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Fractions: Building a Foundation through Conceptual Understanding and Problem Solving

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Introduction

In school math was always a difficult subject for me. My teachers would teach the formula but I never understood the 'why' behind the process. For me as a student, this type of instruction led to confusion and misconceptions. It turned me into the kind of student who tried to memorize the procedures being presented by the teacher. It was not until college that I met a professor who taught about drawing pictures and representations to help aid in understanding various math concepts for students in elementary school. He encouraged us to always come up with various ways to represent situations because in a classroom you will need to think about all of your learners. This way of thinking changed my opinion about math and how it can be taught in the classroom.

Some of my own struggles as a student also plague my students every year. More specifically, I see the struggle students have when we are working with fractions. When we start the concept in class it is like they have never seen a fraction, when in reality they deal with fractions in their everyday lives. For example, fractions can be seen when they share a pizza with their families, when they are baking in the kitchen with their parent, and even when they look at a clock and tell the time. Their inability to connect their experiences to fractions demonstrates that students don't understand how fractions work. Through my research I have learned that fractions involve complex ideas and they require a lot of attention. I have realized that I need to spend more time developing conceptual understanding before moving into procedures. Students need time to manipulate and construct visuals that allow them to understand what a fraction represents. Once students have the understanding they can start to use their knowledge to help solve problems in context.

Rationale

I am a fourth grade teacher at John M. Clayton Elementary School. My school is located in the small town of Dagsboro, Delaware. It is one of eight elementary schools in the Indian River School District. The diversity of almost 600 students in this school is what makes it different from many other elementary schools in the district. John M Clayton Elementary is a Title One School, with 81% of its students receiving free and reduced lunch. The students' diversity can be seen with the following numbers: 1% American Indian, 1% Asian, 23% African American, 44% Hispanic or Latino, and 31% White. There is also a 23% English Language Learner (ELL) population which requires extra services to bridge the language barriers. We are also working with a 21% population of students with an Individualized Education Plan (IEP). With teaching such a diverse group of students I must provide instruction for all of the varying abilities. There are several challenges I face daily in a classroom with these students. As I plan my instruction I have to consider many factors such as, what information they already have about this content, do they have background information on this material, how can I make this relevant to their life so they can be engaged in the learning. With this diversity, my students bring in a range of preparation and aptitude. This requires me as the teacher to be aware of their individual needs.

For the first time this year, our district administered the Smarter Balanced Assessment (SBA) to our students. This new assessment goes well beyond the typical multiple choice assessments that had been given in the past. There are two new components to this assessment. First, students are now being asked to answer extended response questions. Therefore, they can no longer just guess and click on an answer. They need to explain their thinking in a clear and concise manner. The performance task makes up the second component. These tasks challenge students to apply their knowledge to solve complex real-world problems. With the addition of these components the assessment can better measure the student's depth of understanding and ability to apply their knowledge in various situations. This change in assessment has caused teachers in our school to reflect on their practices and realize that our students need to become better critical thinkers and problem solvers. Moving forward I know that I need to focus on conceptual development with my students in all areas of study. I will be able to assess their level of understanding by the way they can apply their knowledge to various situations and problems presented in class.

To better address the CCSS and become better prepared for the SBA my district will be adopting a new math curriculum. Over time this new curriculum will create a bridge across grades K-5. This will help teachers understand what strategies and vocabulary have been taught in the prior year. With my focus on fractions, I looked closely at the new curriculum to determine exactly what my students need to know in order to succeed with it. The district's fraction unit is divided into four different modules. The first module begins with equivalent fractions, which requires the ability to understand how to rename a fraction. Many of my students will struggle through this first module because they don't have the conceptual understanding necessary to understand that two-fourths is equal to four-eighths. I am sure I can teach them a quick algorithm but my students will not understand that process nor will they be able to apply it to other situations. Therefore, my unit will act as a springboard into this first module. In my unit, students will be given the time to really focus on what a fraction means and how to represent one using various models. Having students take this time to develop understanding will allow them to see more success when they start the regular curriculum.

The CCSS places a huge emphasis on the student's ability to understand and apply their knowledge about fractions. Students struggle with fractions every year because they have not been given enough time to develop that understanding. Students need more exposure to various models when learning about fractions. This understanding is built around the development of a unit fraction. It is important that the student understands that the denominator tells how many copies of the unit fraction it takes to make the whole. For example, the unit fraction 1/3 represents a quantity of which it takes 3 to make the whole. They need time to analyze how fractional amounts can be represented in varying ways through the use of various models. For this unit students will only be working with fractions that have a fixed denominator. The purpose of my unit is to provide students with the opportunities to develop their knowledge of fractions through the use of manipulatives and models. They will have exposure to fractions with different denominators as they continue their work in the district curriculum.

I also want this unit to help my students become problem solvers. They will be expected to write their own problems and discuss the process they use to solve each one. I want them to be able to apply this knowledge to various contexts and demonstrate an understanding through the use of pictures and words. From my time in seminar, I have learned the importance of problem solving in math. If a student can read, write, and speak about a concept then you can assess what they understand. In my unit I want the students to use their understanding of fractions to solve word problems, and also be able to write their own.

This unit is not intended to replace the curriculum provided by my district. I want to teach this unit to my students before they begin their studies of fractions in the district curriculum. Taking this extra time will help my students build that deeper understanding of fractions so they can better access the regular curriculum.

Content Objectives

As stated in the Common Core State Standards, one of the critical areas of focus in the fourth grade is fractions. Students are expected to be able to understand how fractions are built from unit fractions and they need to be able to compose and decompose fractions in terms of unit fractions. These understandings then need to be applied to various operations. The objectives of my curriculum unit are to develop the student's conceptual understanding of fractions. With this knowledge I want them to apply it to word problems and be able to clearly articulate and represent the process of adding and subtracting fractions with a fixed denominator.

The "Unit"

The "unit" is an essential understanding in fractions. When working with fractions, it is always important to identify what the unit is. The fraction tells the size of another quantity in relation to the unit. For example, if there are 24 bottles of soda in a case and the unit is one case, then a 6 pack represents one fourth. However, if the unit is one bottle then a 6 pack of sodas represents 6. Another example could be used with time. For example, if the unit is an hour, then a day represents 24. If the unit is changed to a week then a day represents 1/7. Other measurements such as inches/feet/yards and centimeter/meter require students to understand the size of the unit being looked at. The unit allows us to know how to express the relationship between two quantities. These examples show how it is important for the student to identify the unit in the problem. Then numbers are used to express other quantities relative to the unit. Being able to identify the unit and recognize how other quantities are related to it is essential to understanding the meaning of fractions.

Unit Fraction

A unit fraction is any fraction that has a numerator of one. If any whole is broken into *b* equal parts, and then each part equals the unit fraction 1/*b*. The fractions one half, one third, and one fourth are all unit fractions. The denominators of these fractions tell the size of the unit. For example, the denominator of three in one-third tells us that it takes 3 unit fractions to make the whole. Students need opportunities to build this understanding through the use of manipulatives and visual models. Through these practices they need to be able to identify the unit, which continues to build their understanding of the whole.

General Fraction

A general fraction represents multiple copies of the unit fraction. This means that if I have three copies of, say 1/5, then I would have three fifths, expressed symbolically as 3/5. Students need to understand how to look at a general fraction as one number, that expresses the size relation between the fractional amount and the whole. Through the process of decomposing, students will develop the understanding that general fractions are composed of unit fractions.

Adding and Subtracting Fraction with Like Denominators

Adding and subtracting fractions requires the understanding of joining and separating pieces of a whole. In this unit we will only be looking at fractions with a common denominator. As the standard states, students should also be able to use visual models and equations to represent these problems. Student's previous work with equations to solve problems involving whole numbers is applies as they work with equations involving fractions and mixed numbers.

In this unit I will be creating a bank of problems from a scenario that kids would relate to and then change how I use the numbers in each set of problems. I want students to become more comfortable analyzing the word problem and discussing the process they used to solve it. Through this process I want the students to use the concrete and pictorial representations they use when working with fractions. By the end of the unit I want the students to have the ability to write their own word problems.

Background Information

Models of Fractions

Learning about fractions requires students to think about numbers in a different way than when they work with whole numbers. Students have to understand the relative nature of fractions. That is, fractions express the size of a quantity in relation to a unit, which is given or understood in the context. For example, the same fraction of different units will be different. To get ½ of one egg involves breaking the egg; but ½ of one dozen eggs is 6 whole eggs.

The fact that the larger denominator indicates the smaller piece contradicts what students are used to with whole numbers. For example, 5 is greater than 3, but 1/5 of a cake is less than 1/3 of the same cake. I will give students work comparing unit fractions until I am satisfied that they have absorbed this important but not immediately intuitive fact.

To help students develop an understanding about a fraction they need to be given opportunities to see fractions represented in various models. Through these various representations they will start to understand the relationship between the numerator and the denominator.

Area Models

The relationship between the unit fraction and the whole determines how to draw an area model. The use of circular pie pieces, geoboards, paper folding, pattern blocks, and drawings on grids are all good examples of area models. Using these models allows students to partition, or divide, the whole into equal parts. The rectangle below is an area model representing the fraction 1/3 by the shaded region. The denominator tells us that it takes three unit fractions to make the whole.

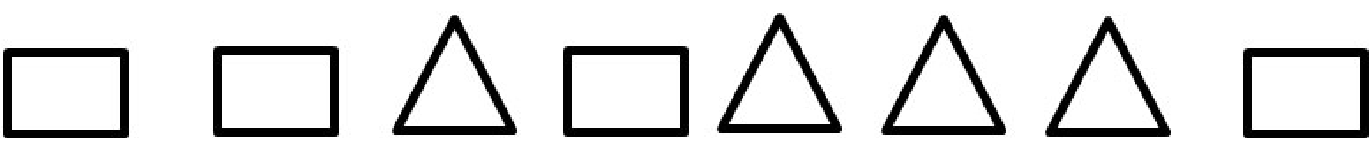
There are some disadvantages to these models because there can be an issue in how the student interprets shape. Another problem that arises is that students do not have a good understanding of area. This issue can be minimized by the use of grids and strips, but it does not completely eliminate them.

Linear Models

When working with linear models the whole is defined by the unit of distance or length. The use of fraction strips, ruler, and folded paper strips are all examples of linear models. In these models, equal quantities are represented by equal distances or lengths. Using number lines helps students understand a fraction as a number (rather than one numeral over another numeral) and helps develop the other fraction concepts. The use of a number line can reinforce the idea that there is always one more fraction to be found between two fractions and that there are fractions greater than one. The zero is particular to the number line and when using other linear models only the total length matters.

Set Models

In set models, the whole is determined by the number of objects in the set. This is an opportunity to consider a fraction as a ratio. Then subsets of the whole can be expressed as fractional parts. Fractions can be depicted using this model in many ways. Any countable object can be used as a set model. For example, if 8 shapes make a whole, then a subset of 4 triangles is 4/8 or 1/2.

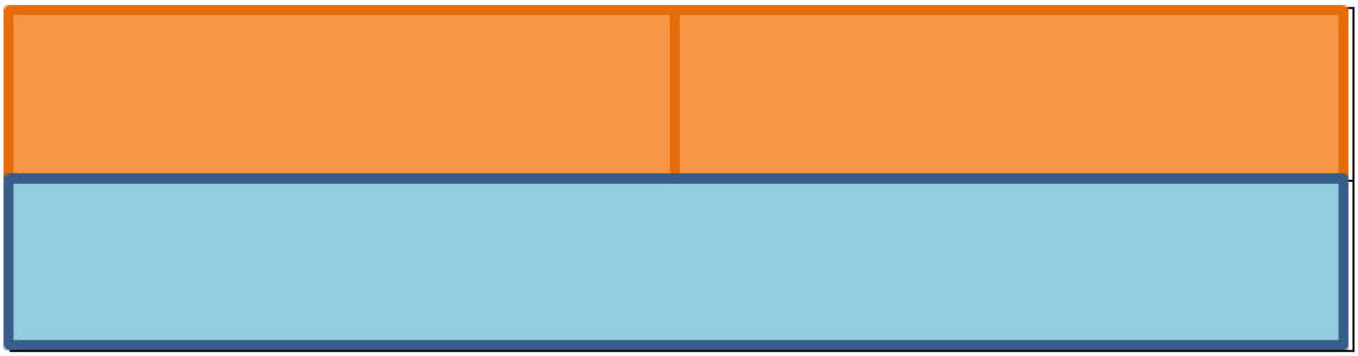


Division in Fractions

Division is an operation that can be related to fractions in two ways. The idea of partitioning is the process of sectioning a shape into equal-sized parts. This strategy can be used with area, linear, and set models.

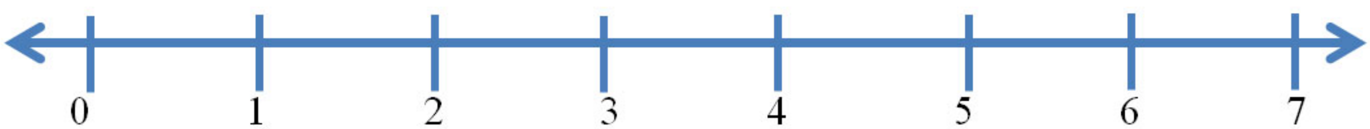
Partition with the Area Model

When partitioning an area model it is important to address two main ideas. Students first need to see that fractions may be formed by taking a region and subdivide that into some number of equal pieces. They need to see that some visuals do not show all of the partitions. In the example below, the orange and blue regions each represent half of the whole colored area, although they have been partitioned in different ways. The orange has been further partitioned, and the blue has not. Each part of the orange area is 1/2 of that part, or 1/4 of the whole colored area. Another concept students need to be aware of is that the number of equal-sized parts that can fit into the unit determines the fractional amount. For the example below students need to be able to divide the blue part into two equal parts to see that the small orange partitioned areas would represent one-fourth. Students who don't have a conceptual understanding may suggest that each of the small partitioned areas represents one-third.

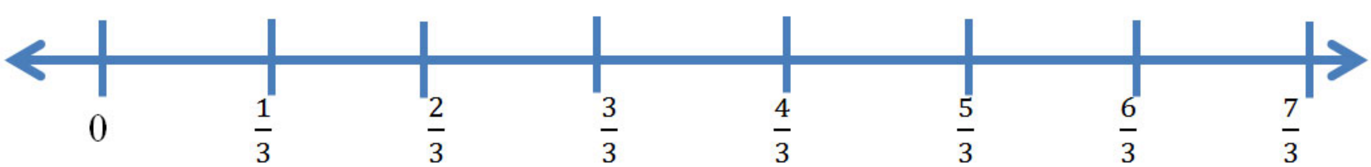


Partition with Linear Models

There are a few things that need to be determined before you start to partition a number line. First, the unit distance must be determined. This can vary from situation to situation. The Measurement principal states: The number labeling a given point tells the distance of that point from the origin/endpoint, as a multiple of the unit distance.



Students need practice labeling number lines in the same fractional increments. This helps students develop the understanding of how general fractions are the multiples of a unit fraction and conceptually understanding that fractions can be greater than one whole.



Partition with Set Models

For example, 12 shapes are partitioned into 4 equal sets – fourths. That would be represented by showing three shapes in each set. Therefore, it is the number of sets that allow us to label each as 1/4, not the number in each set. The second way is through a concept students have learned from a young age, which is sharing. For example, there are 3 people sharing 5 cookies. How many cookies will eat person get? This type of task requires students to think about sharing and also to consider how to share the leftover whole cookies.

Problem Solving

Problem Solving is a process that requires many different skills. As discussed in seminar, the student needs to be able to read and understand the problem. The use of word problems in mathematics can actually help improve the student's ability to read and comprehend. Therefore, it is important I present various types of problems that expose vocabulary to my students.

Through problem solving we want students to develop a sense of perseverance. This skill will develop as the student works on various problems. When developing problems we want to start with a basic problem that models the structure being used. As students become comfortable then we want to take that problem and add more complexity. The teacher's role during this process of problem solving is very important. Students need to be presented with a problem and then given time to work through it. So often teachers want to help the students when they start to struggle when really they need to step away and allow them to persevere.

Having a sense of organization is another skill students need to develop. As Polya states, there are steps that should be followed when solving words problems. First, students need to understand the problem. They should be able to make sense of the question and the numbers being used. Then they need to see how the parts of the problem are connected to each other. At this time they can create a plan of action for the problem. After that they can carry out their plan, which may take more than one trial depending on their success. The last step is to look back and discuss the problem. [1] These are steps that will help organize the student's thinking and allow them to process what the question is asking. For students to become better problem solvers these practices need to be modeled by the teacher.

Taxonomy of Addition and Subtraction Word Problems

In Roger Howe's seminar—"Problem Solving and the Common Core"—we have discussed how problem solving is a skill that can be taught to our students. We did a great deal of work looking at the various structures of addition and subtraction problems. The taxonomy of addition and subtraction problems can be referenced in the mathematics glossary of the Common Core State Standards, which can be found at http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/. To construct word problems it is important that the various types and structures are represented.

Through our work in seminar I have learned how important it is to first start with one step problems. First we would create a bank of problems and then share them out to the other fellows. Then each group would read one of their problems aloud and the others would identify the type of problem. This practice made us become more familiar with the different types and we also became more aware of how to write clearer word problems. After our work with one step problems we would start to create two step problems using the same scenarios. We discussed the fact that you can start setting up these problems with whole numbers but you can substitute other numbers, depending on your grade level.

There are three broad classes of one step problems. Those categories are: change, comparison, and part-part-whole.

Change Problems

Change problems have the student look at how things change over time. This change can either show an increase or a decrease in the initial amount. When constructing problems you can change the structure by altering the unknown. These will be the problem types I use in my unit. The chart below shows the different subtypes with some examples:

Change increase - result unknown	Change decrease – result unknown
Example: *There are 9 pencils in the drawer. Sara placed 2 more pencils in the drawer. How many pencils are now in the drawer?*	Example: *Melanie's cat had 11 kittens. She gave 4 to her friends. How many kittens does she have left?*
Change increase - change unknown	Change decrease – change unknown
Example: *There are 9 pencils in the drawer. Sara placed some more in the drawer and now there are 11 pencils. How many pencils did Sara place in the drawer?*	Example: *Melanie's cat had 11 kittens. She gave some to her friend. She now has 7 kittens left. How many kittens did she have to start with?*
Change increase - start unknown	Change decrease – start unknown
Example: *There are some pencils in the drawer. Sara added 2 more pencils to the drawer. There are now 11 pencils, how many pencils were in the drawer to start?*	Example: *Melanie's cat had kittens. She gave 4 to her friends. She now has 7 kittens left. How many kittens did she have to start with?*

Comparison Problems

When working with comparison problems one of the quantities is being compared as more or less as the other. When constructing problems you can change the structure by altering the unknown. The chart below shows the different subtypes:

Compare more – difference unknown	Compare less – difference unknown
Example: *Tom has 6 pencils. Greg has 10 pencils. How many more pencils does Greg have than Tom?*	Example: *Tom has 6 pencils. Greg has 10 pencils. How many fewer pencils does Tom have then Greg?*
Compare more – bigger unknown	Compare less – bigger unknown
Example: *Greg has 4 more pencils than Tom. Tom has 6 pencils. How Many pencils does Greg have?*	Example: *Tom has 4 fewer pencils than Greg. Tom has 6 pencils. How many pencils does Greg have?*
Compare more – smaller unknown Example: *Greg has four more pencils than Tom. Greg has 10 pencils. How many pencils does Tom have?*	Compare less – smaller unknown Example: *Tom has 4 fewer pencils than Greg. Greg has 10 pencils. How many pencils does Tom have?*

Part -Part Whole Problems

As the name suggests there will be two parts that when put together form the whole. When constructing problems you can change the structure by altering the unknown. The chart below shows the different subtypes:

Part – Part – Whole – total unknown
Example: *There are 6 daisies and 4 roses in a vase. How many total flowers are in the vase?*

Part – Part – Whole – addend unknown
Example: *There 10 flowers in a vase. Six of them are daises and the rest are roses. How many of the flowers are roses?*

Through discussions, we saw that the “key word” strategy for teaching word problems is doomed to fail. This strategy is not fool proof and will create misconceptions for students. Instead I want them to become careful readers and critical thinkers who are able to understand the problem and what is being asked of them. Often times on assessments, students will be given problems where they can be tricked depending on how the key words are being used. For student success the importance needs to be put on the structure of problems and understanding what information has been given and what is still missing.

When working with fractions it is important that my students understand why the procedures for computations with fractions make sense. It may help for fractions to be put into real life contexts that connect with the students. By personalizing the word problems, I want my students to become more engaged in the process. The context should provide meaning to the fractions quantities being used in the problem and the procedures used to solve it. Using scenarios that focus on school events, such as field trips or class parties, and track and field days can serve as engaging contexts for problems.

In my unit I want to create an environment like the one I had in the seminar. I will give my students practice working with the different problem types. First, I will show them the different structures for change problems using whole numbers and then I will have them substitute those numbers with fractions and mixed numbers. This type of work will demonstrate their depth of understanding and allow me to assess where they are in their learning of the content. My unit will focus on the student’s understanding of change increase and change decrease problems.

Strategies

Cooperative Learning

I plan to set up cooperative learning groups in my class. Students start to develop a sense of positive interdependence within their group. They begin to work together because their success stems from all of them. Students also start to develop the ability to work in small groups, which helps with their interpersonal skills.[2] When creating groups in my classroom I consider the student’s abilities. When placing them together I want there to be a good discussion amongst all of the students. These discussions occur throughout the various activities. They allow me to get a quick formative assessment of their understanding. Depending on the task I can have the students work with one partner or with their whole table. As the teacher I am also responsible for making sure these groups stay on task. I do this by asking questions like: How did you come up with answer? How can you support your answer? Does everyone agree at the table? Is there another way to solve this problem? As Sullivan states, “The group becomes greater than the sum of its parts, everyone performs better academically than they would if they worked along.”[3]

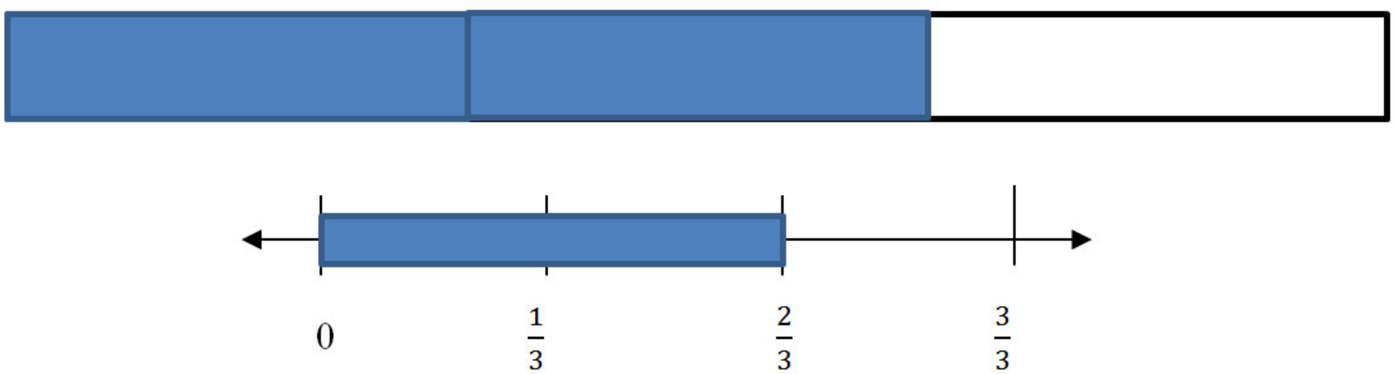
Reflecting on the Lesson

At the end of an activity I think it is important to take the time and reflect. I will give the students either a question or a problem and they answer it on a notecard. There are days when I want them to write their name on the card and turn it in. However, there are times when they don’t write their name and I use their answers to foster a learning opportunity. This activity I call the “perfect wrong answer”. The purpose of this strategy is to have students analyze each other’s work and explain what they see is correct and incorrect. I have found that sometimes children have a better way of explaining concepts than teachers. Since students don’t write their name, there is a sense of safety. As we go through the answers students discuss the strategies being used to solve the problem. This allows students to explore the possibility that problems can be solved in different ways. When we get to a wrong answer we first talk about what that student did correctly. Then we start to discuss what they did incorrectly and I have students explain how to fix the answer. Having my students look at the wrong answer helps clear up misconceptions about the topic, which can prevent further mistakes on similar questions. This conversation is driven by my students and I am just the facilitator. This activity allows the learning to become student centered and driven by their conversations about the content.

Visual Representations

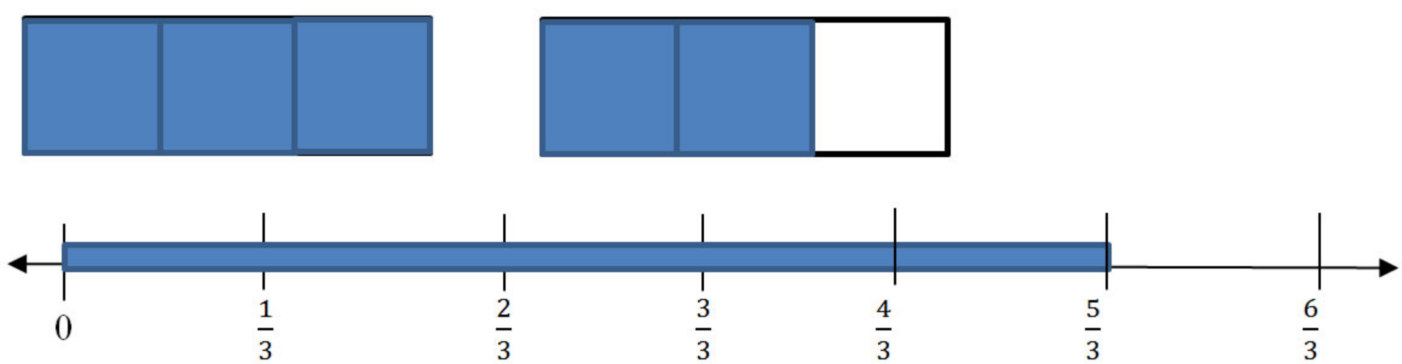
Through the use of visual models I plan on teaching the concepts of fractions through the use of visual models. This continued practice will help in developing a sense about the size of fractions and how fractions can represent quantities greater than a whole.

I will scaffold the questions I ask the students so they start to build connections among unit fractions, proper fractions, and improper fractions. First I would ask them to represent two- thirds on both visuals:



This strategy allows students to connect the idea that a unit fraction is a multiple of a general fraction. From these models students could write the following equations: 1/3 + 1/3 = 2/3 or 2 x 1/3 = 2/3.

Next, I want my students to continue their use of visual models so they can see fractions greater than one whole. For example, I would ask them to represent five-thirds on both visuals. When constructing these models they would discover that in order to represent this amount they would need to draw another whole and/or extend the number line to two.



After constructing these models students could write the following equations: 1/3 + 1/3 1/3 + 1/3 + 1/3 = 5/3 or 5 × 1/3 = 5/3.

Gradual Release Model

The Gradual Release Model discusses how the role of the teacher changes over several lessons. Over a period of time the responsibilities of the teacher will shift towards the students and by the end of the lesson students are responsible for their learning. First there is a focus lesson where the teacher will provide the content to the students and provide a sense of purpose for what they are learning. Then there is guided instruction and during this time students are working on task with teacher prompting and questioning. Next we move into collaborative learning where students will be given opportunity to work together. The last step is where the student works independently on a task. [4] Through the work of this model I want my students to be able to write their own word problems using fractions.

Identifying the Structures of Word Problems

In this unit the focus will be on the change plus and change minus structures of addition and subtraction problems. This decision was based on the idea that this is an introductory unit that sets the foundation of fractions for my students. This understanding will help them as they continue learning about fractions. The other types of problems will be introduced at a later time in my district's curriculum.

Students will first start with solving one step word problems involving whole numbers. I want to spend time talking about the structure of the problems and not have to focus on how to calculate with fractions. Research shows that students are able to formulate new knowledge by modifying and refining the concepts they already know. [5] Then I will transition into fractions with the same denominator. The problems will be about a single scenario but for each question there is a different unknown. For examples of change plus and change minus see the charts below.

Change Plus

Result Unknown	Change Unknown	Start Unknown
Tom ate 3/12 of a cherry pie and Kim ate 5/12 of the same pie. How much cherry pie did Tom and Kim eat?	Tom ate 3/12 of a cherry pie at lunch. Then Kim came in and ate some of the same cherry pie. Between the two of them they ate 8/12 of the pie. How much did Kim eat?	Tom ate some cherry pie at lunch. Kim sat down at the table and ate 5/12 of the cherry pie with her lunch. At the end of lunch they determined that between the two of them they had eaten 8/12 of the cherry pie. How much pie did Tom eat?

Change Minus

Result Unknown	Change Unknown	Start Unknown
Molly measured 6/8 cup of sugar for her cake recipe. The cup fell and 2/8 cup spilled out onto the floor. How much sugar is still in the cup?	Molly measured 6/8 cup of sugar for her cake recipe. Her cup fell and some sugar fell on the floor. Now her cup only has 4/8 cup of sugar left. How much sugar fell out of cup when it hit the floor?	Molly had some sugar set aside for her cake recip. When she turned around she spilled 2/8 cup of sugar on the counter. Now her cup only has 4/8 cup of sugar left. How much sugar did Molly start with?

Once students have become comfortable with this type of problem the number can then change. For instance, you can substitute improper fractions and mixed numbers into the problem and have them solve it. As they prove their work using visual models they will determine that their answers equal more than one. Once students have worked through these problems then they begin writing their own one step problems.

Activities

Lesson 1 – Concept of a Unit

Objective: Students will learn the importance of a unit and how they need to use that information to help them understand and solve a word problem.

For this lesson, there is an assumption that my students have already practiced with word problems with whole numbers and have developed a beginning understanding of the different problem types. As research suggests, students need to understand the problem before they can solve it. Throughout this activity the student will have to continue to reference to the unit being used in the problem.

Students in this activity will be presented with various scenarios. I want them to use visuals to help them understand and then solve the problems. The activity will start with basic questions that ask students to manipulate between the different units being discussed. For example:

Scenario – bottles of soda 6 bottles equal one pack 24 bottles equal one case	
Kerry has 48 bottles of soda. How many cases of soda does Kerry have? How many packs of soda does Kerry have?	John and Tom went to the store and bought 54 bottles of soda. How many cases of soda did John and Tom buy at the store? How many packs of soda did John and Tom buy at the store?

The discussion students have about these problems help in their understanding of a unit. At the end of this lesson I want students to be able to work with partners and answer questions that involve problem solving. The problems I would then present would still be referencing the same scenario.

Kerry decided to buy enough sodas for her class's holiday party. If she purchased 7 packs of soda, how many bottles of soda does Kerry have?	Tom, Kerry, and Sam decided to buy a case of soda to share. Tom drank 2 more than Kerry. Sam drank 5 bottles and Kerry drank 2 less than Sam. What fraction of the case is left?
Tom purchased two six packs of soda. He drank some of the soda and he had 4 bottles leftover. How much of the soda did he drink?	For the birthday party, Kerry purchased one case of bottles. After the birthday party there were a total of three packs left over. How many sodas were drunk at the party?

During this part of the activity I will differentiate my instruction based off the needs of my students. This next scenario can be used for students to work collaboratively or the teacher can pull a small group of students. Throughout these discussions students need to be identifying the unit and how it dictates how to draw the picture. This understanding will help them as they start to learn about the unit fraction in the next lesson.

Scenario – Pizza
Small = 4 slices Medium = 6 slices Large = 8 slices

The boys' soccer team was given a number of large pizzas. If the team ate a total of 24 slices, how many pizzas did the boys eat?

The Pizza Shop decided to make 5 small pizzas. Each pizza was cut into 4 equal slices. If 4 people each purchased 3 slices of pizza, how much of the pizza is left / over?

Kyle brought in 15 pizzas to the lunch room to reward the 4th graders. Each pizza was cut into 8 equal slices. Mrs. Low's and Mr. Jones's homerooms ate the same amount of pizza. Mrs. Campbell's homeroom ate a total of 35 slices. Mrs. Campbell's class ate 10 fewer than Mr. Timmons's class. What fraction of (all) the pizza did Mrs. Low's class eat?

Lesson 2 – Unit Fraction – Fraction Strips

Objective: Students will develop an understanding of what a fraction represents.

Materials: Pre-cut 15-by-2-inch strips of construction paper (at least 6 different colors), large gallon bags to hold the strips, and scissors

Students will first need to create their own set of fraction strips that they will use and refer back to throughout the unit. Each strip of paper will represent a different denominator. For each strip I will have the students cut apart the wholes into the specified fractional part. This allows them to manipulate the pieces and look at each piece as a unit fraction of the whole. For example students will create the following fraction strips:

1 Whole
One-Half One-Half
One-Fourth One-Fourth One-Fourth One-Fourth
One-Eighth One-Eighth One-Eighth One-Eighth One-Eighth One-Eighth One-Eighth One-Eighth
One-third One-third One-third
One-sixth One-sixth One-sixth One-sixth One-sixth One-sixth

Once the fraction strips have been created then I want the students to start manipulating these different amounts. To begin the lesson I would have my students get with a partner. This allows them to both construct a different fraction with the same denominator and then they can combine their pieces to represent amounts greater than one whole. For example, I might ask them to represent two fractions with the same denominators such as 2/6 and 5/6. Once they have been able to represent the amounts correctly I want to show them how they can construct number sentences to illustrate those amounts. In their math journals they can write the following number sentences: $1/6 + 1/6 = 2/6$ and $1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 5/6$. Then I can ask the students to think about what would happen if you put those two fractions together? This question will have students realize that fractions can be larger than one. With their fraction strips the students will be able to first show me the amount using their manipulatives and then we will connect it back to a number sentence. In their journals I want to see the following equation: $1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 + 1/6 = 7/6$ or I can also have students write $2/6 + 5/6 = 7/6$. At this time I will introduce the idea of an improper fraction and how fractions can have numerators greater than the denominator, since we can take any number of copies of a unit fraction. From this number sentence I can ask, "What is another way to write this number sentence?" At this time I would like to have students discuss how multiplication by a whole number can represent this same amount. For example, $7 \times 1/6 = 7/6$. I will give the students several different examples that will require them to model a proper fraction and then an improper fraction. With each problem I will have the students write number sentences to represents the fractional amounts they are representing.

Lesson 3 – Unit Fraction – Pattern Blocks

Objective: Students will investigate the relationships between blocks when different blocks are designated a whole.

Materials Needed: Pattern Blocks, 1 tub containing only hexagons, trapezoids, blue rhombi, and triangles.

It will helpful to first review the names of the pattern blocks and if necessary create a chart that can be used as a reference. Once my students become familiar with the names of the shapes I will identify the hexagon as a whole. Using their pattern block have student determine how many different ways they can cover the hexagon, using only one type of block. They should discover there are three ways, using two trapezoids, three blue rhombi, or six triangles.

At this time discuss what a unit fraction represents. In this case my students will discover that the blue rhombus represents one-third, the trapezoid represents one half, and the triangles represent one-sixth. The denominator of the unit fraction is determined by the number of copies needed to make the whole. At this time I will point out, or have the students observe, that larger denominators means smaller unit fractions. Once this has been established it is important to pose the question, “What would happen if I changed the whole”? I have created some questions that they can work on with a partner to help with this understanding. For example:

Trapezoid = 1 Whole	Triangle = 1 Whole	Rhombus = 1 whole
Write the unit fraction for the following shapes: Triangle = 1/3 Rhombus = 2/3 Hexagon = 2 Trapezoid = 1	Write the unit fraction for the following shapes: Triangle = 1 Rhombus = 2 Hexagon = 6 Trapezoid = 3	Write the unit fraction for the following shapes: Triangle = 1/2 Rhombus = 1 Hexagon = 3 Trapezoid =1 1/2

Then my students can work together as they determine the values of the various pattern blocks. To organize their thinking I would create a graphic organizer like the one below.

	Whole	 Whole	 Whole	 Whole
	1	3	2	6
				
				
				

As the whole changes there need to be a discussion on how that affects the unit fraction. For example, when the rhombus becomes a whole they will find that the triangle, which was one sixth of the hexagon, is only one-half of the rhombus. To aid in their understanding here are some questions I would consider:

- How does the size of the whole affect the fractional value of the pattern blocks?
- What does the unit fraction tell you about the whole?
- What do you notice about the size of the pieces in the whole?
- What does the numerator represent?
- How can you describe the denominator in a fraction?

Lesson 4 – Representing Fractions using Linear and Area Models

Objective: Students will be able to represent fractions using area models and number lines.

Materials: chart size grid paper laminated, dry erase marker, plain white paper

Starting with the area model I want the students to be able to represent various types of fractions. I would refer back to the fraction strips in the previous lesson since they resemble an area model. This is a model that they can recreate on their own as they continue their study of fractions. The first set of fractions will consist of all general fractions less than or equal to one whole. Students can represent the following fractions: 2/3, 3/5, 4/6, 6/8, and 4/10. I would then continue with students modeling improper fractions using the same types of models. Through this discussion, I hope students will come to the understanding that they will need to draw more than one whole for these fractions. Students can then represent the following fractions: 5/3, 6/5, 10/6, 12/8, and 15/10.

Now students need to understand that a number line is just another way to represent fractions. Using the grid paper they will draw number lines to represent various types of fractions. Time needs to be spent on how you partition a number line so there are equal spaces between each fraction amount. I would pick a certain number of squares and make that the whole. For example, I would have my students draw a number line that measures 30 squares in length. Then with the number they will be able to represent the following denominators: 2, 3, 4, 5, 6, 8, and 10. Introducing the denominators in a specific sequence will show the relationships between the fractions. Here is the sequence in which to introduce the denominators:

2, 4, and 8 3 and 6 5 and 10

After the introduction of these denominators I will have them show improper fractions on the number line. At this time students should be practicing the linear model and representing fractions greater than one whole.

Lesson 5 – Problem Solving with Fractions

Objective: Students will learn how to identify the types of word problems based off the taxonomy of addition and subtraction problems.

As stated earlier in the unit, I will have my students focus their attention on one type of problem. When I first introduce the change increase and decrease problems I will use whole numbers. This helps my students because they already have a background with whole numbers so they will be better able to focus on the content of the problems. Therefore, after sufficient time is spent dealing with whole numbers the teacher can substitute in fractions.

When introducing the problems we will first start with proper fractions and then move to improper fractions and mixed numbers. For each problem my students will draw a model and write a number sentence to explain their thinking. The problems can be used for partner practice or independent. In my appendix I have included word problems that can be used for this lesson. I will pose the questions in triples, with any of the three variables unknown, and I will expect my students to be able to do the same thing.

Students will then start to write their own word problems. They will need to continue to use visual model and equations to prove their thinking. My students will get a chance to share their problem with their classmates who will then in return identify the type of problem. All problems created by students can be collected and inserted into a class word problem book.

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The Moscow Puzzles: 359 Mathematical Recreations. New York: C. Scribner's Sons, 1972. An entertaining puzzle book that has an assortment of problem types that will keep the brain engaged.

Appendix - A

Through the work in this unit you will be addressing the 4th grade math Common Core State Standards. The beginning activities in this unit will help students develop their sense of a unit fraction, which addresses *4.NF.3 and 4.NF.4A*. Then students will use this knowledge to help them decompose fractions with same denominator, which addresses *4.NF.3B*. With their understanding of fractions the students will then apply this to real world problems addressing *4.NF.3D*.

Appendix – B

Common Denominator Addition Word Problems

Start Unknown - Tom ate some cherry pie at lunch. Kim later sat down at the table and ate 8/12 of the cherry pie with her lunch. At the end of lunch they determined that between the two of them they had eaten 10/12 of the cherry pie. How much pie did Tom eat?

Start Unknown - Terry made some cupcakes for her son's birthday. On Monday the family ate some cupcakes. Then on Tuesday, the family ate 8/2 cupcakes. Between Monday and Tuesday the family had eaten a total of 6 1/2 cupcakes. How much did the family eat on Monday?

Start Unknown - Before starting the race Nick drank some sports drink. In the middle of the race he stopped and drank 1 3/5 cup of sports drink. At the end of the race he had a total of 3 2/5 cups of sports drink. How many cups of sports drink did Nick drink at the beginning of the race?

Result Unknown - At lunch there were 2 medium pizzas set out for the teachers. Jeff ate 4/6 of one pizza and Jill ate 3/6 of the other pizza. How much pizza did Jeff and Jill eat?

Result Unknown - The recipe called for 3/4 cup of chocolate chips. After mixing the ingredients the baker decided to add 1 1/4 cup more of chocolate chips to the recipe. How much cups of chocolate chips are in the recipe now?

Result Unknown - Derek rode 12 4/5 miles on his bike before lunch and 23 1/5 miles on his bike after lunch. How many miles did Derek ride his bike?

Change Unknown - Kyle brought in a dozen blueberry muffins to class. Mike ate 7/2 muffins. Sue came in and ate some of the muffins as well. Between the two of them they ate 10/2 of the muffins. How much did Sue eat?

Change Unknown - Kim was making a trail mix for the upcoming camping trip. She first added 3 4/5 cups to the mix. Once she got done mixing all of the ingredients she decided to add more cereal. If she used 4 3/5 cups of cereal altogether, how many cups of cereal did she add at the end?

Common Denominator Subtraction Word Problems

Start Unknown - Molly had some sugar set aside for her cake recipe. When she turned around she spilled 2/8 cup of sugar on the counter. Now her cup only has 4/8 cup of sugar left. How many cups of sugar did Molly start with?

Result Unknown - Jesse measured out 5/4 cup of flour for her recipe. She used 3/4 cup of flour for her recipe. How many cups of flour does Jesse have left?

Change Unknown - Allison is making a cake for her kids. She measures out 5/2 cups of flour for the mix. When she pours it into the bowl she spills some of the flour on the counter. Now she only has 2 cups of flour for the cake mix. How many cups of flour spilled out on the counter?

Notes

1. Polya, *How to Solve It*, pages 5-6
2. Keith Sullivan, *The Anti-Bullying Handbook*, page 95
3. Keith Sullivan, *The Anti-Bullying Handbook*, page 95
4. Douglas Fisher, *Effective Use of Gradual Release of Responsibility Model*, pages 1-2
5. Arthur Hyde, *Comprehending Problem Solving*, page 3