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Curriculum Units by Fellows of the National Initiative
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Introduction

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The number line is a central object in mathematics. It is seen in many primary classrooms, and at the other end of the spectrum of mathematical activity, forms the backdrop to many advanced mathematical constructions. It is the key link between geometry and algebra. Yet in comparison to its fundamental and central nature, it plays a remarkably small part in the mathematics curriculum. The goal of this seminar was to consider the ideas inherent to the number line and to think about ways in which it might play a larger role in mathematics instruction. I am delighted that all the Fellows, who teach in grade levels 2nd grade through high school, found creative and productive ways of using the number line in their units.

Carol Boynton plans to make a very long number line that goes to 1000, which her class will share with the whole school. They will place three-digit numbers on it, to bring out the hierarchical nature of decimal expansions – for example, in the number 642 = 600 + 40 + 2, the 600 tells you which of ten equal subintervals (of length 100) between 0 and 1000 you should look, then the 40 tells you of which of ten equal subintervals (or length 10) of the first subinterval you should look, and finally the 2 tells you at end of which of the ten unit intervals in the length 10 interval you land. Creating the full number line to 1000 will also allow her students to see how much larger 100 is than 10, and how very much larger again is 1000. This sense of the rapidity with which numbers grow with the length of their base 10 representation is important, but not easy to acquire, since the notation itself is so compact.

Kathleen Gormley and Jolene Smith will use the number line to teach fractions. It provides a valuable way to visualize all the whole number multiples of a unit fraction 1/d, as evenly spaced points, very much like the whole numbers themselves, except with closer spacing. It also provides a uniform way to think about addition, as putting bars end-to-end, that puts addition of fractions and addition of whole numbers on the same geometric footing, despite the difference in the symbolic manipulations they require. Jolene Smith's unit also connects the number line and fractions to Din´e culture, via the theme of blanket design.

The standard curriculum presents several methods for representing numbers. There is the standard base 10 place value notation for whole numbers. This is later extended to represent decimal fractions by introducing the notorious "decimal point". In between, there is the n/d notation which uses two whole numbers to represent a rational number as a fraction. In addition, the whole numbers are extended to integers by introduction of the – sign. All four of the middle grades teachers had found that these disparate symbolic representation schemes led to balkanization of their students' thinking: each class of number as is conceived of as unrelated to the others. In addition, they sometimes do not think of fractions as numbers, but rather as a pair of numbers that are somehow associated. These Fellows had the insight that the number line offered a means for showing that all the types of numbers, despite their disparate symbolic representations, are in fact

part of a common system, and all live together on the number line. Jade Lee's unit focuses on decimal expansions, and uses the number line to make visible the uniformity of the relationship between adjacent places in a base 10 number, regardless of the position of the decimal point. Jeffrey Rossiter and Aaron Bingea have created coordinated units. The first deals with placing the several types of numbers on the number line, showing how they coordinate with each other. The second uses the length model of addition and subtraction to develop a unified viewpoint toward the various symbolic procedures used to compute sums and differences. Coretta Martin's unit is conceivedin a similar spirit, with the goal of helping her algebra students achieve a stronger and more unified conception of number.

Finally, Klint Kanopka's unit focuses on the process of coordinatization: the preliminary choices, of origin and unit interval, that must be made in order to turn a line into a number line. The goal here is to give his physics students an appreciation of the coordinatization process in physics more broadly, and in particular, to help them deal with 2 and 3 dimensional spaces, and the geometry and algebra of vectors. Taken as a whole, the units demonstrate that there is potential for much greater use of the number line than is currently common, and that it can serve as a unifying feature of mathematics instruction throughout the K-12 curriculum.

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