Curriculum Units by Fellows of the National Initiative
2016 Volume V: The Number Line in the Common Core

Rational Number Placement on the Number Line

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Context

The students whom I've been teaching have always seen the various ways of writing numbers symbolically – fractions, decimals, and integers - as siloed representations of quantities with no relation between them. For instance, they learn counting principles at early ages. This continues to more sophisticated counting principles like money combinations. But later on in their math careers, students learn about decimals and fractions, without making the connections to their previous learning at the primary levels. Many curriculums that I've seen use the area model to represent fractions. This is a fine concept and is important to help students visualize rational numbers however; my students view this representation as independent from their previous learning. Most of my students are left with an incomplete idea of what fractions represent when using the area model. Difficult as it may be, changing from the area-model approach to a number line approach will allow my students to have a fuller idea of basic number sense topics. The number line is the perfect vehicle to give these different representations of numbers a common home.

I work at Lowell Elementary, which is a neighborhood school in a K-8 building located next to Humboldt Park in Chicago. With a 19% African American and 79% Hispanic population, the overwhelming majority of students are low income (95%). We have a large diverse learner population with a similar percentage of students having limited English proficiency. With all of the daily struggles my students face, Lowell is still in good standing and is a Level 1 school. This means that Lowell is exceeding the district's expectations in student tests scores and closing the achievement gap. Lowell also has good attendance and is creating a successful school climate that puts learning first. Students are below average in math attainment in 2015. However, they have made above average growth overall according to the NWEA results.

I found that my students need concrete representations of abstract concepts to solidify their understanding of number sense topics. A tool that is readily available to them, but often forgotten, is the number line. By the 7th grade students are expected to, according to the CCSS, apply previous knowledge of fractions and solve real-world examples using all four operations. Without a stronger foundation on fractions and what they really represent, students will fall sort of being fully able to understand why certain truths are evident and how the rules and formulas are developed. My class is all about the why to a problem, not just getting the answer without making any connections to previous learning, connections that they so desperately need. Classroom routines are usually a shock to the students who have me for the first time. Not only are they unable to explain the mathematics, my students are not used the rigor of thinking and application to connect the dots. The challenge I face is weaning students off of answer driven and very surface-level understanding. My students have a difficult time using rote memorization, let alone being able to apply formulas in slight variations.

Description of Structure of Unit

Major concepts will continue to be introduced throughout the year in the form of Math Talks and other types of discussion-based classroom techniques. This unit will be taught in its entirety and is intended to lay the foundations for establishing a routine after the initial ideas are understood. This unit is remedial in nature and can be taught as intervention as well. But because of its position at the 7th /8th grade level where students are deepening their fluency of number sense topics, this unit will set up my students to be successful mathematical problems solvers in the future. This unit is intended to be used in succession with Aaron Bingea's unit titled: *Adding and Subtracting Rational Numbers on the Number Line.* While my unit focuses on the foundational skill of placing various classes of numbers on the number line, Aaron's unit will deal with the operations of addition and subtraction.

Progression of Concepts

Joint Unit Concept Progression

Unit 1- Rational Number Placement on the Number Line

Concept #1: Establishing the Measurement Principle and Placing Positive Whole Numbers on the Number Line.

Concept #2: Placing Positive/Negative Integers on the Number Line and Introduction of Numbers as Vectors.

Concept #3: Comparing Integers on the Number Line

Concept #4: Introduction to Unit Fractions and Defining General Fractions as Multiples of the Unit Fraction.

Concept #5: Placing Fractions on the Number Line

Concept #6: Placing Decimals on the Number Line Using the Expanded Form

Unit 2- Adding and Subtracting Rational Numbers on the Number Line

Concept #1: Adding and Subtracting in the Context of Length Measurement

Concept #2: Adding and Subtracting Integers as Vectors

Concept #3: Adding and Subtracting Fractions with Like Denominators

Concept #4: Adding and Subtracting Fractions with Unlike Denominators

My unit will not cover the entire list of concepts below. The list is provided to describe the full progression of both units. Before operations can be addressed fully, placement of numbers on the number line is the most important foundational concept. After the list is a narrative of the necessary explorations in which my students will be participating.

Concept #1: Establishing the Measurement Principle and Placing Positive Whole Numbers on the Number Line

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The Measurement Principle

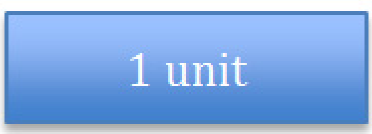
In order to remedy the siloed nature my students' view of a number, and to help them develop a unified sense of number, I plan first to give numbers meaning by exploring where numbers come from. Without exploring this very basic principle, there is no foundation to build on. Before getting to the number line in greater detail, my students will participate in a process of measuring everyday objects, from first principle, without using a standard tool. Each group of students will be asked to measure anything in their book bags. They will be asked to determine how big is their object with no other context besides making meaning for themselves.

Throughout this process, I expect, and will encourage, my students to try to acquire some unit of measure to compare their object to. Students will innately try to discover that their object is 4 fingers, or 3 pen caps in length. During this process, students will not be able to exactly align their “unit” into multiples. The object will be a multiple plus some leftover length. The answers that I will get from the class will be vary from exact to in between unit lengths.

Next, the students will be asked to team up with another group and start to combine their objects. This will allow students to compare how they measured their individual object and come to a conclusion about which unit measurement is most efficient. This should lead them to conclude that we need some sort of standardized unit. The development of the unit is of the upmost importance because this will come into play when whole and rational numbers are developed later in this unit.

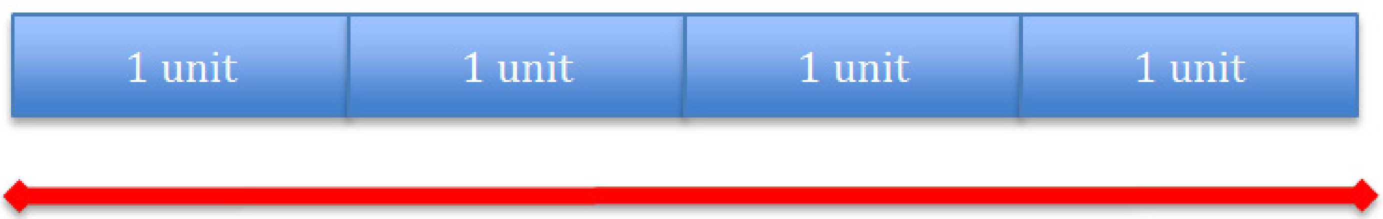
Students will see that numbers are generated from standardized measurement. First, the unit must be introduced, whether the length is a bar or a fixed counting manipulative, it does not matter. Below, I have defined the unit arbitrarily in *Figure 1*. This is compared to other bars that are multiples of that unit length. Students will then repeat the above exercise but using the standard unit.

Figure 1



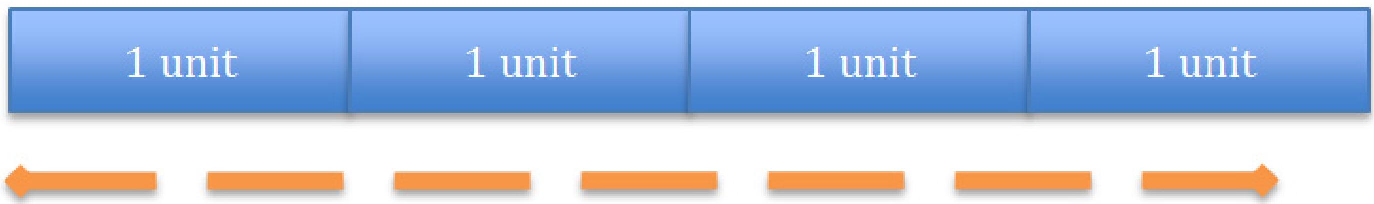
An arbitrary object will be chosen. It can be measured in terms of the unit by putting unit lengths together and lining them up against the object. There are no gaps in between unit lengths. We now can say that this object has a length of 4 units with no leftover lengths.

Figure 2



Next, we can see that the length of the object is defined as the number of unit lengths that fit into it, without overlap or gaps. Later students will see that is exactly the principle for placing numbers on the number line. The broken line below in *Figure 3* has measure not exactly 4 units, but a little less. We know that in terms of the unit measure that this second object is between 3 and 4 units in length.

Figure 3



This leads to error in measurement and may create greater error when measuring more than one object together. This is not an ideal procedure when measuring; we will need take parts of this unit measure to more accurately measure objects.

Placing Positive Whole Numbers on the Number Line

Once students have grasped the measurement principle, they can apply this understanding to the number line. They can use it as a viewpoint from which to see numbers in terms of distance and length. By having students measure objects using standardized units, I have laid the foundation for the number line. The importance of the unit is that it can be used to develop the rest of the whole numbers to the right of zero. We must do the following:

i) Choose an origin. “0”

ii) Choose a unit interval to represent the distance from “0 to 1.”

Now that we have a fixed standard model, we can start to place each whole number as a multiple of that distance. In other words, the location of whole numbers on the number line depends on what is chosen as the 1. Every whole number following 1 is a multiple of the unit distance i.e. the distance from 0 to 1.

Note, this implicitly gives us an orientation. Going to the right of the origin will give birth to positive numbers. Later we will see inversely, that laying off multiples to the left of the origin will give negative us whole numbers. Keeping in mind the measurement principle:

iii) The distance from 0 to n is n times the unit distance.

Along with orientation, this will give us 3,4,5...and -3,-4,-5... as well. This will allow my students to be making the connection between counting number and measurement number.

Concept #2: Placing Positive/Negative Integers on the Number Line and Introduction of Numbers as Vectors.

Placing Positive/Negative Integers on the Number Line

Every integer has an exact location on the number line. Whether the integer to be located is positive or negative, the idea is the same. To locate a number on the number line, my students will simply count the unit intervals from zero in the appropriate direction, as indicated by the sign of the number. This concept will not require a whole lot of time nor attention.

Figure 4 shows locating 4 on the number line. Almost all of my students will be able to do this before entering my class. The next series of steps will be taken to help increase fluency of placement of whole numbers.

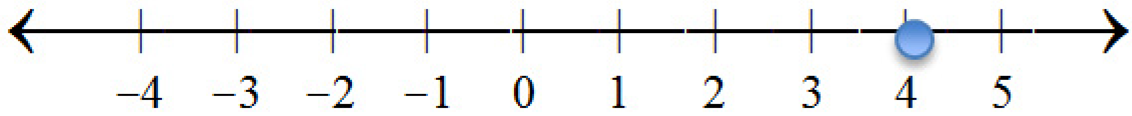


Figure 4

In order for this concept to be relevant and grade appropriate to my students, missing or incomplete information will be provided to them. This will be delivered in two types. First a number line with unlabeled partitions and labeled endpoints will be outlined below. Students will be asked to identify the point with the arrow. Answer to below: -16.

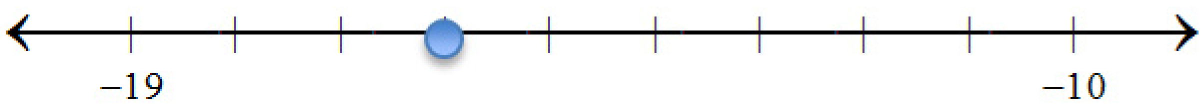


Figure 5

Second, students will be asked to fill in the incomplete sections of the number line. This will increase familiarity of the succession of integers and build off of counting principles that were learned at an earlier age. Through practice of this kind, my students will better understand placement and order of integers.

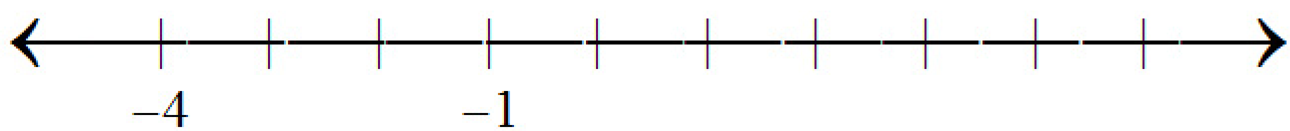


Figure 6

I will adjust the remainder of the activities when appropriate. For instance, I could give my students the same number line and ask them to locate the final integer 3 units in the positive direction from 10. Or I could use the terminology 3 greater than 10. I would like to change the terminology and give prompts that change sign as well. Also, changes in scale might be useful when these are revisited later in the year. These multiple forms of number placement will be a continuing idea that could work its way into a Math Talk. Further extensions and examples of number lines with missing information can be found in the Appendix section labeled A2.

Introduction of Numbers as Vectors

Before moving forward with comparing and ordering whole numbers, I need to lay the foundation of representing whole numbers as vector arrows on the number line. This needs to be established now for operations in the next unit. Numbers have direction and magnitude. *Figure 7* is a representation of these vector arrows pointing to both positive and negative 4. The base of the arrow will start at the origin and end at the number's location. This will help students further understand that numbers are a distance from zero, but not simply a length: an oriented or directed distance, or *vector*.

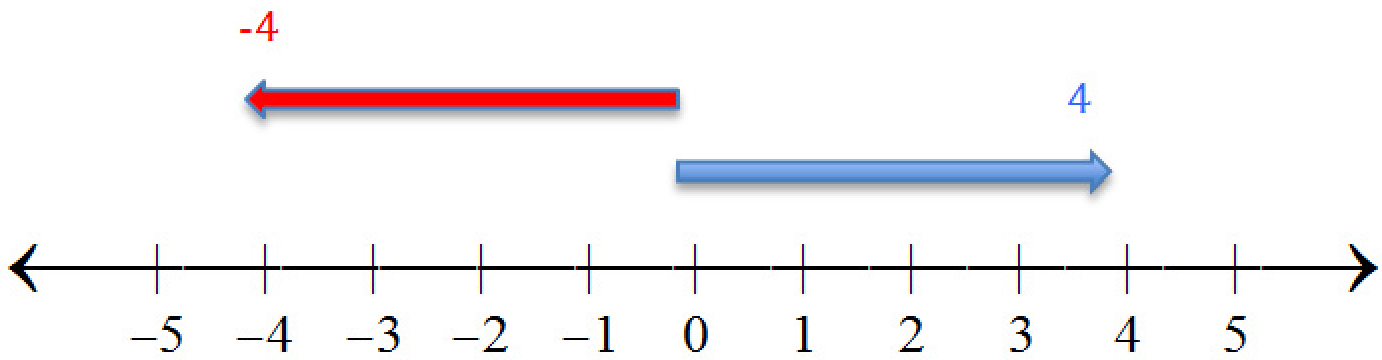


Figure 7

Orientation is something that confuses my students a great deal. Using vector model prompts similar to *Figure 7* and asking my students to express signed numbers as vectors, they will understand that the measurement equivalent of sign is orientation. More vector placements are found in the appendix.

Concept #3: Comparing Integers on the Number Line

When working with number lines, my students will be able to see that these whole numbers inherently have order to them. They will see that the larger number will live to the right of the smaller number. This concept is rarely realized by my students because I have had to explain why -2 is greater than -3. The confusion arises from my students thinking about magnitude. Indeed the magnitude of -3 is greater than -2. I want to promote the idea that -2 is more positive that -3 and I have to be careful with the terminology that I use. Having students compare different types of pairs of numbers on the number line will help them visualize which number is more positive, or to the right. Once they have located these whole numbers, students will be able to place inequality signs between the pairs to formally order them. This activity will be approached in stages. First, students will compare sets of positive numbers.

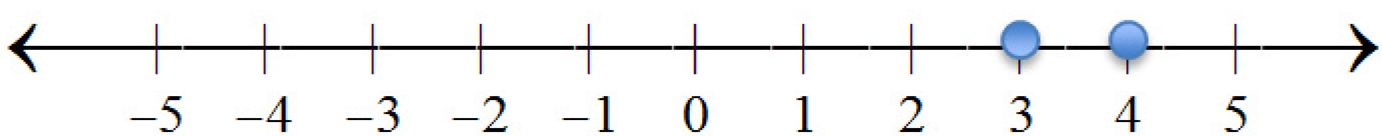


Figure 8

Figure 8 shows a number line with 3 and 4. Conclusion: 3 < 4. Students should interpret this as "4 is to the right of 3" or as being more positive. Next, students will compare one negative and one positive number.

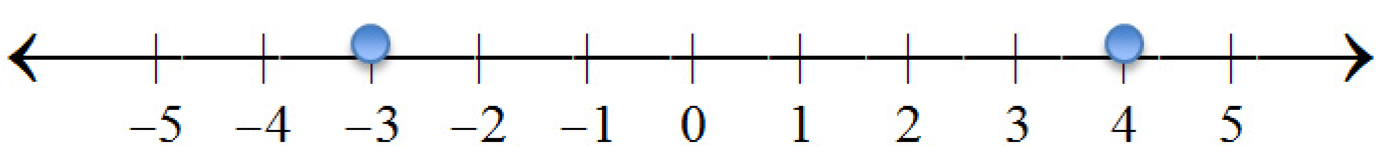


Figure 9

Figure 9 shows a number line with -3 and 4. Conclusion: -3 < 4. Students should interpret this as "4 is to the right of -3." This should cause no trouble, however in terms of magnitude, a more difficult problem would be -4 < 3. I will encourage my students to follow the progression of problems at attack them the same way. Placing or locating the integers first will help minimize errors in thinking. Last, I will have students compare two negative numbers.

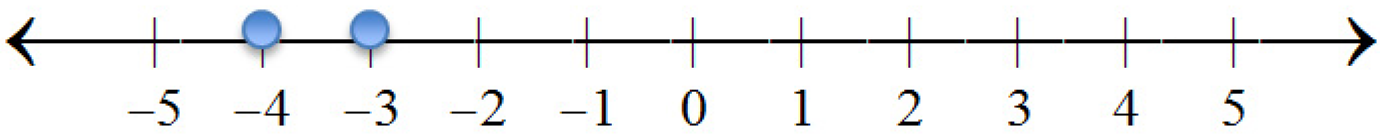


Figure 10

Figure 10 shows a number line with -3 and -4. Conclusion: -3 > -4. Students should interpret this as "-3 is to the right of -4."

The above-mentioned placement and comparison models are taught in 6th grade. However, they are still relevant to explore in this unit. This will help lay the foundation for operations in Aaron's companion unit that mainly deals with addition and subtraction. Various numbers sets for comparison will be found in the appendix. A2

Concept #4: Introduction to Unit Fractions and Defining General Fractions as Multiples of the Unit Fraction

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Introduction to Unit Fractions

My students need to improve their facility with fraction. In a very clear way students need to see [1] that number sense cannot be directly taught. Rather, it emerges from an awareness that there is a connectedness to the subtle relationships among concepts and procedures. This is exactly what my students are lacking. They rarely see the connections between subtle nuances. Doing so will lead to deeper understanding.

Whole numbers with orientation and placement have already been discussed. Hopefully by this point in the unit, my students will confident in placing and comparing integers. As mentioned before, revisits will often take place so that information will be retained throughout the unit and beyond. After the measuring activity using the unit measure, students were left with an incomplete answer to how big their object was. Instead of having a range of distances as outlined in *Figure 3*, we will need a more refined way to measure to make their measurements more exact. We will start to supply this by studying placement of unit fractions on the number line.

A length is *1/d* if n of it makes up the unit length. Students will have to grapple with the unit of partition and the whole. That is, a unit fraction is in some sense a new unit, of which it takes *d* to make the original unit. This will help my students think about size. Especially, how each numerator and denominator controls the size.

As mentioned before, students are taught to see fractions in terms of an area model. This model needs to be transferred to the number line as a distance. This will erase the misunderstanding that fractions are just slices of pizza or portions of a brownie tray. The area model is important for students to understand. However, the number line is much better at putting these concepts together under one roof. To transition my students' thinking about fractions to the number line, they will be asked to create area models of fractions with denominators 2-10. They will then crate a picture similar to *Figure 11* for each different denominator. Students will have to pay particular attention to the size of their intervals on the number line because they have to be the same size. This activity will also assist my students in learning the placement of these fractions later on in the unit.

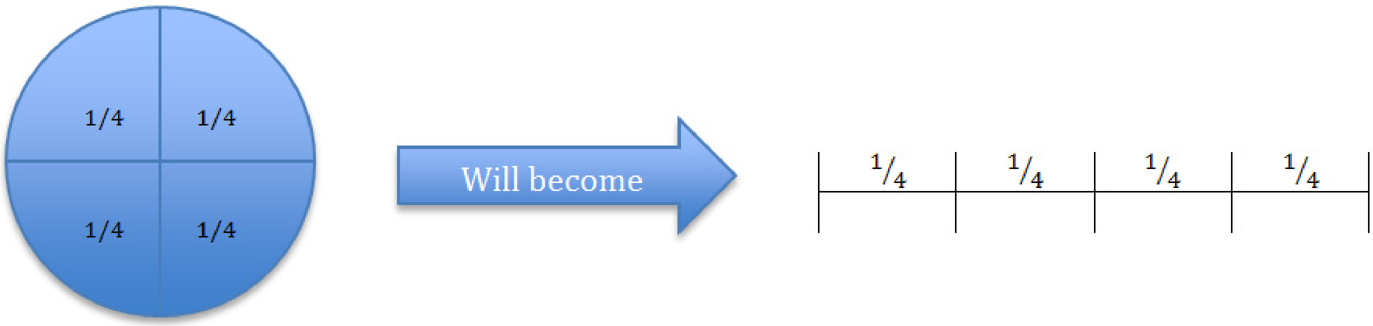


Figure 11

Defining General Fractions as Multiples of the Unit Fraction

My students have been taught at the primary level that a general fraction is the product of a whole number and the unit fraction. That is, the general fraction k/d is k times 1/d or k copies of 1/d. Furthermore, my students will see the connection between the definition and the location of each fraction between 0 and 1.

Below, in *Figures 12-14,* are a few examples of zooming into the unit interval from 0 to 1. Students will be expected to place all unit fraction multiples from halves to tenths. From this point, students should see that the endpoint of the whole number 1 would just be another fraction. For example, 4/4 will be the 1 in *Figure 11*.

Figure 12 – Fourths

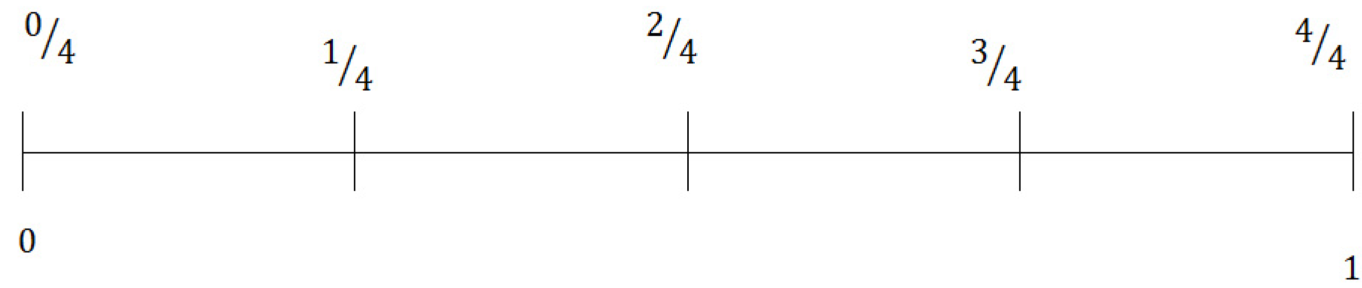


Figure 13 – Fifths

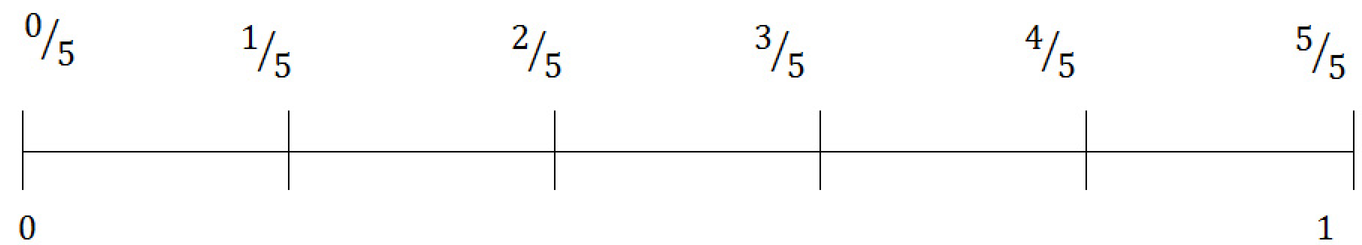
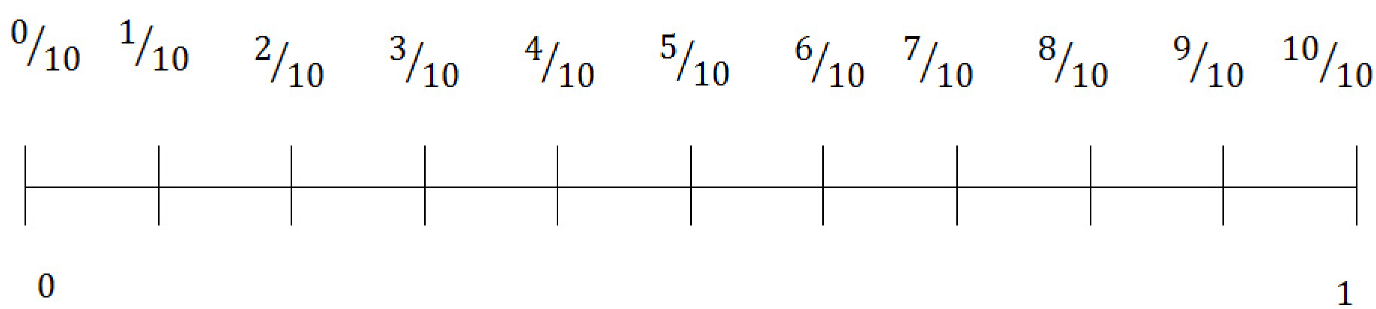


Figure 14 – Tenths



We can expand k times 1/d past the unit interval by letting *k* become greater than *d*. This will allow my students to understand that a fraction still is a distance not just a partition of a whole. Using fourths as an example, we can continue 5/4, 6/4, 7/7… for as long as we want. See *Figure 15below.* I want my students to see that each unit interval gets partitioned into four intervals of length 1/4, just as the unit interval did. This is highlighted in blue below.

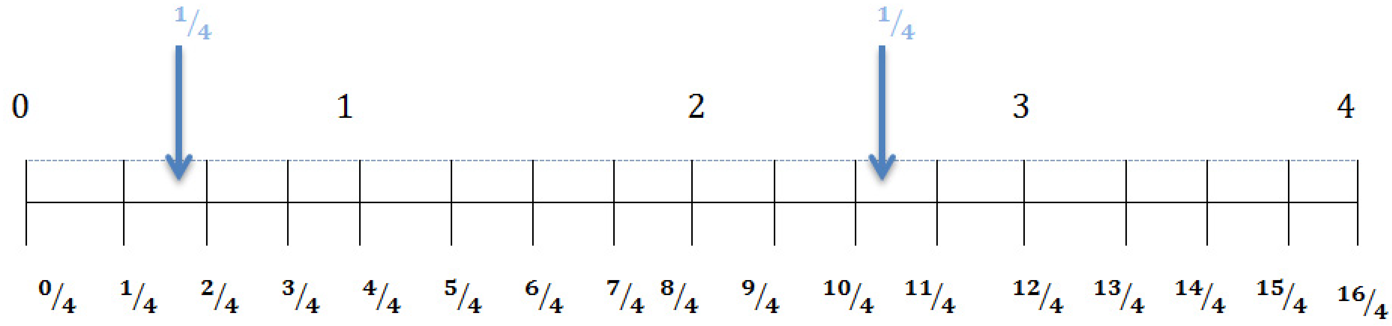


Figure 15

Students will create many examples on their own that mimic *Figure 15.* They will use their own choice of a fixed denominator and expand their models past the unit interv. Students will do this multiple times in groups and alternate the participants so that students can compare models and rely on each other to make meaning for themselves.

After my students have completed all of their transitional "pizza to number line" pictures, I want them to demonstrate how equivalent fractions can be represented. Students will line up two number lines at the same time. For instance, in order for my students to see why 1/5 = 2/10, they will see that both fractions have the same position on the number line. Drawing from our definition of fractions stated above, five collections of 2(1/10) s fill the unit interval. So, 2(1/10) = 1/5. That is of course what you see if you line up the fractioned number lines from fifths and tenths. Students can also use their own artifacts similar to *Figure 15* to represent equivalent fractions past 2.

Furthermore, whole numbers can be represented as a multiple of that unit fraction as well. The fraction 6/3 can represent the whole number two because these lengths line up. The original definition of 6/3 is: 6 x (1/3). Since 3 x (1/3) = 1, by definition of 1/3, we can calculate that 6 x (1/3) = 2 x (3x(1/3)) = 2x1 = 2

Concept #5: Placing Fractions on the Number Line

To get my students to understand how to place fractions, they will go through a series of placement prompts. First, I want them to work together to place multiples of unit fractions between 0 and 1. The general fraction was already defined above and will be practiced by the students using the *Table Mats* activity outlined in the appendix. Below in *Figure 16,* students will be asked to decide what fractions the different colored arrows represent. This will be repeated throughout the activity using a variety of different denominators and arrow locations.

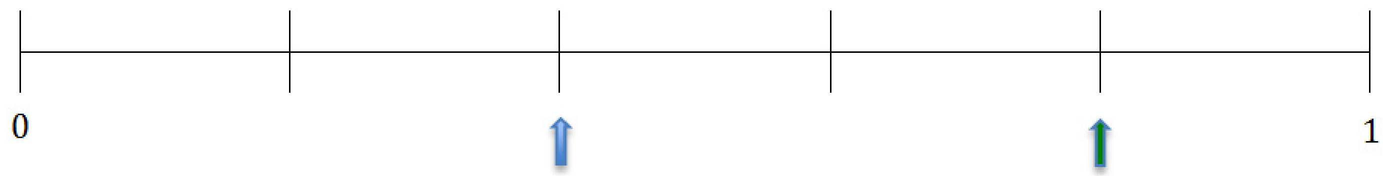


Figure 16

A more difficult problem set will be similar to *Figure 17* and *Figure 18.* This will be useful if a group is progressing quickly through the problem set or are in need of more challenging material.

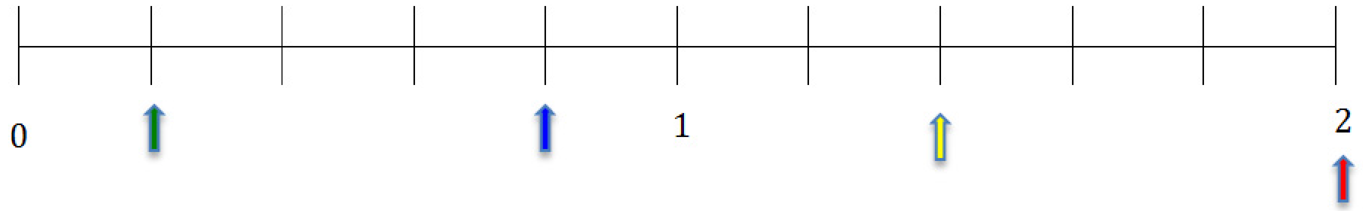


Figure 17

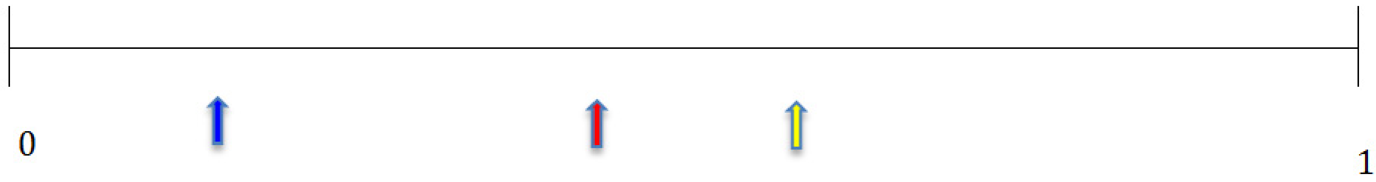


Figure 18

Similar problems to *Figure 18* will have my students use reasoning to justify what each letter represents. These types of problems can be done in a math journal entry or even a Math Talk. I want my students to do this same procedure as before; however there will be some missing information just as there was in the integer placement section of this unit. Missing information will promote justification of what fraction each arrow represents and discussions surrounding these decisions. The students will then create their own number lines with missing fractions on them and trade them with a partner to complete. This will allow for student generated problems that can be later used on a future Math Talk or assessment.

The students will now have examples locating fractions on the number line using approximation. I will use that to approximate placement of 13/5 to illustrate how I can teach a general placement of this improper fraction. First, we know that the nearest whole number can be determined by dividing 5 into 13, getting 2 with a remainder of 3. The leftover will be described as follows: 2 x 5 = 10 and 3 x 5 = 15, so 13/5 = 2 + 3/5 and must be between 2 and 3. As outlined before, the partitions between 2 and 3 will match the denominator and we can split the interval evenly into five parts. The leftover will have to be placed between these whole numbers.

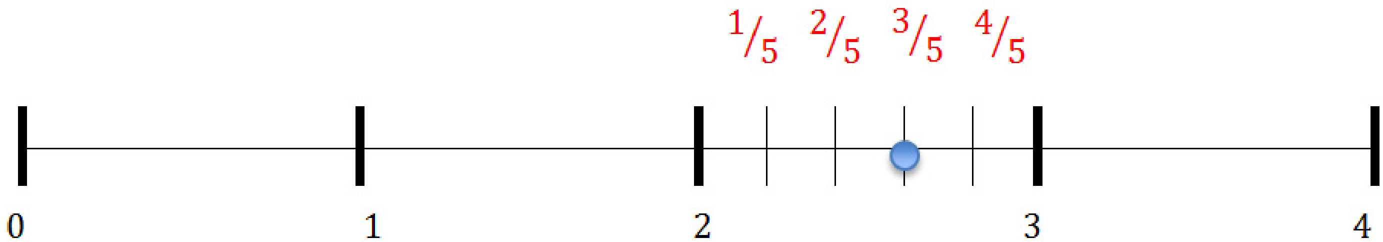


Figure 19

Another strategy for students to use is simply lay off 13 copies of 1/5. As mentioned above in *Figure 15,* this would be an easier method. I would want my students to use both methods during their explanations to the problem sets. Students will be asked to place a series of improper fractions and mixed numbers located in the

Appendix labeled A2.

Orientation and Negative Sign

The above-mentioned activities with fractions have all been on the right side of the number line, or only in the positive direction. Now we need to make use of the entire number line, just as we did for whole number placement. We will simply reflect the positive rational number across zero to obtain the negative number. Reflections show numbers and their opposites. I would want my students to realize that the distance from zero is the same for both some number *d* and *–d.* This can be done as an activity as well with my students. They will be asked to reflect a series of numbers across the zero and can arrive at this conclusion on their own.

My students have a very difficult time placing the negative sign in front of rational numbers. I need to make a series of prompts where negative rational numbers will be seen three different ways. For instance, the fraction -1/3 might be written as (-1)/3 or on rare occasions 1/(-3). Students will see a mix of these signs when they discuss the prompts in class.

Lastly, students will be given a series of fractions with different denominators and be asked to order them. These problem sets can be found in the appendix.

Concept #6: Placing Decimals on the Number Line Using the Expanded Form

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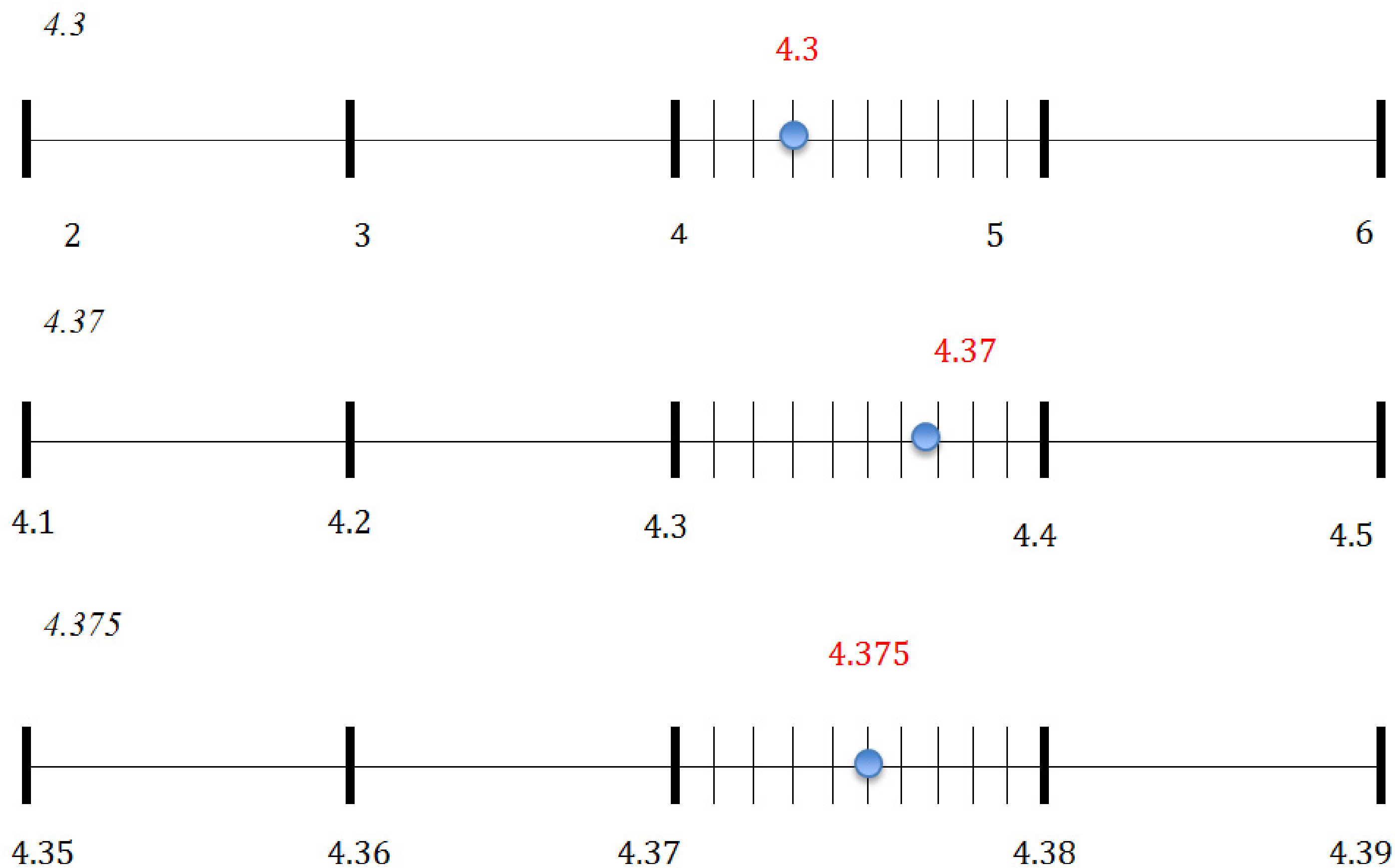
If I can continue the idea that my students need a common vehicle to move them to a better understanding of numbers, decimals will assist in making the connection between location and place value. We can represent each place value as a multiple of a unit fraction. Each successive place is a fraction whose denominator is a power of 10 where decimals "fill in" the number line between the integers.

[2] Three main stages that allow for a more detailed placement procedure will really help my students. At stage one, consecutive whole numbers are place in both positive and negative directions. During stage two, we have decimals in the tenths place and no smaller place. Each whole can broken into 10 smaller interval, each having a length of 1/10 of the unit. Notice, this graphic representation was already observed above and students will make the connections between this particular unit fraction set-up on the number line and place value in base 10. The third stage deals with the hundredths place where each tenths place is further broken into ten equally space partitions. Each interval length will now be 1/100 of a unit long. Going through each stage of placing this decimal expansion, the more precise the placement gets. Zooming into a more specific place on the number line will allow students to clearly see that there are space limitations, however we can measure even more precisely the closer we zoom in. We can repeatedly break each resulting unit into ten equally sized pieces seemingly forever. The point needs to be made here that we don't need very many place values to get a location of a decimal on a number line that is accurate enough for all practical purposes.

In order to locate decimal expansions on the number line we can use a breakdown of a fractional expanded form of 4.375. 4.375 = 4+3/10+7/100+5/1000. There are a series of number lines that are needed here to really bring home this point. In *Figure 20,* students will see that with each decimal place the number line is zoomed in.

The progression of *Figure 20* starts at 4.3, and then zooms in to the next decimal place in the next number line. Each successive decimal place is a new number line. Students will have multiple problems starting with the tenths place. They will be asked to place each number. Next, students will receive a double number line for the hundredths place value. They will then mimic the first two number lines in *Figure 20*. This will continue up through the thousandths place and stop there. It is important for students to see that after a few decimals places, we can be very accurate in our placement.

Figure 20



Note that the scale, i.e., the size of the unit interval, has been multiplied by 10 in passing from each of these lines to the next. Students are expected to transform rational numbers into their decimal expansions. Placement here will help students make connections between the different forms of these numbers and see that they are indeed the same thing. The process of getting from a fraction to a decimal is computationally a good thing to do. Students will practice a series of important math facts throughout this process as well. Long division is a skill where my students will be able to apply simple rules of mathematics.

Strategies

Math Talks

There are four main principles taken from [5]*Intentional Talk* that I follow when first implementing a Math Talk. First, the specific talk should achieve a specific mathematical goal. Well-defined goals will lead to more focused discussions. Second, students should know how to share their own ideas. A formal and transparent framework for professional dialogue to happen should be well known to students well before the first talk takes place. See A1 for the particular norms in my classroom. Third, I orient students to the mathematical ideas generated from the class. Lastly, all ideas are valuable. These cultural norms harness a safe environment for students to focus and critique the ideas of others and not the students themselves.

Whatever the protocol used, I make sure that my talks are consistent in format and in delivery. The idea or style of talk may change, but the groundwork and norms are well laid out at the beginning of each year and class. Talks should range in time from 10-15 minutes, but if they go longer than normal, that's quite acceptable. As long as that first principle remains intact throughout the talk, there is no need to end a productive talk early. As far as frequency of talks concerned, this unit suggests that talks happen regularly three times per week. Prompts should have more than one answer and allow students to make meaning through multiple representations.

After the prompt has been given, I allow for 1-2 minutes of think time. When it is time for whole class sharing I scribe for my students for every talk, putting the name of the student next to the idea. That student owns that idea and it is crucial for students to see where that idea came from.

Due to the large portion of students whose primary language is not English, I make it a point in my class to have an organic, discussion-based approach for the major themes in each new unit. There is a mix of different levels of English fluency as well. [3] The theme of *Trilingualism* needs to be incorporated here into many classrooms as well as my own. There are three languages that need to be taught in school: home language, formal language, and professional language. Home language is language spoken amongst peers within a specific community. Formal language is spoken in a school setting while professional language is spoken on the job site, in this case we can call it the technical language spoken for each specific subject. Teachers should offer guidelines to help students overcome the obstacles in the learning process. Instead of providing the [4] corrective model, teachers need to create a discussion of that nature of language. This will then open up the change of discourse of the "I'm not good at math concept" we are all too familiar with as math educators.

Math Vocabulary with ELL Populations

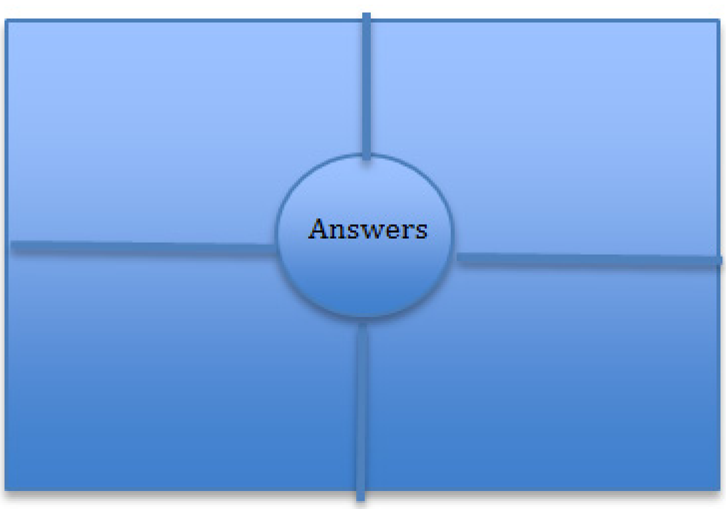
[6] In order to have successful children learning English, and in this case foundations in math, teachers need to instruct subject matter using simplified language with pictures, gestures, and demonstrations. This is exactly where the vocabulary will be so critical to develop deeper conceptual understanding. When students are using the correct terminology, it allows for richer class discussion while maintaining precision in explaining how they arrived at that particular solution. Teachers must respect and admire the language strengths students bring to schools. Only when students are seen as assets to the learning environment, can society bridge the gap between cultures. [7] Children of poverty are able learners and teachers need to realign strategies by differentiating instruction. Vocabulary review along with pictured notes is vital to not only mathematics but other content areas. Teachers need to also accept the language in which students learn.

With each new vocabulary word that my students learn, they will put the following pieces into their notes: pictures, that vocabulary word in Spanish, and of course the definition. This is very important for my students to make the connection to their home language as well as the exact terminology that we use in class. This will help those who struggle with using academic language in their responses to explain how they did a particular problem set. The direction of standardized tests will assess my students on exactness of how they arrived at a problem's answer.

Supporting Activities

1. Math Talk Prompts – Each prompt or problem set should match the figures discussed in the unit. They can be delivered in sets or individually. The format could be a math journal entry, group problem sets, or even as an exit ticket. I encourage you to create problems that fit the needs of your classroom. There are only suggested examples provided below in A2, with differentiated types for whole number, rational number, and decimal prompts
2. Number line partition with unit fractions. Students will create their own number line with the unit fractions with denominators 2-10. This will be used as a reference in their notebooks.
3. Vocabulary List for Unit. Pictures are encouraged and tailored to specific class.
    - Numerator
    - Denominator
    - Greater Than, Less Than
    - Equivalent
    - Unit Fraction
    - Rational Number
    - Improper Fraction
    - Mixed Number
4. Table Mats: Students will have their own workspace on a piece of divided butcher-block paper. Each student will complete the same problem in his or her own workspace. Once the team agrees on the answer, it is placed in the center circle. They raise their hands to signal the teacher to come check the work. After the teacher checks the work to make sure it is accurate, then and only then can the team can move on to the next problem. The teacher is encouraged to only say correct or incorrect to allow students to make meaning on their own. Students are encouraged to work together so that everyone in the group participates and understands. Below is an example of how the paper is divided.

Paper Example:



Appendices

A1

[5] Math Talk Norms to Facilitate Discussion.

- Agree/Disagree with ideas and not the person.
- Allow people to speak for themselves and allow for appropriate think time.
- It's ok to make mistakes and revise our thinking.
- Make sense of mathematics by asking questions to clarify misconceptions.

A2

Whole Number Prompts: Each placement prompt can be given with or without labeled intervals while comparison is encouraged throughout.

- In Relation to Figure 7: Express the following integers as vectors: -7,19,4,12,-2,-5,9,5
- In Relation to Figure 8: Positive Number Pairs for Comparison: 3&5, 6&1, 18&8…Students will be asked to compare each pair using the greater than or less than symbol.
- In Relations to Figure 9: Positive and Negative Number Pair: -3&5, 3&-5 … These can be altered to mimic the pairs previously covered, just changing the sign. I want to choose integer pairs that will challenge the idea of magnitude. Students will be asked to compare each pair using the greater than or less than symbol.
- In Relation to Figure 10: Both Negative Integers. Students will be asked to compare each pair using the greater than or less than symbol.

Rational Number Prompts

- Problem sets should include variations of Figures 16 through 18 with different partitions as well as omitted information.
- In Relation to Figure 19: Improper Fraction Placement: 24/9,21/4,10/3,56/6,32/5,34/7,9/8,11/2
- Rational Numbers Placement with Different Denominators/Signs:
    - 3/8,_2/7,1/3
    - 5/3,7/4,1/7
    - (-4)/7,7/4,(-7)/4

Decimal Prompts

- In Relation to Figure 20: Students will be asked to "zero in" on the following decimals by creating a series of number lines for each place value.
    - 923. First, placing 5, then 5.9, then 5.92, then 5.923
    - 23. First, placing 1, then 1.2, then 1.23
    - 211. First, placing 8, then 8.2, then 8.21, then 8.211

Combined Prompts

- As an extension to the above prompts, students will be asked to place multiple representations of numbers on the number line. This will allow for deeper understanding and help solve the siloed understanding of numbers.

A3

Common Core State Standards and Standards for Math Practice

CCSS

6.NS.C.5 - Understand that positive and negative numbers are used together to describe quantities having opposite directions or values

6.NS.C.6 - Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

7.NS.A.2.B - Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

7.NS.A.2.D - Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

SMP

MP1 – Make sense of problems and persevere in solving them.

MP3 – Construct viable arguments and critique the reasoning of others.

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Notes

1. Bassarear
2. Addison-Wesley
3. Baker
4. Stubbs
5. *Intentional Talk*
6. Wong, Fillmore & Snow
7. Purcell-Gates