Curriculum Units by Fellows of the National Initiative
2015 Volume V: Problem Solving and the Common Core

Creating a Culture of Critical Thinkers in the Mathematics Classroom: Reducing Dependency on Key Words

Curriculum Unit 15.05.06, published September 2015
by Melissa Anderson

Introduction

Creating a culture of critical thinkers is a unit concerned with building confidence in the elementary student's ability to approach written math problems in an analytic, organized, and reasoned manner. Although this unit is intended for the use of third through fifth grade educators, the principle uses are applicable to a wide range of mathematics classrooms. It is the intention of this unit to prepare students to parse and discuss and solve many types of one and two-step, addition and subtraction problems. Through this process of exploring problem scenarios, it is my hope that students will begin to view themselves as expert problem solvers.

As used in my title, "Key Words" is a reference to a method of approaching a written problem wherein students memorize words associated with: addition, subtraction, multiplication, and division. Students are then asked to use those words to assign a computation, or multiple computations to a word problem. Although I have observed that this method occasionally satisfies the problem posed, it is my assessment that students should not become too dependent on it because it can often be misleading. As Roger Howe expresses in Three Pillars of First Grade Mathematics, and Beyond, "Addition is often described as combining, and subtraction as taking away, but the types of situations in which these operations are used are more varied than these brief descriptions would suggest. Mathematics educators have articulated a taxonomy of one-step addition and subtraction word problems."[1] A close study of the taxonomy of word problems as they progress sequentially through Kindergarten and on, advises that relying on key words after Second grade becomes very unhelpful. A popular educational blogger at *The Math Spot*[2] discusses this contentious issue in an entry entitled "Key Words Are Not the Key to Word Problems." She relies on Common Core progression charts in Kindergarten through Third grade to demonstrate her point. After studying the taxonomy of word problems in Kindergarten, she finds that key words within problems are somewhat helpful. She then examines key words used in the types of problems that First graders encounter. The blogger notes that beginning in First grade, the unknowns in a problem may appear in any given part of the expression and that the types of problems students are exposed to vary much more than in Kindergarten. She concludes that again, 'key words' are somewhat helpful but cannot be relied on consistently. The blogger then examines the usefulness of keywords in Second grade. She notes that students begin to encounter word problems with key words that are misleading enough for students to write the wrong expression for a given problem. Her sentiments are echoed by the authors of Teaching Student-Centered Mathematics when they express, "Instead of making meaningless rules, create opportunities for discussing adding and subtracting, using contextual situations."[3]

"Creating a culture of critical thinkers" refers to the style in which I will be approaching word problems in my classroom. The lessons will be anchored in classroom discussions and creating a structured and safe environment in which to explore mathematic possibilities as a whole class, in groups, and individually. Suggestions and tips for modeling the expectations and implementation of a student centered discussion will be a major component of my unit and will be elaborated on and referred to in the 'Strategies' portion of the unit.

Rationale

Demographics

I teach fourth grade at George Washington Carver Elementary School. It has recently become fully accredited, which means that our students have met state standards for academic success in the past several years. Our school is one of very few accredited schools within our district. Nearly 500 students populate our elementary school. Of the students that attend, 100% of them receive free or reduced priced lunches. There is very little diversity within the student makeup. Approximately 95% of students are African-American and fewer than 5% can be described as white or 'other', according to Richmond Public Schools.[4] Our school is referred to as a Title One School, which means that we receive federal funding in the hope that this money will help students meet state and national standards for education each year.

The school is situated in the heart of Richmond, Virginia, and resides within the historic Jackson Ward neighborhood. It is adjacent to the Virginia Museum of Fine Arts and the Science Museum of Virginia. Even with all of the history and opportunity for exploration that surrounds the school, students have not had many experiences outside of their neighborhood and classrooms. Because of this lack of experience, it can sometimes be difficult for students to bring context to our classroom work. Solving word problems can be particularly challenging for my students because the situations depicted in many word problems are unfamiliar to them. They are often unfamiliar with the vocabulary that appears in problems, unrelated to mathematics, as well. A lack of context, limited reading comprehension, and poor understanding of problem solving in general can often lead my students to rely heavily on guessing and looking for "key words" when attempting to solve word problems. The most worrisome issue I encounter in my classroom is the tendency of students to not see themselves as problem solvers. They have not yet developed strong reasoning skills and do not approach problems in a logical way. More often than not, students have been taught since Kindergarten to approach word problems by looking for the 'key words' within the problem. When this approach is unhelpful in satisfying the problem, students become frustrated and begin to form a negative opinion of mathematics. Many are able to solve purely numerical problems.

Example: 800 - 299 = ___

In this form, it is a relatively uncomplicated problem for students. However, when the same problem is expressed in word form, it is much more difficult for students. A good example of this is when the same problem is written as follows:

Syasia has set a goal to read 800 pages this month. If she has already read 299 pages, how many more will she need to read in order to meet her goal?

What I most often see students do is highlight the word “more” in a problem such as this and assume that they need to perform an addition operation. They will then proceed to string any number they can find in the problem, together and add, producing: 800 + 299 = 1,099.

Not only have they performed an incorrect computation: adding when they should have subtracted, but they have ended up with a number larger than the original goal of 800. Occasionally, students will realize their mistake by checking the reasonability of their answer. More often, students will see the incorrect computation as a choice on a standardized test and select it, validating their reliance on ‘key words’. They have not even realized they have selected an inappropriate choice.

Being able to solve word problems with a balanced approach is crucial for students in fourth grade. Not only are students expected to reason with whole numbers in written problems but are also expected to add and subtract fractions and decimals in the same way. Knowing that my students are in need of improving their problem-solving skills has inspired me to write a unit that will encourage and show students how to discuss math problems, specifically word problems, throughout fourth grade and beyond. It is my hope that by the end of this unit students will have a better relationship with mathematics and will begin to reason through problems on their own.

Content

Background

In the Commonwealth of Virginia, students are expected to solve single-step and multi-step addition and subtraction problems with whole numbers by the fourth grade. I have been teaching this content for two years in the same school and I have found it helpful to spend a long time on this standard, as word problems are a particularly difficult concept for my students to grasp. Typically, I spend approximately a week on each aspect of the standard. For example, during the first week of the unit I attempt to teach addition word problems in isolation. During the next phase of the unit, I would spend another week concerned with subtraction word problems. Through my research, I have concluded that it would be more conducive to student understanding of addition and subtraction to discuss addition and subtraction together, as they are linked by an inverse relationship. It is very important for students to recognize this connection.

This unit draws heavily on an understanding of the taxonomy of several types of word problems. More information on taxonomy of the specific problems I have chosen will be discussed under the following sub-headings, under Content: One-Step Problems and Two-Step Problems. It is also important to note that this unit is being taught with the assumption that students are already competent in multi-digit addition and subtraction computations. If students are also struggling with basic mechanical errors in their work, this unit may be overwhelming.

Also included is a set of word problems that will target a specific issue: misleading ‘key words’ in written math problems. These problem sets can be found in the appendix. To help students become more analytical, I have constructed a set of word problems that intentionally include misleading “key words” that may guide students to perform the wrong computation. For example, a problem will use the words “greater than” or “in all”, which students typically associate with addition, when in fact the problem requires subtraction. Another example would be when the words “less than” appear but it is necessary to add to find the correct answer, rather than subtract. These issues often appear in comparison problems, but ‘inconsistent’ key words can also be found in other types. What I hope students will conclude after closely studying these problems is that the “key word” approach that they often depend on becomes very unaccommodating as they encounter more rigorous problems of a wide variety, and beyond that, that a careful reading and analysis can lead to their desired results.

My wish is that, from a close examination of several types of problems, my students will gain an understanding of the relationships among variables in a problem. I would like them to focus more on analyzing problems rather than looking for specific words. What I mean by that is that instead of asking students to focus on “key words”, I would like them to focus on what relationships can be identified in each problem. For example, I want students to start asking them selves, “What part is missing in this problem?” or, “Am I comparing anything in this problem? If so, which part is greater, which is smaller, what is unidentified?” I do not think it is necessary to explicitly teach students the taxonomy of word problems. I believe it is more beneficial for students to create their own meanings and types by solving many different word problems and sorting them by their own definitions and categories. This will be elaborated on more in the “Activities” section. Below is a figure that discusses the taxonomy of one-step addition and subtraction problems. It provides an example of each type of problem. The scenarios that are discussed in my unit and presented in the appendix have been constructed to reflect the types of problems my students may encounter on standardized tests. I will discuss a few types of problems that appear in the chart in detail and give a brief explanation of the relationships I want students to identify. I have designed sets of problems with fixed variables and set scenarios in order for students to easily see the relationship among each variable in a problem.

General Taxonomy of One-Step Addition and Subtraction Word Problems

Figure 1

	RESULT UNKOWN	**CHANGE UNKOWN**	**START UNKOWN**
ADD TO	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 =?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 +? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 =5
TAKE FROM	Five apples were on the table. I ate two apples. How many apples are on the table now 5-2 =?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 –? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?? -2 = 3
	TOTAL UNKOWN	**ADDEND UNKOWN**	**BOTH ADDENDS UNKOWN**
PUT TOGETHER	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 =?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 +? = 5, 5-3 =?	Grandma has five flowers. How many can she put in the red vase and how many in her blue vase? 5 = 0 + 5, 5 + 0 5 = 1 +4, 5 = 4 +1 5 = 2 + 3, 5 = 3 + 2

COMPARE GREATER	**DIFFERENCE UNKOWN**	**BIGGER UNKOWN**	**SMALLER UNKOWN**
	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy 5 – 2 =? 2 +? = 5	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? 2 + 3 =?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have have? 5 – 3 =?, ? + 3 = 5
COMPARE FEWER	**DIFFERENCE UNKOWN**	**BIGGER UNKOWN**	**SMALLER UNKOWN**
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have then Julie? 5 – 2 =? 2 +? = 5	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 =? 3 + 2 = ?	? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 =? ? + 3 = 5

In each type (shown as a row), any one of the three quantities in the situation can be unknown, leading to the subtypes shown in each cell of the table. The table also shows some important language variants, which, while mathematically the same, require separate attention. Other descriptions of the situations may use somewhat different names. This chart is adapted from CCSS, p. 88, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33[5]

One-Step Problems

Add To

Add To problems involve an addition expression but to solve them may require subtraction. When referring to the 'Add to' category in figure 1, it can be noted that there are three types of problems within this category: Result Unknown, Change Unknown, and Start Unknown. I have chosen one scenario and a fixed set of variables to demonstrate the relationship between each type of problem. The scenario describes a girl's height and how much she grew in one summer. I have added an extra layer of difficulty by referring to some variables in one problem in inches and feet, and then using just inches in other problems. Students will have to recognize the relationship between the units in order to successfully solve these problems.

Result Unknown

Aleah was 39 inches tall at the end of the school year. Over the summer she grew 4 inches. How tall is she now?

In my classroom, I will start my unit by opening up a discussion of this type of problem, as students have typically encountered 'Add To, Result Unknown' problems by the fourth grade. All necessary information is given in this problem and I feel that my students will feel confident in their ability to solve this type. This particular problem is the most basic 'Add to' problem and can be solved by taking the initial height of '39' inches and then adding the '4' inches that Aleah grew over the summer. By combining the two known amounts, the 'Result Unknown' can be found: 39 + 4 = 43. After discussing a few problems of this type with students, I will facilitate a discussion wherein I ask students, “Can all word problems be solved by adding the numbers you find?” This is when I would move students into a slightly more difficult problem type: ‘Add to, Change Unknown’. Using the same scenario, I would ask students what they notice in the new problem and what information is needed to solve it.

Change Unknown

Aleah was 3'3" tall in the Spring. Now, in the Fall she is 43 inches tall. How much did she grow over the summer?

By slightly changing the ‘result unknown’ problem students are familiar with to a ‘change unknown’ problem, it is my hope that they will understand the relationship between the two word problems and readily identify the unknowns.

Initial Unknown

An ‘Add to, Initial Unknown’ problem can easily be created by changing the information given in the scenario and can be discussed much like the first two.

Example: Aleah is now 3’7” tall. and She grew by 4 inches over the summer. How tall was she in the spring?

Take From

After discussing ‘Add To’ problems with students, I would begin to present them with both ‘Add to’ and ‘Take from’ problems in order for them to explore the inverse relationship between the two operations. ‘Take from’ problems are expressed with a subtraction. When the ‘Take From’ category is studied in figure 1, it can be noted that there are three types. As with Add to problems, the three categories are: Result Unknown, Change Unknown, and Initial Unknown. To demonstrate the relationship among the three types of ‘Take From’ problems, I have chosen a new scenario with fixed variables to discuss. In this new scenario, a girl has a collection of desert plants that she wishes to share with her friend.

Result Unknown

There were 40 albums in Aniyah's collection. If she gave James 25, how many does she have left for herself?

This problem is the most basic of the ‘result unknown’ problems. The operation seems obvious enough to students that there will be little confusion regarding which should be used. To solve this problem and find the result unknown, the student will need to subtract 25 from 40. This should be expressed as 40 - 25 = 15, with the Result Unknown being 15.

Change Unknown

Aniyah now has 15 albums. She used to have 40 in her collection that she wanted to share with James. How many did she give to James?

By slightly changing the information given in this problem, a new problem can be examined and compared to the last. My goal is for students to understand the connection between the ‘Result Unknown’ and ‘Change Unknown’ through individual examination and whole class discussion.

Initial Unknown

Aniyah had a collection of albums. She gave 25 albums to James. Aniyah now has 15 albums left. How many albums did Aniyah have in her collection originally?

An initial unknown problem, although slightly more difficult for students, can easily be constructed by adjusting the given variables and feeding them into this third scenario problem. Students will be expected to compare this problem with other 'Take From' and 'Add to' problems through whole class discussion, and cooperative learning in pairs in groups. Ideas on how set up this cooperative learning and discussion will be discussed within the 'Strategies' heading.

Compare

Just as for change problems, compare problems can be divided into two main types and six subcategories in all. 'Compare Greater' refers to problems that use the term "more, higher, longer, greater, heavier, etc. 'Compare Fewer' refers to problems wherein the term "less, lower, smaller, lighter, shorter, etc." is used and the known value is smaller than the missing value. They can be described as problems wherein one quantity or unit is compared to another. The subcategories of Comparison type problems will be described below.

I plan to spend a longer amount of time discussing 'Compare' problems because I notice that my students struggle with this type more frequently. I believe it may be helpful to paraphrase the first few problems as change problems to help students understand the meaning of comparison language before presenting them with the compare type. The scenario in the Compare problems I have constructed describe a boy and a girl who are comparing the number of donuts they have. They are constructed in a way that shows the relationship between the larger number, the smaller number, and the difference between each value As an extra layer of complexity, I have used two different units to describe donuts: base ten wherein an amount of donuts is referred to, and base 12 wherein donuts are described in terms of dozens. These problems can technically be described as two-step problems. It will be necessary for students to know how to convert between the two units (dozens and single donuts) to solve the problems.

Compare Greater-Smaller Unknown

If Eden gets 2 dozen more donuts, she will have 38, just like Quan'ye. How many does she have now?

In this problem students will need to understand that a dozen refers to 12. If I feel that converting between units will be too taxing on students, I will chose to leave the two-step aspect out of the problems. For example, instead of using the term 'two-dozen', I would stick to explicitly stating how many donuts Eden had '24 more'. Given in the problem is how many more donuts Quan'ye has than Eden, which is 2 dozen more. We also know that Quan'ye has 38 donuts. In order to solve this problem the students will need to subtract rather than add to find the solution. This will be a key point in classroom discussion of this problem. Students who are conditioned to associate the word 'more' with addition will have a difficult time figuring out this problem. The solution to this problem can be expressed as 38 - 24 = 14.

Compare Greater- Difference Unknown

Quan'ye has 38 donuts and Eden has 14. How many more does Quan'ye have than Eden?

By changing the status of the three variables within the problem, a slightly different problem type can be created. Class discussion on the relationship between the 'Smaller Unknown' and 'Difference Unknown' will be held after introducing the two problems.

Compare Greater-Larger Unknown

Eden has 14 donuts. If Quan'ye has 2 dozen more, how many does Quan'ye have?

Changing the two variables whose values are given in the scenario created this third problem. Students will be asked to compare and contrast the three ‘Compare Greater’ problems and make a conclusion about the relationship among the three. By this point in the unit, it is expected that students will have a strong understanding of how class discussions work and will be expected to reason through their conclusion by using specific vocabulary and logical ‘language’. For example, their thoughts both verbally and on paper should be expressed something like this, “I know this is the answer because___________”, or “I know I need to subtract because__________.”

Compare Fewer-Smaller Unknown

Eden has 2 dozen fewer donuts than Quan'ye. If he has 38, how many does Eden have?

The given amounts in this problem are the difference (2 dozen fewer) and how many donuts Quan’ye has (38). In this case the word ‘fewer’ is consistent with the ‘key word’ method because it refers to the actual operation required: subtraction. For many of the problems presented, this is not the case. The solution to the problem can be represented numerically as 38 - 24 = 14

Compare Fewer- Difference Unknown

Eden has 14 donuts and Quan'ye has 38. How many fewer does Eden have?

By slightly changing the known variables from the ‘Smaller Unknown’ problem, a slightly more complex problem can be examined and discussed. Students will now be able to ‘compare and contrast’ both Compare Greater and Compare Fewer problems within the same scenario. Comparison of both types of ‘Compare’ problems will come after a very thorough discussion of both Change problems and ‘Compare Greater’ problems.

Compare Fewer- Larger Unknown

Eden has 2 dozen fewer donuts than Quan'ye. If she has 14 donuts how many does Quan’ye have?

Above is another example of how one scenario, with fixed variables, can be used to create and discuss many types of compare problems.

Put Together

This type of problem is often referred to as a Part-Part-Whole Problem. There are two types of ‘Put Together’ problems: Part Unknown and Whole Unknown. It should also be noted that the parts within the problem may be different. For example, if the problem is referring to a basket of apples, one part of the apples could be green and the rest red. Put Together problems are set up so that in one problem either the whole or one part will be missing. The order of the parts is irrelevant. I have created a new scenario with fixed variables wherein two boys are selling headphones. A boy is selling two different types of headphones for a fundraiser.

Part Unknown

Dylan is selling a total of 75 headphones for a fundraiser. If he has 19 'Beats' headphones to sell and the rest are iPhone brand, how many iPhone brand does he have to sell?

In this problem one part of the whole is missing, the whole being all headphones together. The parts we know are that there are a total of 75 headphones being sold, and that Dylan has 19 beats headphones to sell. In order to find the part unknown, students will need to subtract the part known (19) from the Whole (75). This can be expressed as 75 - 19 = 56. Possible aspects to discuss with students are the different parts of the problems, different units used, and if the units matter in finding a solution.

Whole Unknown

Dylan is selling beats and iPhone brand headphones for a fundraiser. Dylan has 19 'Beats' brand to sell and 56 iPhone brand to sell. How many headphones does he have to sell in all?

In this particular problem, both parts are known: Dylan’s Beats brand headphones (19) and Jeremiah’s 56 iPhone brand headphones. When added together, the whole can be found. This problem can now be expressed numerically as 19 + 56 = 75.

Two-Step Problems

Two-step word problems can usefully be described as being composed of two one-step problems. There are close to 200 possible combinations that can be made by choosing one-step problems and joining them. I have chosen a few to discuss to give examples of how one-step problems can be combined to create multistep problems. Although there are many varied types of problems, the goal of my unit is not for students to categorize problems by type, but rather to learn to analyze the relationships in each problem based on what they know about one-step problems. I will introduce Two-Step problems after I feel that a majority of my students are comfortable navigating and reasoning through one-step problems. I expect that this would be about two weeks into the unit.

Take From, Result Unknown and Take From, Result Unknown

There were 120 students on a bus headed home from school. At the first stop, 65 kids got off. At the second stop, 25 got off. How many are still on the bus?

A similarly basic two-step problem would take on the form: Add to, Result Unknown and Add to, Result Unknown.

I used this example to show a relatively simple way to combine two problems. This type of two-step problem may provide a sense of ease for students who are unfamiliar with multi-step problems. This problem type will be where I start the discussion of multi-step problems. In this case, two ‘take from’ problems with a ‘result unknown’ in each, are combined. In order for students to solve this problem they must take the initial amount (120) and subtract the first (65) kids who get off. Students can express this symbolically as 120 - 65 = 55. Now that students know the missing variable from the first problem, they are in a position to feed that value into the second part of the problem to find the second result unknown. This can be expressed as 55 - 25 = 30.

Add to, Result Unknown –Compare Greater, Difference Unknown

There were 55 fourth graders and 65 fifth graders in attendance on the first day of school. On the second day of school there were 10 fewer fourth and fifth graders combined at school. How many students were in attendance on the second day?

In order to solve this problem a student must combine the initial amount given (55) fourth graders and the change (65) fifth graders within the first step of the problem to find the combined total. This can be expressed as 55 +65 = 120. When students have solved the first problem, they are now ready to feed the variable they obtained: (120) into the second part of the problem. If students now know how many students were in attendance on the first day of school and also know that there were (10) less on the second day, they can find out how many were in attendance on day two with the expression 120 - 10 = 110. After solving this problem, it is my hope that students will draw the connection between the two problems and discover that one is needed to solve the other. It is also helpful if they can conclude that multi-step problems are made up mostly of the many types of problems we have already discussed over the past two weeks. This conclusion will provide students with a stronger confidence when reasoning through multi-step problems. I would now like to share a couple of multi-step problems involving comparison, as they seem to be particularly difficult for students to grasp, exceedingly so when more than two units are being compared.

Compare Greater, Smaller Unknown and Compare Fewer, Difference Unknown

Eden had 50 donuts, which was 30 more than Quan'ye had. If Tayquawn had 12 donuts, how many fewer did he have than Quan'ye?

Students must find the relationship between how many donuts Eden has and how many Quan'ye has, then apply that relationship to solve the second part of the problem. The solution to this can be expressed as 50 - 30 = 20. Now that students have obtained the amount of donuts Quan'ye has, they are set up to find how many more Quan'ye has than Tayquawn, who has 12. The representation of this would look like 20 - 12 = 8. We now know the difference between how many donuts Quan'ye has and how many Tayquawn has. Possible points of class discussion might be identifying the similarities between these two-step compare problems and the original one-step compare problems they have been introduced to.

Compare Greater, Smaller Unknown and Part-Part-Whole, Part, Unknown

Eden and Quan'ye ate a combined 25 donut holes at Sugar Shack in a donut-eating contest. Eden chose to eat sprinkle donut holes, and Quan'ye ate chocolate ones. If Eden ate 15 sprinkle donuts holes on her own, how many more sprinkle donut holes did she eat than Quan'ye ate chocolate ones?

In this problem, the numbers of donut holes that each contestant eats are being compared. There is also a different section of the problem that asks the problem solver to describe the missing part, in a 'part-part-whole' problem. The solution to the first part can be expressed as 25 - 15 = 10 and then the difference between the parts is 25 – 10 = 25.

Add To, Result Unknown, Add to, Result Unknown

Ton'ye had 12 Beyoncé songs on her iPod. She got 10 more Beyoncé songs at her birthday party in the afternoon. Then her father gave her 9 at dinner that evening.

How many Beyoncé songs did Ton’ye have then?

This problem is two-step because the reader is given the initial amount (12) and asked to add (10) more to find the first result of (22): 12 + 10 = 22. After finding the first ‘result unknown’, the student can plug the value of (22) into the next part of the problem in order to find a solution. Now, the student must add the (9) songs given after dinner to the initial amount (22) and combine them to find the final result unknown. 22 + 9 = 31

Add to, Initial Unknown, Add to, Initial Unknown

Ton’ye had some Beyoncé songs on her iPod. She received 10 more Beyoncé songs at her birthday that afternoon and 9 from her father at dinner. Now she has 31 Beyoncé songs. How many did she have before?

This is a variation of the ‘Add To, Result Unknown, Add To, Result Unknown’ problem discussed above. By slightly changing the given variables, a new problem was created in relation to the first. Now, the students will have to figure out how many songs Ton’ye initially had on her iPod. I hope students will easily draw a conclusion about the connection between problems.

Add to, Result unknown, Add to, Change Unknown

Ton’ye had 12 Beyoncé songs on her iPod. She received 10 more Beyoncé songs at her birthday party in the afternoon. Then her father gave her more Beyoncé songs at dinner that evening. Then she had 31 Beyoncé songs. How many songs did her father give her?

Similar to the first two examples, this version has been created using the same number values and subject. To solve this problem, students have already seen that to find the ‘Result Unknown’ in the beginning, they will express a problem 12 + 10 = 22. Students are now in a good position to find the ‘Change Unknown by taking the total of 31 songs and subtracting the 22 that are known. Although students are subtracting, the problem can be expressed as 22 + 9 = 31.

Add to, Change Unknown, Change Plus, Start Unknown

Ton’ye had 12 Beyoncé songs on her iPod. She received more Beyoncé songs at her birthday party in the afternoon. Then her father gave her 9 more songs at dinner that evening. Then she had 31 Beyoncé songs. How many songs did she get at her party?

There are a couple ways students can solve this problem. Starting from the last part of the problem and working their way back through the given information is one way to do this. The student can take the final value of (31) and subtract (9) songs that Ton’ye received at dinner to find the ‘Change Unknown’ of the first problem. That looks like this 31 - 9 = 22. Now that students know how many songs Ton’ye had before dinner (22) and how many she had originally, the student can now find out how many she received at her party with the expression 12 + 10 = 22.

Add To, Result Unknown, Compare, Difference Unknown

Ton’ye had 12 Beyoncé songs. Shantoya has 19 Beyoncé songs. Then Ton’ye got 10 more Beyoncé songs on her birthday. How many more songs does Ton’ye have than Shantoya?

This problem is similar to the first couple in this scenario, but now the students must compare songs Ton’ye had to the number of songs Shantoya had. This can be done by calculating how many total songs Ton'ye had 12 + 10 = 22. Now that the total amount of songs Ton'ye has is known, it can be compared to the amount of songs Shantoya had. This is expressed as 22 - 19 = 3. A related compare problem can be constructed by concealing how many total Beyoncé songs Shantoya had and providing the information about the total songs Ton'ye had. That problem is described below.

Add To, Start Unknown, Compare More, Larger Unknown

Ton'ye had some Beyoncé songs. Shantoya had 7 more Beyoncé songs than Shantoya. Then, Ton'ye received 10 Beyoncé songs for her birthday. Now she has 22 songs.

How many songs does Shantoya have?

Three-Step problems will be discussed as an extension to the unit if I feel confident that students can navigate one and two-step problems with certainty. Essentially, Three-Step Problems involve joining several types of one-step problems, much like in two-step problems.

Progression of Unit

I will divide my unit into two distinct phases: One-step and Two-Step. I will introduce one-step problems first because they are relatively simple and most students will be familiar with them. I plan to spend two weeks presenting as many as possible one-step problem types using a variety of scenarios. Students will thoroughly examine the many types of problems through discussions, drawings, categorizing, and creating their own one-step problems. However, students will not be expected to categorize the types of problems according to a taxonomy chart. Instead they will categorize problems by their own definitions, after carefully studying many types. Two-Step problems will be introduced in much the same way in the following week and two more weeks will be dedicated to solving multi-step problems.

Below is a sketch of the progression of my unit.

Week One and Two

The first week will be spent finding out what students know about solving written math problems and getting a feel for each student approaches word problems. Students will also begin discussion one-step problem types. Students will be presented with a one-step problem with single-digit values. I will give students a blank piece of paper and observe how they approach the problem individually. I will then collect the work and use it as an initial assessment of what students know about problem solving. This week will also be used to establish the expectations for a classroom discussion. The following guidelines will be established: exhibiting respect by talking only when it is their turn, reacting positively and politely to responses they do not agree with, etc. Ideas on how to set up these discussions will be provided below under 'Methodology' and 'Whole Class Discussions'. During the second week students will begin discussing part-part-whole and comparison problems. Students will be asked to compare all three problem-types. They will be asked to discuss what is different about each, what is the same, etc. After thorough discussion, I plan to assign specific terminology to use when talking about problem types. For example, they may label problems "addition problems", "subtraction problems", "comparing". It is not necessary to ask students to memorize these problem types or use the language expressed in figure 1. The specific terminology chosen will be used solely to make discussion and sorting more accessible for students.

Week Three and Four

After I observe that a majority of students can reason through one-step problems and present their solutions logically, I will begin discussion of two-step problems in the same way problems were discussed in week one and two. Students will hold discussions; sort problems, and draw conclusions about what they observe about problems. During the progression of this unit, students will be asked to create several of their own one and two-step problems that will be published upon the conclusion of our word problem unit. More details about this project can be found under the "Activities" heading.

Strategies

Facilitating Class Discussion

Often, I find that during whole-class discussions around two-thirds of students are engaged at any given time. My goal is to hold meaningful discussions in my classroom wherein all of my students are actively participating and contributing. Through my research I have found a great reference for opening up effective class discussions. A model in which every student is active is referred to 'Total Participation' by authors Himmele and Himmele.[6] In their publication of Total Participation Techniques: Making Every Student an Active Learner, they have articulated several distinct categories for garnering student engagement. Within each category, they have described exercises for teachers to implement in their own classrooms. I will describe a few I plan to use during this unit. The categories I will reference are "On-the- Spots", "Hold Ups", and "Movement". The following are methods as described in Himmele and Himmele's publication and are paraphrased.

On-the-Spots

This category is intended for use when the teacher notices that a majority of students are becoming lethargic or disengaged from the class discussion.

Quick Draw: The teacher selects a concept students have been discussing. Then, students create a visual about what the concept means to them. The students then share with partners and the whole class.

Chalkboard Splash: During this activity, the teacher will write a question on the board, and then students will 'splash' the board with their responses. When students have recorded their thoughts, call students up to the board to make connections between responses and discuss the similarities and differences in their thinking.

Hold-Ups

These strategies should be used to promote whole class participation and on-task behaviors during a class discussion.

Whiteboard Response: Students can use paper, whiteboards, thumbs etc. to share their responses to questions at the same time. If students are positioned in a precise way, only the teacher will get to view their response. When used in my classroom, it has helped more quiet and shy students to contribute to discussion.

Number Card Hold-Up: To use this method, provide students with index sized cards labeled (0-9).

Students will use the numbers either individually or together to share their responses. Himmele and Himmele suggest using the following questions to garner responses:

Which number is greater? Which of these numbers is the least in value? What is the sum of these two numbers? What is the difference in these two numbers? What is the product of these two numbers? Himmele, Pérsida; Himmele, William (2011-07-21).

Involving Movement

Adding movement to class discussion is intended to give students more energy and get them excited about the topic being discussed.

Line-Ups: After posing a question to students, the teacher will ask students to jot down their thoughts and responses on a piece of paper. Students are called to line up in two parallel lines facing each other. This will create an opportunity for students to pair up with the person directly in front of them. After these two partners have shared their responses, the teacher can either ask students to share out what they and their partner discussed or to rotate and discuss further with a new partner.

Bounce Cards: In this exercise, students are presented with a bounce card. A template for a 'Bounce Card' can be found in the appendix. 'Bouncing' in this activity refers to taking a person's idea and bouncing off it or 'piggy backing' on it. Guiding questions will be provided on the each card for student use. Students will use the cards to talk about what they agree on, what they disagree on, to make connections, and to summarize their partner's ideas. I will model this activity regularly and very explicitly to students.

Methodology

The problem-solving model my students will use is referred to as the 'UPS, Check' method. The acronym stands for "Understand, Plan, Solve, Check." This is a common approach to problem solving that is drawn from George Polya's method for problem solving in his 1945 publication of How to Solve It. In his book, Polya identifies four basic principles for problem solving, " Principle 1: Understand the Problem, Principle 2: Devise a Plan, Principle 3: Carry out the plan, Principle 4: Look Back."[7] A Graphic organizer for this method will be provided in the appendix.

Understanding: I will explicitly model this step throughout the unit as this step is commonly skipped and overlooked by my students. I would like them to be conscious that 'Understanding' in the context of a word problem refers to the following:

1) Comprehending the words in the problem.

2) Identifying the ‘given variables’ and the ‘unknowns’ in a problem.

3) Determining what information is needed to solve the problem.

4) Restating the question in their own words.

Planning: This will be modeled regularly for students. They will learn that planning must be done for every problem and that it will involve all or a few of the following elements:

1) Making lists.

2) Eliminating unreasonable answers.

3) Using a model/Drawing a picture.

Solving: Students will also review what happens in this part of the problem-solving process. This can look different for each student. To solve problems, students will be asked to:

1) Persevere through the problem solving process, even when situations are tricky, meticulous, and difficult.

2) Write an expression for their problem.

3) Show work and be able to walk someone else through the way they solved the problem.

Checking: Students often circle an answer when they have computed a number without considering if their choice makes sense. There will be a huge emphasis on this step in my classroom, as I want students to reason through their problem solving both verbally and on paper. This step will involve:

1) Looking for technical errors within their work.

2) Rereading the question to check for the reasonability of their answer.

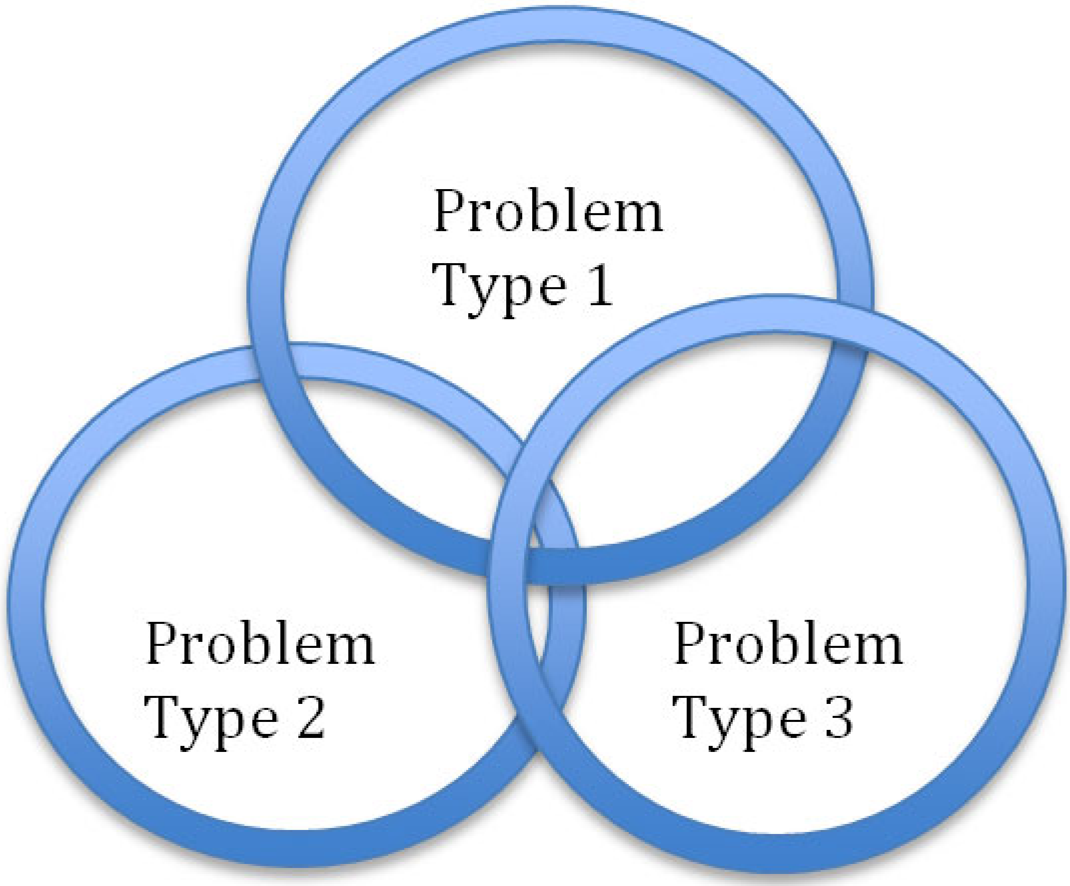
3) Explaining to others how they arrived at their answer.

Activities

Activity 1: Discussing Problems using Graphic Organizers

Throughout this unit, I will be using several graphic organizers to discuss word problems. The use of these graphic organizers could possibly be used for a few specific student activities or implemented on a regular basis.

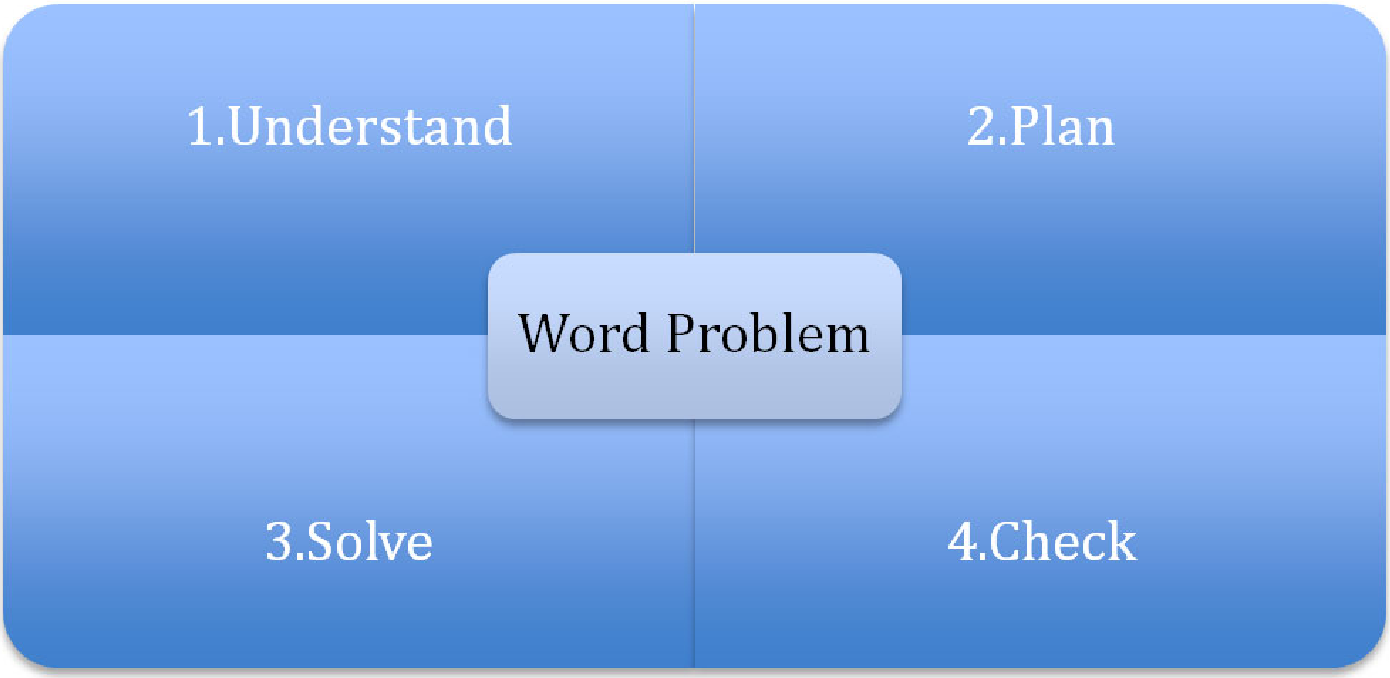
Venn Diagrams: Students can use this classic organizer to compare and contrast problem types they encounter. I plan to use a Venn diagram that includes three circles, rather than one. It can be used in the traditional way, or students can also place problems they feel cannot be qualified on the outside of the circles. An example of how to arrange circles or build an image for student use is below. I have purposely left the headings blank because I would like students to define their own titles based on what they observe about problems.



Problems that students do not feel belong within the confines of the Venn diagram can be placed on the outside.

UPS, Check Problem Solving Boards:

Students will be expected to use this format for problem solving for all written problems they encounter throughout the school year. It is my hope that by the end of the school year, they will be able to draw their own problem solving boards. This diagram is directly related to the methodology I discussed above. An example of how to organize and layout the graphic is provided below and a blank template for students will appear in the appendix.



Activity 2: Sorting Problems

Students will sort problems many times throughout this unit. I plan to use a Venn diagram to help students sort word problems by the types they have already identified. Students will be divided into small groups consisting of 3-5 students. Each group will be given a bag that contains word problems written on thin strips of paper. In order to differentiate this activity, some bags will contain only one-step problems, some will contain both one and two-step problems. As a group, students will read through the problems and categorize them in as many groups as they can, based on qualities they find within problems that are alike, different, or that do not fit into a category at all. At this stage students will only sort problems by their own definitions. As the unit progresses, and student understanding of word problems begins to develop, it may be appropriate to ask students to use specific terminology to categorize word problems but not necessarily the terminology in figure 1.

Activity 3: Word Problem Project

This project will be worked on throughout the entire unit. In small groups, students will be guided in writing their own one and two-step addition and subtraction word problems. After conferencing several times with the teacher and students in their groups, students will get to publish their word problems for the class to view. Students will be able to publish in the format of their choice. The guide must include the following: a guide for solving word problems (tips and suggestions for solving problems), a graphic organizer, and at least ten word problems.

Appendix A: Collection of Word Problems

One-Step Addition and Subtraction Word Problems

1. Myona collected 75 'One Direction' stickers. She gave Ton'ye 36 of her stickers. How many 'One Direction' stickers does Myona now have?
2. Myona gave Ton'ye 36 'One Direction' stickers. Now she has 45. How many stickers did Ton'ye have before?
3. Ton'ye has 19 'Justin Bieber' stickers. Then she buys 18 more 'Justin Bieber' stickers. How many stickers does Ton'ye have now?
4. Ton'ye has 37 'Justin Bieber' stickers and 36 'One Direction' stickers. How many stickers does Ton'ye have?
5. James' class has 4 fewer students than Qualee's class. James' class has 26 students. How many students are in Qualee's class?
6. 6. Since the first day of school, 2 students have left Qualee's class. Qualees's class now has 28 students. How many students were in Qualee's class on the first day of school?
7. 17 students in Mr. Freeman's class buy pretzels on the field trip to the Siegel Center. In Ms. Anderson's class, 9 more students buy pretzels than in Mr. Freeman's class. How many students in Ms. Anderson's class buy pretzels?
8. 8.There were 32 students in Mr. Osborne's class eating lunch. Then, students from Ms. Davis and Ms. Anderson's class joined them. Now there are 97 total students eating lunch. How many students combined from Ms. Davis and Ms. Anderson's class joined Mr. Osborne's class?
9. There were 67 'Panda Pack' bagged lunches at the start of lunch. After the fourth graders ate some, 39 'Panda Packs' remained. How many 'Panda Packs were eaten by the fourth graders?
10. At the 'Father Daughter Dance', 59 students bought ice cream sundaes and 39 students bought popcorn. How many more students bought ice-cream sundaes than popcorn?
11. Cameron's book has 38 more pages than Jeremiah's book. There are 49 pages in Jeremiah's book. How many pages are in Cameron's book?
12. There are 17 oak trees on Carver's playground. Volunteers from Carverponics will plant 29 poplar trees today. How many more poplar trees than oak trees will there be on the playground?
13. 13. Marlin had 49 dimes and 45 pennies in his bank. Valencia gave him 38 pennies and 11 quarters. How many pennies does Marlin have now?
14. Tayquawn had 29 Pokémon cards. Aniyah has 17 more than Tayquawn. How many Pokémon cards does she have?
15. It was a rainy summer week. To pass the time, Dylan read 13 books. He read 6 magazines, 3 novels, and ___ comic books. How many comic books did he read?

Two-Step Addition and Subtraction Word Problems

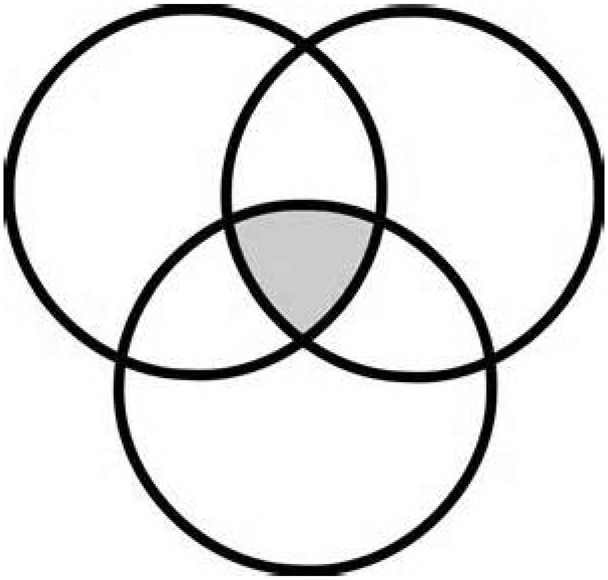
1. Dylan read 36 pages from Diary of a Wimpy Kid on Monday morning. He read 24 pages from the same book on Monday afternoon. After dinner he read 15 more pages from Diary of a Wimpy Kid. How many pages did Dylan read on Monday?
2. Dylan read some pages from Diary of a Wimpy Kid on Monday morning. He read 24 more pages on the same afternoon. Then, after dinner he read 15 more. If he read a total of 75 pages on Monday, how many pages did he read in the morning?
3. Dylan read 36 pages from Diary of a Wimpy Kid on Monday morning. He read some pages in the

afternoon. After dinner Dylan read 15 pages from the 'Wimpy Kid' book. If he read 75 pages on Monday, how many of the pages were read in the afternoon?

4. 4. Dylan read 36 pages from his 'Wimpy Kid' book on Monday morning. In the afternoon he read 24 pages. He also read some pages after dinner. If he read 75 pages from Diary of a Wimpy Kid on Monday, how many of those pages were read after dinner?
5. By Monday afternoon, Dylan had read 60 pages from Diary of a Wimpy Kid. On the same day his friend James had read 55 pages by the afternoon and put the book back on the shelf. After dinner Dylan read 15 more pages. How many more pages did Dylan read on Monday than James did?
6. On Monday morning Dylan read some pages from his 'Wimpy Kid' book. His friend James read 20 pages fewer than he did. Then, Dylan read 39 more pages during the course of the day. If Dylan has now read 75 pages, how many has James read?
7. On Monday, Dylan read 75 pages. James read 20 pages fewer than Dylan. James read 15 fewer pages than Quan'ye. How many pages did Quan'ye read?
8. When I woke up, it was 60 degrees. During the day, the temperature increased by 30 degrees and then it began to decrease. When I went to bed, it was 50 degrees. What was the total amount of temperature change during the day?
9. The temperature from morning to noon increased by 30 degrees and slowly began to decrease. Near bedtime, the temperature was 50 degrees. If there was a total fluctuation in temperature of 70 degrees that day, what did the temperature read in the morning?
10. For the school dance, Cameron's mom baked 96 cupcakes. If she sold 36 of them in the morning, and baked 24 more for the afternoon, how many cupcakes would she have?
11. On Saturday morning, Sweet Frog Frozen Yogurt had 26 toppings to choose from. By the afternoon, they ran out of 15 of the toppings. The storeowner sent an employee to retrieve 20 more toppings for the yogurt store. How many toppings was the store stocked with when the employee returned?
12. At the Virginia State Fair there are 59 fair goers in line for cotton candy. A family of 8 grew restless and stepped out of line. A Boy Scout troop of 12 members, stepped into the cotton candy line when they saw the family leave. How many fair goers are now in line for cotton candy?
13. There are 63 folks standing in line for cotton candy at the fair. There are 16 fewer fair goers in line for the bumper cars. There are 9 fewer folks in line for the Ferris wheel than in line for bumper cars. How many folks are in line for the bumper cars?
14. There are 38 fair goers in line for the bumper cars. There are 9 more people waiting in line for the Ferris wheel than there are people in line for the bumper cars. If there are 16 more people waiting in line for cotton candy than the bumper cars, how many are in line for cotton candy?
15. A waiter at Ihop had 21 customers he was waiting on. If 10 of his customers left when they were through with breakfast and 23 more customers arrived, how many customers would he be waiting on?

Appendix B: Templates for Graphic Organizers

Venn Diagram



Bounce Card

Below is an example of questions/tasks that appear on a ‘Bounce Card’ created by Himmele and Himmele in Total Participation Techniques: Making Every Student an Active Learner

Bounce

Take what your classmate(s) said and bounce an idea off of it. For example, you can start your sentences with—

"That reminds me of ..."
"I agree, because ..."
"True. Another example is when ..."
"That's a great point ..."

Sum it up: Rephrase what was just said in a shorter way.

Appendix C: Implementing District Standards

In Virginia, students are expected to solve single-step and multi-step addition and subtraction problems with whole numbers by the fourth grade. This unit addresses the following Virginia Standard of Learning in mathematics.[8]

4.4 The student will solve single-step and multistep addition, subtraction, and multiplication problems with whole numbers.

Bibliography

A., Van De Walle John, Karen S. Karp, LouAnn H. Lovin, and Jennifer M. Bay-Williams. "Chapter 9." In *Teaching Student-centered Mathematics*.

"About RPS." About RPS. Accessed July 15, 2015. http://web.richmond.k12.va.us/AboutRPS.aspx.

Himmele, Pérsida, and William Himmele. *Total Participation Techniques: Making Every Student an Active Learner*. Alexandria, VA: ASCD, 2011.

Howe, Roger E. *Three Pillars of First Grade Mathematics, and Beyond*.

"Keywords Are NOT the Key to Word Problems." *The Math Spot* (blog), December 6, 2014. Accessed June 12, 2015. http://polkadotsandteachingtots.blogspot.com.

Pólya, George. *How to Solve It; a New Aspect of Mathematical Method*. Garden City, NY: Doubleday, 1957.

"Preparing America's Students for Success." Home. Accessed July 20, 2015. http://www.corestandards.org/.

"Virginia Department of Education." VDOE. Accessed July 31, 2015. http://www.doe.virginia.gov/.

Notes

1. Howe, Roger E. *Three Pillars of First Grade Mathematics, and Beyond*.
2. "Keywords Are NOT the Key to Word Problems." *The Math Spot* (blog), December 6, 2014. Accessed June 12, 2015. http://polkadotsandteachingtots.blogspot.com.
3. A., Van De Walle John, Karen S. Karp, LouAnn H. Lovin, and Jennifer M. Bay-Williams. "Chapter 9." In *Teaching Student-centered Mathematics*.
4. "About RPS." About RPS. Accessed July 15, 2015. http://web.richmond.k12.va.us/AboutRPS.aspx.
5. "Preparing America's Students for Success." Home. Accessed July 20, 2015. http://www.corestandards.org/.
6. Himmele, Pérsida, and William Himmele. *Total Participation Techniques: Making Every Student an Active Learner*. Alexandria, VA: ASCD, 2011.
7. Pólya, George. *How to Solve It; a New Aspect of Mathematical Method*. Garden City, NY: Doubleday, 1957.
8. "Virginia Department of Education." VDOE. Accessed July 31, 2015. http://www.doe.virginia.gov/.