Yale National Initiative
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Curriculum Units by Fellows of the National Initiative
2016 Volume V: The Number Line in the Common Core

The Starting Line-Up: Analyzing the Number Line to Conceptualize Foundational Skills for Algebra

Curriculum Unit 16.05.06, published September 2016
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Introduction

This is an introductory unit designed for my 9th grade Algebra 1 classes. The majority of students who are taking my class have not taken pre-algebra and only took middle school math, though a handful of the students took middle school pre-algebra or algebra. The unit, however, addresses concepts that should be prior knowledge to all students from the elementary years but is not. It would, therefore, be immediately applicable as an introductory unit in any secondary high school mathematics course. The unit, with very little modification, would also fit well into an elementary level class. The complexity of questions and techniques can be adjusted depending on the age or ability level of the students.

I work at a small comprehensive high school where the student body is incredibly diverse. My classroom contains students who represent a full range of academic abilities. In addition, students come from a variety of backgrounds that reflect the demographics of Washington DC as a whole. There are a large number of recent immigrants and English language learners, from a wide variety of countries and cultures. This enormous variety of backgrounds and skill levels is at the front of my mind as I design this unit.

I believe this unit will be extremely helpful for the students that I teach. I teach in the District of Columbia public school system, an enormous and diverse urban district that serves the entire city. Based on the data received from my beginning of the year assessments over the past two years at DCPS, a large majority of the students entering into the ninth grade are performing below grade-level in mathematics. While some students are able to answer simple questions that only show understanding at the abstract level of the math involved, they still have not fully conceptualized foundational math skills. In the 9th grade the students will be asked not only to answer high-level questions about Algebra but also to write explanations about their mathematical thinking in both brief and extended constructed responses in preparation for the standardized PARCC examination. I believe that if we start off the year with this unit where we connect students' prior knowledge of these skills through the study of the number line, we will build a solid conceptual foundation for their continued success throughout the school year.

The focus of the unit, "The Starting Line-Up: analyzing the number line to connect foundational skills to Algebra", develops what is considered to be prior knowledge, things the students should already know coming into my class, to the number line. These skills include adding and subtracting, using signed whole numbers, and understanding fractions. During this review I will use the number line and associated manipulatives as the primary tools to ensure that students have an abstract as well as a conceptual understanding of these foundational skills through discussion. Many students already have an abstract understanding of the skills, as they are able to solve math problem sets in isolation to one concept and without any application. The plan is to push the students towards a conceptual understanding where they are able to apply their learning to a variety of situations and contexts.

The unit will start with ensuring student understanding of numbers and the conceptualization of the addition and subtraction operations. Once the basic understanding of these operations exists, I will transfer these skills to the number line to re-explain by addressing the ideas of placing, shifting, and reflecting numbers on the number line. Having the visualization of these operations will increase student conceptualization of the operations and they will be in the beginning stages of understanding the effects of shifts and reflections when graphing in Algebra. Additionally, the number line will assist students in recalling the mental math skills needed to increase mathematical fluency. Such mental skills include identifying large shifts when skip counting, and using friendly numbers.

After this introductory work, we can begin using numbers to solve equations. We will begin our understanding by representing base-10 numbers by using base ten bars and transferring them to the number line for addition and subtraction. By transferring back and forth from base-10 blocks and base-10 bars to ten strips, students will begin to understand the connection between basic operations, measurement, and graphing. This connection is not often addressed by teachers and can lead to misconceptions in Algebra I when solving and graphing algebraic equations.

I will progress through the instruction using the base-ten connection to addition and subtraction on the number line to encourage continued conceptual understanding from whole numbers to representing fractions by demonstrating the parallels of the arithmetic of fractions and the arithmetic of whole numbers. Finally, I will incorporate solving problems with signed numbers on the number line by acknowledging the idea of direction and its relation to positive and negative numbers on the number line. At the end of the unit through the use of the number line model, students will have the conceptual understanding of numbers and the number line needed to solve problems that require addition and subtraction of signed, whole numbers and fractions in the Algebra I course.

In each lesson, students will use learning strategies such as the use of manipulatives to work from concrete to pictorial to abstract comprehension of the operations on the number line. I will encourage active learning through student-to-student discourse and intentional math discussions in the classroom, employing the use of Socratic Seminars. These discussions increase student understanding and they also prepare students to express their understandings both orally and collaboratively. It will be important to plan high-level questions with an understanding of how to structure the conversations to encourage debate while knowing what to listen for, what to highlight, and what misconceptions to address. It will be important for students to hear a wide variety of strategies and thought patterns as they gain their own conceptual understanding of mathematics.

Content Background

The Measurement Principle and Placing Whole Numbers on the Number Line

The number line is the primary model used in this unit to increase students' number sense. The number line model assists students' conceptualization of numbers and the relationships between them. The conceptualization comes from the student ability to visualize the numbers, the position of said numbers, and the operations done to them. This model will develop gradually so students can develop this understanding at any level.

In order to re-teach whole number placement onto the number line in a way that students may conceptualize the process, they must first look at numbers as a measurement of length. The linear number line model assists students in understanding numbers by way of the measurement principle. According to the measurement principle, in order to determine a number, an origin has to be determined in addition to a unit interval before you can label points by numbers.

The Measurement Principle: The number labeling a point tells how far the point is from the origin/endpoint, as a multiple of the unit distance.

The endpoint itself is labeled 0, since it is at 0 distance from itself. The unit interval [0, 1] has unit length, and 1 is at distance 1 from 0. Then 2 is twice as far from 0 as 1 is, so the interval [0, 2] is composed of two intervals of unit length, namely [0, 1] and [1, 2] and similarly with larger whole numbers. That is to say, the label of a point is determined by the ratio of the unit length to the distance from the point to the origin.

Traditionally in the math classroom, students have made connections between numbers and the number line by means of counting up from left to right without a conceptual understanding of the number placement. Instead I will address counting to students by discussing the origin and units of measurement being used to deepen understanding. I will initially address the geometry of the line, discussing the distance and origin. Next, we will discuss the orientation or direction of the line. This is done by determining the "ends" of the lines in regard to the direction of positive or negative movement. Finally, numbers can be placed on the number line accordingly.

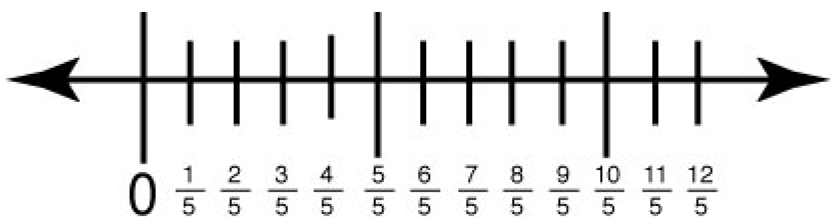
Fractions as Ratios/Placement of Fractions

When looking at the conceptualization of fractions as ratios, students will use the number line as a model to strengthen their understanding. Students must look at fractions as numbers. The students will use the number line to see fractions as numbers and begin by again looking at the origin, 0 and looking at the interval from 0 to 1, where 1 represents the unit whole. Teachers will instruct students on how to partition and place fractions on to the number line. They partition, place, count, and compare fractions. The conceptualization comes into place as students connect the measurement principle to the distances between fractions.

> "The number 1/2 goes in the middle of the unit interval, so that [0, 1/2] and [1/2, 1] have the same length, which is 1/2. The label 1/2 goes with the middle point, because then the point 1 is two times as far from 0 as 1/2 is. Similarly, the points 1/3 and 2/3 are the points that partition the interval [0, 1] into three equal subintervals, so that each interval has 1/3 the length of [0, 1]. The label 1/3 goes with the interval that starts at 0, since the point 1 will then be 3 times as far from 0

as is 1/3. The label 2/3 goes on the other 1/3-division point, since there are two intervals of length 1/3 between it and 0, or in other words, it is 2 times as far from 0 as 1/3 is. Similar reasoning serves to locate fractions with larger denominators on the number line" [1]

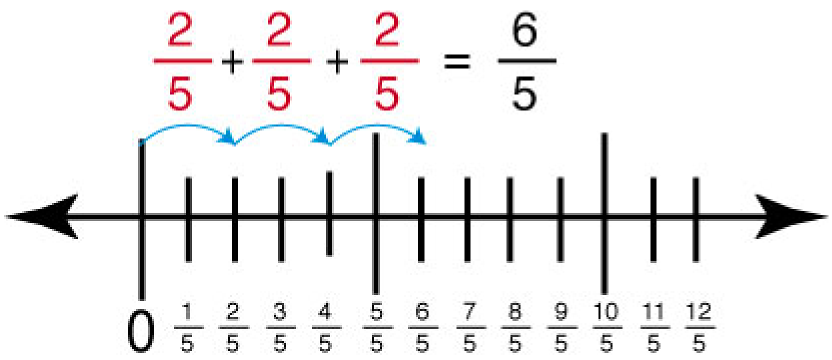
I will work carefully with my students to make sure they know how to place fractions on the number line. This is not understood by many students and is an important misconception to address since students are often only taught to work with fractions in isolation. Students need to understand that placing fractions involves the same process as placing whole numbers. If students understand how a fraction n/d on a number line with a fixed denominator will create a variety of points with equal intervals the length 1/d. This visual will help generate a deeper more conceptualized understanding of fractions on the number line as they will see it's similarity to the placement on whole numbers on the number line. Once they reach this point of conceptualization, it will be easier for students to be able to compare, contrast, and perform operations with fractions.

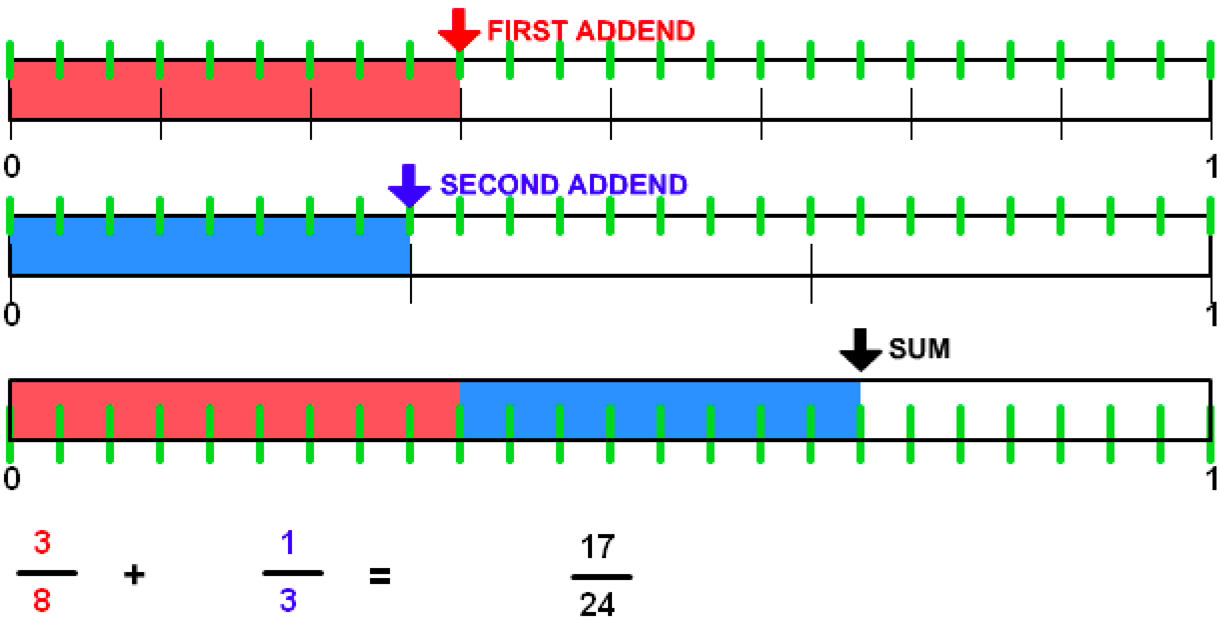


I am eager as a teacher to revise my student's conceptual understanding of fractions to measurement because the previous memorization and visual aids have not been successful in the long term. The linear model is more effective than the frequently used pie model because of the support of unitization. When my students can visualize the unit fraction as the basic structure for all numbers and fractions on the number line, I believe that they will gain conceptual understanding of the fractions. This model also lends itself to instruction that can interchange representations from concrete to pictorial to abstract, which differs from previous student learning.

Adding on Number Line as Combining

With the number line model, the students are at the point where they are able to accurately envision numbers on the line as distances from the origin. It is now appropriate to use representations of addition on the number line for computation. To begin conceptualization, students will use bar models on the number line to represent numbers (or lengths) and then begin the process of putting the lengths together. The concept being taught to students is the idea of seeing addition as the combining of lengths. It is important next to make the connection to the measurement principle so students understand the connection to measurement by addressing the numbers and the units. This is visualized by discussing the unit length, its connection to the given number or length, and the combination of two or more lengths, followed by measurement to reach your answer or total number of units. This will continue to be refined when discussing fractions by using the fractional unit. It is important with fractional addition that students understand that geometrically the process is the same and even when a unit is defined the bar lengths may differ in length.



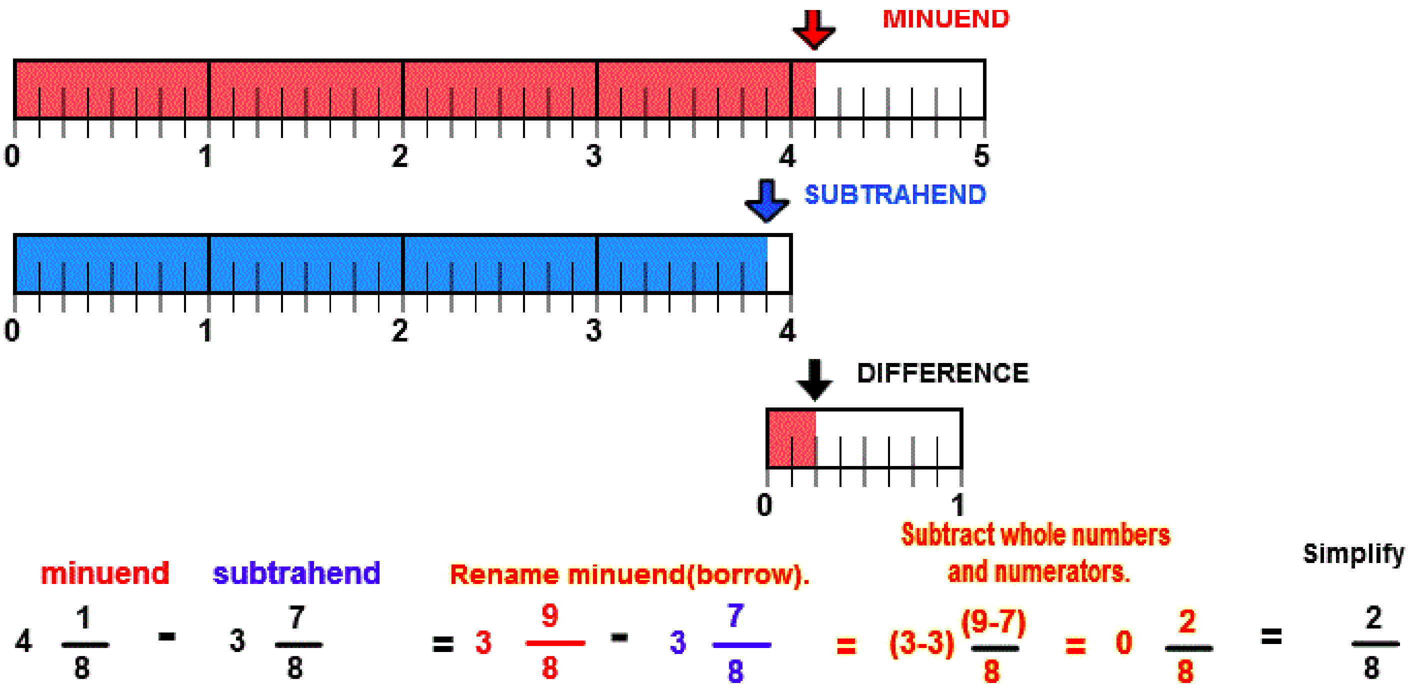


Here we have divided the unit interval into intervals of length 1/24. This lets us see that each 1/3 of the interval is 8/24, and each 1/8 of the interval is 3/24. This means that 3/8 + 1/3 = (3x3)/24 + 8/24 = 9/24 + 8/24 = 17/24. The number line does the computation for us!

Next students will use the number line as the arena for solving addition problems with attention brought to orientation and direction when working with signed numbers. I am in hopes that using the number line for addition will continue to support students as they develop deeper number sense and number relationships. Previous student learning has relied on simple models and memorization that often resulted in a low level of conceptual understanding and computational facility needed in higher grades. The number line is a simple tool that if used after students understand the measurement principle, will assist in creating a conceptual and procedural understanding due to the strong grasp of position and measurement in addition.

Subtracting on Number Line as Comparing

Subtracting on the number line will be done similarly to addition, but instead of combining lengths, students will compare the lengths. When introducing subtraction, students must concretely see the bars but instead of putting the bars together they must line the bars up in order to compare them. Students will continue to bear in mind the measurement principle when comparing the two bars distances from the origin. It is important to show students how to use the number line to measure the differences in the unit lengths. This can be looked at by the subtraction of the units or finding the “missing part” of the two lengths. Once this is understood, I will continue to practice subtraction with students but with the introduction of vector addition of signed numbers on the number line. This is the point where addition and subtraction start to become unified.



Students have traditionally been taught to see subtraction as taking one number away from another, as opposed to comparing two numbers. This basic understanding does not lead to a conceptualization of subtraction and leads to misconceptions when students are working with fractions and signed numbers. The number line model allows students to continue to see how numbers and operations are all connected to measurement. By transitioning their thinking from separation to comparison, the students will have an easier time understanding subtraction as a translation on the number line, which will ultimately have a greater impact on their ability to make connections from arithmetic to Algebra.

Content Objectives

Established Goals and Understandings

This unit will address several standards in the Common Core State Standards, the Mathematical Practice standards, and the course objectives. Since the skills being addressed are prior knowledge for the high school students and the unit will serve as a review, the Common Core State Standards addressed come from the earlier grades. The focus area of these standards being Operations and Algebraic Thinking as well as Numbers and Operations in Base Ten. The standard goals include developing an understanding of numbers, identifying units, developing an understanding of fractions as numbers, and representing and solving problems involving addition and subtraction.[2]

The unit will be based around "provocative questions that will foster inquiry, understanding, and transfer of learning". [3] These essential questions will include: *What is a number? How do you define addition? How do you define subtraction? How does the number line connect everything we know about the operations?*

This instructional unit pushes students to acquire key knowledge and key skills in order to be prepared for the remainder of the academic year. Using number lines, Cuisenaire rods, base ten blocks, technology and active learning, students will be able to accomplish the following objectives in sequential order:

- Learn that numbers on the number line increase from left to right (addressing the direction/orientation/geometry of the number line)

- Learn the measurement principle

- Conceptualize counting by continuing the idea that each successive number refers to a quantity that is one more unit than the previous number/recognizing that the number before any given number is one less than that number

- Practice placing whole numbers on the number line (with a variety of units) and explaining why the numbers are being placed where they are (demonstrating the conceptualization of unitization)

- Partition the number line to demonstrate an understanding of fractions

- Reason with fraction placement, referencing the origin and distance

- Compare fractions using the measurement principle

- Discuss addition as putting lengths together (concrete)

- Demonstrate conceptualization of addition by discussing addition and it's connection to measurement by addressing the numbers and the unit

- Demonstrate conceptualization of addition of fractions by discussing addition and it's connection to measurement by addressing the fractions and the fractional unit

- Use the number line as the actual model for solving problems with addition

- Discuss the idea of orientation and how that impacts distance (particularly with signed numbers)

- Discuss subtraction as comparing lengths (concrete)

- Demonstrate conceptualization of subtraction by discussing subtraction and it's connection to measurement by subtracting the numbers and the unit and measuring the difference

- Demonstrate conceptualization of subtraction of fractions by discussing subtraction and it's connection to measurement by addressing the fractions and the fractional unit

- Use the number line as the actual model for solving problems with addition and subtraction

- Discuss the idea of orientation and how that impacts distance (particularly with signed numbers) and how this idea is unified with both addition and subtraction

If students master the skills needed within these objectives, they should maintain a stronger conceptual understanding of the fractions, addition, and subtraction for their continued use in Algebra.

It will be necessary to administer diagnostic exams before the Unit to identify student areas of need and to create supplemental work for students who have already mastered both an abstract and conceptual understanding of these skills. The students will receive a diagnostic exam to include addition and subtraction, fractions and percentages, decimals, and order of operations. To supplement learners working below or above the addition and subtraction stage, they will be working on iReady, a computer software that is adaptive that will target students directly where learning is needed.

Teaching Strategies

Vocabulary Instruction Using Cornell Notes

Cornell Notes are a tool that will be beneficial in teaching the vocabulary required for this unit. The Cornell Note style of note taking requires students to take notes but also generate questions and summaries to ensure their understanding. The Cornell Note taking system assists students in "reviewing their notes and make deeper understandings after the learning has taken place".[4] In the article "Narrowing the Language Gap: The Case for Explicit Vocabulary Instruction", Kevin Feldman and Kate Kinsella break down the importance of vocabulary instruction in the classroom. They discuss how it is important to focus on vocabulary instruction and take vocabulary beyond just looking up definitions in a dictionary. I agree that it is important to explicitly describe vocabulary, to make sure no student homes in on a wrong student-given answer, in a "who knows what this means?" situation.

Direct vocabulary instruction is seen in such small quantities in Mathematics classrooms across the United States. In all subjects, it is vital for vocabulary instruction to take place, particularly in mathematics where terms are often foreign to the students outside of the classroom setting. Vocabulary is a key element to the curriculum and without content-based vocabulary instruction; students may not understand entire units and the concepts within them. Feldman and Kinsella brought up great points in this article about why we should not have students use the dictionary to learn vocabulary. Dictionary definitions tend to be very confusing and skip words that make the definition too obvious. That is the opposite of the type of definitions teachers should use when explicitly teaching vocabulary. In addition to the confusing wording and the length of these definitions, there is also no way for teachers to ensure students actually understand the usage and meaning of the words.

The Cornell Note-taking model however is a strong and suitable for all learners. The model starts with discussions around the vocabulary. It is important for language learners to develop vocabulary by hearing more vocabulary both by reading aloud themselves and by hearing read aloud readings where teachers emphasize important vocabulary. The model then moves towards the idea of directly teaching specific words, which is one of the most important keys to successful vocabulary instruction. The most important tools for all students are the strategies for word learning. These skills in breaking down and analyzing parts of words for meaning will assist all students as they continue their learning through high school, college, and adulthood. Finally, fostering opportunities for students to work with vocabulary through designing questions and reflecting back on their questions to summarize their learning will support their vocabulary acquisition.

Using Manipulatives

The usage of manipulatives is beneficial for learners when teaching this unit. Students who are visual learners or do better with concrete examples will strive by using physical objectives as facilitators to learning. There are a variety of different objects that were developed to help instruct students with addition and subtraction. Manipulatives allow learners to get more hands on to gain a greater grasp of the concepts. Research from both learning theory and classroom studies shows that using manipulatives to help teach math can positively affect student learning. This is true for students at all levels and of all abilities. It is also true for almost every topic covered in elementary school mathematics curricula. Papert calls manipulatives "objects to think with." Incorporating manipulatives into mathematics lessons in meaningful ways helps students grasp concepts with greater ease, making teaching most effective."[5] This connects to this unit because the addition and subtraction skills can be modeled with manipulatives and will be a great tool in solitary or in conjunction with another teaching strategy.

Blended Learning

A teaching strategy that is vital to this curriculum unit is the incorporation of technology through blended learning. To many, literacy is defined as the ability to read and write. In schools, literacy is focused on the ability to read books, write papers, and understand underlying concepts such as phonics, syntax, semantics, and so on. According to the article "Confronting the Challenges of Participatory Culture", many adults while very knowledgeable about traditional means of literacy they "lack basic information that would help them deal with both the expanding media options and the breakdown of traditional gatekeeping functions". With the expansion of gatekeeping literacies through the introduction of new Medias and technologies within the classroom, the definition of literacy in turn has expanded.

New media consists of all the culture changes that came about after the changes in technology the latter half of the twentieth century. Today's youth live within this new media culture. Youth see culture as their ability to access all things at all times. Because of the importance of the new media culture, the definition of literacy should expand to include the elements of the new media culture. According to Jenkins, the new media culture "shifts the focus of literacy from one of individual expression to community involvement… the new literacies almost all involve social skills developed through collaboration and networking which build on the foundation of traditional literacy, research skills, technical skills, and critical analysis skills taught in the classroom". [6] With this change of definition to the term literacy, literacy education in the classroom must change.

Going forward, it will be increasingly important to consider the new media and participatory cultures as we think of education. In order for students to be successful in a new media literate world, they must learn these skills in school to better prepare students for their future in the new media technology driven world. I plan to implement a blending learning model within this unit by using interactive websites, virtual manipulatives, and the adaptive software iReady as tools for review, reinforcement, and modification purposes.

Intentional Math Talks

The key to ensure that students understand these retaught skills will be to engage them in intentional class discussions. I want to ensure that my students are confident and competent when engaging in class discussions. I ensure students are confident by never discouraging student's attempts. I don't ignore their errors but I make sure that I commend the students for fixing a mistake. I also make an attempt to make sure questions for guided discussions are crafted with clarity and vocabulary that is appropriate for my students. This promotes student competency. At times I may repeat the key points so students are certain of the discussion topics and are more comfortable with talking about them.

I believe that academic talk is a type of presentation where you discuss an academic idea. I sometimes engage in an academic talk with students at the beginning of units for students to self-discover the unit. I include evidence checks throughout our talks to ensure that students are indeed responding to my instruction. (Appendix A)

Socratic Seminar

The Socratic Seminar is a strategy used to facilitate in-depth, meaningful conversations between students. The strategy allows students to discuss and question the work they complete while collaborating to reach conclusions on the topic at hand. Socratic Seminars are based on analyzing text and higher-level problems in math by raising questions around them and working as one cohesive unit to find solutions.

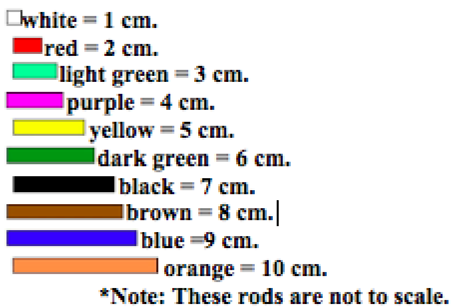
The key to creating successful Socratic Seminar questions is to develop a question that does not necessarily have one "correct" answer, rather a question that encourages debate and dialogue. According the *AVID Write Path Mathematics* text, "Eventually, when participants realize that the leader is not looking for the "right answers" but instead is encouraging them to think out loud and to openly exchange ideas, they discover the excitement of exploring important issues through shared inquiry. This excitement creates willing participants, eager to examine ideas in a rigorous and thoughtful manner".[7] The idea is for the facilitator to question students based on their learning to think about the "why" and "how" of mathematics. A good question provided by the facilitator should encourage the seminar participants to explore the topic in their minds in order to come up with even more questions for discussion. "In Socratic Seminar, participants share with the leader the responsibility for the quality of the seminar. Good seminars occur when participants study the text closely in advance, listen actively, share their ideas and questions in response to the ideas and questions in response to the ideas and questions of others, and search for evidence in the text to support their ideas". With the use of Socratic Seminars, students are able to develop thoughts and ideas to increase conceptualization.

Socratic Seminars are extremely beneficial in the Mathematics classroom. Socratic Seminars allow students to use their social skills in an academic world while simultaneously increasing their questioning and reasoning skills. Socratic Seminars also allow students to hear a variety of responses, strategies, and techniques to encourage a collaborative learning experience opposed to one that is teacher lead. Students who may struggle with expressing Mathematics in a written form are able to express their arguments and connections orally. Working in heterogeneous groups allows students who may have a misconception about a topic, work with their peers to understand and work through the content. The expectation is that all strategies and answers, whether right or wrong, are acknowledged and respected during the discussion. While the teacher will prepare questions to continue the conversation, it important for the Socratic Seminar to be student-led with little teacher input. As a result with this work, not only will student confidence grow when it comes to discussing mathematics, additionally students should have an easier time responding to brief and extended response questions on assessments.

Classroom Activities

Cuisenaire Rods/Manipulatives

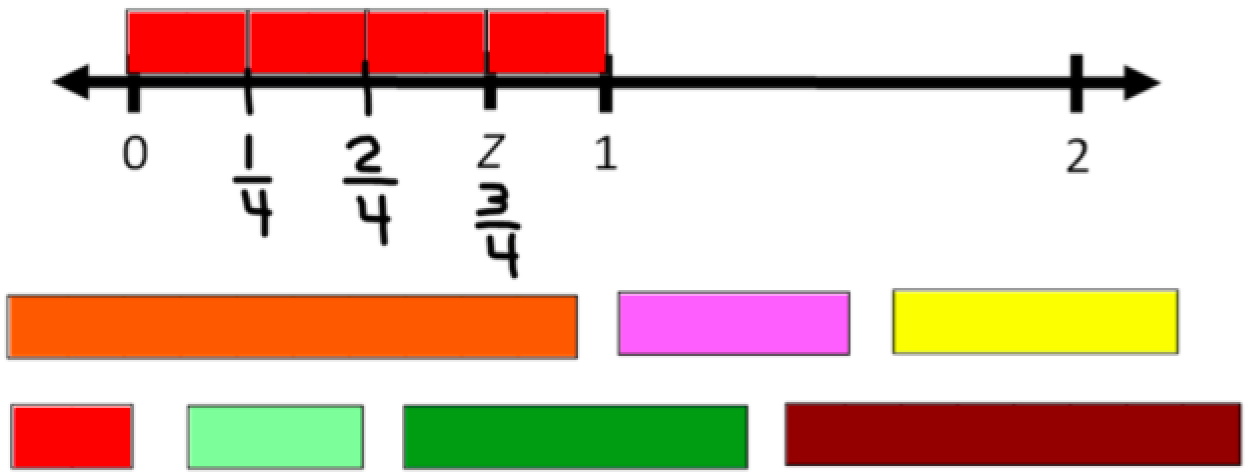
Manipulatives will be used daily during lessons. Cuisenaire Rods are the key tool utilized in this topic as they align perfectly to the number line. Cuisenaire Rods are useful because they allow the students to have visual representations of the numbers and their relationships with one another. They are multi-colored for visual conceptualization and help to engage students in the learning. There are ten different types of Cuisenaire Rods.



These rods when lined up along the number line will give the students a deeper understanding of the mathematical properties. Not only will this help with addition and subtraction of whole numbers, I can use the Cuisenaire Rods to represent fractions as well.

The first topic to discuss when using the Cuisenaire Rods is determining the unit value. When we discuss unit value, it will be important that students understand the Cuisenaire Rod sizes and their relative size compared to another rod. It will be important to determine what the value of each color rod is how that will impact our work with the manipulatives. Once this is determined, students will be able to complete addition and subtraction of whole numbers with the Cuisenaire Rods along a number line. (Appendix B)

When I begin work with the addition and subtraction of fractions, we must first model what fractions look like with Cuisenaire Rods. During instruction, students will first use the rods to generate as many models they can for a variety of fractions: ½, ⅓, ¼, 1/10. This will lead us into the conversation about the unit value and determining the value of the varied colored rods according to the unit value. Students will then use their Cuisenaire Rod models to create number lines on paper.



Students can continue this work with the Cuisenaire Rods and with online virtual models (Appendix C) to ensure conceptual understanding before they begin adding and subtracting on the number line.

Field Work

Student fieldwork will occur at the beginning of our learning. On two separate trips, students will be creating number lines from maps of our neighborhood and the local metro system. These maps will be tied into daily lessons when addressing unit value, signed numbers, and the varied operations.

During the first activity, students will be creating a map of 5th Street. When creating the map of 5th Street, students will mark as the 5th & E Street intersection where the school is located as the origin. From that point, each block will be labeled as one unit. During our walk, students will also be required to note the distance from block to block as well as the time it takes to walk from block to block. When mapping, we will note any major landmarks and where they are located. Once all data is collected, students will return to class and share their findings to create one class map of the neighborhood. The map will be linear and have unit markings that we will address when going through problems throughout the unit. The students will complete the same process separately on the metro train. Students will travel the Red Line metro line to create a map, identify distance and speed, mark landmarks, and determine each stop as a unit. We will again create a classroom map that becomes our “real-world” number line.

Once the class maps are created, students will use them as a reference to answer and create real-world application problems. Street blocks and metro stops will be used as units and the unit values will vary depending on if the students are addressing the amount of stops or blocks, the distance between them or the time it takes to travel between points. I will generate example problems for the students but will push the students to do the lift of applying the maps to their addition and subtraction learning.

Discussions

We will engage in Socratic Seminar style conversations when addressing signed numbers with addition and subtraction. Students will sit in a circle to discuss what they have learned about adding and subtracting numbers. Questions to be discussed are not limited to but include:

- What is a number?

- What does it mean to add?
- What does it mean to subtract?
- Where are negative numbers located on a horizontal number line?
- Where are negative numbers located on a vertical number line?
- What is the opposite of 2?
- What is the opposite of 0?
- Describe the relationship between 10 and -10.

Students will be pushed to spend at least one minute discussing each topic. Students additionally will have white boards on their laps in the event they would like to provide a visual to support their comments in the conversation. During the conversations, teachers should be monitoring participation and noting commonalities and unaddressed misconceptions during the conversation. The teacher should attempt to stay silent throughout these conversations allowing students to work through each question using math talk sentence stems. (Appendix D)

Appendix A

Socratic Seminar Student Performance Rubric

	5	4	3	2	Total
Behavior	-Patient with differing opinions. -Asks for clarification. -Brings others into the dialogue. -Very focused on the dialogue.	-Respectful. -Comments, but does not attempt to involve others. -Generally focused.	-Participates but shows impatience. -Some focus. -Engages in “sidebar” conversations.	-Disrespectful. -Argumentative. -Does not participate.	
Speaking	-Speaks to all participants. -Articulates -Takes a leadership role without monopolizing the discussion.	-Speaks to most participants. -Attempts to move on to new ideas. -Tends to “ramble on” after making a point.	-Speaks too softly. -Needs prompting to get involved. -Has no sustainable point; uses “sound bites.” -Monopolizes the discussion.	-Reluctant to speak. -Comments do not support point.	
Reasoning	-Cites relevant text/notes. -Relates topic to outside knowledge and other topics. -Makes connections between own thoughts and others’. -Willing to take an alternate viewpoint. -Asks questions to further dialogue.	-Makes limited connections to others’ ideas. -Some intriguing points that merit reaction. -Some references to text.	-Accurate on minor points, but misses the main point. -No prior learning support; “talking off the top of your head.” -Refuses to acknowledge alternate viewpoints.	-Illogical comments. -Ignores the movement of the seminar.	

Listening	-Writes down comments, questions, and ideas. -Builds on other's ideas & gives others credit.	-Generally attentive and focused. -Responds thoughtfully. -Takes *some* notes.	-Appears disconnected. -Takes limited notes.	-Inattentive. -Comments show lack of understanding. -Takes no notes.

Appendix B

Foundational Work with Cuisenaire Rods

1. Use the rods and the rod-length comparison chart above to answer the following questions:

a. If the white rod is 1 then the __________ rod is 7

b. If the white rod is 2 then the __________ rod is 14

c. If the __________ rod is 1 then the orange rod is 5.

Orange Rod = 5				
1	1	1	1	1

What color rod fits here? _________

d. If the __________ rod is 1 then the blue rod is 3. Sketch and label two trains to show this.

Appendix C

Cuisenaire Rods- Online Virtual Models

- http://www.mathplayground.com/mathbars.html
- http://www.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modeling-fractions-with-cuisenaire-rods/
- http://teachertech.rice.edu/Participants/silha/Lessons/cuisen2.html

Appendix D

Math Talk Question Stems:

- I think __________ because _____________.
- My first step is ____________.
- I still have a question about ________.
- I learned _______ when _______.
- _____ is important because _______.
- If ________ then _________.
- The answer is ________ because _______. A better strategy would be ________ because _______.
- The factors that are most important are _______ because _______.
- I predict that _______ .
- I believe that ________ will happen because ________.
- _______ and _______ are similar because _______. ______ and _______ are different because ______.
- Another way to look at _______ is _______.
- Another example is _________.
- This reminds me of _________ because _________.
- I believe this is true because ______.
- I agree with ______ because ______.
- I disagree with _______ because _______.
- ____'s idea reminds me of ___________.
- I solved the problem by ________.
- I proved my thinking by ________.

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