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Personalizing Problem Solving

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Introduction

A common question posed by students in mathematics classrooms is, "When are we going to use this in real life?" The possible answers to that question will vary. Preventing that question from being asked is a difficult task, but many math teachers, myself included, have a habit of promoting the idea that mathematically rich situations already exist and surround us everywhere. Thus, the challenge is backing up the common assertion that math can be relevant, familiar and relatable to students.

The Common Core State Standards (CCSS) Math Practice Standard 1, "Make sense of problems and persevere in solving them," describes the expected outcomes educators should seek as they develop mathematically proficient students.[1] Developing problem solvers is a complex task. Polya states that helping the student is the most important, and often not easy, task of a teacher.[2] In an ideal situation, students entering high school would have developed the craft of problem solving from kindergarten through eighth grade. However, the transition to Common Core in many schools is an often bumpy shift that sometimes results with students entering math classes without the prerequisite background knowledge expected by curricula designed to emphasize the Common Core standards of mathematical practice.

The target audience of this curriculum unit will consists primarily of freshmen who will be taking the entry level integrated CCSS Math 1 course, a significant proportion of students identified to have learning disabilities with IEPs, English Language Learners, and possibly a few upperclassmen who have previously failed Algebra 1 and Geometry. The primary objective of this curriculum unit is to provide students with the skills necessary to comprehend and problem solve with word problems framed within a scenario, to have the skills necessary to identify the known and unknown values beginning with one-step multiplication and division problems with a gradual progression to two-step. An emphasis is placed on analyzing and interpreting calculations in the context of the word problems to determine if their answers are reasonable. Extensions of the problems will lead students to creating variable and linear models and functions. Also, a discussion regarding adaptation of word problems to produce multi-step problems involving all four basic mathematical operations, as well as extensions of word problems and situations using algebraic expressions, equations, and inequalities is included.

Rationale

As our district transitions away from the traditional Algebra 1 – Geometry – Algebra 2 foundational mathematics sequence towards an integrated curricular pathway based upon the Common Core State Standards (CCSS),[3] there are a variety of challenges in my mathematics classroom. In the first year of implementation of a new Common Core State Standards Integrated Math Curriculum (CCIM-1), I experienced more students struggling to adapt to an increased expectation of active engagement, discussion, and questioning in the mathematics classroom. A primary struggle by students with the new curriculum, the Mathematics Vision Project (MVP) Integrated 1 Curriculum,[4] was the inaccessibility of text heavy scenarios and the expectations of student participation in order to drive thought and discussion that would help develop, solidify, and practice students' mathematical understanding. Many of my students were unable to connect or relate to the contextual scenarios that framed each module in the curriculum. As a result of being unable to visualize and comprehend each fictitious scenario, student access to the mathematical concepts, intuitions, and learning objectives were significantly inhibited. Students were unable to connect with the MVP text heavy situational problems and tasks that were meant to elicit mathematical reasoning.

Most students entering the Integrated Math 1 course will have taken a Common Core Middle School Grade 8 course the previous year. However, the courses they are coming from have various course titles, utilize multiple different curricula, and students most likely have not received a full multi-year curriculum experience with Common Core aligned instruction. The CCSS Integrated Math 1 course is the lowest mathematics course offered to students in my district, and freshmen are defaulted into the course regardless of failing their prior course.

Last year, I team taught two sections of the CCSS Integrated Math 1 course with a special education co-teacher. The goal of the teaming was to provide opportunities for students designated as special education to be mainstreamed into general education classrooms. Both my co-teacher and I shared all classroom responsibilities including instruction, circulation amongst students, and assigning course grades. The student population in the team-taught class included approximately 25% of students with Individualized Educational Plans (IEPs) who were mainstreamed into general education classes. Approximately one third of students, coming from both general and special education, had reading levels far below grade level and were enrolled in corrective reading courses, and many were English Language Learners as well. Additionally, approximately one third of the students were identified as higher risk for not succeeding by their middle schools and were in a study skills class that provided social and emotional instruction in addition to academic support. Even though I will be team teaching, the materials designed for this unit are intended to be adaptable to any classroom.

Background and Demographics

William C. Overfelt High School is located in the heart of the East Side in San José, California. Overfelt's notorious and generally negative reputation with the general public is inherited from a past where gangs, low academic achievement rates, neighborhood safety, minority population, and high proportion of low socioeconomic status households were dominant features. The disparity is only magnified by the school's close proximity to the Silicon Valley. Approximately 80% of the student population identifies as Hispanic or Latino and approximately 90% of students qualify for Federal Free or Reduced Lunch.[5]

Currently, Overfelt is undergoing a transformation as the students, faculty, and community continually maintain a strong familial environment at our school campus that strives to emphasize a college going culture through challenging academic expectations, building student involvement through empowerment, and creating a safe space where everyone feels at home when at school. In recent years, the school culture and student academic accomplishments are in strong contrast to the perceptions and stereotypes that once defined the school. Overfelt has produced the most Latino high school student graduates and one of the highest proportion of Latino graduates out of any school in the East Side Union High School District.

One measure of the evolution of Overfelt can be viewed through the mathematics course offerings. When I started teaching at Overfelt six years ago, there was only one class section of Advanced Placement (AP) Calculus AB students. In recent years, approximately four times the number of students have been enrolled in AP mathematics courses each year, including Calculus AB, Calculus BC and Statistics. Despite gains in enrollment in higher level mathematics courses, there has been limited success for students who reach the courses. Furthermore, a far greater number of students are not succeeding in the foundational mathematics courses. Success in mathematics is elusive for a large proportion of students at Overfelt at all levels, but particularly in the freshmen entry-level integrated math course. Heterogeneous assignment of students entering the CCSS Math 1 course can result in classrooms with wide ranges of student prior knowledge, abilities in computational fluency, and attitudes towards mathematics. Students who did not succeed in middle school mathematics courses struggled immensely in my class with the new curriculum. In its first year of implementation, the percentage of students successfully completing Integrated Math 1 with a C or higher in my classes was below 40%.

To adapt to the curricular shifts due to the adoption of the Common Core State Standards, Overfelt refined its long term 5-year school vision. This curriculum unit is designed with Overfelt's three primary instructional goals: (1) Implement student centered instruction that aligns to the common core and develops our graduate outcomes, (2) 25% of instructional time is teacher–led and 75% of instructional time is student-centered and spent on student exploration. (3) Assignments are "loose," allowing students to build upon directions and create unique products to show learning. The goal is to have all students graduate with the critical and creating thinking skills as well as the resilience to succeed in college and careers. Simply stated, we want our students to have the option to get to and through college and/or thrive in their careers.

Personalization of Problems

One of the biggest difficulties I have seen in trying to teach problem solving can be attributed to the problems themselves. Frequently, I witness students who are resistant to any word problem for a multitude of factors. Sentences and paragraphs are seen as belonging to English and history classrooms. A common complaint regarding word problems is that the situations they describe are unfamiliar to my students and possess little to no meaning for them.

The nature of problem context and how this context is attended to in instruction are

important considerations for word problems to make more meaningful

contributions to students' learning of mathematics.[6]

Often I find myself struggling to help students visualize a situational story problem in the Integrated Math classes through the Advanced Placement courses despite the intent of the authors to provide real-life contexts that are also culturally sensitive to students' backgrounds. Personalization of one-step and two-step word problems can be an effective strategy to increase engagement among Hispanic middle school students, particularly males.[7] Research within the United States, as well as internationally, indicate that personalizing mathematics instruction can be an effective strategy to increase student mathematics performance.[8]

Participatory Budgeting at Overfelt

The inspiration for the theme of the collection of word problems for this unit (See Appendix A) draws from recent events at Overfelt. In the Fall of 2014, Overfelt's chapter of the Californians for Justice (CFJ), a student organization promoting youth and minority involvement in social justice causes in California, worked with our school site council and principal to organize a community based participatory budgeting program to determine how $50,000 of the school's discretionary general budget would be spent. The premise is simple: Allow those who will benefit the ability to choose how to spend our school's money. The primary conditions were that projects to be funded needed to support Overfelt students and be within budget.

Our principal was willing to have Overfelt be the first public school in California to attempt participatory budgeting. What arose from this idea was dubbed Royals Rise Up!.[9] Within the process, students, staff members, family, and community stakeholders attended public brainstorming sessions to develop project ideas that would address a need at Overfelt. There were nine finalist project ideas that were presented to the public with estimated costs and impacts promoted on campus and online (See Appendix A). After a public online vote, winning projects totaling up to $50,000 would be funded in the next year. Over one third of Overfelt's students voted for projects to be funded in the first year.

As part of the process, stakeholder input was a key factor in determining the projected estimated costs for each project idea. The mathematical richness of this situation is evident, with the potential analysis of each individual project idea, combinations of project ideas, decisions on spending in total and within each project, as well as a variety of factors that can be considered when determining the number of students and potential impact of each project can have on the Overfelt community. Drawing upon a real-life scenario actively occurring and affecting the students attending Overfelt will hopefully increase interest in the context and the mathematics.

Content Objectives

As I have written it, this unit assumes that, students enter with some prior knowledge and abilities to evaluate expressions using the order of operations, practice with representing and writing expressions, and to represent linear patterns or contexts from word problems. This unit asks students to solve a variety of one-step and two-step addition and subtractions problems as described in Glossary 1 and one-step and two-step multiplication and division problems as described in Glossary 2 of the Common Core State Standards.[10] The primary curriculum utilized for the Integrated Math 1 course consists of mathematical concepts that are intended to be developed, solidified and practiced as students' analyze and progress through various scenario based word problems. Based on prior experiences with my students, this unit serves as a means to develop the problem solving skills and strategies for word problems and allow students to build confidence in skills that are essential for success.

The unit begins by setting up and solving a variety of one-step multiplication and division problems, with a particular emphasis on group size and comparison type problems. Through adaptation of the problems by varying different known and unknown quantities, students will be guided towards seeing how to use expressions involving variables to represent situations, and the value of doing so. As different constraints are introduced, the purpose for representing situations as expressions, equations and inequalities are further developed. Through discussion of solutions in context of the scenarios, students will develop the problem solving skills that are necessary for success throughout the CCIM courses. Included with the discussion of the general mathematical content skills students are expected to gain are examples of personalized problems based upon the participatory budget context at Overfelt and strategies that assist with the progression of ideas and concepts throughout the unit.

Multiplication and Division One-Step Problem Taxonomy

The Common Core State Standards for Mathematics (CCSS- M) courses emphasizes computational fluency with the four basic operations using whole and rational numbers beginning in the late elementary grade levels and throughout middle school. An expectation of students entering the high school CCSS Integrated Math 1 (CCIM-I) course is a level of computational fluency with whole numbers and fractions using the four basic operations, exponents, and radicals. However, my past experience included classes where a majority of students did not have positive experiences and prior success in their previous middle school math courses. This unit begins by reinforcing the computational skills while utilizing the Royals Rise Up! personalized scenario to frame the problems. By starting the unit with a focus on multiplication and division one-step problems, more students should be able to meet the entry threshold,

The taxonomy of the common multiplication and division situations sorts problems into nine different classifications, as outlined from the Mathematics Glossary, Table 2, from the Common Core State Standards for Mathematics.[11] There are three major categories: problems with equal groups, arrays and area, and multiplicative comparison. Each of these categories is broken down based on the unknown quantity that the problem is seeking to find.

Equal Groups Problems

For problems with Equal Groups, the three basic components are the total product, the size of a group, and number of groups. Varying the unknown produces three types of problems.

Unknown Product/Total

Twelve packages of pencils are purchased. If each package contains 8 pencils, how many pencils are there in total?

In this example the number of groups is 12, and the number in each group is 8. The unknown is the total number of pencils, which allows us to set up the general structure: 12 × 8 = ?

Unknown Group size

Twelve packages of pencils are purchased. There are a total of 96 pencils. Each package contains the same number of pencils. How many pencils are in each package?

In this example, the number of groups (i.e., packages) is 12, and the total number of pencils is 96. The unknown is the number of pencils per group, which is the solution to the equation: ? × 12 = 96

Unknown Number of Groups

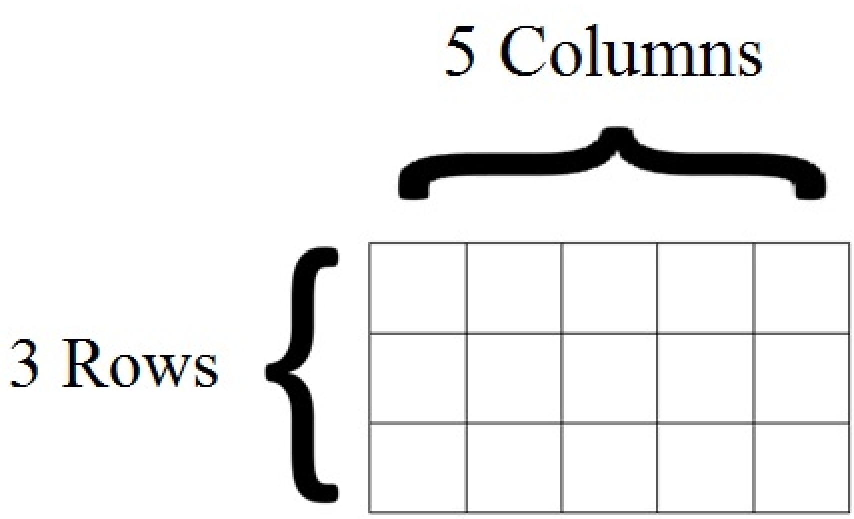
Pencils are solid in packages of 8. If you bought 96 pencils, how many packages did you buy?

In this example, the known values are the group size, 8, and the total amount, 96. The unknown is the number of groups, which is the solution to the equation: 8 × ? = 96.

Arrays or Area Problems

Multiplication and division problems involving arrays can include situations where the unknown quantity is number of rows, columns, or the total array. As outlined in the CCSS Glossary Table 2, area problems are arrays of squares that have been pushed together so that there are no-gaps or overlays (See Figure 1).

Figure 1: Array with 3 Rows by 5 Columns



Array Size or Area Unknown

There are 5 rows and 6 columns of desks in a classroom. How many desks are there in total?

We know both the size of the rows, 5, and the columns, 6. To find the total number of desks in the array, we can use the equation: 5 × 6 = ?

Length/Row Unknown

There are 6 columns of desks and a total of 30 desks. Each column has the same number of desks. How many desks are in each column?

In this situation, we know the number of columns, 6, and the total number of desks, 30. The unknown quantity, how many desks in each column is the number of rows, which is the solution to the equation: ? × 6 = 30.

Height/Column Unknown

There are 5 rows of desks and a total of 30 desks. If each row has the same number of desks, how many columns of desks are there?

In this situation, we know the number of rows, 5, and the total number of desks, 30. The unknown quantity is the number of columns, which is the solution to the equation: 5 × ? = 30.

Multiplicative Comparison Problems

Comparison problems with multiplication and division use the concept of factors and ratios to define the relationship between two quantities. For compare problems, a ratio and quantity may be given in order for a smaller or larger quantity to be found or two quantities may be given in order to find a ratio.

Unknown Smaller Quantity

A gold cell phone costs $600. If a silver cell phone costs three times less than a gold phone. How much does the silver cell phone cost?

We know the greater quantity, the gold phone cost of $600, and are aware that the unknown amount is three times less than the greater amount.

We are able to set up the multiplication structure: Unknown × Factor = Greater; ? × 3 = $600.

Unknown Larger Quantity

A silver cell phone costs $200. A gold cell phone costs three times as much as the silver phone. How much does the gold phone cost?

We know the smaller quantity, the $200 silver phone, and the comparison factor, 3 times the silver, to find a larger unknown quantity, which can utilize the structure: Smaller × Factor = Unknown; $200 × 3 = ?

Unknown Ratio

A silver phone costs $200 and a gold phone costs $600. How many times more does the gold phone cost compared to the silver phone?

Knowing both the smaller amount, $200, and greater amount, $600, we look for the comparison ratio with the structure: Smaller × Unknown = Greater; $200 × ? = $600

Personalization of One-Step Problems

One context can be adapted into multiple problem types by simply changing what information is given and what information is unknown. Providing students practice with variations of the same scenario, especially if it is more familiar will potentially reduce student misunderstanding or confusion about the context and enable students to spend more time analyzing and solving problems. The budgeting personalized scenario I will be using best suits the equal group types and comparison types of one-step multiplication and division problems. The six problems that fall into these categories will be emphasized in the problem collection.

For example, examining the cost of just one of the types of uniforms can yield nearly all of the equal group type of problems.

Name Brand uniforms cost $32 each.

Question A: How many uniforms can be purchased with $10,000?

Question B: How much do 300 name brand uniforms cost?

Question C: If we want 450 uniforms, and we can spend at most $10,000, what is the most we can spend per uniform?

The brand of uniforms is intentionally generic here. In class, the brand names will be replaced with actual brands familiar to the students or actual brands that are being considered. In question A, the underlined dollar value can be changed to produce new equal groups – quantity unknown problems. In question B, the number of uniforms can be adjusted to produce new equal groups – total unknown problems. In question C, we can vary the number of uniforms we want and the total amount of money we can spend to suggest a price point for the uniforms. With question A, we can compare the number of uniforms that can be purchased with the number of athletes we need to outfit. Question B can lead to comparison of the product with our budget constraint. Questions C can lead students into comparing their result with the other options.

Comparison type problems lend themselves to the scenario when comparing the cost between uniforms. For example extending the situation to more options through questioning can look like:

Question D: T-shirts cost one fourth as much as the $32 Name Brand uniforms. How much do t-shirts cost?
Question E: Name Brand uniforms cost how many times more than T-shirt uniforms?

The example above demonstrates how the problems themselves can introduce new considerations. However, depending on the relative preparedness of my students and their responsiveness to the initial problem context, I may choose to gradually reveal the three main options for uniforms at once, or allow students to problem solve their way to the new options. I would consider intentionally withholding information as students begin working within the context, in order to prevent some students from becoming too overwhelmed with the scenario. I anticipate some students who will complete these initial problems more quickly, and I would be able to provide them additional practice with variations of the problems by changing the values, givens, and unknowns while my co-teacher and I are able to help all students access the problems.

Two-Step Multiplication and Division Problems

It is expected that students entering the Integrated Math 1 class as freshmen would have prior experience evaluating two-step and multistep expressions, as well as solving two-step and possibly multistep equations. However, frequently these skills requiring inverse operations and "undoing" of problems are taught in a context-free manner.

From the nine types of common one-step multiplication and division problems, we can conceivably construct up to 81 different types of two step multiplication and division problems. It would be difficult, and inefficient, to provide students with examples and practice regarding each specific type of combination of problems. The goal of the next set of problems within this unit is to provide several two-step problems, of varying types, that will allow students to develop problem solving strategies.

Personalization of Two-Step Multiplication Problems

As students access the one-step equal groups and comparison multiplication problems, introducing additional considerations and constraints to the scenario will lead to two-step multiplication and division problems. In the one-step example problems discussed above, the only consideration was cost. However, in reality there are numerous factors that should be considered before a definitive decision is reached.

Once students are demonstrating success while working with one-step problems, adaptation of the problems to yield two-step problems could include the following scenario:

Quality Scenario

The uniforms being purchased can be reused for multiple years, depending on the quality of the material we select. Based on previous experiences, we know how long each brand of uniform should last. We can use this information to build new one-step problems and begin extending the scenario into two-step word problems.

For example: Consider the two options for uniform types. Option A: The Name Brand uniform costs $32 each. Each uniform can be used for 6 years. Option B: The Decent uniform costs $24 each. Each uniform can be used for 4 years. Option C: T-shirts with school logos for $8 each. Each uniform can only be used for 1 year. They must be repurchased annually.

One-Step Problem: If we assume that Name brand uniforms last 6 years, what is the cost per year of the $32 name brand uniforms?

In this example, the question allows students to reason about a group size - units unknown problem. Adaptation of this problem is possible by changing the number of years the uniforms are expected to last. By decreasing the number of years, students will see the cost per year increase. Increasing the number of years will bring the cost per year down. This problem can also be used to consider the other brands under consideration with their respective use periods, which all can be modified to form a larger set of problems.

Two-Step Problem: Suppose that the t-shirts last only 1 year and we need 450 uniforms. If we expect the same number of athletes for the next 4 years, how much money will we spend on t-shirts after 4 years?

This two-step problem is a combination of two group size – product unknown problems. The first computation could be either considering the cost of 450 uniforms for one year, which is the calculation 450 x $8 = $3600 and using the product, multiplying by the number of years 4, leading to the value $3600 x 4 = $14,400. Students are expected to have prior knowledge reasoning with inequalities. An intention of this problem is for students to reflect upon their solutions in the context of the scenario. Since our budget constraints are limited to $10,000, students would expect questions such as,

Question: Will we have enough money to buy enough t-shirts for 4 years at the current cost per t-shirt? Why or why not?

How much more money do we need for us to afford t-shirts for 4 years?

Two-Step Problem: If spend $10,000 on name brand uniforms and each uniform is used for 6 years, how many total students would have worn an updated uniform after 6 years?

This is a two-step multiplication problem where we examine how many uniforms we can afford for one year, but also consider how many students will benefit from the new uniforms. The first step is determining how many uniforms can be afforded, which is a group – size unknown problem, since we know the total amount we can spend, $10,000 and we know the cost of each uniform, $32 for name brand uniforms. This will yield the equation: $32 × number of uniforms = $10,000.

Taking the result of the first step, we then have a group – total unknown problem, since we found the number of uniforms, we then need to multiply by the number of years that each uniform will be used, 6 years, to find the total number of students that will be able to use the uniforms: "number of uniforms available × 6 years = total number of students

When working with these problems, the units of the various components of each problem may need adjustment. As with the most recent example above, in the first step, we were finding the number of uniforms, which would indicate the units, uniforms, which represents a discrete value. However in the second step, the number of uniforms found in step-one of the problem need to be considered with units representing the number of uniforms available per year. This enables the multiplication with the time of 6 years to yield a result that represents the quantity of students that will be outfitted with the same uniforms after 6 years.

Algebraic Representations of Word Problems

Not all problems need to be represented using algebraic symbols, expressions, or equations to be solved. Guess and check can be an efficient strategy that allows students to understand the quantitative, numerical, relationships within word problem solving.[12] "Guess and check is a powerful problem-solving strategy that can connect a conceptual understanding of word problems with a symbolic representation."

Many of the one-step multiplication word problems discussed in the taxonomy and examples above can be completed without writing any calculations down on paper. In my classes, students are never discouraged from mentally computing their answers, nor will they be forced to show their work in the early parts of the unit. However, by introducing additional considerations and adjusting the constraints, one-step problems can be gradually adjusted to necessitate students' eventual use of symbolic and variable representations of the problems. As students encounter and progress through different types of word problems, the use of variables can be encouraged by changing values frequently, and asking students to make predictions and generalize patterns.

Utilizing the Uniform Scenario, the problem can be extended further: Ms. Gonzalez thinks that there will be more athletes next year because there will be new uniforms. We know that a decent uniform costs $24 each, but there is also an initial set-up cost of $250 that will be charged to print all of the uniforms.

Questions: How much money will 200 uniforms cost? How much money will 250 uniforms cost? 375 uniforms? 450 uniforms? Is there a rule that can be used to describe the cost for any number, *n,* uniforms? How many uniforms can we afford with a $10,000 budget?

Introducing variables in conjunction with word problems can lead students to better understand their purpose and representations.[13] In the context of the scenario, the beginning questions require multiplication and addition in order to determine the total costs. Once problems involve even larger numbers, the calculations are the same level of difficulty, but with multiple steps, the repetitive computations may allow some students to notice patterns that can lead to generalizations and formulations of rules, eventually leading to a linear model. Based upon prior knowledge expectations for this unit, some students will be able to construct a linear variable expression. Nevertheless, past classroom experiences suggest that if students were expected to respond to the prompt, "Write an equation representing the cost of *n* uniforms," a majority of students will not be able to independently construct the equation. I anticipate needing to supplement this unit with direct instruction on specific topics as needed.

Extending the Scenario

This curriculum unit was designed with the intention to grab students' attention by using a context that directly impacts their experience and opportunities at Overfelt High School. Beginning with a lower than grade level entry threshold that is relatively basic in concept for students entering ninth grade, with the consideration of a wide range of student abilities, the connection with the context and problems could potentially draw students in. Also, the intentional use of friendlier whole numbers to begin the problems will allow students to practice the four step problem solving process. Multiplication and division one-step and two-step problems allow students to reach numerical solutions quickly. Emphasizing the fourth step in problem solving, students will continually revisit and look and consider their numerical answers in the context of the scenario.

Within the collection of problems, the questioning employed by the teacher can assist students with the conceptual development and understanding of purpose for variables, linear expressions, linear equations, and inequalities. By varying the questioning, the scenario can be easily extended to analyze and compare multiple linear situations within the uniform costs.

For example, by providing more information and using new questions for the same general scenario with uniform costs, but modifying the questioning, we can easily create multi-step word problems within the context that require addition, subtraction, multiplication and division. Furthermore, we can make open ended problems that can elicit strategies such as guess and check and variable representation of word problems as linear expressions (see Appendix C).

Example: Ms. Gonzalez and Mr. Delgado informed us that we do not need to buy only one type of uniform. We can choose two different types of uniforms if we want, but we still have only $10,000. Option A: The Name Brand uniform costs $32 each. Option B: Decent uniform costs $24 each. Option C: T-shirt with school logo costs $8 each.

Questions: If we buy 200 Name Brand uniforms, how many Decent uniforms can we buy? If choose to buy T-shirts and Decent uniforms, how many of each can we buy and still be within budget?

A benefit to using a real-life scenario is that real-life situations are often messy. The messiness of a problem can be introduced mathematically to the scenario as seen with the constraints on the budget and the lifespan of the uniforms. In addition to mathematical computations, the real-life nature of the participatory budgeting context must consider the personal and community values that influenced the initial project proposals.

Project Based Learning Activity

Overfelt's initial excursion into community budgeting highlighted a potential exemplary context to allow students the opportunities to develop a broad range of conceptual and computational mathematic topics included in the Integrated Math 1 course. The real-life scenario will be included as an ongoing theme that will culminate with a Project Based Learning activity based on the Buck Institute of Education (BIE) model.[14]

Students will have the opportunity to create their own project proposals for future Royals Rise Up! Events. A major component of the Royals Rise Up! process is finding support for one's own projects. Students will utilize mathematical problem solving skills to form talking points and arguments that emphasize their project's budgets, the projected number of students and community that will be positively impacted, and considerations of various constraints and benefits. See Appendix F for project outline.

Reading Problems With Intention

Though the context is personalized to allow students more familiarity with the word problems, the ability to read and comprehend is a vital skill for mathematical word problem solving. Often, a large number of students in my classes were English Language Learners. One strategy I used to assist students with word problem comprehension in all of my math classes, from Algebra 1 through calculus, is to structure the reading of each problem to ensure students understand what is being asked.

When reading word problems or any text-heavy scenario, I would begin by having the problem read aloud in its entirety by either student volunteers or myself. Reading a problem aloud while following along will give the first opportunity for students to process the word problem. My students often need to have problem read through multiple times. I would often ask students to think about "What is going on in this problem?," "What are we being asked to find?," and "What do we know?."

Often, I see my students look at word problems and read through it once, if at all. This would be followed by students taking any numbers given in the problem in order to perform haphazard computations. A necessary component in developing problem solving is understanding the problem, and this may require rereading the problem, perhaps several times. By rereading after prompting students with questions about what the problem is asking, there is an opportunity to read with the goal of identifying key features along the way. In my class, when we read a problem a second time through as a class, I will model how to analyze the word problem by highlighting key vocabulary, terms, and values in the problem while working under a document camera. I will also point out which key terms in a problem provide useful information when solving a problem. Once students understand what the problem is asking, I would then allow students time to work on finding solutions.

Differentiation of Problem Sets

The students in my classes include mainstreamed special education students, English Language Learners, and students with a range of prior knowledge and skills. In my classes, I anticipate a wide range of student mathematics abilities. The initial goal of this unit is to provide a low access threshold to students with a wide range of mathematical understanding and abilities. Using the problem structures described in Appendix D, three sets of problems can be created by simply changing the price points of the various options. Though the most realistic values for the problem would be those found in the highest level of difficulty set, I would begin by using the standard set of problems because the purpose of this unit is to entice student participation and interaction with the scenario. Once interested in the problem is established using the less intimidating prices (i.e. whole numbers), I would progressively increase the complexity and difficulty of the problems to use more real-life numerical values.

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Appendix A: Royals Rise Up! Project Proposals

Project Title & Idea with Estimated Costs

Get Techy: offering career and technical courses ($25000)

College Dream: Free College Trips for students ($20,000)

Career Trip: Hire a career center technician ($43,000 and additional cost of supplies)

Community Building: School activities for entire community ($30,000)

Free Printing: Two printer/copier stations on campus free for students to use ($10,000 to $12,000 and additional cost of supplies)

New Athlete Uniforms: Update uniforms to better present and represent Overfelt ($10,000)

Youth Drivers: Bring back Drivers' Education, free behind the wheel training and permit exam ($15,000)

Royals Love Tacos: Overfelt Taco Truck for student run business ($50,000)

More Tables of Campus: Additional seating in quad areas ($10,500 for 15 tables + maintenance costs)

Appendix B: Example Scenario

One of the projects selected through the Royals Rise Up! public vote last year was "New Athlete Uniforms: New uniforms to better present and represent Overfelt," which set aside $10,000 of the budget to be spent towards purchasing new uniforms for athletes from all sports. There are 15 teams at Overfelt that would like to update their uniforms. The athletic directors, Ms. Gonzalez and Mr. Delgado need your help to make decisions before they spend any money. They started looking and were given the following prices the uniform printing company.

Option A: Name Brand uniforms that cost $32 each.

Option B: Decent uniforms, good but not as fancy, that cost $24 each.

Option C: T-shirts with school logos for $8 each.

Underlined terms and numbers can be changed to create new problems of the same structure and allows for personalization of the problems.

One-step Multiplication/Division Word Problems

The following one-step word problems are examples based on the information presented in the new uniform story.

Equal Groups Problems

Unknown Product/Total

If there are 450 athletes that need new uniforms, how many Name brand uniforms can be purchased? Will we be within our budget? If 14 sports each get $700, how much money is spent? Do we have enough money?

Unknown Group size

If we spend $10,000, how many tee shirt uniforms can we purchase? If the 14 sports each got equal amounts of the $10,000, how much does each sport get?

Unknown Number of Groups

If we spend $9 for each tee shirt, how many new uniforms can we purchase with $10,000? If we give each sport $800, how many sports will get to spend the $10,000? Will we be able to cover all the sports?

Multiplication Comparison Problems

Unknown Smaller Quantity

T-shirts cost one fourth as much as the $32 Name Brand uniforms. How much do t-shirts cost?

Unknown Larger Quantity

If the Decent uniforms cost 3 times more than the $8 tee shirts, how much do the Decent Brand uniforms cost?

Unknown Ratio

If the tee shirts cost $8 each and the Name Brand uniforms cost $32 each, how many times as much do the Name Brand uniforms cost than tee shirts?

Appendix C: Extensions of the Scenarios – Initial Cost

Including initial costs to the scenario will increase the difficult of problems from being one-step multiplication and division problems to two-step problems including all four basic mathematical operations.

Option A: Name brand uniforms cost approximately $38. The company is offering free designing, but $75 shipping of the uniforms.

Option B: Decent uniforms are cheaper and cost approximately $24. There is also $250 set-up cost for designing the new uniforms and shipping the uniforms.

Option C: T-shirts with Overfelt logos for $8 each, no cost for printing set-up or shipping.

Example Questions

The following are example questions based on the modified scenario that includes an initial cost.

How many uniforms can be purchased if we select Option B?

What is the maximum number of uniforms that can be purchased with Option A?

Write an expression that models the cost of *n* uniforms for Option A/B/C.

How many more uniforms can we purchase with $10,000 from Option A compared to Option B?

How many times more t-shirts can we afford compared to the number of decent brand uniforms?

Appendix D: Differentiation of Problems

Standard Level of Difficulty

These problems are the same as found in Appendix A and all students would be expected to solve one-step and two-step multiplication problems using these values.

Option A: Name Brand uniforms that cost $32 each.

Option B: Decent uniforms, but not as fancy, that cost $24 each.

Option C: T-shirts with school logos for $8 each.

Moderate Level of Difficulty

Prices of each item may include cents that are multiples of 25.

Option A: Name Brand uniforms that cost $31.50 each.

Option B: Decent uniforms, but not as fancy, that cost $21.75 each.

Option C: T-shirts with school logos for $6 each.

Higher Level of Difficulty

This situation is the most realistic; where price points are broken down to the cent in order to determine the best deal.

Option A: Name Brand uniforms that cost $31.58 each.

Option B: Decent uniforms, but not as fancy, that cost $22.86 each.

Option C: T-shirts with school logos for $7.92 each.

Appendix E

Extending the scenario to include combinations of uniform types leads to the mathematical development of writing, solving, and graphing linear equations and inequalities with two variables.

Two Variable Equation Example

The company selling uniforms allows us to buy two types of uniforms. Some athletes like the Decent uniforms while others like the Name Brand uniforms. A Decent uniform costs $24 and a Decent uniform costs $32. Ms. Gonzalez and Mr. Delgado adjust their estimate of uniforms that we need to buy to be at least 350 uniforms with the $10,000.

Example Questions

The primary question for this situation: Will we be within our budget if we wanted to buy Decent and Name Brand uniform types for a total of 350 students?

The following questions are some of the questions that can be used as scaffolds to assist students with answering the primary question.

What is the least amount we should plan on spending?

What is the most amount of money we should plan on spending?

How many uniforms could we buy if we choose only the Decent uniform type?

How many uniforms could we buy if we choose only the Name Brand uniform type?

How much would it cost if we chose 100 Name Brand uniforms and 250 Decent uniforms with the deal?

If we wanted at least 200 Name Brand uniforms, would we have enough money to buy Name Brand uniforms for the remaining number of students?

If we wanted 350 uniforms total, are there at least 3 combinations of Decent and Name Brand uniforms that would be ***below budget*** based on the funding constraint? Justify your answer.

Can you find any combinations of Decent and Name Brand uniforms that would be ***just right*** for the budget, making sure that every dollar is spent? Justify your answer.

By using different combinations of uniform brand types, varying the quantities, and varying the prices, this context can be a starting point towards discussions on two-variable equations and inequalities and an introduction to systems of linear equations.

Appendix F: Project Based Learning Activity Outline

The following is the outline for a Project Based Learning Activity intended to be a culminating project.

Launch – Entry Event

Potential activities to begin the unit include having a presentation by Californians for Justice about Participatory Budgeting, showing Participatory Budgeting video,[15] and have a mini-vote based off of the Royals Rise Up! 2015 finalist proposals.

Driving Question

"If you had $50,000 to improve Overfelt, what would be the best way to spend the money?"

Products

Each group of three students will create a project proposal that meets the criteria for the Royals Rise Up program. The project proposals will need to include projected budgets, which will need to have inequalities demonstrating monetary constraints and possible limits to number of persons that the project will positively impact.

Public Audience

Students will give a 5 minute presentation to a panel of stakeholders in the Overfelt community with the initial proposal for a project to be funded in the next cycle of Royals Rise Up. Stakeholders will include Overfelt students, parents, faculty, board members, school site council.

Outcomes

By having students present their work to the Overfelt community, students have the opportunity to display and share their mathematical learning through explaining how they feel money at Overfelt can be utilized. The Overfelt community and its supporters have a history of supporting student ideas, regardless of the practicality or status quo. This will promote civic engagement starting at Overfelt. Also, students may have the opportunity to enter their projects for consideration in the next round of participatory budgeting.

Appendix G: Academic Standards

East Side Union High School District Essential Learning Outcomes

Students will be able to understand the use of variables, inequalities, linear equations, and linear inequalities given a story context and use units as a tool for understanding problems.

Correlation to Common Core Mathematics Standards

The primary goal of this unit is to provide students with the practice and development of problem solving skills within a personalized context. This addresses the standards: Defining quantities and interpreting expressions (N.Q.2, A.SSE.1) and Interpreting expressions and using units to understand problems (A.SSE.1, N.Q.1).

As students reason through problems and provide justification for their work, they will address the standards:
Explaining each step in the process of solving an equation (A.REI.1) and
Using units as a way to understand problems (N.Q.1)

If choosing to use the extension of scenarios, additional standards will be addressed, including:

Writing inequalities to fit a context (A.REI.1, A.REI.3)

Reasoning about inequalities and the properties of inequalities (A.REI.1, A.REI.3)

Solving linear inequalities and representing the solution (A.REI.1, A.REI.3)

An introduction to representing constraints with systems of inequalities (A.CED.3)

Writing and solving equations in two variables (A.CED.2, A.CED.4)

Writing and graphing linear inequalities in two variables (A.CED.2, A.REI.12)

Notes

1. Standards for Mathematical Practice
2. Polya, How to Solve It.
3. Appendix A: Designing High School Mathematics Courses Based on the Common Core State Standards., Common Core State Standards For Mathematics, page 4.
4. Mathematics Vision Project - https://www.mathematicsvisionproject.org/secondary-mathematics-i1.html
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6. Chapman. “Educational Studies in Mathematics,” 214.
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8. Ku and H. Sullivan, Educational Technology Research and Development
9. http://www.participatorybudgeting.org
10. http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/
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