YALE NATIONAL INITIATIVE
to strengthen teaching in public schools®

Curriculum Units by Fellows of the National Initiative
2015 Volume V: Problem Solving and the Common Core

Introduction

by Roger E. Howe, William R. Kenan, Jr. Professor Emeritus of Mathematics

The work in the seminar "Problem Solving and the Common Core" was centered on the taxonomies of one-step addition and subtraction problems and of one-step multiplication and division problems, given in Tables I and II of the Mathematics Glossary of the Common Core State Standards for Mathematics (CCSSM) (www.corestandards.org/Ma). We started by familiarizing ourselves with the addition and subtraction problem types. We constructed examples of problems of the fourteen types distinguished by the taxonomy. We shared and analyzed some of the examples, and we discussed how wording varied from type to type. From this we progressed to constructing, sharing, solving and analyzing two-step addition and subtraction problems, then three step. We went through a similar process with multiplication and division problems.

Studying word problem through the taxonomies provided some valuable insights. One is, it makes visible the complexity of the territory of word problems, in particular, the large number of different kinds of problems. (The number of types of two-step problems is in the hundreds, and the number of three-step problems is in the thousands.). The implication of this is that simple-minded approaches to word problems, such as using key words, are doomed to failure. For students to succeed at word problems, they must learn to read problems carefully and analyze the relationships between the various quantities given. Mastering word problems is genuine higher order thinking. A second insight is that the taxonomies can give teachers insight into the complexity, and increase their skill in navigating the territory of word problems, and in helping their students learn to do the same.

We ended by considering problems that combined types using any of the four operations. We observed that these multi-operation problems were more or less typical algebra problems. We discussed different approaches to solving such problems. We explored the standard approach through algebra: defining variables, converting the information in the problem to equations, and solving the equations. We also saw that many of the problems could be solved by what might be called arithmetic techniques, in which problem analysis allowed one to successively isolate the unknown quantity and find its value. In particular, we discussed examples of the classical {\it method of false position}, which was widely used before symbolic algebra was invented (mainly by Francois Viete in the late 16th century). We also discussed how the Singapore bar model method could be used to approach these problems.

The units are ordered by grade level, with the early grades coming first. Most of the units from the seminar make use of the addition/subtraction taxonomy. The first four units, which are for first and second grade, focus almost entirely on one-step addition and subtraction problems, with carefully articulated mixes of the various types of such problems as described by the taxonomy. The units of Joshua Lerner and Melissa Anderson use the one-step taxonomy to build competence with multi-step problems. Problem solving ideas

are combined with an introduction to fractions in Melissa Grise's unit, with proportional reasoning in Aaron Bingea's unit, and with algebra in the units of Lawrence Yee and Hilary Waldo. The final unit, by Klint Kanopka, deals with estimation problems, in contexts where much of the information needed for solving is not given in the problem, but must be reconstructed by the student. All the units emphasize the need to for students to devote careful attention to understanding the problem, and to the relationship between the quantities involved, in order to arrive at a successful solution strategy.

Roger E. Howe

https://teachers.yale.edu
©2023 by the Yale-New Haven Teachers Institute, Yale University, All Rights Reserved. Yale National Initiative®, Yale-New Haven Teachers Institute®, On Common Ground®, and League of Teachers Institutes® are registered trademarks of Yale University.
For terms of use visit https://teachers.yale.edu/terms_of_use