Yale National Initiative to strengthen teaching in public schools®

Curriculum Units by Fellows of the National Initiative
2016 Volume V: The Number Line in the Common Core

Adding and Subtracting Rational Numbers on the Number Line

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Introduction

The National Assessment of Educational Progress revealed some alarming deficits in students' ability to compute and reason with numbers. The performance on one particular test item highlights this deficit.

> "Students were asked to estimate the answer to 12/13 + 7/8. The choices were 1, 2, 19, 21, and, "I don't know." Only 24 percent of the thirteen-year-olds responding chose the correct answer, 2. Fifty-five percent of the thirteen-year-olds selected 19 or 21-they added either the numerators or the denominators." (Bezuk, Cramer 1989)

This example resonates with my experience teaching middle school mathematics. The most significant obstacle I face as a teacher is that my students lack a complete understanding of number and basic operations. As a means of coping with these deficits students have learned and become dependent on tricks and algorithms. This unit simply but boldly aims to once and for all give students a unifying way of conceptualizing numbers and operations.

Context

This next year I will be teaching one section of seventh grade math and two sections of eighth grade math at Brentano Math and Science Academy in Chicago's Logan Square neighborhood. The school qualifies for title 1 funding and has an enrollment that is predominately Hispanic. According to a district assessment that tests student performance on grade-level Common Core standards in each area, my incoming students consistently perform significantly lowest in the area of number systems. It is in this area where students are required to reason and operate with fractions, decimals, and negative numbers. My students' deficit in this area is apparent not only on tests, but is a persistent obstacle that impacts their day-to-day performance in my classroom.

When students are asked to operate with unfamiliar rational numbers they cling to previously learned algorithms without a conceptual understanding of what is actually happening with the numbers. Their shaky grasp of foundational number and operation concepts are major contributors to an overall intolerance and avoidance of working on any task involving these unfamiliar numbers. This creates recurrent problems, both for them individually and for the class as a whole, since our middle school curriculum assumes and requires a fundamental and flexible understanding of numbers. I have never been able to successfully implement a thorough intervention to remediate these needed foundational concepts. This unit will aim to formally and methodically address the fundamental issue of my students' patchy and deficient number and operation sense via the use and study of the number line.

Rationale

Description of Unit Structure

This unit will fill a deficit that currently exists in my curriculum, which is the lack of a cohesive and complete progression of concepts that deal with the arithmetic of rational numbers. The progression of concepts will all be delivered in the unifying context of the number line. As expressed earlier, my students lack a sound way of thinking about operating with rational numbers.

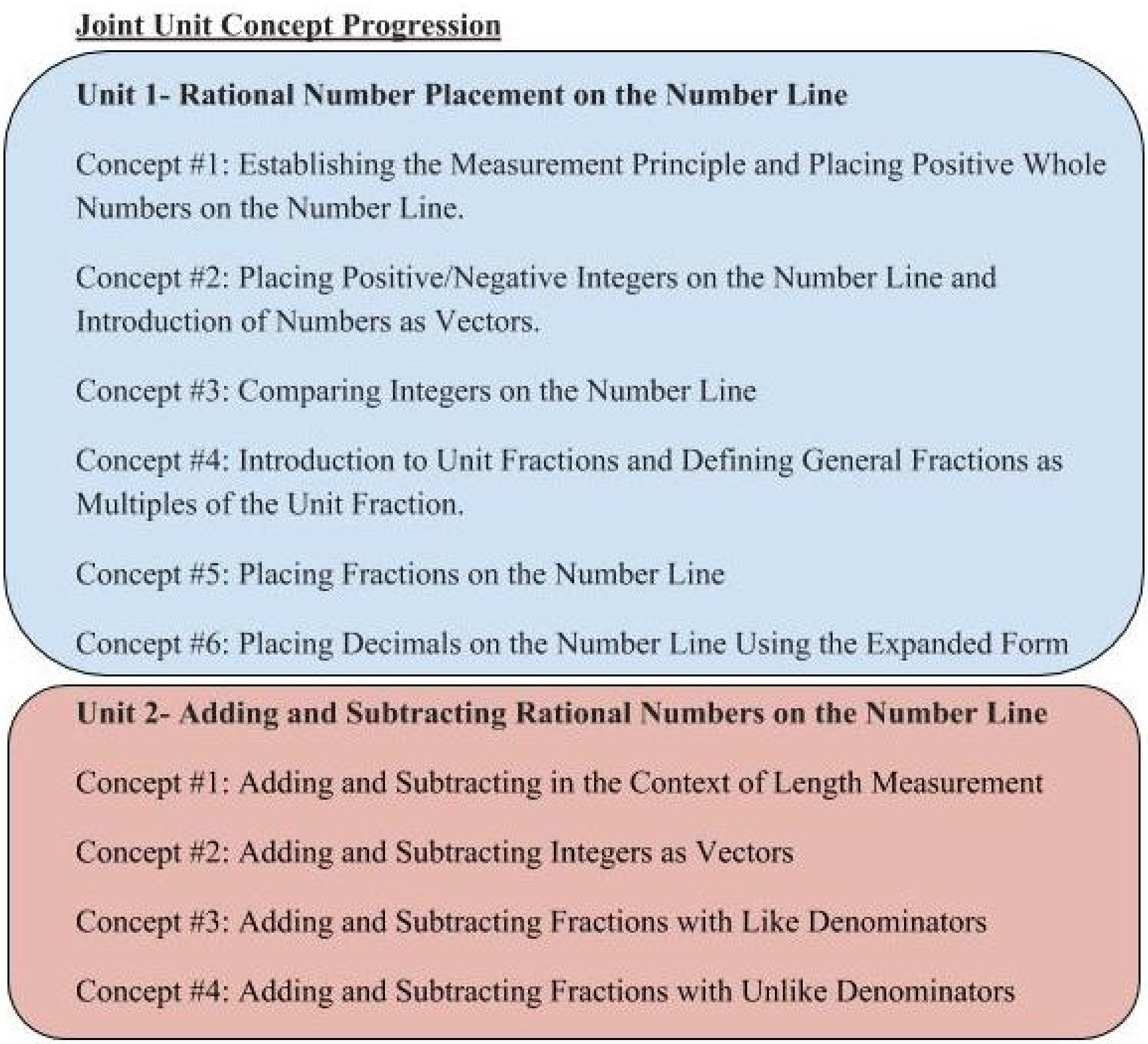


Figure 1

This is now very clear and well organized.

As seen in the figure above, my unit, highlighted in red, will be used in succession with Jeff Rossiter's unit, *Placing Rational Numbers on the Number Line*, highlighted in blue. The concepts covered in the first unit will focus on the realization and placement of rational numbers on the number line. It will be prerequisite that students first have a unified understanding of what rational numbers are and how they can be represented on the number line before moving on to adding and subtracting of rational numbers, the focus of my unit.

For this unit, the flow of concepts will be detailed in the Unit Concepts section along with a thorough description of the essential understandings for each concept. Then, an explanation of how the unit will be implemented is laid out in the Teaching Approach and Activities sections.

Why the Number Line

The use of the number line is fundamental to this unit and is intended to provide an anchoring context in which students can see and represent numbers and then make sense of operations. This unit will only address the making sense of addition and subtraction of rational numbers and will prepare my students to address multiplication and division later in the year. By "making sense", I am referring to a student's understanding and ability to visualize what is happening in the addition and subtraction of rational numbers. By seventh grade, most of my students exhibit many gross misconceptions about addition and subtraction. They rely on algorithms and calculators, especially when it comes to operating with integers and fractions. These misunderstandings are highlighted when they are asked questions such as, "Does your answer make sense?", " How do you know the sum/difference is accurate?"," Can you draw a representation of your computation?" In order to represent the addition or subtraction of whole numbers, students often employ previously learned devices such as adding or crossing out tally marks or finger counting. These methods fail them when, for example, they are required to add fractions with unlike denominators or subtract a negative from a positive. Students seldom demonstrate a conceptual understanding and are only able to operate in the abstract using algorithms. When they are explicitly prompted to demonstrate such understanding they again rely on representations that demonstrate a limited mastery of the concepts. Some students use money to reason with decimals, pie or tile representations for fractions, or different colored chips for signed numbers, but herein lies the problem. Students lack a unified way to think about these different varieties of number and therefore view adding and subtracting of different types as very different tasks. This disjointed understanding is due to the fact that they have not come to realize how operating with these different types of numbers should be viewed as similar tasks that follow similar logic.

The number line ultimately serves the purpose of giving students the opportunity to operate with these different forms of number in a uniform way. As a result of the preceding unit, students will have a unified understanding of the different types of number in terms of distance on a number line. This is prerequisite and serves as a foundation to build on for this unit. In the following section I will lay out how each concept will be taught with the number line, what key language will be used, and what essential points need to be brought to light in order for students to reach a complete understanding.

Unit Concepts

The progression of concepts addressed in this unit is as follows:

Concept #1: Adding and Subtracting in the Context of Length Measurement

Concept #2: Adding and Subtracting Integers as Vectors

Concept #3: Adding and Subtracting Fractions with Like Denominators

Concept #4: Adding and Subtracting Fractions with Unlike Denominators

Concept #1: Adding and Subtracting in the Context of Length Measurement

Addition

In my experience, by middle school, students are well aware of the elementary principle that addition, at least of whole numbers, can be seen as the act of putting things together. To ensure that all students have a firm grasp and are able to articulate this concept in the most general sense, I will start this unit by using arbitrary bars of various lengths length in the absence of units and number. Students generally have little experience with this type of model. Usually students rely on sets of tally marks or counting objects to show addition in the context of whole number addition. Students will count one set of objects and the count on to the second set to find the total. Consequentially, this limited experience causes some students to narrowly see addition as the act of counting on. This device falls short when students are then asked to add non-whole numbers, as it is difficult to represent and count on with fractions and decimals. Instead, the discussion of addition using length of bars rather than sets of objects allows us to make the more general and transferrable point that no matter what quantities we are using, fraction or whole number, addition is a matter of putting quantities together. Also the use of bars is purposeful in that it allows students to make an easier jump to seeing numbers in terms of linear measurement on the number line. See the Figure 2.

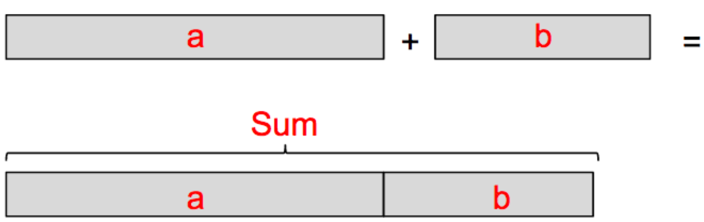


Figure 2

To find the sum of bar a and bar b we simply put them together and take the total length. This geometric representation forces students to view addition as: the "putting together" of things instead of just a symbol that signals us to count on. (Howe 2016)

Commutative Property

At this point in the progression it will be useful to show the commutative property of addition. See Figure 3.

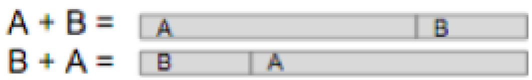


Figure 3

Figure 3 invites students to grapple with the fact that it does not matter which order bar *a* is put together with bar *b*, they will always equal the same thing, thus making a geometric argument for the commutative property. This simple illustration will be useful for the entire year of study because they provide an agreeable and clear way of viewing this property, making it easier to draw upon when we need it again in topics like algebra.

Subtraction

Students often hold a narrow view of subtraction as only the act of “taking away”. Students must also see that subtraction has a lot to do with comparing. To solidify this understanding in the absence of numbers, I will show them subtraction in terms of lining two bars up and comparing the lengths. See figure 4 below.

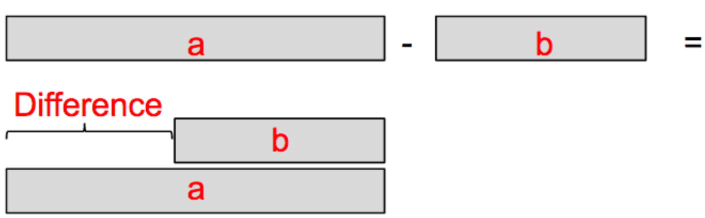


Figure 4

If we compare bar *a* with bar *b,* we can easily see the difference as the overhanging piece represented. Students must be aware that the bars are to be stacked so that they are evenly lined up at one end to make an accurate comparison. This condition needs to be firmly established, because it will be used again when we advance to seeing the subtraction of numbers on a number line.

I do not expect to spend more than two or three class sessions solidifying these concepts. My experience leads me to believe that this will mostly be intuitive and easy to grasp for a middle school audience. Although seemingly basic, these ideas will be used as a basis for reasoning and a way of thinking about addition and subtraction that is symbol independent. This idea of putting together and comparing will be the common language used throughout this unit. It is important to note that some students will have a very narrow and ingrained view of addition and subtraction. Students rely on counting and taking away strategies, which are no longer useful once they are required to operate with different denominators, decimals, or signed numbers.

When students demonstrate an understanding of this “putting together” and “comparing the difference” of bars, I will then task them with this same reasoning but on the number line, giving these bars a numerical value in terms of length. Finally I will transition the bar representation of numbers to vector representations, preparing them for the next concept.

Concept # 2: Adding and Subtracting Integers as Vectors

Adding Integers

Established in the prerequisite unit, *Rational Number Placement on the Number Line*, and again in Concept 1 of this unit, students will already have an understanding of representing positive and negative numbers as magnitudes of distance with an orientation. Students should then make the connection that adding and subtracting positive and negative numbers is just a matter of putting together and comparing these numbers on a number line in relation to the origin. Consider Figure 5 for an example of both adding two numbers with the same sign and adding two numbers with different signs.

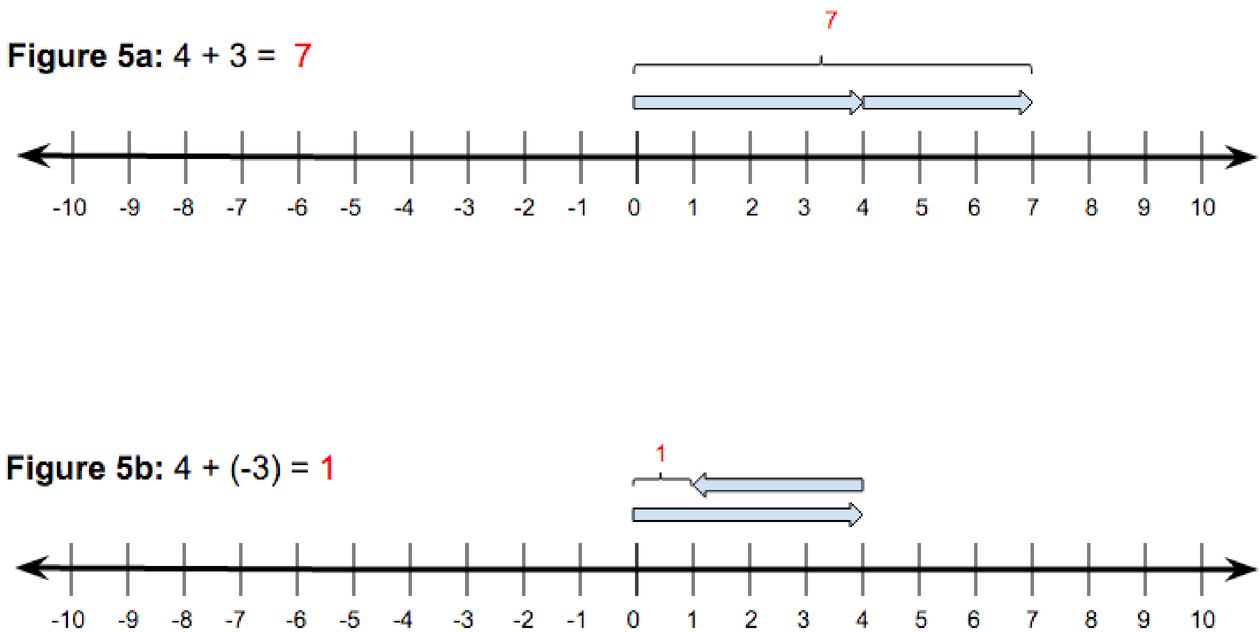
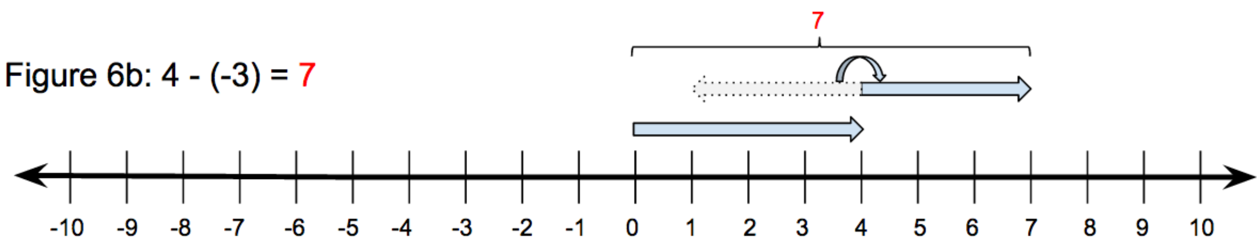


Figure 5

I find that it is essential to use careful and consistent language in discussing these vector representations. For Figure 5a, we move 4 units in the positive direction to the right so we place a vector starting at the origin and extend it to 4 in the positive direction. The plus sign signals the putting together of the following number in its original orientation which is again in the positive direction since it is a positive 3. Now we can see that the combined distance, stretching from the origin to the endpoint of the second vector is 7. The language of movement is key here. For example, “We move 4 in the positive direction and then 3 more in the positive direction.” This explanation gives further justification for the vector representations. In Figure 5b the vector positive 4 is put together with a negative 3 which students will know as a vector length of 3 units directed to the left. Because we are adding, we put them together by starting the vector at the end of the 4. This vector is stacked above the first one, which affords us to see both vectors separately. The distance from the endpoint of the second vector to the origin, 1, is then the sum. These basic examples of addition and subtraction are where precise language needs to be pinned down. Students should be able to easily agree with these representations and the accompanying explanation. Students will exercise this idea with multiple examples in order to adopt this new way of thinking. This will finally provide a sound frame of reference when operating with positive and negative numbers in the abstract, which traditionally for my students is a point of struggle.

Subtracting Integers

When moving to subtracting integers, students should use the same logic we used when subtracting bars. We line them up and then compare. Then to build from this logic, we need to use some additional language to explain what is happening in relation to the number line.



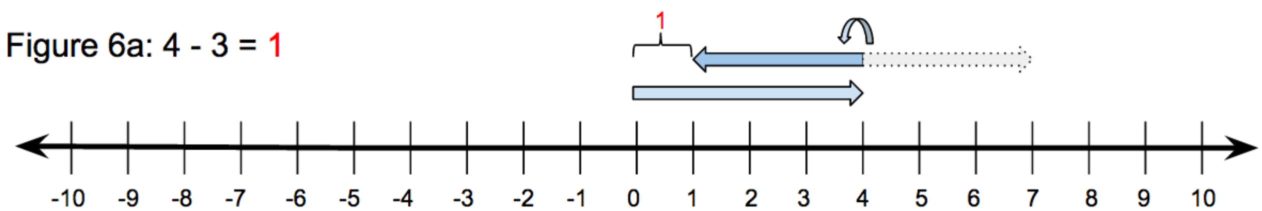


Figure 6

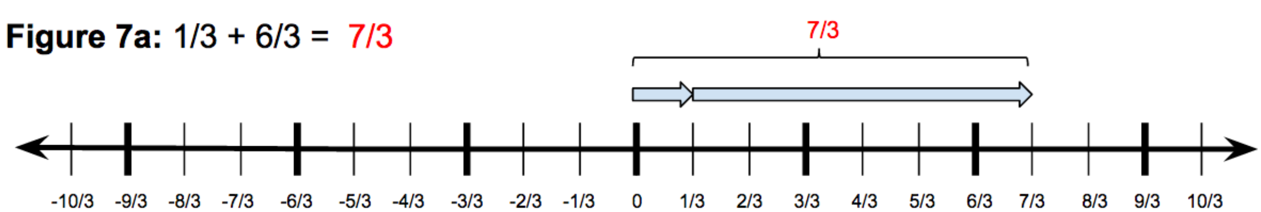
In Figure 6a, we bring 4 minus 3 to the number line. A positive vector starting at the origin represents 4 and a vector with a length of 3 is combined only this time with subtraction. Before the 3 is combined, we change its orientation since we are subtracting it. Subtraction amounts to addition of the negative. Now we compare the endpoint of the second vector to the origin for the remaining distance, which is the difference. After enough experience with seeing subtraction as changing the orientation of the second number before adding, students should realize that this is the same action as adding the opposite number, seen in Figure 5a. This is the key takeaway of this lesson.

Subtracting a negative number as seen in Figure 6b follows this same logic. -3 is seen as a vector directed to the left, but then, we change the orientation to the opposite direction since we are subtracting it. After flipping the -3 to a +3, we add it to the +4 in the familiar way. The distance of the end point of the second vector to the origin is then the difference, 7. Again the point here is to not rely on other devices for understanding this. The subtraction sign should be viewed as flipping the orientation or the ensuing number. If students are relying on an understanding where subtracting is simply taking away, it becomes difficult to conceptualize the taking away of a negative quantity. The point needs to be made that subtraction does not necessarily mean “go left on the number line”, rather it signals us to take the opposite of the subtrahend and then add. If it is viewed this way, we can make a stronger connection of addition to subtraction. Good! Students will ultimately reach the conclusion that subtraction is just addition of the additive inverse, and in this sense, subtraction is absorbed into addition.

Subtracting signed integers, in my experience is a complicated concept for students to grasp and becomes more so when signed fractions are involved. It will be important that students see many examples of the different variations of subtracting integers: subtracting a positive from a positive, positive from a negative, negative from a negative, and negative from a positive. Students should be able to see the same task happening and unify their understanding of subtracting integers no matter what combination they are presented with. Further, the goal is that students understand that just as adding and subtracting positive numbers extends uniformly to adding and subtracting positive rationals, adding and subtracting integers extends uniformly to adding and subtracting all signed rationals.

Concept #3: Adding and Subtracting Fractions with Like Denominators

In the prerequisite unit, students will have mastered representing all rational numbers. In concepts 1 and 2 of this unit we have established the essential understandings of adding and subtracting integers on a number line. Therefore adding and subtracting fractions with like denominators should be a matter of calling on these same understandings. It is the same thing, but with a smaller unit. Here it is essential that students make the connection that what we do when we operate with fractions is the same as what we do when we operate with integers. It is a matter of putting quantities together. See Figure 7a and 7b for examples of addition and subtraction.



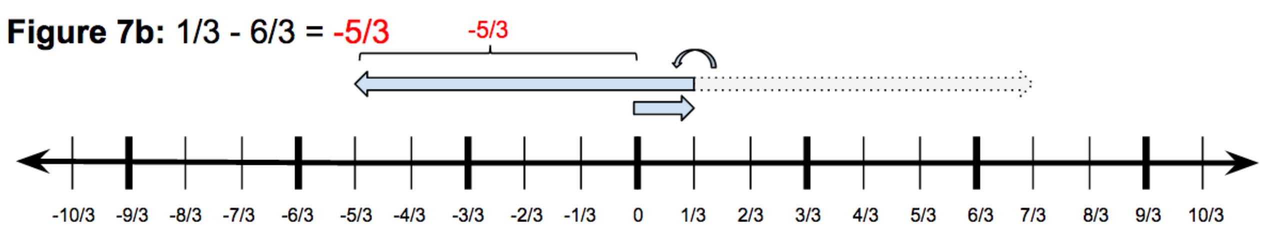


Figure 7

Concept #4: Adding and Subtracting Fractions with Unlike Denominators

At this point in the unit we will address the challenging concept of adding and subtracting fractions. The number line is an excellent place to deal with this concept, because it exhibits the need to find a uniform unit. Finding the sum or difference then becomes a matter of subdividing the unit in a way where the intervals express a common denominator. The issue of renaming, or what is usually called making equivalent fractions, requires separate consideration before moving on to adding and subtracting so that students do not blindly depend on methods that allow them to simply find common denominators and then operate.

Renaming Fractions

The essential understanding to reach regarding renaming fractions is that it is a matter of renaming a quantity, by expressing it in terms of a different unit. Students often demonstrate a deficient understanding of equivalent fractions, because of their incomplete understanding of the abstract concept of equivalency. (Hung-Hsi Wu 2013) When discussing this concept as renaming it is making the obvious and correct implication that the quantity of the fraction remains unchanged and we are just giving it a new name by using different terms. Students are generally familiar with the method of creating equivalent fractions by multiplying the numerator and denominator by the same factor. However this procedure is often done in the absence of reasoning that the physical quantity remains the same. Renaming fractions on the number line allows students to create equivalent fractions while still reasoning about quantity. Just as in the prerequisite unit, where student understanding of fractions was shepherded from area models to the number line, we must guide the renaming of fractions in the same way.

First we need to develop the idea of what it means to be the same, because for many students this is not obvious. To do this we can start with area model representation of fraction in terms of a unit rectangle. See Figure 8. Vertical lines to show fourths first divide the rectangle. When referring to each piece, it is important to use the language that was established in the first unit. Each piece represents 1 of something that takes 4 of to make a whole. (Gross 2012)

To rename this fraction in different terms we can introduce the term "subdivision". As seen in figure 8, the horizontal line shows that the unit was divided into half, thus subdividing each fourth piece into two equal parts. Students will be able to now see that the unit is divided into 8 equal pieces. They are eighths of the whole. Finally I will draw students to attend to the 1/4 pieces and bring them to the realization that we can rename them in terms of eighths. It is then visually justified that 1/4 is the same as 2/8. Although basic, this illustration is important in framing the discussion of renaming fractions as the task of expressing the same quantity in different terms.

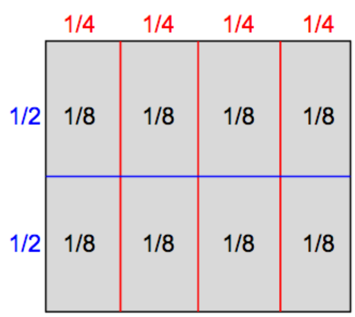


Figure 8

Once students are thoroughly comfortable with the above example of subdividing area models, I will help my students see how renaming fractions works on a number line. See Figure 9. This area model demonstrates the same logic as in Figure 8 by starting with an initial unit divided into thirds and then subdividing each piece into fourths, creating twelfths. Again, it is important to emphasize that all the thirds of fourths are equal. So, since 12 of them make the whole (the unit interval), they are twelfths. We can also conclude that:

$$\left(\frac{1}{3}\right) \times \left(\frac{1}{4}\right) = \frac{1}{12}$$

This representation in Figure 9 is one with which students do not have much experience, but it is an important step towards helping them see equivalent fractions on a number line. When we subdivided by using parallel lines we essentially divided the unit by length, which is the same task as dividing a unit interval on a line. We can then shrink these area representations down to the number line, preserving the connection of renaming the fraction of an area and renaming fraction of length on a line.

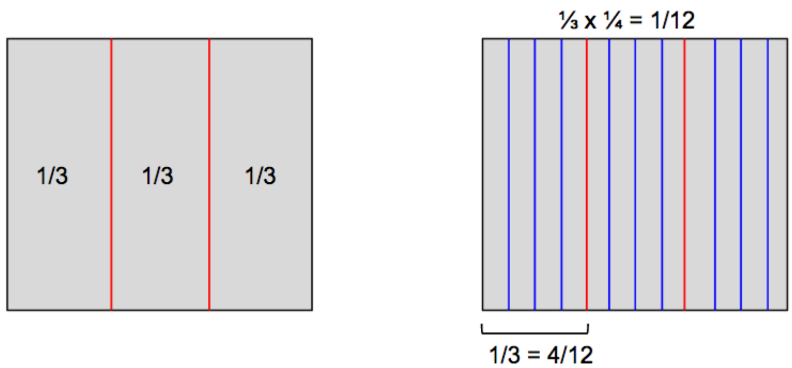


Figure 9

After we have established the idea of subdivision to rename fractions in different terms and carefully shepherded this idea to a line, we can apply this logic on the number line. Instead of subdividing areas, on the number line, we are tasked with subdividing each length interval. See Figure 10. Here 1/3 can be subdivided into each 2 equal parts to make intervals with a length of 1/6. If instead, we subdivide an interval of length 1/3 into 3 equal parts, we will make lengths of 1/9, and so forth.

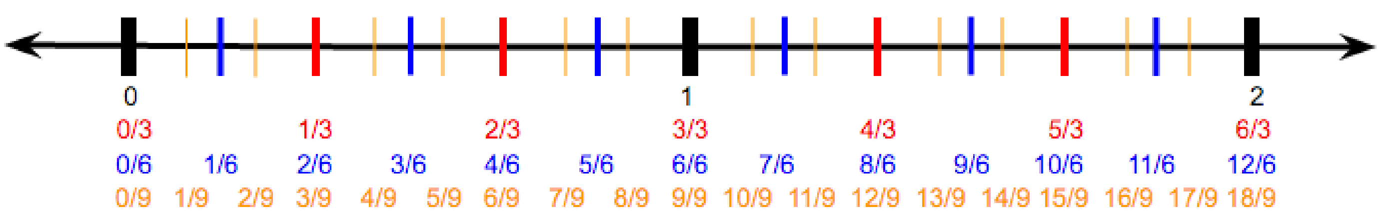


Figure 10

As with the area models, the number line affords us with an agreeable illustration of fractions with different names that represent the same quantity. With enough experience renaming fractions in this way, students will notice that by subdividing any given interval by any number, *d,* you are multiplying the original number of intervals by that number, *d,* making the new denominator. By highlighting the equivalent fractions, we can generalize to the following statement, which we will call the “Renaming Principle”:

For any fraction, a/b = ad/bd, for any whole number *d.*

Finding Common Denominators

After students have a firm grasp of seeing and creating equivalent fractions on a number line we will deal with the issue of fractions with different denominators, so that eventually we can add and subtract with them. I will start by creating a need for a common denominator by presenting students with a number line model like figure 11a. Here we can see both 1/3 and 1/2 on the same line creating intervals of different lengths. Students should see uniformity and symmetry with the intervals and will be prompted as part of a number talk to try to find the lengths of these undefined intervals. After having them grapple with this issue, we can eventually arrive at the notion that in order to find these distances we need to find common denominators by renaming.

Figure 11a

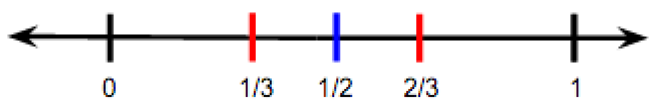


Figure 11b

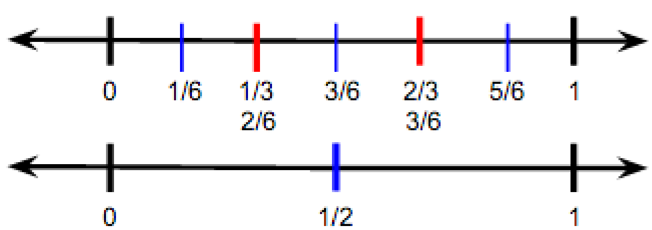


Figure 11

It will be helpful to address this concept with two number lines with the different fractions represented on each. See Figure 11b. By subdividing the intervals of the 1/3 intervals by the denominator of 1/2, we will arrive at a common denominator of 6. Here we can see that 3/6 lines up perfectly with 1/2, therefore 1/2 can be renamed as 3/6. This can also be done by subdividing the 1/2 intervals by 3. Because a given fraction can be renamed to have a denominator of any multiple of the original denominator, two fractions can be renamed to have a common denominator. In particular the common denominator can be the product of the two original denominators. The principle is defined more concisely as:

Given any 2 fractions *a/b + b/d* both can be renamed as fractions with the same denominator. In particular: *a/b=ad/bd* and *c/d=bc/bd*.

So that students become more familiar with this idea, they will engage in multiple tasks where they will be required to find common denominators for fractions with different numbers.

The main point here is that by renaming fractions with different denominators, there will always be a common denominator. This varies from a more typical treatment of common denominators by not pushing students to find the least common denominator. In my experience, students often latch on to the idea that a least common denominator needs to be found in order to do anything with different denominators. This effort takes the focus away from the main point of simply finding a common denominator. The least common denominator is a refinement that can be reached once students completely understand the concept and benefit of renaming fractions in like terms.

It is my expectation that asking students to add and subtract fractions with unlike denominators will be an easy next step to take. This is only because we have progressed so carefully through a progression of understandings that get students to realize the process of finding common denominators while always thinking about quantity in terms of the number line. It is my hope that students will ultimately utilize the concepts and skills addressed throughout the entire unit to do this task. They will have first to find a common denominator by renaming each fraction by subdividing the intervals on the number line so they can properly place both vector lengths on the same number line and then follow the already known routine for adding and subtracting fractions with the same denominator. Although tedious, part of the point of doing this on the number line is that it will ground their conceptual understanding of this process with a sound representation so that they can better reason about computing fractions abstractly. It should help them see that it really is the same process as in adding whole numbers. The difference is only in the symbolic representation.

Teaching Approach

Number Talks

For this and the preceding unit, the content will all be delivered in some variation of a number talk. This is a routine that will last five to ten minutes at the start of a class period. The strategy follows a basic format of presenting a prompt, allowing time for students to think, soliciting solutions, and finally discussing the rationale behind the offered solutions and coming to conclusions as a class. It should be noted that number talks are more typically done with prompts that cover general computational and mental math strategies. The prompts given in this unit will have the narrower focus of representing and interpreting adding and subtracting rational numbers via the number line. These talks will serve the purpose of eliciting different ways of making sense of number concepts, require students to justify their ideas using sound reasoning, and help each other reach better understanding through student led discourse. These prompts will vary in their delivery in terms of what is given and what is being asked, but will have the aim of making sense of various representations of adding and subtracting rational numbers.

Presentation of Prompts

Throughout the unit, I will show my students a variety of different prompts to elicit discussion around the desired content for that particular day. The majority of prompts will be projected at the front of the room. Students will respond on paper where there will be blank number lines for students to answer the prompt and explain their thinking. For certain number talks I will provide parallel practice prompts below so that students can practice and master a given concept after the whole group discussion. It will be important to provide students standard number lines to make sure the intervals are accurate. When students are familiar with salient structures of the number lines, I will then encourage them to create their own number lines to respond to prompts. Their written work will be stored in student portfolios so that students can keep track of their learning and refer back to notes throughout the unit.

I expect the entire unit to last 5 to 6 weeks depending on how quickly students demonstrate mastery of the concepts. In total, this unit and the prerequisite unit should last 10 to 12 weeks. Upon completion of the two units, we will consider more advanced number concepts on the number line including multiplication, division, and irrational numbers.

Prompts will vary by including or excluding certain information on the number line and by asking students to perform different tasks. For a complete set of prompts and other lesson materials that will be used for this unit see Appendix 2 for the website to visit. Below is a description of the types of prompts that will be given to teach various concepts.

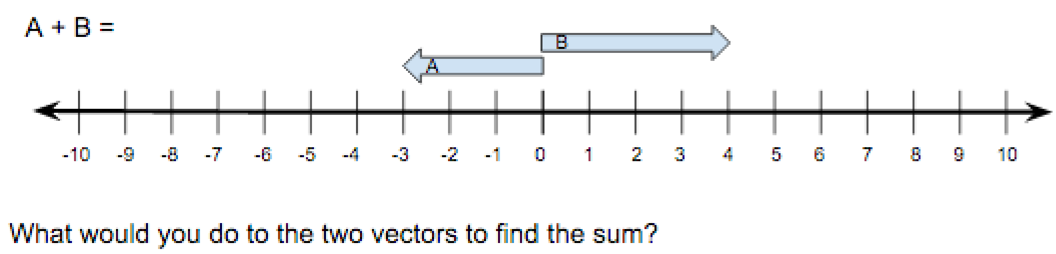


Figure 13

Figure 13 prompts students to "do" something with the vectors in order to represent the sum of two quantities. First students must figure what each vector represents by using the number line to measure while attending to the vector orientation. The question is purposefully vague to allow students the opportunity to discuss different ways of seeing the sum as positive 1.

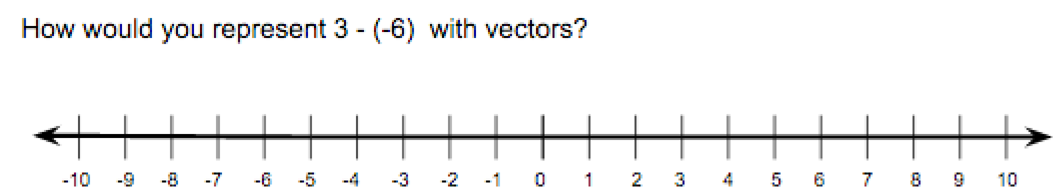


Figure 14

Figure 14 is an extension of the prompt in figure 13. Here students will attempt to draw the vectors on the number line independently. From here students will discuss how they represented each part of the expression. This task requires students take each part of the expression and visualize what is implied, bringing the abstract into the context of the number line.

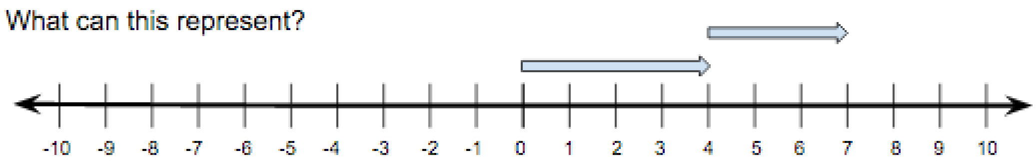


Figure 15

Figure 15 requires students to do the reverse task presented in figure 14. It is important to note that the question encourages more than one answer by asking: What can this represent? Students should see that the representation could either show 4 + 3 or 4 – (-3). This type of prompt should also be used to highlight the connection between adding and subtracting the negative.

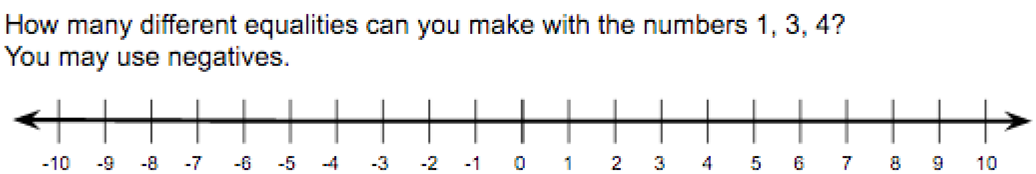


Figure 16

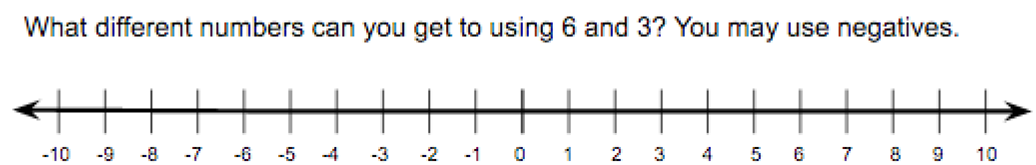


Figure 17

Figures 16 and 17 give students the task of experimenting by using different combinations of numbers to add and subtract. The value of this prompt is that it allows for multiple solutions and provides opportunities for students to see the impact of adding and subtracting negatives or positives. With multiple prompts of these and similar types, I will push students to generalize and find the total number of possibilities for each situation.

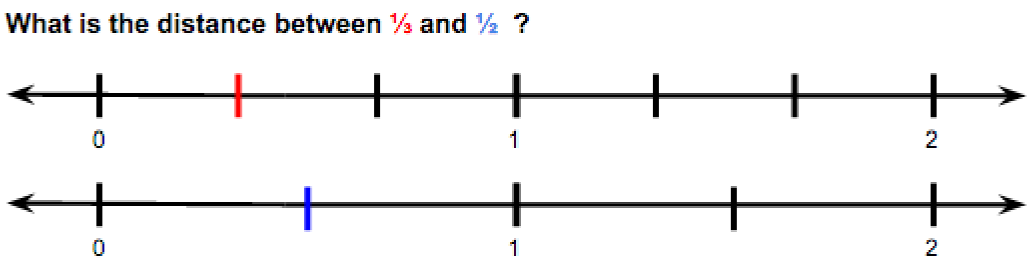


Figure 18

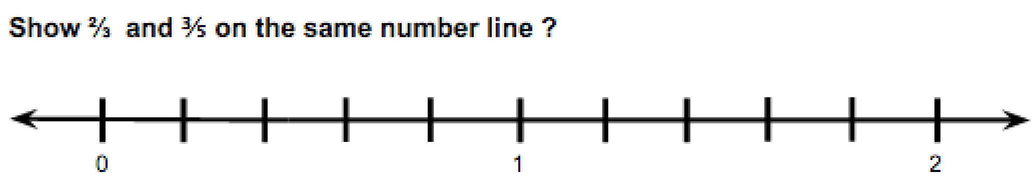


Figure 19

In Figures 18 and 19 students are prompted to grapple with different fraction concepts. In order to answer each question, students will have to manipulate the given number line by adding new interval marks. To answer this prompt they must address the equal subdivision of a unit in order to determine distance.

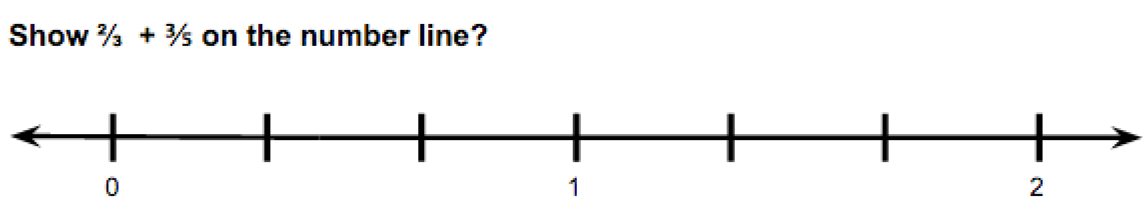


Figure 20

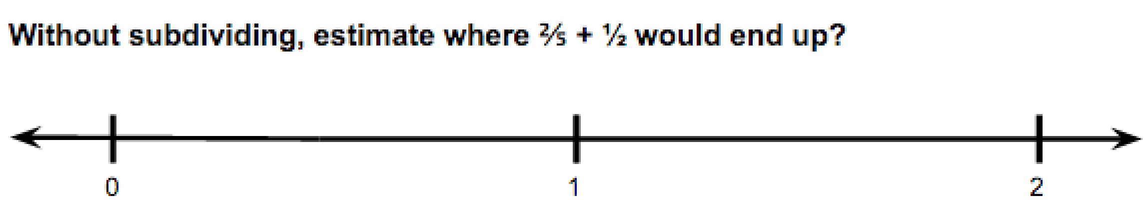


Figure 21

Figures 20 and 21 get students to visualize operating with rational numbers. Figure 20 encourages students to complete the more formal process of renaming fractions in order to find a common denominator before adding. While Figure 21 requires students to estimate and use their knowledge of fractions on the number line to find a sum. In figure 21 it will be important to prompt students to justify why they chose their placement on the number line.

Activities

Assessment

In order to gauge how well my students grasp the concepts addressed in the unit, I will administer a pre test. This will be a written test and will include a sampling of prompts that I will use throughout the unit. They will require students to offer written responses. The pretest data will allow me to determine the most common misconceptions and the range of ability levels. This information will be important when planning discussion groups and the amount of time that I should plan to spend on each concept. This same assessment will be administered at the end of the unit in order to gauge which concepts need to be revisited to reach mastery. It is important to note again that a sufficient mastery of concepts addressed in the preceding unit must be reached before moving on to this unit.

In order to raise the stakes, hold students accountable and gauge student understanding throughout the unit I will give weekly concept checks as formative assessment. These will be short checks that require students to demonstrate their understanding of the week's concepts. In general they will ask students to represent an addition or subtraction sentence on the number line or generate an expression for a given number line model. This task will be accompanied by a prompt that requires students to justify their answer. In my experience, these concept checks are needed to reinforce to students the importance of daily full participation. When students know they will be asked to demonstrate their learning at the end of the week, I find that they are more motivated to offer their ideas and attempt to make sense of what other students say. It is also an important opportunity for students to track their progress through the concepts.

Problem solving contexts

Adding and subtracting signed numbers is a major work of the middle school math curriculum. In order to supplement their work with the number line, I will also present students with problem solving tasks where they can use the number line to solve them. These problems will give them a chance to deal with adding and subtracting integers in contexts that support their understanding. In order for problem contexts to be supportive they should (1) have a meaningful origin, (2) include two distinct objects or movements to act as positive and negative integers and (3) result in a net value. Some good contexts include elevation compared to sea level, movement back and forth on a city street, the rise and fall of the balance of a bank account and movement up and down a football field. To detail how a problem solving context could support concept development of adding and subtracting integers, I will describe an activity that I will return to throughout the unit: the hot air balloon game.

Hot Air Balloon Game

One useful problem solving activity that I will incorporate throughout the unit is the Hot Air Balloon Game (NCTM 2016). The premise of the game requires students to imagine having a basket with a bunch of attached hot air balloons hovering in the air at a starting altitude. Students will understand this starting altitude as the origin. To visualize this, students will have paper cut outs of hot air balloons set next to a vertical number line. The game begins once students start drawing cards that direct them to add or take away helium balloons and sandbags. Although not scientifically accurate, the assumption will be made that for every helium balloon added the basket will rise one unit and for one taken away it will fall one unit. Sandbags, if added, will be assumed to cause the basket to fall in altitude by one unit and if taken away will cause the basket to raise one unit. Partners take turn drawing their cards that either say plus or minus a given number of balloons or sandbags. Students are then forced to reason with what the action will ultimately do to their basket in terms of the vertical number line. The value in this context is that it provides a concrete object to associate with positive and negative numbers. Students can see positive numbers as balloons that cause an increase in value and negative numbers as sandbags that cause a decrease in value when added. Students will also benefit from being able to easily conclude that if sandbags are taken away, the basket will rise. This supports the abstract concept that if negative numbers are taken away it results in positive movement on the number line.

It should also be noted that the number line represented in a vertical fashion supports students understanding of value going up and down. A common point of confusion for my students is the fact that the numbers on a horizontal number line seemingly increase going to the right of the origin and to the left. In my experience, this muddles students' understanding of what up and down mean in certain word problems. The vertical number line better supports their understanding of up and down and will afford them an easier visualization of adding and subtracting integers that they can then translate to the horizontal number line.

Appendix 1 - Standards

This unit has been written in accordance to the Common Core State Standards, which have been adopted by Chicago Public Schools. Specifically this unit will cover a set of standards within the strand of number systems.

CCSS.MATH.CONTENT.6.NS.C.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

CCSS.MATH.CONTENT.6.NS.C.6.A

Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., -(-3) = 3, and that 0 is its own opposite.

CCSS.MATH.CONTENT.6.NS.C.6.C

Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

CCSS.MATH.CONTENT.7.NS.A.1

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

CCSS.MATH.CONTENT.8.NS.A.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

Appendix 2- Unit Material

For copies of all unit material that will be used in this unit, visit the website www.mrbingea.blogspot.com. This website contains a page where I will my store updated prompts, notes on how to present certain number talks, student materials, examples of student work, and assessments used throughout the unit.

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