YALE NATIONAL INITIATIVE
to strengthen teaching in public schools®

Curriculum Units by Fellows of the National Initiative
2017 Volume V: From Arithmetic to Algebra: Variables, Word Problems, Fractions and the Rules

Solving Problems Multiple Ways: Using Arithmetic and Literature - Hooray!

Curriculum Unit 17.05.02, published September 2017
by Brittany McCann

Introduction

"If The Words Don't Add Up, It's Usually Because the Truth Wasn't Included In The Equation". I found this quotation on Facebook and instantly shared it with my friends. Two days later, our school was having a fundraiser at a local bookstore and Sharon Flake was doing a reading and signing for her novel, *The Skin I'm In*. The next day, there was an informational meeting about the Yale National Initiative where I met with previous fellows and talked about applying. One of the former fellows wrote an entire unit using *The Skin I'm In.* From that moment, my goal was to become a fellow and write a unit marrying problem solving through literature with problem solving in mathematics.

My School

According to data from the Pittsburgh Public Schools' website for Pittsburgh Colfax, there were over 900 enrolled students as of October 2016. Approximately half were Caucasian, a quarter were African American, and a quarter were Asian, Multiracial, or Hispanic. The students who leave for Greenway, another Pittsburgh Public School across town, to receive gifted services have been predominantly white students from privileged socioeconomic backgrounds. Out of the 52 students enrolled in my two sections of math this past school year, 17 went to Greenway, and they were all Caucasian. For the past few years, my school has had the largest achievement gap in the district. In our school-wide improvement plan, the number one goal for at least the last two years has been to close the achievement gap. I want this unit to serve that goal.

Rationale

This unit is intended for my 4th grade "Magic Monday" students. In the 2015-2016 school year, students identified as being gifted received gifted services though our school building. In the 2016-2017 school year, these identified students were bussed across town every Monday to receive gifted services at a school called Greenway. Knowing of this change, I spent the summer in between the two school years going to many

training sessions about Project Based Learning. If almost one-third of the class was going to Greenway, I was determined to bring Greenway to my classroom. I declared these days, “Magic Monday” and told the students who visually and verbally expressed their feelings of being left out that first day that we were going to do something different on these days. Students were initially skeptical, but on the next Monday as students gathered their things to leave for Greenway, my “Magic Monday” students were happy to get to work on their projects in our classroom.

Content Objectives

There are numerous Common Core Standards for fourth grade mathematics. However, they all relate to the core operations of arithmetic: addition/subtraction and multiplication/division.

To make this unit cross-curricular, I want to incorporate the novel *The Skin I’m In* by Sharon Flake to provide context for real-life problem solving scenarios. The novel revolves around problematic situations repeatedly encountered by its protagonist, Maleeka. For example, she gets bullied by her peers for her clothes or skin color. Maleeka is quick to react to these situations.

My unit will work towards blending math and literature in a new, interesting way. We will confront race and self-esteem issues through the novel and discussions. My lessons will start with a read aloud of *The Skin I’m In.* We will pause when we get to a problem for Maleeka. Students will first need to identify the problem. After we have identified the problem, we will brainstorm possible solutions as a class, and then discover how Maleeka chose to solve her problem. We will follow this up by expanding mathematical knowledge, learning and practicing multiple ways to solve word problems. I have created a word problem bank of one and two-step math problems, and after each read aloud, students will work through a selected few of these problems just as we did with the read-aloud. First, they will identify the problem, and then possible solutions. Problems will steadily get more complex as the unit continues.

Problem Solving

Students can have difficulty deciding which operation to use when solving word problems. For the early lessons of this unit, a lot of time will be spent upon understanding the problem, including identifying the operation. I will start with simple examples and move to more complex problems with distractors and irrelevant information, in order to make isolating key features more of an issue for students. After students have success with identifying the problem, we can look back at the previous examples, and new ones too, to select a strategy to solve the problem. Once students select a strategy, they will be asked to defend the strategy by answering the question, “What makes you say that?”.

In 1945, George Polya published the book *How To Solve It* in which he identifies the four basic principles for problem solving. I will not be teaching these steps as a recipe to follow, but this strategy will be summarized as an experience. First, students must understand the problem. Polya taught teachers to ask students questions such as:

> Do you understand all the words used in stating the problem?

What are you asked to find or show?

Can you restate the problem in your own words?

Is there enough information to enable you to find a solution?

Second, Polya mentions that there are many reasonable ways to solve problems. The skill of choosing an appropriate strategy is best learned by solving many problems.[1] Third, carry out the plan. Students will be told to persist with the plan that they have chosen. If it does not work they can simply choose another strategy. I will remind students that this is even how professionals handle problem solving. Finally, students will look back and reflect on what they have done. By discussing what worked and what did not, students will be able to predict a strategy to use on future problems.

Because my students can solve basic addition and subtraction problems with numbers less than ten with ease, I will focus a lot of my time and attention to problem solving. Most of my students simply look for two numbers in a word problem and then a keyword for how to solve. I'm not sure why, but there is always a competitive nature among students for who can solve the question first. There is no trophy or prize for solving a problem first, and I make sure to reiterate that to my students daily. I will include problems that have multiple numbers, some of which are irrelevant, in order to make my students work to understand the problem and pick out the important information.

Addition/Subtraction

Writing Problems

When writing addition and subtraction problems, I must be mindful of the Common Core Taxonomy of 14 types of one step addition/subtraction problems.[2] They fall into three main categories: change, compare, and part-part-whole. I have included samples of each in the problem bank in the appendix.

When writing problems, I'm including keywords that students would typically assume would mean one operation. For example, students have previously been taught when a problem uses the word "more" they need to add. I will disprove this strategy by writing a problem such as, "Maleeka had some pencils. Her friend gave her three more. Now she has 7. How many pencils did Maleeka start with?" In using the keyword strategy, students would add 7 and 3 to answer 10. I will use this example and reread the problem as a class. I will emphasize, "Maleeka had some pencils - since we have answered this as 10, I'll say ten instead. Now, Maleeka had 10 pencils, her friend gave her three more, and now she has 7." Then I will have a puzzled look on my face and ask the class, "Does this make sense?". This will lead to a discussion on keywords and if they help us understand the problem.

Change problems start with a certain number of items, then they either get more of that item or give some of that item away. There are two types of change problems, increase and decrease, which we will refer to as change plus and change minus. It should be noted that in Table 1, they are also called add to/take away. There are three possible unknowns in the problem: the start, the change, or the result. The formulas for change problems are as follows:

start + change = result (increase problems)

start - change = result (decrease problems)

In comparison problems, there are two people who have the same type of item and the problem will say that one person has more, or less, of the items. There are two types of compare problems, more and less. There are three possible unknowns in the problem: the smaller number, the larger number, or the difference. The formulas for compare problems are as follows:

smaller number + difference = larger number

larger number - smaller number = difference

larger number - difference = smaller number

In part-part-whole problems, there is a given total of items and two parts that make up that total. In part-part-whole problems, there are only two possibilities for unknown (one part or the whole). The formulas for part-part-whole problems are as follows:

part + part = whole (whole unknown)

whole - known part = unknown part

I created the following chart and found it helpful when creating one-step addition and subtraction problems. It is a checklist to ensure that I was including each type of problem when writing problems. When writing two step problems, I number each problem, and put the problem number in the box for each of the single step parts of the problem. For example, if problem number one was a change plus, result unknown and change minus, start unknown, I would put a number one under two-step for each type. This way, I can keep track of which types of two-step problems I have created. There are a lot of possibilities!

Type of Problem	One Step	Two Step
Change plus, result unknown		
Change plus, start unknown		
Change plus, change unknown		
Change minus, result unknown		
Change minus, start unknown		
Change minus, change unknown		
Compare more, greater unknown		
Compare more, smaller unknown		
Compare more, difference unknown		
Compare less, greater unknown		
Compare less, smaller unknown		
Compare less, difference unknown		
Part-part-whole, whole unknown		
Part-part-whole, part(s) unknown		

In his paper *The Most Important Thing for Your Child to Learn about Arithmetic,*[3]Roger Howe describes the five stages of the base ten place value system. The first stage is the standard form of writing numbers, the second is expanded form, the third stage factors each piece into its digit times a base ten unit, the fourth stage exhibits base ten units as products of several factors of ten (or as powers of 10), and the fifth stage makes a connection to algebra in revealing that base ten notation is a very compact way of representing numbers as

“polynomials in 10”. By following these stages, I can take my students from a basic understanding of writing numbers and using expanded form to identify and isolate the base ten pieces of a number to justify the addition/subtraction algorithm (add/subtract the ones, add/subtract the tens, add/subtract the hundreds, regroup when needed). Because my students are fourth graders, we will only be working on stages 2 and 3. This means that my students can write number in standard form (first stage). For example, the number three hundred twenty-five can be written as 325. In using this number as an example, my students will review expanded form (second stage) by breaking 325 into 300 + 20 + 5. To move their thinking into the third stage, we will first review place value by identifying the place of each of the digits. I remind my students that place is a location (ones, tens, hundreds, etc). What digit is in the ones place? 5! What is its value? 5! Now, we can represent this as 5 x 1. What digit is in the tens place? 2! What is its value? 20! Now, we can represent this as 2 x 10. What digit is in the hundreds place? 3! What is its value? 300! Now, we can represent this as 3 x 100. So this number is a sum of three hundreds, two tens, and five ones. I can anticipate that this will help students when it comes to computation and solving the problems.

Sample 1-Step Addition/Subtraction Problems

(Change plus, result unknown)

Miss Saunders loves to wear pant suits! She had 12 pant suits hanging in her closet. When she went back to school shopping, she bought 7 more and put them in her closet. How many pant suits does Miss Saunders have in her closet?

(Change plus, start unknown)

1st period, Maleeka had some of Char’s clothes in her locker. 2nd period, Char gave Maleeka two more clothing items. Now Maleeka has 7 clothing items in her locker. How many items were in Maleeka’s locker 1st period?

(Change plus, change unknown)

At lunchtime, Maleeka had 13 carrots. Her friend gave her some more. Now she has 21. How many carrots did her friend give her?

(Change minus, result unknown)

Char bought a pantsuit for three hundred dollars. Now she is offering to sell it to you for fifty dollars. How much money off the original price are you saving?

(Change minus, start unknown)

John-John had some buttons on his shirt. He played with them so much, two came off! Now he has 8 buttons on his shirt. How many buttons were originally on his shirt?

(Change minus, change unknown)

4th period, Maleeka has 7 clothing items from Char in her locker. 5th period, Char takes some of the items back. Now, Maleeka only has one shirt in her locker. It is Char’s. How many clothing items did Char take back?

(Compare more, greater unknown)

John-John is only 48 inches tall. Caleb is 14 inches taller than John-John. How tall is Caleb?

(Compare more, smaller unknown)

Miss Saunders is six feet tall. If she is two feet taller than Tai, how tall is Tai?

(Compare more, difference unknown)

At recess, Maleeka was playing with 2 friends and Char was playing with 10. How many more people were playing with Char than Maleeka?

(Compare less, greater unknown)

At lunchtime, Char was upset with Maleeka. Char had three cookies, and that was one less than Maleeka! How many cookies does Maleeka have?

(Compare less, smaller unknown)

Maleeka loves to write and draw! She grabbed a big box of crayons to work on a project in art class. John-John had fifty crayons in his box of crayons. When Maleeka counted her crayons, she realized that she had ten fewer crayons than John-John in her box. How many crayons does Maleeka have in her crayon box?

(Compare less, difference unknown)

The twins share a locker. They have 4 clothing items from Char in their locker together. Maleeka only has one shirt from Char in her locker. How many fewer pieces of clothing items from Char does Maleeka have in her locker?

(Part-part-whole, part unknown)

Char lives at home with her older sister Juju. They each have their own closet. Together, they have exactly 123 clothing items hanging in the closets. If Char has 57 clothing items in her closet, how many clothing items are in Juju's closet?

(Part-part-whole, whole unknown)

Miss Saunders and Tai are trying to count all of the Language Arts textbooks between their rooms. Miss Saunders has 22 and Tai has 24. How many books do they have together?

Solving Problems

When asked to find: the total, the sum, how many in all, how many altogether, etc - and when all the items in the problem have the same units, use addition.[4] Having the same units is very important because numbers do not always refer to the same thing. If I said 3 + 4 = 2 in my classroom, my students would be quick to correct my calculation. However, if I said 3 dimes plus 4 nickels is the same as or equal to 2 quarters, I would be correct. In my experience, students have a lot of trouble with units in word problems. If the problem is in minutes, but requires an answer in hours, the problem now has one extra conversion step. Howe[5] breaks down addition even further to three key steps: break each number into its base ten pieces, add each pair of pieces of the same order of magnitude, and recombine the sums into a base ten number. He states that it

may seem like a lengthy process, but it's very close to the standard algorithm

For my unit, I also plan to discuss the commutative rule for addition, which says that for any two numbers a and b, a + b = b + a. With the overall theme of the unit of problem solving in different ways, I want students to know that 1+2=3 and 2+1=3. Students will understand that this rule applies for multiplication as well as addition, but not subtraction or division. With Common Core Standards, students in fourth grade must fluently add and subtract multi-digit numbers using the standard algorithm (4.NBT.4). In our curriculum, Everyday Math 4, students are taught a plethora of strategies for adding numbers in third grade. For my unit, I do not care which strategy they choose, but they must be able to add digits to the millions place value.

Moving to subtraction, I will make sure my students develop the understanding that a subtraction problem can be thought of as a missing addend problem. When asked to find: how much more, how much less, how much larger, how much smaller, how many more, how many fewer, the difference, the balance, how much is left, how far above, how far below, how much further, etc – (and when all the items in the problem are the same or have the same units), use subtraction.[6] As explained above, having the same units is very important. With Common Core Standards, students in fourth grade must fluently add and subtract multi-digit numbers using the standard algorithm (4.NBT.4). In our curriculum, Everyday Math 4, students are taught a plethora of strategies for subtracting numbers in third grade. For my unit, I do not care which strategy they choose, but they must be able to subtract numbers as large as millions.

In starting with addition and subtraction, I want my struggling students to find success early on in the process. This will require very simple addition and subtraction problems in the beginning before moving onto more complex and/or two-step problems.

Multiplication/Division

Writing Problems

When writing multiplication and division problems, I will be guided by the 9 part taxonomy of Table 2 in the Glossary of the Common Core Standards for Mathematics.[7] The problems fall into three categories: equal groups, area/arrays, and comparison. I have included samples of each in the problem bank.

Equal group problems involve repeating the same number a given number of times. In equal groups problems, there are three potential unknowns: 1st factor (number of groups), 2nd factor (number in each group), or product.

Array and area problems involve rows and columns and organizing items so that they form a rectangle. In array/area problems, there are three possible unknowns are: columns, rows, or total.

Comparison problems use the key word “times” indicating how many times larger/smaller one quantity is in relation to another quantity of the same type. In comparison multiplication problems, as with the other two types, there are three possible unknowns: smaller quantity, larger quantity, or ratio between of them.

I used a similar method to keep track of multiplication/division problems as described above for addition/subtraction problems. I created the following chart for the various types of multiplication/division problems, both one step and two step. It is a checklist to ensure that I was including each type of problem when writing problems. When writing two step problems, I again numbered them, and used the number to the problem I wrote to record the types that it involved. For example, if problem number one was an equal groups,

product unknown and arrays, columns unknown, I would put a number one under two-step for each type.

Type of problem	One step	Two step
Equal groups, # of groups unknown		
Equal groups, # in group unknown		
Equal groups, product unknown		
Arrays, columns unknown		
Arrays, rows unknown		
Arrays, total unknown		
Comparison, smaller unknown		
Comparison, larger unknown		
Comparison, factor unknown		

Sample 1-Step Multiplication/Division Problems

(Equal groups, product unknown)

Char says that she has to look at Miss Saunders's face for forty-five minutes every day. If we have school five days a week, how many minutes a week will Char look at Miss Saunders?

(Equal groups, number in each group unknown)

Char has 9 outfits to bring to school. She wants to share them equally with Maleeka, Riana, and Raise. How many outfits will each girl receive?

(Equal groups, number of groups unknown)

Mr. Pajolli received a shipment of 100 boxes of pencils for the teachers. He wants to give each teacher 10 boxes of pencils for their classroom. How many teachers can get 10 boxes of pencils?

(Arrays, column size unknown)

Miss Saunders arranges the 30 desks in her room into 5 equal rows. How many desks are in each row?

(Arrays, row size unknown)

In Tai's room, she arranges her 30 desks into 6 columns. How many desks are in each column?

(Arrays, total unknown)

In the school auditorium, the seats are arranged in three sections. Each section has 10 seats wide and 30 seats deep. How many seats are in each section?

(Comparison, smaller unknown)

At school, Char proudly exclaims, "My outfit costs $300, which is 100 times more than Maleeka's." If Char is correct, how much is Maleeka's outfit?

(Comparison, larger unknown)

Maleeka's mother has been saving money in two jars - one for the lottery and one for new clothes. In the lottery jar, she has saved $30. However, the new clothes jar as three times as much as the lottery jar. How much money is in the new clothes jar?

(Comparison, ratio unknown)

At Maleeka's school, there are 750 students. At the school across town, there are 250 students. How many times more students are at Maleeka's school than the school across town?

Solving Problems

In third grade, there is an emphasis on mastering the multiplication table, but students learn many strategies for the ones they struggle with. However, a resource available to students can be a multiplication grid. This can help them do the computation for a problem once they have identified how to solve it. When you are asked to find the product, the total, how many in all, how many together, etc - and when you have groups of individual items, use multiplication.[8] In the fourth grade, students are later introduced to multi-digit multiplication and learn multiple strategies to solve those problems.

Students in fourth grade have a limited proficiency in division. Later on in the school year, they will learn strategies for a long division, but for this unit, I will focus on basic division problems with no remainder. I want my students to understand to use division when they are given the total number of items and a number of groups and need to find how many items in each group, or given the total number of items and the number of items in each group and need to find out how many groups there are.[9] There are two main ways to think about division: sharing (aka partitive) model and the measurement (aka quotative) model. In division as sharing, one wants to make a given number of equal pieces from some initial amount. For example, one wants to share 9 cookies among 3 friends. In division as measurement, one has a large quantity and a smaller quantity and wants to know how many of the smaller can be made from the larger. Division is also a "missing factor problem" and 24 / 4 can be thought of the same as asking "What, when multiplied by 4, produces 24". This is also the basis for describing division as "unmultiplication" or "the inverse of multiplication". From the missing factor point of view, division sits rather uncomfortably with whole numbers, because often there is no whole number that can be the missing factor (25 / 6 for example).

2-Step Problems

Since there are so many possibilities for two-step problems, I will only give four examples, to give the flavor of what I have in mind.

(Part-part-whole, whole unknown and change minus, result unknown)

While working in the office, Maleeka found her permanent record. Two years ago, she attended 167 days of school. Last year, she attended 182 days of school. If there were a total of 364 days of school in those two years, how many days of school did Maleeka miss?

(Equal groups, product unknown and change minus, result unknown)

Momma sent Maleeka to the store to buy 3 six packs of soda that cost $6 each (including tax). If momma gave her a $20 bill, how much change will Maleeka get in return?

(Arrays, total unknown and change plus, change unknown)

Miss Brady's detention room is very small. She has 5 rows of desks with 3 desks in each row. If 18 students showed up for detention, how many desks does Miss Brady need so that everyone has a seat?

(Equal groups, product unknown and change plus, result unknown)

Juju, Char's sister, loves to have parties, and Char loves money. Last weekend, Char spent all of the money in her piggy bank buying new clothes. If Char helps out at the party, she gets $5 per hour from her sister. Char worked five hours last weekend and put that money in her piggy bank. She plans to make $50 for the party this weekend and put it all in her piggy bank. How much money should Char have in her piggy bank at the end of the weekend?

Extensions

Starting with problems students are familiar with, I will change a problem slightly to create a new type of word problem.

Example:

Original problem:

Char says that she has to look at Miss Saunders's face for forty-five minutes every day. If we have school five days a week, how many minutes a week will Char look at Miss Saunders?

Extension:

Char says that she has to look at Miss Saunders's face for forty-five minutes every day. If we have school five days a week, how many hours per week will Char look at Miss Saunders?

Many students will need to reread this problem to catch the subtle difference. In the original problem, students need to answer in minutes. In the extension, they need to answer in hours.

Changing the units in a problem creates a two-step problem from a one-step problem. Students should know how to solve the first problem in previous lessons, but now they have to convert the minutes to hours.

Teaching Strategies

Whole Group Discussion

Before starting a whole group discussion on problem solving, I plan to first have the students sit for 1-3 minutes in silent reflection on the question and gather their thoughts. This way, once the discussion starts, each student can bring an idea to the group.

I've found that students need direct instruction about how to have a class discussion. My favorite way to model this is by choosing a silly topic and modeling the correct and incorrect way to contribute to a discussion. For example, if I start class by telling the students, "Barbie is the greatest toy ever created!", there is always one student who will disagree and voice their opinion. When I reply with, "I'm the teacher, so

what I say is right!" the students get even more excited for this discussion. This is a perfect place to pause and talk about positive talk. In my reply, I was not adding anything to the discussion. We also discuss that it is perfectly ok to disagree in a discussion, but you need to state why you disagree. As a class, we make sentence models for how to begin sentences in a class discussion. They may change slightly, but they typically stay within the same four phrases for accountable talk:

"I agree with you because…"

"I disagree with you because…"

"I would like to add on to that point…"

"I have a question about what you said…"

In the beginning, the teacher needs to lead the discussion and make sure everyone gets a turn to speak. As time goes on, the teacher can release some of his or her tasks to the students (taking notes about what is said, keeping track of who has had a turn to share their thoughts, calling on students, etc).

"What Makes You Say That?"

In his book *Making Thinking Visible,* Ron Ritchhart describes the "What makes you say that?" (WMYST?) routine. This routine helps students identify the basis for their thinking by asking them to elaborate on the thinking that lies behind their responses.[10] It seems easy enough, but when used consistently in the classroom it creates an atmosphere of using evidence for reasoning. I like to use this routine in my classroom because it encourages the students to justify their answers. There is a common joke in my classroom that you cannot simply choose a number from the sky - you have to be able to backup your answer. Richart states that this question (WMYST?) should not sound like a challenge or a test, but convey a curiosity regarding how the learner is constructing understanding of a complex idea.[11] In my classroom, I have it on a poster and model it from day one, sometimes even pointing to it to remind students I'm looking for their thinking. As time goes on, I will start to hear it in student conversations when doing group work or whole class discussions.

Teaching the Teacher

When I was a student, one of my favorite things do was to catch an error made by the teacher. Now that I am the teacher, one of my favorite things to do is sprinkle in errors into random lessons so that my students can catch them too!

These errors have to be with skills that the students have had a lot of practice with and are nearing mastery. This gives the teacher time to discover common misconceptions and present them to the class. I always make sure that I explicitly teach common misconceptions, and as a class we discuss why they are incorrect.

My favorite example is with subtraction. In third grade, students spend weeks learning subtraction strategies. In the fourth grade, they should have subtraction mastered, but many still struggle. I would write the following problem on the board and talk through my steps:

4714

-2533

———

2221

“Ok, I’m going to start on the right. Four minus three is one, one minus three is two, seven minus five is two, and four minus two is two. So my answer is two thousand, two hundred twenty-one.” Almost instantly, a student should catch my error (one minus three is not two). This can start a class discussion and students can teach the teacher. This is extremely rewarding when students start to repeat the skills they have learned during class lessons. In fourth grade, students are allowed to use calculators. I tell the students to always check their work with a calculator, but to solve it first without one. If no students catch the error, I will have them check the teacher’s work with a calculator.

A side note: the more animated the teacher is, the more excited the students get. I like to pump up my acting skills and make claims like, “The chalk did it, not me!” and “The calculator must be broken!” It also gives students, especially ones who struggle, a chance to see that teachers make mistakes too!

Classroom Activities

Below is my tentative schedule for teaching this unit. Note that I am only teaching this on “Magic Monday”, but it could easily be adapted to a daily lesson instead of weekly. The following schedule is very ambitious and I will adjust it as needed as per the needs of my students. Some lessons may need two days instead of one.

Day 1:

The Skin I’m In Read-aloud: Chapters 1-3

Skill: Discuss the four operations - What do we look for in problems?

Day 2:

The Skin I’m In Read-aloud: Chapters 4-6

Skill: How would you solve the sample problems? Why?

Day 3:

The Skin I’m In Read-aloud: Chapters 7-9

Skill: Solving one step problems.

Day 4:

The Skin I'm In Read-aloud: Chapters 10-12

Skill: Strategy - solving addition problems to the millions place value.

Day 5:

The Skin I'm In Read-aloud: Chapters 13-15

Skill: Strategy - solving subtraction problems to the millions place value.

Day 6:

The Skin I'm In Read-aloud: Chapters 16-19

Skill: Solving one and two step problems

Day 7:

The Skin I'm In Read-aloud: Chapters 20-22

Skill: Solving one and two step problems

Day 8:

The Skin I'm In Read-aloud: Chapters 23-25

Skill: Solving two-step problems. Is there another way to solve?

Day 9:

The Skin I'm In Read-aloud: Chapters 26-28

Skill: Solving problems another way - creating another solution after one has been given.

Day 10:

The Skin I'm In Read-aloud: Chapters 29-32

Skill: Solving problems another way - creating another solution after one has been given.

Identifying Problems and How to Solve (Day 2)

After reading our chapters from *The Skin I'm In* and discussing what Maleeka's problems were and how she could have solved them, I will start by telling the class, "Just like we identified and provided ways for Maleeka to solve her problems, today we are going to use the same strategy to solve math problems." My current plan is to hand write the problems on large note cards and project them onto the board for the class to see. Using whole group discussion, students will silently read the word problem and reflect for 1-3 minutes before the discussion can begin. First, students must identify the problem. We are going to write a box in the corner of the note card with a question mark to clarify the problem. After we identify the problem, we will collectively decide what operation will be used to solve the problem. Once we decide on the operation, I will ask my favorite question "What makes you say that?" and students will defend their answer by stating the problem in

their own words. Once we do a few together, I will pass out the remaining note cards to the students and have them work in teams to identify the problem and how to solve it. I will remind students that they are not solving the problem, but working to understand the problem. Students will need to identify what operation they should use to solve the problem. Students will present their note card to the class. This is where the teacher can check for understanding. By starting with something very easy, I will give my struggling students a way to have success early on.

Solving 1 Step Problems (Day 3)

After reviewing the previous lesson on identifying the problem and the operation to solve, students will revisit the same notecards. I can tell my students that most of the work is already done! Fanning out the notecards, I will ask a student to choose one to put on the board. We will review the problem and operation chosen during the last lesson. I will set up the problem, but I will ask the students how to write it (vertically or horizontally) and if the order in which I write the numbers matters. The problems will be chosen to start out with simple, easy to solve numbers, and simple statements, and then increase in difficulty. The students will be given new problems, so not all of them are repeats from the previous lesson. Students will practice identifying the problem and operation needed to solve before solving the problem using arithmetic. This strategy can also work for two-step problems.

Subtraction Manipulatives (Day 5)

A common misconception for my students is that they assume the commutative rule applies to subtraction. At this point in the school year, we will have already reviewed and hopefully mastered place value. I want to line students up in the front of the classroom standing on a chair and quickly have the students identify where they are in the line (ones, tens, hundreds). Those students will also be told to pick a digit from 0-9, and that will determine the number of base ten blocks they will hold (if the student in the tens place chooses 7, they will take 7 base ten rods which represent 70, etc). Next, I will take three more volunteers and have them stand in front of the chairs and complete the same process. As a class, we will determine if this is a solvable problem - the goal will be for them to recognize that the larger number needs to be on top. We will solve the problem as a class (possibly completing more than one at a time so that we can cover regrouping and subtraction with zeroes) so the students can visually see what to do when it is not a simple single digit subtraction with positive numbers. This could easily be adapted to use for addition.

Finding Another Way to Solve Problems (Day 9)

At the conclusion of my unit, I do not want to give them a traditional test. However, at this point, they should have mastered the experience of problem solving to the point where they can identify the problem, select a strategy to solve the problem, carry out the strategy, and evaluate their answer. We will have also discussed other ways to solve multi-step problems and even touched upon using algebra and variables as another method. To engage a higher level of thinking, I will provide a packet of various two and three step word problems based upon the entire book *The Skin I'm In*. On each page, I will have solved the problem one way and their task will be to work with a partner to solve the problem in another way. However, on some of the problems, I will use common misconceptions from this unit to solve problems incorrectly. For example, if students are simply looking for keywords, they may interpret a subtraction problem as addition. So, I will solve the problem incorrectly. When a student discovers this, either by reviewing my work or noticing their answer and my answer do not match, I will stop the class and have everyone focus on that problem. I will have the student explain their theory for why the teacher's answer is incorrect. If they would like, the student can solve the problem correctly on the board for the class, or they can choose a volunteer from the class. This will

create excitement for solving the rest of the problems and checking the teacher's work for errors.

Resources

Bluman, Allan G. *Math word problems demystified*. New York: McGraw-Hill, 2011.

Cross, Christopher T., and Woods, Taniesha A.. Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity. Washington: National Academies Press, 2009. Accessed July 20, 2017. ProQuest Ebook Central.

Flake, Sharon. *The skin i'm in*. New York: Jump at the Sun/Hyperion Paperbacks for Children, 2007.

Howe, Roger. "Pages 107-114." In *The Twenty-third ICMI Study: Primary Mathematics Study on Whole Numbers, Proceedings*. China, Macao. June 2015. Accessed August 14, 2017. http://www.umac.mo/fed/ICMI23/doc/Proceedings_ICMI_STUDY_23_final.pdf.

"Mathematics Glossary » Table 1." Mathematics Glossary » Table 1 | Common Core State Standards Initiative. Accessed July 20, 2017. http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/.

"Mathematics Glossary » Table 2." Mathematics Glossary » Table 2 | Common Core State Standards Initiative. Accessed July 20, 2017. http://www.corestandards.org/Math/Content/mathematics-glossary/Table-2/.

Pólya, George. *How to solve it: a new aspect of mathematical method*. Princeton: Princeton Univ. Press, 2014.

Ritchhart, Ron, Mark Church, Karin Morrison, and David N. Perkins. *Making thinking visible: how to promote engagement, understanding, and independence for all learners*. San Francisco, CA: Jossey-Bass, 2011.

Appendix

Standards

This unit focuses on two of the Common Core Standards for Mathematics:

4.OA.3: Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.NBT.4: Fluently add and subtract multi-digit whole numbers using the standard algorithm.

Endnotes

1. George Pólya, *How to solve it: a new aspect of mathematical method* (Princeton: Princeton Univ. Press, 2014).
2. "Mathematics Glossary » Table 1," Mathematics Glossary » Table 1 | Common Core State Standards Initiative, accessed July 24, 2017, http://www.corestandards.org/Math/Content/mathematics-glossary/Table-1/.
3. Howe, Roger. "Pages 107-114." In *The Twenty-third ICMI Study: Primary Mathematics Study on Whole Numbers, Proceedings*. China, Macao. June 2015. Accessed August 14, 2017. http://www.umac.mo/fed/ICMI23/doc/Proceedings_ICMI_STUDY_23_final.pdf.
4. Allan G. Bluman and Allan G. Bluman, *Math word problems demystified*(New York: McGraw-Hill Professional, 2011), 20.
5. Howe, Roger. "Pages 107-114." In *The Twenty-third ICMI Study: Primary Mathematics Study on Whole Numbers, Proceedings*. China, Macao. June 2015. Accessed August 14, 2017. http://www.umac.mo/fed/ICMI23/doc/Proceedings_ICMI_STUDY_23_final.pdf.
6. Allan G. Bluman and Allan G. Bluman, *Math word problems demystified*(New York: McGraw-Hill Professional, 2011), 21.
7. "Mathematics Glossary » Table 2," Mathematics Glossary » Table 2 | Common Core State Standards Initiative, accessed July 24, 2017, http://www.corestandards.org/Math/Content/mathematics-glossary/Table-2/.
8. Allan G. Bluman and Allan G. Bluman, *Math word problems demystified*(New York: McGraw-Hill Professional, 2011), 21.
9. Allan G. Bluman and Allan G. Bluman, *Math word problems demystified*(New York: McGraw-Hill Professional, 2011), 22.
10. Ron Ritchhart, Mark Church, and Karin Morrison, *Making thinking visible: How to promote engagement, understanding, and independence for all learners*(San Francisco: Jossey-Bass, 2011), 165.
11. Ron Ritchhart, Mark Church, and Karin Morrison, *Making thinking visible: How to promote engagement, understanding, and independence for all learners*(San Francisco: Jossey-Bass, 2011), 167.