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Moving from Rods to Number Lines to Understand Fractions

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Introduction

The Common Core Mathematics standard states students should be able to understand a fraction as a number on the number line and represent fractions on a number line diagram. In my 14 years of teaching third grade, I have had difficulty explaining this concept adequately to my students! Many of my students think of a fraction as a combination of two whole numbers. The most popular strategies involve folding paper, identifying shaded portions on a partitioned shape, or placing fractions on a number line. All of these activities are a beginning, yet I know now that I can do better. My intent for this unit is to shepherd fractions from an area model to the number line as I begin to explain and represent fractions as a distance or measurement. As useful as the area models are for developing fractional understanding, students begin to over generalize and pigeon hole their understanding of a fraction as a piece of something instead of understanding that a fraction is a number in and of itself and that a fraction can be greater than 1.

In order to gain a thorough understanding of mathematical ideas, students need to be able to make connections and integrate their learning of concepts in a variety of ways. I will start this unit by allowing students to explore with manipulatives to put together and compare trains of blocks and rods of various lengths. Through the use of manipulatives, students can connect ideas to gain a deeper understanding. Student's achievement grows when they have access to manipulatives and are explicitly taught how these manipulatives can assist their learning and how they connect to other representations. Manipulatives not only increase understanding but also allow students to make sense of problems, make and test conjectures about a problem, and justify their solutions, just to name a few benefits. I explain the importance of the use of manipulatives here because too many times by third grade, these enriching tools become scarce. This unit will include developing number sense through the use of base ten blocks and Cuisenaire rods. These tools are an excellent way to bring students from whole number understanding to the understanding of fractions. Using these rods will enable students to begin to make connections to measurement and lengths. Students will use the rods to measure a large variety of items found in and around the classroom. They will be putting the rods together and modeling addition. I will then lead them to actual measurement activities using specific units. During their investigations, students will begin to realize the need for numbers in between whole numbers. Our discussions of fractions will begin and since students discovered authentically the need for these numbers, they will have a connection to their learning. This unit will progress from recognizing fractions as a number and placing them on a number line. I will also begin to explain how to discover equivalent fractions.

Students will obtain an understanding of fractions and this will aid them as they move into the arithmetic, (adding, subtracting, multiplying, and dividing) of fractions, however this will not be addressed in this unit.

Demographics

The Red Clay Consolidated School District is located in Northern New Castle County, Delaware with a combination of urban and suburban settings. Some of its elementary schools are located in the heart of the largest city in the state. The district comprises 28 schools with approximately 1000 teachers. It services over 16,000 students. Of those students, 27% are African American, 4% are Asian, 20% are Hispanic, and 49% are White. Students' needs vary, with almost 15% receiving Special Education Services and 10% receiving English Language support. In addition, 41% of the students come from families with low incomes.

Highlands Elementary is an urban school in the city of Wilmington, Delaware. We are a small K-5 school with an enrollment of about 300 students. Eighty-six percent of our student's make-up belong to ethnic minority populations, while 92% of the students fall into the classification of low socio-economic status. I am a third grade teacher with a class size varying between 20-26 students who are representative of the make-up of the school. Recently I have been designated as an inclusion teacher. I collaborate with Special Education teachers who come in to assist during the English/Language Arts block and during Mathematics instructions. I am the primary teacher responsible for instruction of all, general education and special education, students for all other subjects. Highlands recently has been designated as a Priority School by the Delaware State Department of Education. This designation is the result of lower than average standardized assessment scores. In the last year, we have undergone administration changes and about a 50% turnover in the teaching staff. We will be given specific proficiency targets developed by DEDOE and must meet those targets by the end of the 2017-2018 school year.

Using rods and ten blocks to recognize lengths

In order to build some background understanding for students, I plan to begin with having students work with Cuisenaire rods, base ten blocks, and other tools for connecting length with counting. In the initial experiences my students will not assign numbers to the rods but I will develop the idea that putting rods together is a version of addition. The act of placing the rods end to end without gaps will be reinforced, I will explicitly teach this process to my students so that they will be able to use the manipulatives in a mathematical way that will build their conceptual understanding that we are creating measurement sticks. This activity will allow my students to visualize varying lengths and begin to join lengths and compare lengths without assigning values. As students work with the manipulatives, I will encourage them to describe their trains in terms of how many units they have assembled. "My train is 5 units in length", will enable them to make the connection between length and number. Once students become comfortable with this connection between number and length, the concepts of joining trains end-to-end as addition will give them visual and concrete meaning. Using base ten pieces provides for an excellent transition from single-digit addition to multiple-digit numbers.

In our seminar we distinguished five Stages of Addition.

Stage 1 Putting bars together;

Stage 2 Measuring against a unit length;

Stage 3 Addition on the number line (number ray);

Stage 4 Vector addition of signed numbers on a number line;

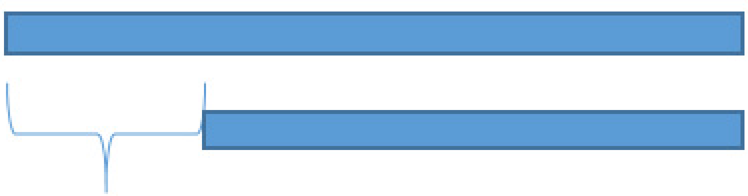
Stage 5 Addition as a translation of the number line.

My unit will focus on Stages 1, 2 and 3.

Stage 1: Putting lengths/ bars together

Length measurement has its own notion of addition and we shall begin by putting lengths together without worrying about the number, allowing students to compare lengths and make observations about this.

a.  putting two lengths together (combining- analog of addition)



b. comparing lengths (analog of subtraction)

As students engage with the manipulatives, they are building understanding of addition properties, and also that numbers can also describe length measurement.

Stage 2: Measuring against a unit length, connect numbers to counting via measurement

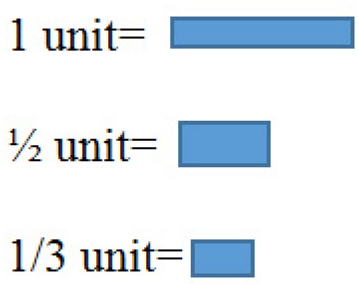
At this stage, a unit is designated and then addition and subtraction can be assigned numbers by measuring in terms of the unit. The operations of addition and subtraction of lengths can be checked to agree with the already learned ideas in terms of cardinality of sets.

 = 1 unit

A visual representation of 5+3=8

There are five units that are placed in a train or end to end. Then we place the three units on the end of the train to create a length of 8 units. Five units added to three units will create a length of 8 units.

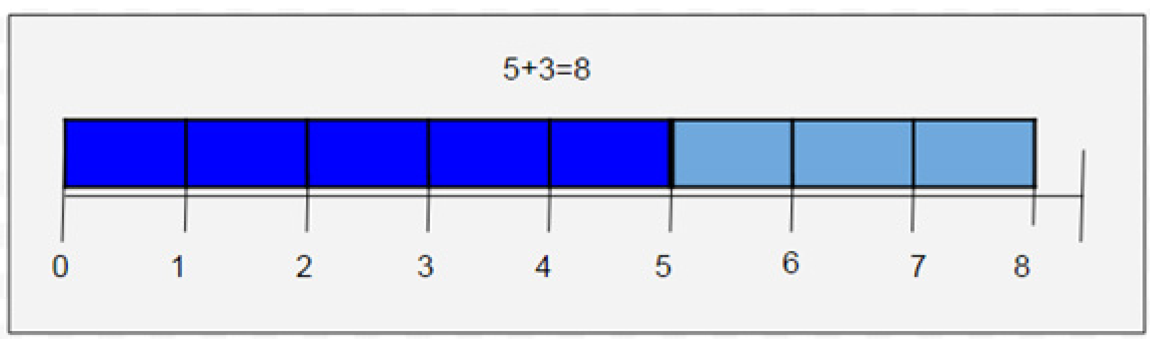
This can be refined to fractions by creating fractional units. Using Cuisenaire rods, I will establish one rod as the whole, and students will then investigate which rod they will need 2 of to cover the whole unit, I will explain that since we need two units to cover the whole, this new rod is ½ the size of the whole unit. We will repeat the process with the unit fractions 1/3 and ¼. At this point, I will emphasize the relationship between one whole unit and two equal units of ½ to cover the whole unit. This can be extended to then include equivalent fractions for later lessons, however I will not be addressing equivalency in this unit.



Geometrically the process is the same, as the bars don't care how long they are.

Stage 3 Addition on the number line (number ray)

The rods we are using do not have an intrinsic value. As we place them together and place them on a number line, value is assigned. I will be using number lines that correspond with the Cuisenaire rods. This will allow the bars to form a standard unit for the students to compare their trains to the numbers on the number line. In this stage the number line will create a standard model and an origin will be chosen. As the origin is determined, the rods then become a measurement from the origin. The number line then will give a standard exemplar for each length. So for the expression 5+3, take a train of 5 units then place the next train of 3 where the train of 2 ends. You have now created an addition model combining 5 to 3 end to end. The unit will measure 5 units from the origin. This models the principle of slide, as you place one train onto the end of an existing train along the number line.



Using base ten blocks for addition and subtraction

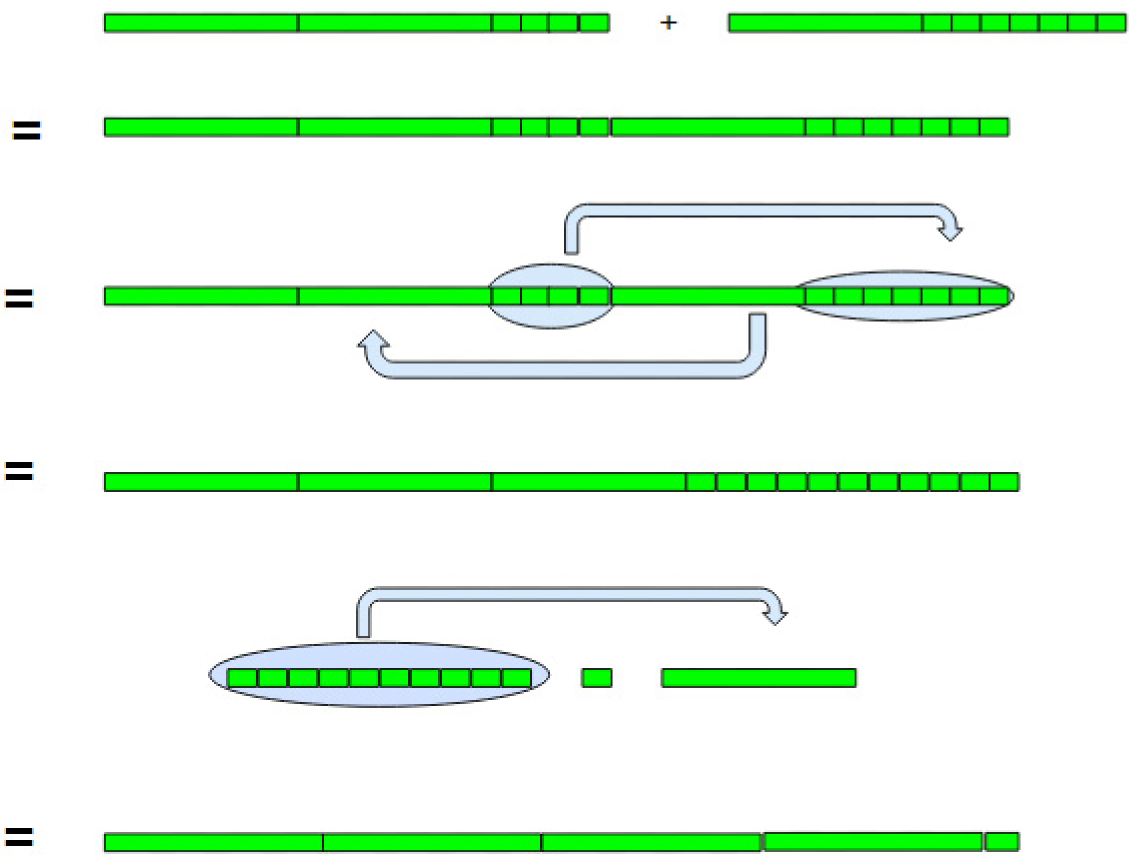
Now that students have made a connection between number and measurement, students can use base ten pieces to carry on this process as place value is uncovered as well as reinforcing procedures for adding across decades. They can do this by combining "trains" of base ten blocks. Suppose they want to add two two-digit numbers. Students can create a representation of the first two-digit number using base ten pieces and place the piece end to end to make a "train" of blocks with a given length. We will adopt a standard form for such trains, with the 10-rods on the left and the ones blocks on the right. Similarly, they will make a second train for the second addend. Next they will place the second train at the end of the first train. We will observe that the combined train is not in standard form, and will discuss what to do to put it in standard form. Students will then physically rearrange the pieces of the combined trains, grouping the tens pieces together on the left side and the ones cubes to the right, keeping the pieces, tens and ones, all grouped together without any gaps in a train. This movement exactly represents the symbolic procedures that many students have memorized and many have not assigned conceptual knowledge to yet. Students will make a train using base ten pieces to represent the expression: 24 + 17.

24+17= (20 + 4) + (10 + 7)

= 20 + (4 + (10 + 7)) = 20 + ((4 + 10) + 7) = 20 + ((10 + 4) + 7)) = 20 + (10 + (4+ 7))

= (20 +10) + (4 + 7) = 30 + 11= 30 + (10 + 1) = (30 + 10) + 1=

40 + 1= 41.



Students rearranged the trains of tens and ones. Once this was accomplished, they then saw they had eleven “ones” pieces and could make another ten train. Trading in ten of these “ones” pieces for a ten piece parallels the procedure for the algorithm. Students will notice that the rearranged and regrouped train is the same length as the first train, which may not be evident in this diagram. The final train exhibits the answer in standard base 10 form.

Recognize that the size of numbers can correspond to length on a ruler

The bars and trains students have been making do not have an intrinsic location in space, they essentially are just floating around until they are assigned an origin. Deciding on where the trains begin and assigning that place a value of 0 will create a number line. The number line will then give a standard exemplar for each length. As students begin to measure a variety of items throughout the classroom, I will point out the connection between their trains and a ruler. I will make sure to encourage students to measure longer items as well to allow them to be accustomed to working with larger numbers. In preparation for larger numbers, I intend to create a 100 train by gluing ten "ten's" pieces together. It is not practical to have every student have a "1000" train, however having one for the class to use will be invaluable, I will use a meter stick for actual measurement. This will allow students begin to formulate a visualization as well as a conceptual knowledge of the magnitude of the number 1000 in relation to 1. Students should independently discover the ease of using larger trains in measuring larger items. As they record their measurements, students should consistently say, "my item is so many cubes long." If they are two-digit measurements, I will have them report how many 10s and how many ones, and then state standard name of the length. Additionally, if they are three-digit measurements, I will have them report how many 100s, how many 10s and how many ones, and then state standard name of the length. This reinforces the importance of the unit used. Inevitably students will measure items that will not produce an exact whole number. Typically, students will report this as "about 21 units long". A better description of this situation would be, "my object is between 21 and 22 units long.". This language will naturally point out the need for numbers different from whole numbers in the realm of measurement. This then will set up nicely the need for fractions and point out that fractions are not a combination of two different whole numbers but a number between two numbers.

What is a fraction?

Fractions can be a stumbling block for many students. There are two points that can be relevant to this:

i) Students need to revise their understanding of what a number is: from a counting or additive conception to a ratio or multiplicative conception. A number line is a description of the relative size of a given quantity, as measured by another quantity of the same type, which thereby functions as the unit. In mathematics education literature, this is sometimes referred to as the transition from additive thinking to multiplicative thinking.

ii) In contrast to the situation for whole numbers, we do not have unique names for rational numbers; the number indicated by any given fraction is unchanged if both the numerator and denominator are multiplied by the same factor; and doing computations with fractions often hinges on selecting an appropriate representation, which may involve changing from the representation with which one it initially presented. In particular, to compare or to add or subtract two fractions, it is often necessary to replace the given fractions with equivalent fractions that have the same denominator.[1]

A fraction is a rational number, which means it can be written as a ratio of a whole number, a relationship between two numbers expressing how many times the first number contains the second number. Common

Core defines fractions in two steps;

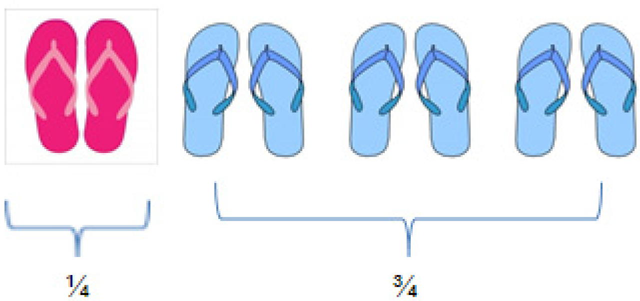
i) understand a unit fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts, or equivalently, as a quantity of which it takes *b* copies to make the whole;

ii) understand a general fraction *a/b* as the quantity formed by a parts of size 1/*b*.

A key understanding that will need to be reinforced is that for a fraction, the larger the denominator, the smaller the pieces. There are many models of fractions and I will show them examples using all of the models. Students develop misconceptions about fractions when they think of the numerator and the denominator as separate values; it is difficult for them to see that *2/3* is one number. For this reason, I will give a variety of examples in order for them to create a connection between fractions and other forms of measurement. We call an hour *1/24* of a day because there are 24 hours in one day. Other examples could be a day is *1/7* of a week, since there are 7 days in a week. A cup is *1/4* of a quart since there are 4 cups in a quart.

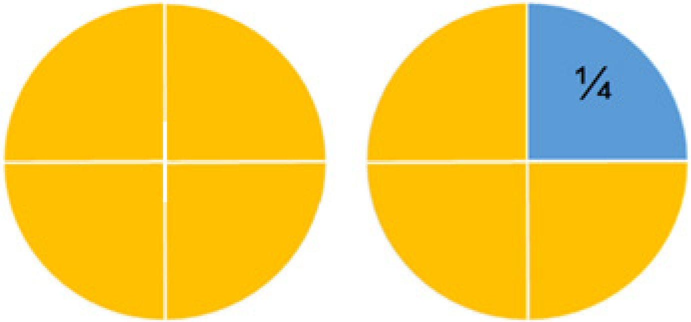
Set Model

The set model uses a set, meaning a designated piece, to express a fraction. The denominator tells how many total items are contained in the set while the numerator tells how many items are being represented by some qualifier. For example, my set contains 4 pair of flip flops, 1 pair of flip flops are red and the remaining three pair of flip flops are blue. I would then write the fraction *1/4* of the four flip flops are red. I will emphasize that the *1/4* functions as a size comparison. If you were to have four times as many of the red flip flop, you would have as many as are in the whole set of flip flops.



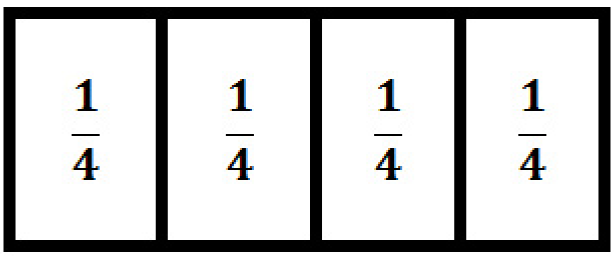
Circular Model

The circular model is a model many third graders see often. Problems usually involve dividing up a pizza into a number of pieces that would represent the denominator. The numerator could be explained as eaten pieces. For example, I brought home a large pizza that was cut into 4 equal pieces. I ate 3 pieces. How much of the pizza was left? The numerator would be the 1 piece left and the denominator would be the total of 4 equal pieces total.



Area Model

The area model has a determined unit, that is most commonly a rectangle due to the ease with which you can partition it evenly and can represent many real-life objects like chocolate bars. The rectangle is partitioned into *d* equal parts. This rectangle has been partitioned into 4 equal columns and each column shows ¼ unit of the rectangle.



One valuable feature of the rectangle area model is that the unit rectangle can be divided or partitioned into equal pieces, *1/d*, of determined size vertically and then further divided or partitioned into another set of equal pieces, *1/b*, of determined size horizontally which gives an excellent way to talk about renaming fractions, and will ease students into multiplication of fractions at a later date.

Linear Model

The linear model is represented on a number line, (or more accurately stated a half ray). This model reinforces the notion of length or distance, and most importantly there is a unit of length. The points on the line are labeled by the Measurement Principle.

The Measurement Principle states,

"The number labeling a point tells how far the point is from the origin/endpoint, as a multiple of the unit distance".[2]

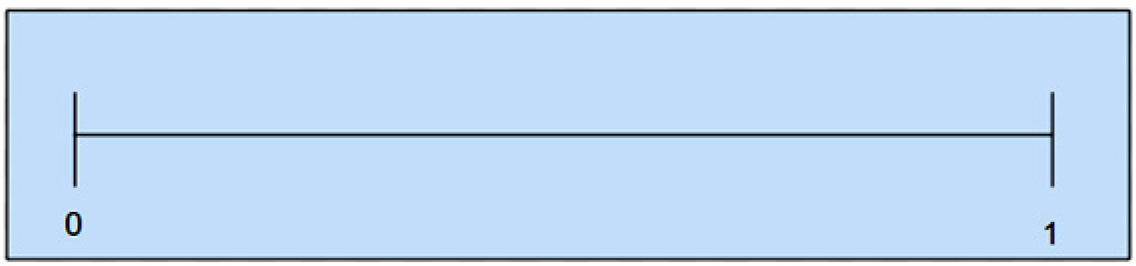
The end point of the number line is labeled 0 showing that there is 0 distance from this point. The unit interval, (0,1) has a unit length and the point labeled "1" is at distance 1 from 0. The 2 is then placed at the point that is twice a far from 0 as is 1. Therefore, the interval (0,2) is made up of two intervals of unit length, (0,1) and (1,2).

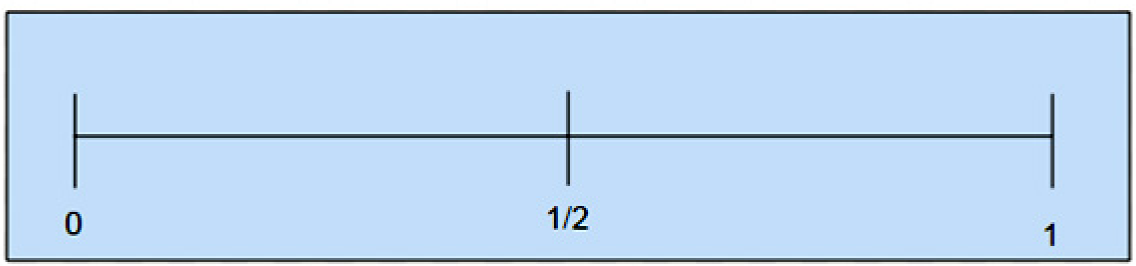
Fractions on the Number Line

Once the Measurement Principle has been explained, students should be ready to begin to understand that a fraction is a number and has a place on the number line. In elementary classrooms we technically do not work on a full number line since we only work with positive numbers and only need to go in one direction, positive. We can call this a number ray, a half line with an origin, which we label with a 0, and that travels in one direction from that point. This number ray essentially is just like a ruler. On a ruler, the end point is 0, and the point labeled 1 defines a standard unit of measure, perhaps an inch or a centimeter. Number lines do not need to have a standard unit, and can have an arbitrarily chosen unit to fulfill the desired need.

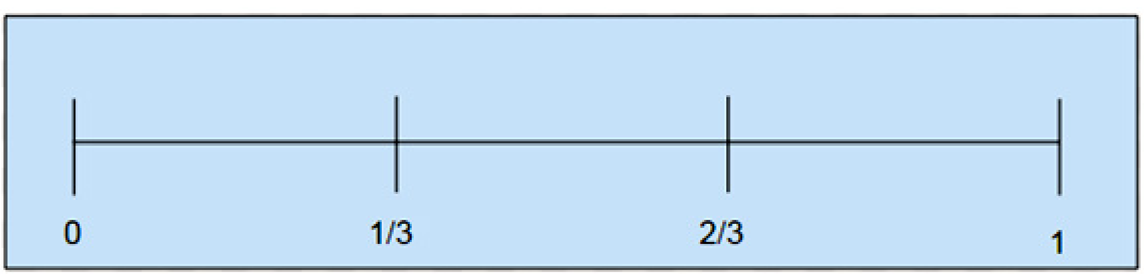
When students decide on the unit distance, the placement of all other numbers on the ray is determined. The number placed on the ray labels the distance that point is from the endpoint or origin. As I explained above, students should be given an opportunity to explore using rods and measuring items to help them develop an understanding of this principle. This should give them a better grasp as to why the numbers are placed on the number line and can begin to transfer this knowledge to the placement of fractions.

A good place to begin is with the fraction *1/2*. A defining property of *1/2* is that 2 of it will make 1. On a number line that has an endpoint defined with 0 and a 1 placed one unit distance from the endpoint, *1/2* will be place so that if you traveled from 0 to 1 you will have traveled that distance two times.

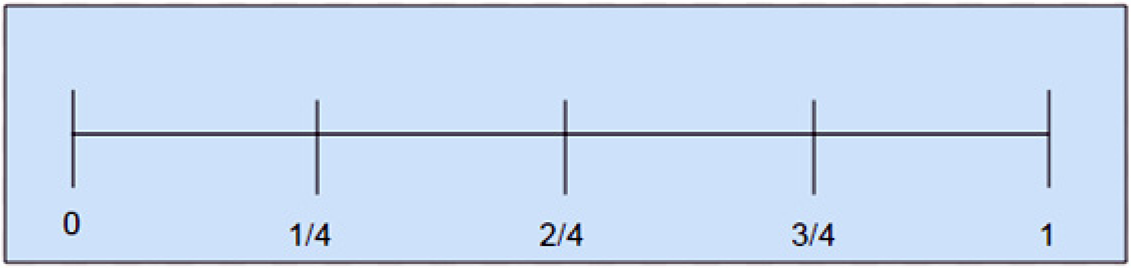




This same principle governs the placement of the other unit fractions. A defining property of *1/3* is that 3 of it make up the unit, so *1/3* should be at a distance from the origin that if you go that distance three times you get to 1.

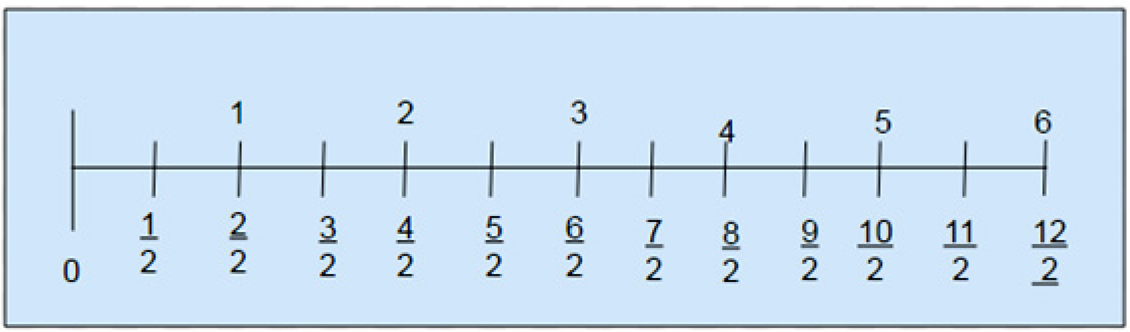


You can follow the same principle and place *1/4* is that 4 of it make up the unit, so *1/4* should be at a distance from the origin that if you go that distance four times you get to 1.



A good teaching sequence using these diagrams would be, show the number line for 1/2., set it up for 1/3, and let students work on it and discuss it, draw it, and then repeat for ¼. If ¼ works fairly quickly, you can ask about 1/5.

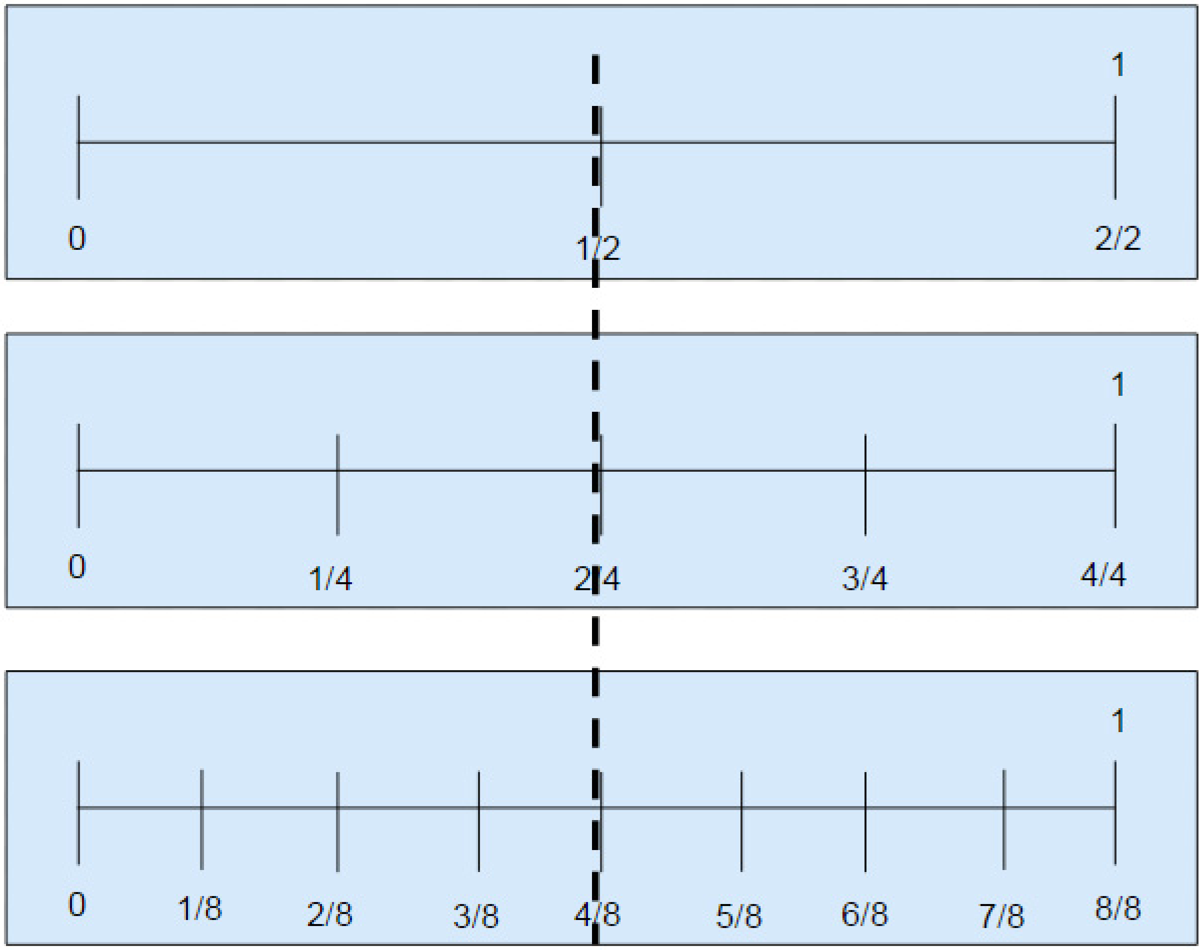
It is important to show students number lines that extend past 1. Students will need to see that fractions can be larger than 1. These pictures may help to strengthen the understanding that each unit fraction is indeed a new unit, that provides a finer means of measuring that extends the system of whole numbers.



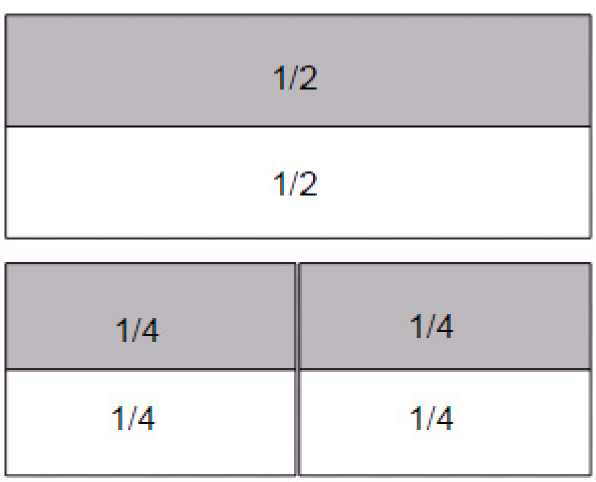
Equivalent Fractions

Equivalent fractions are fractions that represent the same number. Equivalent fractions represent the same point on a number line. When students model *1/2 = 2/4* on the number line, both fractions represent the same length unit. You can find an equivalent fraction for any given fraction by multiplying or dividing both the numerator and the denominator by the same number. “This is the Identity Property of Multiplication, which states that *Nx1 = N*. For instance, since *2/2 = 1* and *1/2 X 2/2=2/4*, then *1/2* and *2/4* are both equivalent fractions.”[3]

We will investigate several different strategies for helping students to grasp the principle stated formally above. Below is a diagram that shows equivalent fractions for *1/2*. Number lines can be used to show equivalence by adding partitioning lines. In order to find an equivalent fraction for *1/2*, the distance between 0 and *1/2* could be divided in half the distance and the distance between *1/2* and 1 could also be divided in half. Now the distance between 0 and 1 is divided into four equal lengths, so each of these is *1/4*. Students will be able to see that *1/2* can be also called *2/4* because two of the lengths of *2/4* make the whole, so *2/4* satisfies the definition of *1/2*.



Equivalent fractions can also be found using the area model. When a rectangle is divided into two equal parts and one part is shaded, one of those parts is *1/2*. When the rectangle is divided in half again, creating four equal parts, students can see that each of the *1/2*s is made of *2 1/4*s. This means that *2/3* equals *1/2*. These fractions are the same because they represent parts of the rectangle of equal sizes.



Teaching Strategies

Problem Solving Strategies

Math Problem Solving Circles

I had a big ah-ha moment as I wrote this unit. I frequently use literature circles in my English/Language Arts classes. Literature circles are an excellent strategy that infuses student centered inquiry with collaborative learning. Using this strategy encourages students to take responsibility for their learning based on the plan and choices they make. Students choose their own reading materials, lead the discussion, and engage with the texts and each other in a positive, authentic way. Why can't we create the same experience in math class?

I have now developed Math Problem Solving Circles where small groups of students engage with a problem and have designated roles that will help students to make sense of the problem, facilitate discourse, and decide which tools or strategies to use to help solve the problem. I create student groups of 3 to 4 students, then present the problem, and assign roles. I have developed four roles to best suit my classroom needs, they are; Reporter, Questioner, Strategizer, and Reflector. Each role has a purpose, yet the purpose is not to isolate the problem into pieces, the purpose is to provide all students the opportunity to access a problem, comprehend what the problems is asking, discuss a variety of possible strategies, and make connections to other mathematical concepts. The following descriptions provide a starting point and have suggestions for students' engagement. Students will be using a KWCSR chart while participating in a Math Problem Solving Circle. What a KWCSR is, is explained in detail below.

REPORTER: The reporter's role is to read the problem to the group and to fill in the "What do we KNOW" portion of the chart. The reporter will also lead the group's presentation to the rest of the class.

QUESTIONER: The questioner's role is to lead a discussion about what the problem is asking, what needs to be solved. The questioner should fill in the "What do we WANT to know" portion of the chart. The questioner should be asking throughout the process, does this solution make sense? Are we solving the question the problem is asking? Have we used the correct units? Do we need more information? Have we ever solved a problem like this before?

STRATEGIZER: the strategizer's role is to lead the discussion about which strategies would be most efficient to solve the problem. The strategizer should fill in the "What STRATEGIES will we use?" portion of the chart. They will help to ensure that group members are using a variety of strategies.

REFLECTOR: the reflector's role is to lead the discussion about following the plan the group is setting up and to think about its efficacy. Some questions the reflector can ask the group are; Will this plan help find a solution to the problem? Is there another way we could solve this problem? If there are multiple ways to solve the problem, is one way better than the other? The reflector should fill in the "What is the ANSWER?" section of the chart. After the problem is solved, the reflector will help lead the discussion about what each group member learned and how this new information can be used again by asking questions like, What have we learned? How can we use this information again? I believe this type of approach to problem solving will enable more students to become successful as they tap into their collective knowledge. I believe a barrier to problem solving for many students is the lack of experience and the issue for students of not knowing where to start.

Problem Solving Process

In my experience, when I talk about problem solving, many of my colleagues think I am speaking of word problems. I take a minute to explain to them the difference; word problems are math exercises that embed numeric equations into a variety of questions, and problem solving involves implicitly teaching students strategies to solve a variety of problems. There are a set of steps that students need to follow in order to become successful when they begin the problem solving process. Both have value in a mathematics classroom and should be used to cement understanding.

In my classroom, I have found that there are seven strategies that are appropriate and useful to my students: draw a picture, look for patterns, make a chart or graph, guess and check, work backwards, make a list, choose an operation. Each strategy is introduced along with several problems that lend themselves to that specific strategy. I also provide my students with a graphic organizer to help them organize and make sense of the problems. I am not a big fan of teaching key words because there are always a few problems that do not fit the key word rules and I think this also teaches students to focus on a set of words and not to think holistically of the problem. This approach is both inadequate and misleading.

Understand the Question: students need to read the questions carefully and develop an understanding of what the question is asking. Many misconceptions and errors began when students answer a different question than what was being asked.

Choose a Plan: as students begin to work with the problem, they need to decide which strategy will best aid them.

Try your Plan: this is the place in the problem solving process that students put their ideas into action. They are thinking about each step as they proceed and continue or make changes if necessary.

Check your Answer: Once students come to a solution they need to ensure their response is accurate. They should ask themselves some questions to guide their thinking. Did you answer the question that was asked? Does your answer make sense? Did you remember to use the correct units? Then they should redo the problem another way and try to get the same answer and check your math work for small errors.

After the solution has been determined students should then, Reflect: Think about what you have done and what you have learned. Also, students should ask themselves if there is anything they are still confused about.

Understanding the Problem using a KWCSR

Using the Standards for Mathematical Practice as a guide, I have worked to develop strategies that aid my students as they make sense of problems and persevere in solving them. My students use a revised KWL form specifically adapted to help in my math classroom. We call the graphic organizer a KWCSR chart. The K section asks students, What do you KNOW about the problem? This enables students to clarify the information within the problem and provides them a place to record information they will need to solve the problem. They must also make decisions to justify what information is needed to solve the problem and what information is superfluous. The W section asks students, What do I WANT to find out? Many times my students get confused as to what they are actually being asked in the problem and this gives them a place to write it down and focus on what they are solving. The C section asks students; Are there any CONDITIONS, rules or tricks I need to look out for? The S section asks students to list two to three STRATEGIES that they believe will help them solve this problem. Multiple strategies are listed so students know if one strategy is not working they can try another. The R section is the place where students REFLECT on their solution strategy and record their answer. In this section students will review their solution and make connections to other problems they have seen. I was finding that many of my students would work hard to solve a problem and then never finalize their work. This space reminds them to refer back to the W section and make sure they have answered the question they were asked.

Small Groups and Centers

Small groups and centers allow me the opportunity to differentiate the learning process and provide remediation to some students and enrichment to others. I try to commit one day a week to this strategy as it allows students the chance to apply and practice skills. Centers should provide meaningful, independent work for the students and should be open-ended in order to provide students with multiple entry points and solutions. It is important to set up routines in order to build independence and insure engagement.

Gallery Walks

Students will work in groups of 3 or 4 and will be presented with a problem to solve. The problems should be written at the top of large poster paper. Place the posters around the room. Students work together to solve the problem. After a designated time period (depending on the difficulty of the problems, I go for about 5 to 10 minutes), groups move to another poster. Students review the previous solution and then solve the problem using a different strategy. I usually allow for 3 rotations and then groups will present the problems to the class and a discussion will take place about the variety of strategies.

Classroom Activities

Activity One- Investigating Measurement

Essential Question: What will you do when an object measures in between two whole numbers?

Enduring Understanding: Students will be able to use rods to measure a variety of classroom objects. Students will be able to generate measurement data. Students will understand that a fraction is a number on the number line.

Procedure: Students will be given an assortment of rods in varying lengths. These rods could be Cuisenaire rods, base ten blocks, or any other type of manipulative that can be put together to form chains. Students will locate up to twelve classroom objects to measure. Students will focus on making chains of blocks and determining the lengths of the objects. The objects will include both lengths that are whole numbers of units and lengths that are not whole numbers of units. I will lead students in discussions about the lengths of their objects inquiring whether or not the length is nearly a whole number or between whole numbers. I will frame their responses to say, “My pencil is more than 7 units and less than 8 units.”. This will focus the students thinking to realize that there is a need for some type of unit in between the whole numbers. A sample recording sheet can be found in Appendix A.

Assessment: Exit Ticket- In your journal, answer the following prompt: What do we do when we measure something and it is in between two whole numbers?

Activity Two-Multiple Models of Fractions

Essential Question: What is a fraction? How can I use a number line to understand fractions?

Enduring Understanding: Students will be able to use multiple representations for fractions, including pictorial representations of the set model, circular model, area, model, and number line.

Procedure: Using our interactive math journals, students will create visual representations of unit fractions. As students move between the various representations, they will need to construct an understanding the relationship between the models. We will focus on unit fractions with denominators of 2, 3, 4, 6, and 8. Students will complete the worksheet located in Appendix B.

Activity Three-Equivalent fractions

Essential Questions: How can I find equivalent fractions? How can I show equivalent fractions using different models?

Enduring Understanding: Students will be able to understand what makes a fraction equivalent. Students will be able to find an equivalent fraction.

Procedure: Using paper strips that are equal in length, students will create fractions strip models. One strip will remain unfolded to represent one whole. Then students will color each of the remaining strips a different color, there is not a specific color scheme that must be followed I will choose primary colors. Students will color a strip blue and then will fold the strip in to two equal parts and label each part *1/2* and cut the strip on the fold. They will follow this procedure of coloring, making folds, labeling, and cutting for each unit fraction in our lesson; *1/3,1/4,1/6,1/8,1/12*. I chose to use these fractions because we can investigate many equivalent combinations with these. After students create their fraction strip models they will investigate finding equivalent fractions and complete the worksheet in Appendix C. Students will then use their math journal to place equivalent fractions on a number line. Using their fraction strips, students will partition one number line at a time with two different fractions. (A sample of this activity can be found in Appendix D, however many more variations of this activity can be completed.) Students will discover equivalent fractions and will label the lengths on the number line.

Appendix A

Name: ____

What did you measure? How many units? Was it exact or more than?

1.
2.
3.
4.
5.
6.

7.
8.
9.
10.
11.
12.

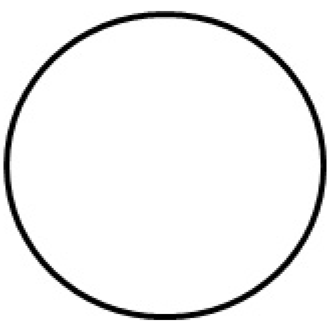
Appendix B

Name: ________________

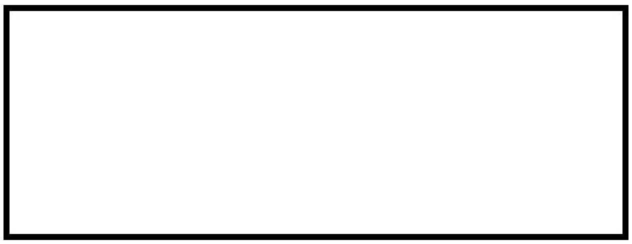
1/2

Set Model

Circular Model



Area Model



Number Line

Appendix C

Name: ________________

Using your fraction strips, find equivalent fractions for each fraction listed below and write the fraction in the next box. There definitely is always more than one.

1/2
1/3
3/4
4/6
3/8
10/12

Appendix D

Name: __________________

Use your fraction strips to partition each number line with the given fractions. Record all of the equivalent fractions you discover.

1/2 and 1/4

1/3 and 1/6

Appendix E

Ten Essential Strategies for Supporting Fraction Sense

Strategy 1: Provide opportunities for students to work with irregularly partitioned and unpartitioned, areas, lengths, and number lines.

Strategy 2: Provide opportunities for students to investigate, assess, and refine mathematical “rules” and generalizations.

Strategy 3: Provide opportunities for students to recognize equivalent fractions as different ways to name to same quantity.

Strategy 4: Provide opportunities for students to work with changing units.

Strategy 5: Provide opportunities for students to develop their understanding of the importance of context in fraction comparison tasks.

Strategy 6: Provide opportunities for students to translate between fraction and decimal notation.

Strategy7: Provide opportunities for students to translate between different fraction representations.

Strategy 8: Provide students with multiple strategies for comparing and reasoning about fractions.

Strategy 9: Provide opportunities for students to engage in mathematical discourse and share and discuss their mathematical ideas, even those that may not be fully formed or completely accurate.

Strategy 10: Provide opportunities for students to build on their reasoning and sense making skills about fractions by working with a variety of manipulatives and tools, such as Cuisenaire rods, Pattern blocks, fraction kits, and ordinary items from their lives.[4]

Appendix F

Vocabulary list

Word	Definition of term
partition	
equal parts	
fraction	
equivalent	
equivalence	
reasonable	
number line	

unit
unit fraction
interval
origin
endpoint
numerator
denominator
compare
equal to =
less than ˂
more than ˃
half/ halves
third
fourth
area model
benchmark
common denominator
common numerator

Appendix G

KWCSR

KNOW
WANT
CONDITIONS
STRATEGIES
REFLECT

Appendix H

Math Problem Solving Circles

Create student groups of 3 to 4 students. Present the problem and assign jobs.

Jobs: Reporter, Questioner, Strategizer, Reflector

Reporter:

- Before: Discuss what we know about the problem. What are the variables?
- After: Reports the group’s solution and reflections to the class.

Questioner:

- Before: Discuss what the problem is asking, what needs to be solved. Do you need any more information?
- After: Lead discussion in group, does our answer makes sense? Did we answer the question that was asked?

Strategizer:

- Before: What strategies would you use for this problem? Discuss what other problems you have solved before that remind you of this problem. What strategies were successful?

Reflector:

- Before: Discuss your thoughts about the plan you have set up? Will this plan help you find the answer? Why or why not?
- After: What have we learned? How can we use this information is again?

Reporter

Read the problem to the group.

Before you solve: Begin to discuss what you know about the problem. Talk about the math facts that you see in the problem. Do you understand the problem?

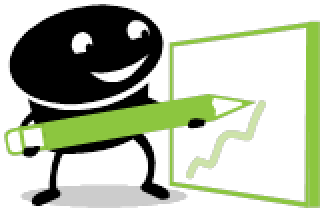
After you solve: Report the group’s solution and reflections to the class.

Questioner

Before you solve: After reading the problem, lead the discussion about what the problem is asking you to solve. Do you need any more information? What units will your answer use? Do you understand the problem?

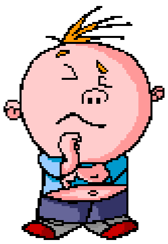
After you solve: Lead discussion in group, does our answer makes sense? Did we answer the question that was asked?

Strategizer



Before you solve: Lead the discussion, does this look like other problems? Begin to list which strategies may work for this problem. Which strategies would work best for this problem?

Reflector



Before you solve: Does our plan make sense? What could go wrong?

After you solve: What have we learned? How can we use this information is again?

Implementing Common Core State Standards

CC.3.NF.1- Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.

CC.3.NF.2a- Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line.

CC.3.NF.2b- Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

CC.3.NF.3a- Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

CC.3.NF.3b- Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

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Endnotes

1. (R. E. Howe n.d.)
2. (R. E. Howe n.d.)
3. (McNamara and Shaughnessy 2010)
4. (McNamara and Shaughnessy 2010)