YALE NATIONAL INITIATIVE
to strengthen teaching in public schools®

Curriculum Units by Fellows of the National Initiative
2016 Volume V: The Number Line in the Common Core

Using the Number Line in Second Grade

Curriculum Unit 16.05.02, published September 2016
by Carol Boynton

Introduction

For many second graders, the beginning of the year is a time for revisiting knowledge and skills learned in first grade. The summer months away from mathematics instruction along with little opportunity for practice and guidance sometimes means a loss of strong understanding of concepts. Addition and subtraction of numbers to 100 is a Common Core Standard for first-grade students, although many struggle with this fundamental concept even throughout second grade. This curriculum unit is designed to increase my students' understanding of the number line as a tool to help them master two-digit addition and subtraction, improve their base-ten number sense and increase their fluency in mathematical operations.

Edgewood Magnet School in New Haven, Connecticut is an arts magnet school, integrating the arts across the curriculum. Students in this school are encouraged to use the strategies of observation, interpretation, and analysis to increase their thinking skills in every subject. With this mission, the teachers and students use unique and exciting approaches to “the basics” and work together to ensure that all students are learning successfully. This unit is designed with the school's mission and vision in mind.

Our district mathematics curriculum for Kindergarten through Grade 5, *Math in Focus Singapore Math,* is one based on the highly successful mathematics instruction model which introduces students to concepts through the CPA approach – using *concrete*, then *pictorial*, and finally *abstract* activities to introduce, practice and master mathematical concepts. Although the early activities in this program include an introduction to the relationship between counting and addition and subtraction, the first weeks of the second grade school year are an important time to remind and reintroduce this fundamental understanding.

In this curriculum unit, students will begin their work with the number line as they learn to connect measurement with counting. By using rods and cube train manipulations and transferring the bars to the number line, students will see the relationship between length and counting. The start of the school year will include a review of addition and subtraction using small numbers, sums equal to or less than ten, to help students become familiar with this connection and begin to actually use the number line as a tool just as they use counters, cubes, and base-ten blocks as tools. Making numbers and combining numbers using ten frames shows values concretely and helps students write equations, using the abstract mode of conceptual understanding.

The students will then move on to higher addition facts that include regrouping, recognizing the connection to the base ten structure and specifically understanding that the teen numbers are ten and some more. Once they are proficient with teen numbers, the students will then be ready to focus on the multiples of ten - twenty, thirty, forty, fifty and so on - applying the same strategies for regrouping they've already mastered.

Math in Focus for Grade Two does not include the number line as a tool for addition and subtraction but instead uses a hundred board (tens stacked on tens) to help students with the notion of our base ten system. Students can easily see that counting up by tens starting at 23 means the pattern will continue on the same vertical path to 33, then 43, to 53 and so on. What they do not see visually is the linear jumps up the number line and the length connection to the counting by tens. This focus of this unit will help students recognize the relationship between bar models, number sentences, and number patterns.

Through the *Every Day Counts Calendar Math* program, an enrichment component to the *Math in Focus* curriculum, the students build a number line. The Counting Tape, as it is called, is a linear timeline for recording each passing day of school in second grade. As students watch one new square being added to the counting tape each day and see the number for that day of school recorded, they become familiar with increasing quantities and the numbers that represent them. Counting the groups of ten and the extra ones reinforces students' understanding of place value. Every ten days, the color of the square alternates (orange to yellow) to help students see patterns between decades. From the 101st day on, the students see the pattern from the first 100 days repeated. This process helps students develop a connection between counting and length; a good start to the concrete step of learning.

For the duration of this unit and in addition to the Counting Tape, we make use of the open hallway outside of the classroom to create an interactive number line, taking advantage of the longer distance than our room offers, writing number sentences to solve and creating challenging problems to share with classmates. Maybe we will even be able to answer the question "How long is a thousand line?"

Getting Started Right is Important

As was mentioned earlier, before students can grasp the second grade Common Core State Standards in Mathematics (CCSSM), they must first be prepared and have a firm grip on the standards in place for first grade. Because all later mathematics builds on this earlier learning, it is important that students have a good start. In particular, students should have a good start in arithmetic.[1] They need to develop a broad conceptual understanding of the operations of addition and subtraction such that it can ultimately be applied across contexts; they should recognize the connection of arithmetic to geometry through measurement

In Roger Howe's paper *Three Pillars of First Grade Mathematics, and Beyond,* he identifies and explains the "three main ingredients that are key to starting off right."[2]

I) A robust understanding of the operations of addition and subtraction.

II) An approach to arithmetic computation that intertwined place value with the addition/subtraction facts.

III) Making connections between counting number and measurement number.

Addition and subtraction are introduced in Kindergarten. Students use a variety of manipulatives to develop an understanding of putting together, addition, and taking away, subtraction. This practice prepares children with experiences of not only using the materials but learning the vocabulary necessary to discuss and further understand the operations of addition and subtraction. Upon entering first grade, students begin the intensive instruction in mathematics, with the CCSSM using place value understanding and properties of operations to add and subtract within 100 (1.NBT.C.4). This expands to adding and subtract within 1000 once the students move on to second grade (2.NBT.B.7).

Linear Measurement

Students first see numbers as counting numbers and then generally next as linear measurement. The key features of a number line are that it has:

1) order, and

2) distance

A next step to turning a line into a number line, known as coordinatization, you must:

1) choose an origin,

2) choose unit distance a positive direction,

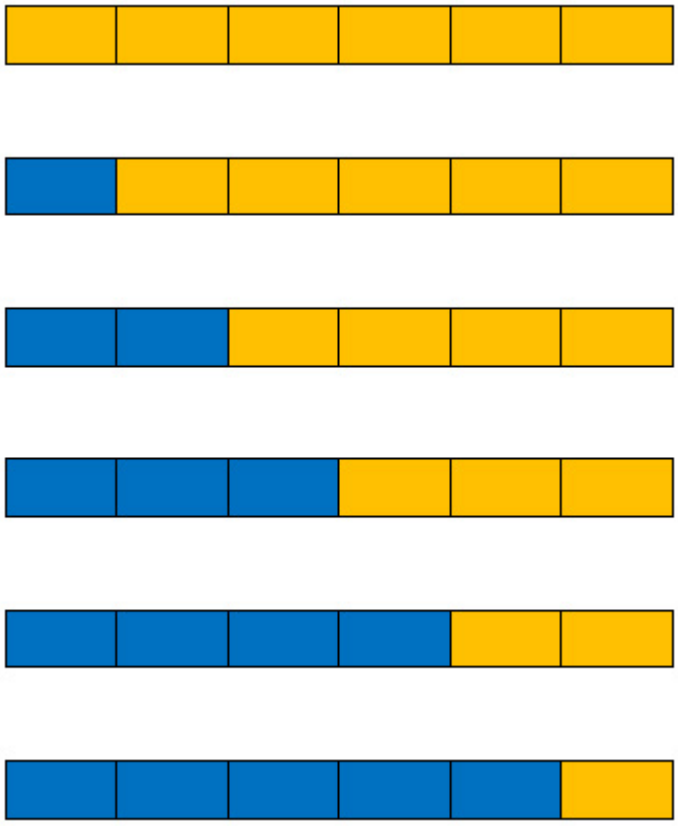
3) label other points with the number that as the ratio of the distance of the point to zero, + if it is in the same direction, - if in the other direction.

Which all explains the Measurement principle: the number labeling a point on the number line tells you the distance of that point from 0, the origin, as a multiple of the unit distance.

Because second grade in many ways continues the first grade mathematics learning, it is important to continue to focus on the Three Pillars listed above. This unit focuses particularly on the third pillar: Making Connections Between Counting Number and Measurement Number, specifically linear measurement. Linear measurement, the measurement of length or distance, would probably be considered the most basic and the simplest types of measurement. A ruler is what we often think of when considering linear measurement. Young students should use a variety of manipulatives to work with and practice this concept, connecting cubes, rods and cubes. This is an important time and step to take with students as they begin to think about length or distance using counting numbers.

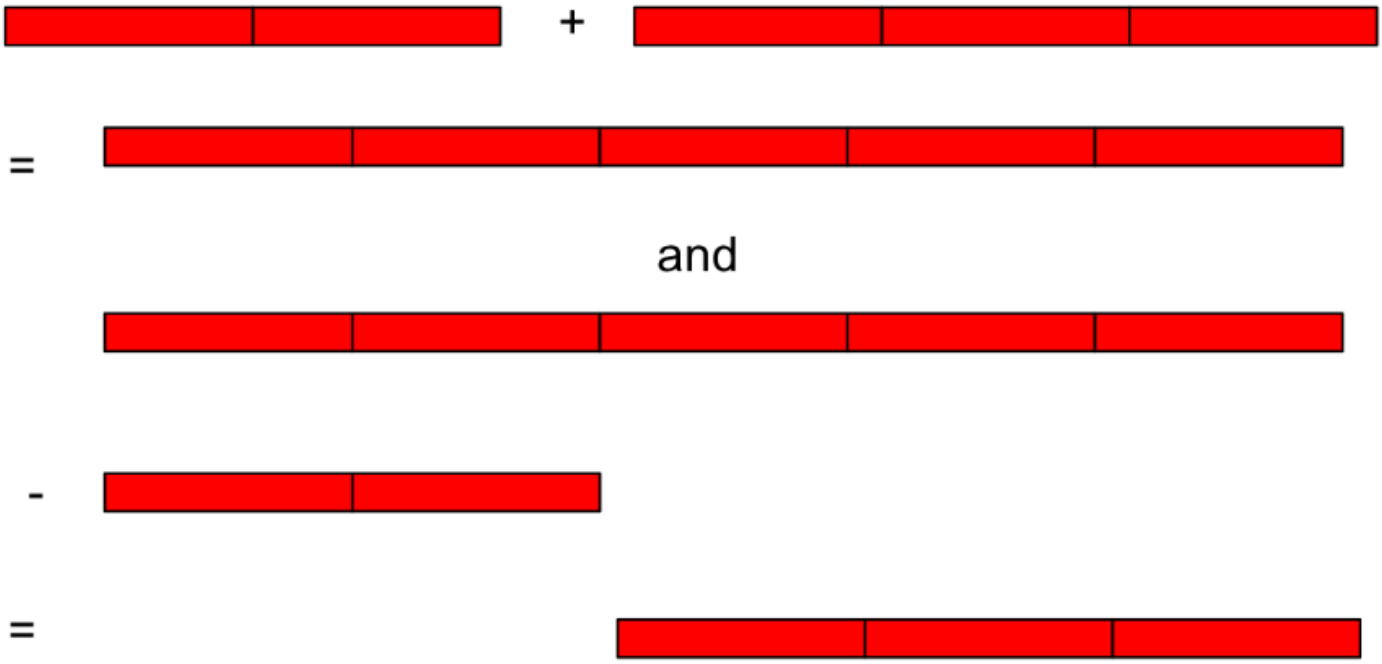
Rods and cubes are a basic mathematics materials used by students in early grades. Interlocking cubes can be made into trains to show a number and a length. Unifix cubes are one type but any connecting cubes of that are of the same size and come in different colors would serve the same purpose. Students will readily see that although the composition of a number changes, the total number of cubes remains the same and, important to emphasize, the length also remains the same. Using the bar length of 6 as an example for how students can use this strategy to gain understanding. The use of the length model for addition shows all the ways of decomposing a number into a sum, by showing a bar length of 6 above bars of lengths 1 and 5, above

bars of lengths 2 and 4, above bars of lengths 3 and 3, above bars of lengths 4 and 2, above bars of lengths 5 and 1.



Cuisenaire rods are a set of 10 colored rods ranging in lengths from one unit to ten. They are useful in this unit for students to investigate and practice combining and comparing lengths, pre-numerical addition and subtraction. They will notice that some collection of the rods, when placed end-to-end, can match a length of another set of rods placed end to end. With guidance, students can learn that one of the rods, likely the smallest which is actually a cube, can become the designated unit. This can be used to measure the other rods to determine the number of small rods make up the longer rod. This number is the length of that rod. Students will begin to realize that the length of the rod made by placing two rods end to end is the sum of the lengths of the individual rods. Lengths can also be subtracted by comparing the rods. By putting the rods side by side, only end lined up and then measuring to "extra" length of the longer rod as compared to the shorter rod, students will find the difference.

Example of 2 + 3 = 5, and 5 - 2 = 3



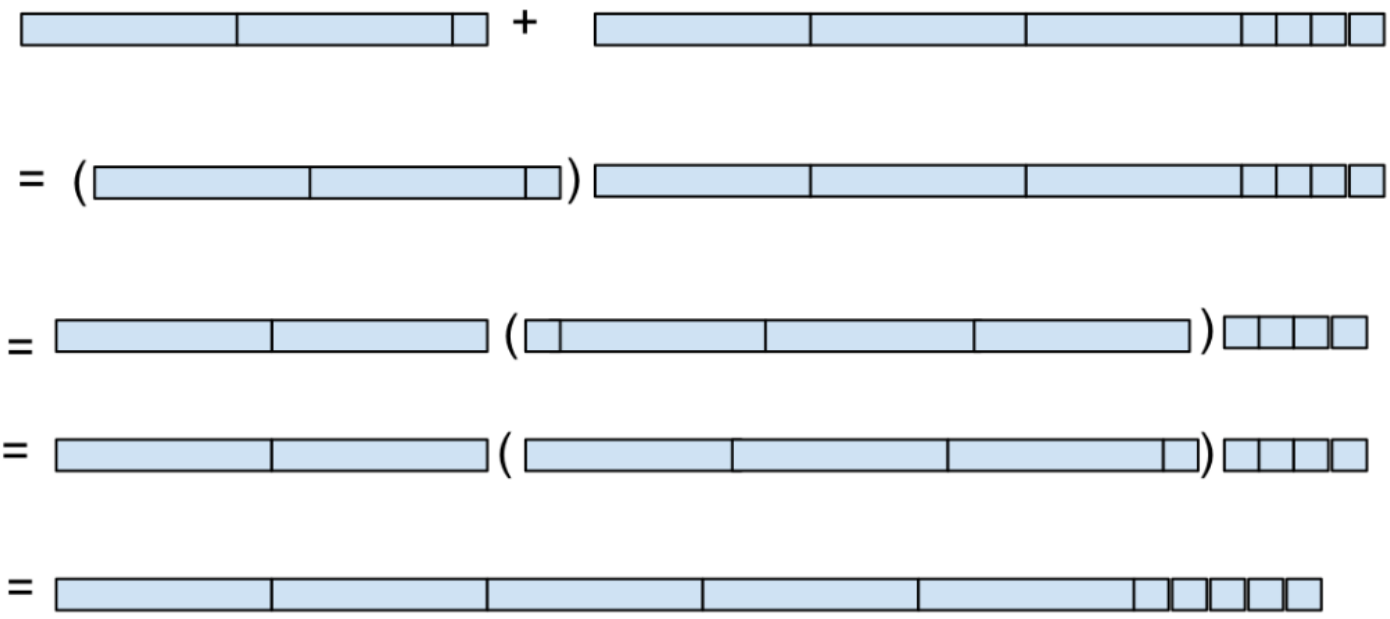
Other ways for students to practice linear measurement could include activities such as using a pencil to see how many it takes to equal the desk top, the chalk board or other choices around the classroom. As they become good at this idea, connecting length and number and practice joining rods end to end, they are taking the step toward having a concrete understanding of how the base ten structure of our base ten system works.

Base ten blocks can be introduced, with each cube representing 1and rods of length 10 representing 10. These pieces can be assembled to represent any two-digit number. To add together two-digit numbers, the 10 rods and cubes can be lined up in trains arranged in the standard arrangement, 10-rods on the left and cubes on the right. Once they are in place, the blocks can be rearranged to show that the numbers are decomposed into their base ten components and added, the recombined.

21 + 34 = (20 + 1) + (30 + 4)

= 20 + (1 + (30 + 4)) = 20 + ((1 + 30) + 4) = 20 + ((30 + 1) + 4) = 20 + (30 + (1 + 4))

= (20 + 30) + (1 + 4) = 50 + 5 = 55



This example shows that first we put the trains together and see the length combined. Then we notice that the one-cube from the first addend and the ten rods from the second addend are in the wrong places or order. Using the Commutative property, reverse the order of the one-cubes and ten-rods. Now the trains are one long length and looked at as a whole.

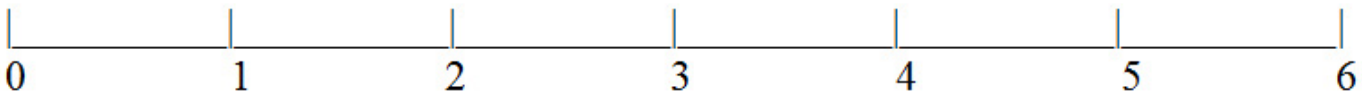
After this practice, combining two-digit numbers, students can work with three digit numbers to perform the same process. To simplify management, 100 flats are formed as a square which cannot convey the sense of length when three-digit numbers are combined. Showing the length by using rods would demonstrate how much larger 100 is in length to 10, and similarly, although not easily done, a 1000 rod would provide an indication of magnitude that a thousand cube does not. For this more dramatic visual, the use of the school hallway, and likely some borrowed materials, can allow enough room for the 1000 length to be seen. The differences in length when moving from 2-digit numbers to 3-digit and finally to 4-digit numbers will be quite evident!

The Number Line

As defined above, the basic principle that determines a number line is:

The number that labels a point tells the distance of that point from the origin (the endpoint), as a multiple of the unit length.

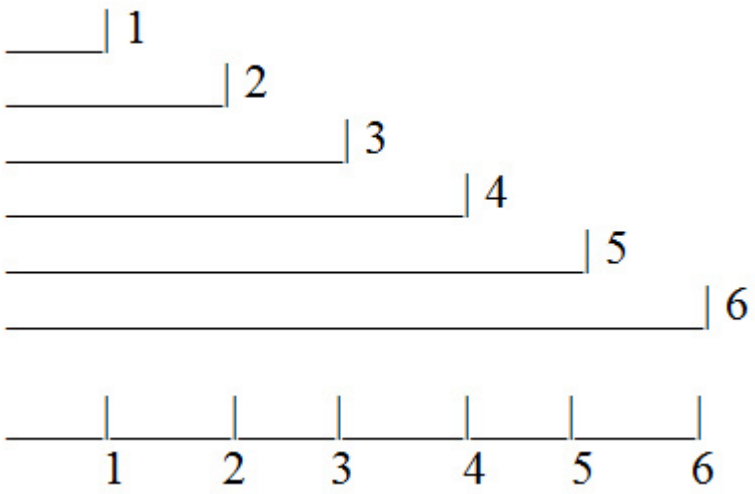
Students in first and second grade will be using what is called the number ray which is the positive half of the number line. It of course is essentially a ruler with the zero point, or origin, at one end and extending in the positive (right) direction.



With the introduction of the number line, bars of various lengths, such as the Cuisenaire rods, can be used to show the connection of counting to length. The unit length of the rods will be equal to the unit length on the number line. This practice will demonstrate visually that the numbers on a number line are describing distances. Numbers on the number line are often referred to as points on the number line. Points do not provide meanings for adding and subtracting. How can you add one point and another point and get a third point? This is only possible if the points are actually end-points of distances from zero created by length.[3]

Our brains seem to be wired to see things, not lengths. This makes seeing length/distance on the number line difficult. Because of this visual difficulty for students (and others), the National Research Council reports conclude that number line are not appropriate for PK, K or even grade 1. The CCSSM is consistent with these recommendations, first introducing the number lines in grade 2.

As mentioned before, rulers has the same structure as a number line. One strategy to help students see and use the distances is to have them draw one unit length and write a 1 after it, then very close below draw a two length units with a 2 after it, then three length units followed by a 3, and so on. They can think of a ruler as all of these lengths pushed together to make a single line with all of these lengths on it; the number of lengths so far is noted at the end-point of each length.



Visual models are one of the central core ideas and practices of the CCSSM. Research shows that simple math drawings that students can make and use in their own way in problem solving and explaining of thinking increases and improves their understanding.[4]

Mathematical Practices

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Encouraging these practices during this unit is important to consider as they support the strategies that are applied and the CCSSM that are targeted. As the students become aware of the practices they are using, they will become more proficient at using them consistently. The eight practices are as follows:

Make sense of problems and persevere in solving them. Students should start by explaining the problem to themselves and looking for entry points to its solution. Young students can rely on concrete objects and pictures to help conceptualize and solve a problem and answer the question “Does this make sense?”

Reason abstractly and quantitatively. When students look at a problem, they should be able to break it apart and show it symbolically, with pictures, or in any way other than the standard algorithm. Conversely, if students are working a problem, they should be able to apply the “math work” to the situation.

Construct viable arguments and critique the reasoning of others. Students should be able to talk about math, using mathematical language, to support or oppose the work of others.

Model with mathematics. Students use math to solve real-world problems, organize data, and understand the world around them.

Use appropriate tools strategically. Students can select the appropriate math tool to use and use it correctly to solve problem.

Attend to precision. Students speak and solve mathematics with exactness and meticulousness.

Look for and make use of structure. Find patterns and repeated reasoning that can help solve more complex problems. For young students this might be recognizing fact families, inverses, or the distributive property. As students get older, they can break apart problems and numbers into familiar relationships.

Look for and express regularity in repeated reasoning. Keep an eye on the big picture while working out the details of the problem. You don’t want kids that can solve the one problem you’ve given them; you want students who can generalize their thinking.

Teaching Strategies

The approaches for this curriculum unit vary to reflect the learning styles of all students.

The general format is based on the workshop model. The concepts and skills are taught through a series of mini-lessons focused on the objective with the following methods used throughout:

Experiential Learning: Young students should to begin with hands-on learning. During the concrete models phase, students work with manipulatives, they discover and practice decomposing numbers, comparing lengths, and connecting rods to the number line. This open learning allows students to see the changes and connections that are happening. This type of learning is an important first step.

Differentiated Instruction: Lessons and activities will be designed to maximize learning. The students will use a variety of approaches, working sometimes individually and sometimes in small groups, determined by the complexity of the work and level of understanding. Some students will move more quickly as they master skills and some will need more opportunities for practice.

Cooperative Learning: The students will be given opportunities to work as cooperative groups to use the number line as the tool to show addition and subtraction as distance. This strategy will allow students to work

collaboratively taking on various roles necessary to complete the work, with a focus on success for all. This strategy helps students support each other as learners.

Classroom Activities

Activity One - Cuisenaire Rod Exploration

Objective: Students will recognize the connection between length and number.

Procedure: Students will work with Cuisenaire rods to match and compare a unit length (the white rod/cube) with rods of other lengths. Through practice and guidance, students will see that a number of unit lengths are assembled to match another rod and the number of units used to match it is the length of the rod. Placing rods end to end creates a new length (addition) and comparing two different lengths by putting rods side by side illustrates the difference (subtraction).

Mini-lesson: Present the Cuisenaire rods to the students. Explain that these rods belong together in a set and they each have their own color name. Show that the rods are designed to compare length by creating the "staircase" arrangement, in the following order:

White

Red

Light green

Purple

Yellow

Dark green

Black

Brown

Blue

Orange

Prepare bins of Cuisenaire rods, enough for each student to have access to a complete set of rods plus a number of extra white rods. Group students in small groups of 2 or 3 for the activity and practice. Allow students recreate the staircase and spend some time exploring with the materials. After a few minutes, students should be ready for guided instruction.

Students, with their partners, will show the number lengths of the red rod up to the orange rod by placing the corresponding number of units (white rods) directly next the chosen colored rod, demonstrating the connection between length and number. What are they noticing as they work together? Write some of their

comments on chart paper or the board.

Allow students to discover combinations of rods with unit measurement as the number. Guiding questions to ask as students explore: How many units does it take to make this length? How many more units do we need to make this length? Let's compare the red and light green. How are they different? By how much? Compare two other colored rods and describe what you discover.

Activity Two – Decomposing Base Ten Components

Objective: Students will use base ten blocks to demonstrate the Commutative Rule

Procedure: Student will move from Cuisenaire rods to base ten blocks, particularly to unit cubes and ten rods. Students will learn to assemble the rods in trains to find the sum of two digit numbers and rearrange the train components to practice decomposing base ten components - collecting together all the ten rods followed by the ones, standard order. This will illustrate the Commutative Rule.

Mini-lesson: Present the base ten blocks and review their use as mathematics manipulatives. Review that the ten rods are the same length as ten unit cubes pushed together. Practice making 2-digit numbers for students to identify how many tens and how many ones to show a number. The rods and cubes should always be shown in a linear manner, not side by side. Students are connecting number and length so be consistent with demonstrations and displays of number.

The example below (with a corresponding image above) gives the written example of how students should be thinking about decomposing numbers. This is a complex visual and not intended for 2nd grade students to understand on its own or create. It should be used by the instructor to ensure that the steps to the Commutative Rule are followed. Carefully walk through each step of this problem with students using their collection of base ten blocks. Students will move the ten rods and unit cubes in this sequence to see how grouping tens together and ones together. Demonstrate that the length remains the same as the ten rods and unit cubes are collected together to show 20 + 30 and 1 + 4.

$21 + 34 = (20 + 1) + (30 + 4)$

$= 20 + (1 + (30 + 4)) = 20 + ((1 + 30) + 4) = 20 + ((30 + 1) + 4) = 20 + (30 + (1 + 4))$

$= (20 + 30) + (1 + 4) = 50 + 5 = 55$

Students will practice this activity using the following problems:

$16 + 43 = (10 + 6) + (40 + 3)$

$28 + 21 = (20 + 8) + (20 + 1)$

$35 + 42 = (30 + 5) + (40 + 2)$

Students will work on problems that do not require regrouping, or making new tens. Once students are comfortable decomposing numbers and demonstrating that length connects to the sum, more challenging problems will be introduced. Students should recognize that 10 unit cubes will become a new ten rod. Use the same rule and process to decompose the numbers.

14 + 38 = (10 + 4) + (10 + 8)

27 + 36 = (20 + 7) + (30 + 6)

These more challenging examples should only be used if the students show great understanding and can explain their thinking. The practice of making new tens is an additional component to solving the next level of problems.

Activity Three – Connecting Counting Number to Measurement

Objective: Students will use correlate base ten blocks to the number line

Procedure: After learning to combine rods and compare rods, students can begin to work with the number line. They will connect the counting number to measurement on the number line, continuing to use the base ten blocks. By placing ten rods and unit cubes directly on the number line, students will see the number/length correlation. They will learn that numbers on the number line describe distance. Number lines can be made to about 60 or 70 depending on desk/table top size that the students will be working on. Sentence strips are made of sturdy card stock and are long enough to create number lines.

Mini-Lesson: Show the number line which corresponds directly to the base ten blocks. Unit cubes should be the unit length (from 0 to 1) and it would follow that 10 rods would be the distance 10 on the number line. Demonstrate that 24 in base ten blocks, when placed on the number line is 20 for the 2 tens and 4 for the unit cubes, showing the distance on the number line as 24. Demonstrate several examples.

Students will work in groups of 2-3 and practice comparing number on the number line to length. The instructor will call out numbers and students will demonstrate understanding by placing rods and unit cubes on the line, 10's first followed by ones. The next steps of this activity combine the understanding from Activity Two, the Commutative Rule. The instructor gives two 2-digit numbers for students to place on the number line. They students will then apply the Rule and show the sum of the two numbers. The example problems in the previous activity will work with this activity as well.

Activity Four: School-wide Number Line

Objective: Students will connect length with number using the distance available in our school hallway

Procedure: Students will take the hundreds chart apart by tens and place the sections of 10 end to end to demonstrate length to see how much longer 100 is that 10. Students will repeat this same idea in the hallway of the school to show how much farther away 1000 is from 100 by creating a school-wide number line with the origin at our classroom.

Mini-lesson: Using the 100-chart similar to the one below, show that for ease of use, we place our tens on top of each other so they are together up to 100. Demonstrate with a larger version, that if we cut the "decades" apart, we can create a 100-number line instead. Students will need to help by attaching one decade to the next with tape and holding the example as it grows longer.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

31 32 33 34 35 36 37 38 39 40
41 42 43 44 45 46 47 48 49 50
51 52 53 54 55 56 57 58 59 60
61 62 63 64 65 66 67 68 69 70
71 72 73 74 75 76 77 78 79 80
81 82 83 84 85 86 87 88 89 90
91 92 93 94 95 96 97 98 99 100

Students will use their 100-chart to create a number line. They can attach their pieces to lengths of adding tape that unrolls as they attach a new decade. This "number line" can be rolled up and kept for future activities.

Additionally during this activity, a number line with the unit measure of 1" will begin in the hallway directly outside of the classroom and continue down the corridor ending at 1000. The unit measure can be adjusted based on the length of the hallway and total distance available.

The completed hallway number line will demonstrate the concept of number meaning a distance from the origin, in this case, our classroom. This number line can become the tool for many activities. Placing students in locations that are a distance of 10 apart, then a distance of 20, and so on up to a distance of 100; solving addition and subtraction problems; reinforcing place value and other ideas that students may generate.

Resources

Common Core State Standards for Mathematics,

http://www.corestandards.org/the-standards/mathematics.

Fong, Ho Kheong. *Math in Focus Singapore Math by Marshall Cavendish*. Final ed. Singapore: Marshall Cavendish Education, 2015.

Fuson, Karen C. *Math Expressions*. 2011 ed. Orlando, FL: Houghton Mifflin Harcourt, 2011.

Fuson, Karen C (in press). *Building on Howe's Three Pillars in Kindergarten to Grade 6 Classrooms,* In Y. Li, J. Lewis and J. Madden (Eds.), Mathematics Matters in Education – Essays in Honor of Roger Howe, Cham, Switzerland, 2016

Gillespie, Janet G. and Patsy F. Kanter, *Everyday Counts Calendar Math, Grade 2,* Great Source Education Group, Wilmington, MA, 2005

Howe, Roger. *Three Pillars of First Grade Mathematics and Beyond*

Howe, Roger, *The Most Important Thing for Your Child to Learn about Arithmetic*

Howe, Roger and Harold Reiter, *The Five Stages of Place Value*

Ma, L. *Knowing and Teaching Elementary Mathematics*, Erlbaum Associates, Mahwah, NJ, 1999.

Endnotes

1. Roger Howe, Three Pillars of First Grade Mathematics, 1
2. Howe, Three Pillars, 1
3. Karen C. Fuson, Building on Howe's Three Pillars in Kindergarten to Grade 6 Classrooms, 16
4. Fuson, 2

Appendix - Implementing District Standards

The New Haven Public School District follows the Common Core State Standards for Mathematics. The following standards are targeted in this curriculum unit:

2.NBT.B.5

Number & Operations in Base Ten

Use place value understanding and properties of operations to add and subtract. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Students will demonstrate this fluency in Activities Two and Three as they use the number line to solve 2-digit addition and subtraction problems. Students will use the strategy of decomposing numbers and demonstrating the use of the Commutative Rule as they show understanding of place value through the operations of addition and subtraction.

2.MD.B.5

Measurement & Data *Relate addition and subtraction to length.* Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

The focus of this unit is to demonstrate the connection between number and length. The students will use drawings and the number line (a form of ruler) to solve equations with numbers within 100. Students will be creating their own number lines to use with the activities in this unit and learning to apply the concept of numbers on a number line describing distance.