part-Whole / Whole Unknown

Sam has 4 cookies for lunch. He has 2 more for dinner. How many cookies does Sam have?

Part-Part-Whole / Whole Unknown

Sam ate 6 cookies today. He had 4 of them for lunch. How many did he have for dinner?

Appendix B – Implementing Common Core Standards

This unit integrates, quite naturally, literacy and math. Both reading and writing are essential parts of the students' ability to solve word problems involving both addition and subtraction.

Students will work most specifically toward the Common Core State Standard in Mathematics, 2.OA.A.1 which states that second graders should, by the end of the year, be able to "represent and solve problems involving addition and subtraction within 100 to solve one- step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions. During this unit students will begin solving and crafting word problems with numbers to 10, advance to numbers to 20 and continue on

to the 100's and up to 1000 as they master place value concepts.

This unit also addresses the Language Arts Common Core State Standards of Reading Informational Text, RI.2.1, in which students work on locating key ideas and details by asking and answering such questions as who, what, where, when, why, and how to demonstrate understanding of key details in a text. Throughout this unit on word problems, students will be working asking these questions as they determine what information the math story is providing. As they begin to write their own word problems, they will need to consider these questions to craft a meaningful story for the text of their problem.

Notes

1. Common Core State Standards for Mathematics, http://www.corestandards.org/the-standards/mathematics.
2. Common Core State Standards for Mathematics
3. Ho Kheong Fong, *Math in Focus Singapore Math by Marshall Cavendish*, 8.
4. Roger Howe, *Three Pillars of First Grade Mathematics and Beyond,* 2.
5. Roger Howe, *Three Pillars of First Grade Mathematics and Beyond,* 1. Common Core State Standards for Mathematics
6. Roger Howe, *Three Pillars of First Grade Mathematics and Beyond,* 2