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"Simplifying" the Issues with Expressions

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Introduction

I have been teaching at Central Jr. and Sr. High School, a school that goes from 7-12 grade, for two years. I was lucky enough to have taught ninth grade and then eighth grade the following year. Many would not think of this as beneficial to their career, but being able to understand what kind of support students need the most when they get to high school allows me to attack these issues when they are in the eighth grade. There was not much difference between eighth graders and ninth graders in the way that they thought about math. Many of them believed math was something they would never understand. That pessimism came from their elementary and middle school experiences. When teaching students who have gone through a great deal of trauma in their lives and who feel as if they have no way of amounting to anything, you must find ways to engage them or hear them out, as a first step towards clarifying misconceptions they might have had when they were younger. My students have a great deal of potential, but because they have not had anyone to motivate them and give them a voice of their own in math, they end up struggling through the subject and ending up even farther behind.

Content Objectives

Oklahoma state standards are leaning more towards creating problem solvers and literate math students. For me as an educator, bringing out those skills and having them think through the relevance of mathematical concepts in the classroom is sometimes a challenge. Since math is a subject that is thought of by many people as being only procedural, I would like to help my students see that there is more to the subject than just formulas and steps. It is about ideas. To help guide the unit, the standards that will be addressed from the Oklahoma Academic Standards are:

(1) Use substitution to simplify and evaluate algebraic expressions; and

(2) Justify steps in generating equivalent expressions by identifying the properties used, including the properties of operations (associative, commutative, and distributive laws) and the order of operations,

including grouping symbols.

In order to not only fill in these standard gaps, but also clear up other misconceptions, Yale National Initiative (YNI) fellows have developed units that complement each other within the topics of translating word phrases, simplifying expressions, and solving equations. The curriculum units are written by 2017 math fellows Rachelle Soroten, Jeffrey Rossiter, and Sally Yoo, and are closely related with this one. These units, depicted in the flow chart (see Figure 1 below), will not be following the traditional textbook way of teaching these topics. In our YNI seminar, Roger Howe, seminar leader, saw a connection between the four units that were being created. He believes that, in order to get students motivated to translate verbal phrases into expressions, they need context through the word problems, which is the basis of Rachelle's unit, *Formulating Algebraic Equations from Word Problems.* Once they understand how to translate word problems into simple expressions, they will go into the various ways to translate word phrases that will become more complex through Jeffrey's unit, *Introduction of the variable by forming and interpreting expressions.* That will lead into this unit, the third section of the chart below, where students will be able to use their knowledge about writing algebraic expressions to simplify them. Finally, they will be able to put all of these skills from the previous units together in Sally's unit, *Making Sense of Solving Equations through Word Problems - The Cornerstone of Algebra,* to be able to solve algebraic equations.

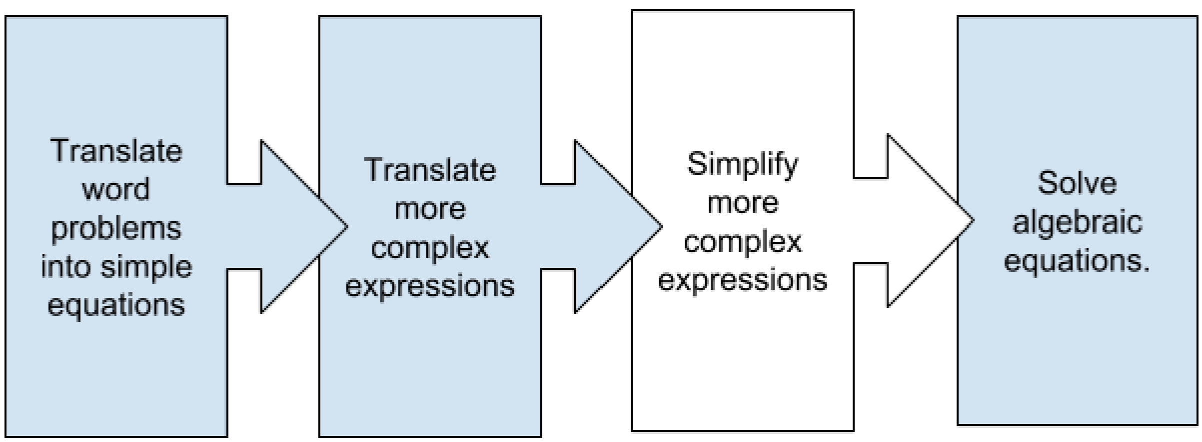

Figure 1

Rationale

The focus for this unit will be on giving my students a clear understanding of how to manipulate compound expressions and reduce them in order to create an equivalent expression in standard form. My goal is to avoid from the beginning misunderstandings that students may have about the meaning of expressions and their relationships. Teaching expressions to students in the ninth grade has been a challenge because they did not have the foundation they needed to steer clear of misconceptions while simplifying expressions. I have often seen students turn a two-term algebraic expression into either a one term algebraic expression or a one term numeric "answer". While teaching eighth graders expressions, we faced the same issues. The only way to stop this from continuing in high school and to enable students to begin building on their math skills for college is to catch these mistakes when they are learning pre-algebra.

The main mathematical theme of this unit is that any first order expression in some *variable*, say *x*, can be simplified using the rules of arithmetic, to an equivalent expression of the form *ax+b*, known as *standard form*, where *a* and *b* are constants (meaning, known numbers in any particular problem). But in general, it cannot be made any simpler. Indeed, two expressions *ax+b* and *a'x+b'* are equivalent only when *a* = *a'* and *b* = *b'*. The condition for equivalence is: two expressions in the *variable* x are equivalent if, whenever a particular number is substituted for *x*, the two expressions evaluate to the same number. So if *ax+b* and *a'x+b'* are equivalent, then setting *x* = 0 would give the condition b = b'. Then setting *x* = 1 gives the condition *a+b* = *a'+b'*. Since we already know that *b* = *b'*, we can subtract these equal quantities from both sides of the equation to conclude that *a=a'* also.

One of the main issues that students have while working with expressions is that they try to simplify algebraic expressions down to one term even when it is not possible. In 1998, a study was conducted of four seventh grade math teachers.[1] The strategies the teachers used when teaching how to simplify algebraic expressions, and how aware the teachers were of the common mistakes their students tended to make were observed, recorded, and investigated. In four separate teaching situations, it was observed that there are "different sources to students' tendency to conjoin open expressions".[2] When students conjoin open expressions, an expression that includes combinations of numerical terms, algebraic terms and operations, they are calculating for one numerical or algebraic term. This being said, if they simplify an expression and end up with the open expression 3x + 5, they do not believe this is the answer because there remains an operation and it looks as if it is not finished. They do not understand that this expression does not and cannot have a specific value unless numbers are assigned for x. What they tend to do is add the two terms as if the 5 also has the *variable*, x, to get 3x + 5x and say it is the same as 8x. Another way students conjoin these open expressions is by removing the *variable* from the term 3x to get 3 and adding 3 + 5 to get 8. To test if this is correct, students should substitute various values for x in order to see if they get the same answer from the two expressions. It should be made clear that the role of the *variable* is to represent an unknown number and when in an algebraic expression, it allows us to identify equivalent expressions. For two expressions, if substituting in the same value for the *variable* in both always results in the same numerical value, then the expressions are *equivalent*. The topic of the role of *variables* in expressions is covered more deeply in the unit of Jeffrey Rossiter.

One of the reasons students want to conjoin, or oversimplify, open expressions is because "students face cognitive difficulty in accepting lack of closure".[3] When students start learning math, they are taught that when they see the addition sign between two or more terms, they must add the terms together in order to get an answer. This might not always be the case. As they get to higher levels of math, it is crucial for them to be able to identify not only what the addition symbol means, but all the symbols of the operations, and especially the equal sign, which has been shown to be often misinterpreted.[4] When they see an equal sign, they are accustomed to computing for one specific value, not finding equivalence through other forms of expressions. This could probably be avoided, by using the equal sign more flexibly in elementary school. Students should be able to notice the difference between the case of 5 + 2 and 5x + 2. Being that in elementary school when they saw 5 + 2 it meant that they had 7, when they see 5x + 2 in middle or high school they feel as though that open expression is incomplete. To discourage this misunderstanding, they must get used to the idea of using substitution to check their work in order to see if they are correct. By substituting values for the *variable*, they will be able to see if the expressions are equivalent. I have concluded that I must not skip these crucial steps in our teaching and assume that students understand or have been taught the material as clearly as they should have been, especially in math, because it is a new language that can get lost in translation. I will try to listen to them carefully and when they make these mistakes of "finishing expressions," focus on understanding their thinking in order to fix it. I am afraid that if I do not, they will go back to what is comfortable to them and continue making these errors.

In schools, teachers teach “syntactics (the study of rules governing the behavior of systems, without referring to meaning)” rather than “semantics (the study of meaning)”.[5] If students do not learn the meaning behind the symbols, they will not be able to understand why and how the operations are able to do what they do, which causes them to not understand why they cannot “finish expressions” the way their minds want them to. I will aim to provide a combination of both syntax and semantics when teaching mathematical concepts to our students, whether they are simple or more complex. If they have the background knowledge they need when solving problems in math, as they reach higher levels, if they have forgotten the “rules” of certain topics, they will still have the knowledge to be able to get back to that “rule” in order to solve the problem. This split focus between semantics and syntax will be most helpful while covering the rules of arithmetic, which students know as the Properties of Operations. These Properties should not simply be memorized by students as a list of rules when first learning them. They should be clear on what each of these rules mean in order to understand how and why they can manipulate numbers.

As students get to more complex expressions, they tend to struggle with the simplification process. In textbooks, students are always advised to combine like terms when they simplify. However, it is never made clear that “combining like terms” is an application of the Distributive Rule. This is where they manipulate an expression, for example 12x+25-3x-3, to have the “like terms,” which are the terms that have the same *variable* raised to the same power, next to each other, 12x-3x+25-3 to be able to add or subtract them easier and get an expression in standard form, 9x+22. The confusion with a problem like this is usually related to the subtraction symbol. When students manipulate the expression, they do not realize that the subtraction symbol should be interpreted to mean that they are adding the additive inverse of the indicated term. The result of this is that they often write the expression as 12x+3x-25-3, keeping the signs where they were while permuting the symbolic terms, instead of moving the signs as integral parts of the whole symbolic term. Another way that students struggle working with simplification is when an expression involves *variables* raised to different powers. Although my unit focuses on linear expressions, it is crucial that I touch upon the fact that a *variable* raised to the power of one, x, is not the same as a *variable* raised to the power of two, x^2, because when we substitute a number greater than 1 for x, and then substitute the same number in x^2, we will get different answers, and see that they are not equivalent. For example, if $x = 2$, then $x = 2$, but $x^2 = 2^2 = 2\times2 = 4$.

Math Background

Translating Verbal Statements to Find Equivalent Expressions

This unit will begin with translating verbal phrases into algebraic expressions, meaning students will be creating expressions from a series of words by distinguishing operations and how they should be set up. Before beginning to translate verbal statements into equivalent expressions, students will have learned the language necessary to convert, for example “twice a number plus seven” into an algebraic expression, as well as understand that algebraic expressions are directions to be able to compute. Students will be able to recognize that “twice a number” means 2 times some number that students can choose. This will be represented by some *variable*, a letter that represents the unknown number, and “plus seven” means that you are adding 7 to that term. Since both terms are in the form of one conjoined statement, they will know that they have to include both terms to create one linear expression in the form “ax + b”, where a and b are any numbers. In this case, the expression would be 2x + 7. Since they will also understand that 2x + 7 can be read as picking a number x multiplying it by 2 and adding 7, when finding expressions that are equivalent, if when they compute the expressions for any choice of number for x, they produce the same result, then they are indeed equivalent. For algebraic expressions to be equivalent, students should be able to choose any value for the *variable* x, substitute this value in the two expressions, compute the resulting numerical value, and they will always get the same answer. This unit will cover the more complex algebraic expressions that will require the use of multiple operations at one time. For example, I would like my students to be able to simplify an expression like 3((2x+7) – (4x – 5)) + 6(x – 5). It will be important to give clear instructions on translating the simple numerical and algebraic word phrases before moving on because then they will struggle with the more complex expressions being represented in this unit. Although students will have been introduced to the terminology involved in this topic, it will still be necessary to touch upon the vocabulary as a reminder. Some of this vocabulary includes expression, algebraic expression, equivalent, coefficient, constant, and *variable*.

In order to ensure that students understand the concept, it is necessary to scaffold. There are many types of complex word phrases, but they will be categorized into three groups, Tier 1, Tier 2, and Tier 3, to show growth in complexity. An example from each group would be,

Tier 1	Tier 2	Tier 3
Let *c* be the number of counties in Wisconsin. Oklahoma has 4 less than 6 times the number of counties in Wisconsin. How many counties does Oklahoma have? a) 4-6c b) 6c-4 c) 2(3c-2) d) 2c	Five times the sum of a number and four a) 5s+4 b) 5(s+4) c) 5s+20 d) 9s	The product of three-fourths and eight more than twice a number, plus five and one half a) ¾•8+2z+5 ½ b) 6+2z+5 ½ c) ¾(2z+8)+5 ½ d) ¾z+6+5 ½ e) 3/2 z + 11 ½

Notice that each tier has multiple steps, but as the students go from one to the next, the questions will gradually become harder. This will allow them to work their way up to more complicated expressions. The question that will accompany this table, or each question separately, is “Determine which expressions are equivalent to the verbal statements. State which expression most closely represents the verbal phrase.” In the Tier 1 problem, students are given the *variable* that they should use in the expression in order for them to understand that it should be representing the number of counties in Wisconsin. It is necessary for students to understand the idea behind “less than” so that they know they are taking 4 away from the term 6c, which is representing “6 times the number of counties in Wisconsin,” to get the full expression 6c-4, answer choice b). Since the question also asks to determine which expressions are equivalent, there is another answer choice that will answer this question and that is answer choice c). Students may not know how to use the distributive rule just yet, but they will have been introduced to substitution earlier in the year. This will allow them to be able to substitute the *variablec* with any number to find which expressions are equivalent. For example, if c = 3, in answer choice b) students would get 6(3)-4=18-4=14. With this information and their knowledge on the meaning of equivalent expressions, students will be able to find the answer choice that is also equal to 14 in order to choose the correct equivalent expression. The answer choices were specifically created in order to catch any misconceptions that students might have while translating word phrases. The first answer choice touches on the mistake that students make when they see the phrase “less than.” They know it means to subtract, but they have a hard time realizing that it actually means that something is 4 units smaller than the original. The last answer choice addresses the misconception of conjoining terms that cannot be subtracted from one another.

Once students have chosen their equivalent expressions, a follow up question can be asked. An example of a follow up question is, "If Wisconsin has 72 counties and Oklahoma has 77 counties, is this correct? Show your work to justify your answer." This touches on the process of substitution to check their work. Since they know that Wisconsin has 72 counties and in the expression *c* represents the number of counties in Wisconsin, they can substitute the 72 for the *c* to show Oklahoma would have 6(72)-4=432-4=428 counties, which is not true. To challenge students and allow them to formulate a variety of answers within the class, I will also ask them to formulate an expression that would give the correct answer. Students would have to define a *variable* for the number of counties that Wisconsin has and begin manipulating numbers and operations to create an expression that would equal 77, for example let c represent the number of counties in Wisconsin then 2c – 67 or c+5would equal the number of counties in Oklahoma. Although the problems in Tier 2 and Tier 3 are more complex, the same steps and thought processes are followed when creating the answer choices.

Justifying Equivalence Using the Properties of the Operations

Once students have a grasp on what these more complex algebraic expressions look like, they will be introduced to manipulating them using the rules of arithmetic. These rules are also known as the Properties of Operations. They give us a clear understanding of the formal properties of the operations, addition, subtraction, multiplication, and division, and why manipulations work better with some of these operations than others. These rules also allow the students to understand that when certain rules are applied, alone or together, an expression can be transformed into a different, but equivalent one. Usually in the classroom, students are given these sets of rules as a list that shows them how to perform them in order to create equivalent expressions but never go into detail as to why it all works. To clear up these misunderstandings, this unit will be focusing on the "why" aspect of the rules.

In order to explain the Rules of Arithmetic, which are listed in Appendix 2, I will attempt the same strategy as is used to simplify expressions in a math problem. Before getting into the "why" we must first introduce the proper vocabulary that will be addressed in this section of the unit. They include the Commutative Rule, the Associative Rule, the Identity Rule and the Inverse Rule for each operation (i.e., addition and multiplication) and the Distributive Rule that connects the two.

The Commutative Rule of Addition is a rule that students know to be true, but might not officially know it as being the Commutative Rule. It explains that when we add any two numbers, we can reverse their order and still get the same answer. This is the same for the Commutative Rule of Multiplication, any order we multiply two numbers, we get the same answer, for example if we multiply 10 by 5 that will give us 50. If we multiply 5 by 10, we will still get 50. Since they both give us the product of 50, it can be written that 10•5=5•10.

The next rule to be discussed is the Associative Rule. It states that when adding three numbers, if we move the parentheses to change the numbers that will be added first, we can still get the same result. For example, if we add 9, 7, and 3, we can group them as follows, (9+7)+3. To solve this, we must add 9 and 7 first to get 16 then add 3 to that sum which will get us 19. If we must the parentheses to add 7 and 3 first, then 9 to that sum, the new expression would be (9+7)+3. The sum of the numbers in the parentheses would be 10 and after adding 9 we will also get 19. This shows us that the two expressions are equivalent, (9+7)+3=9+(7+3). This is also true when multiplying three numbers.

Once my students understand the Commutative and Associative Rules of Addition and Multiplication, I will introduce them to the Identity Rule. The Identity Rule of Addition states that when we add any number to 0, we get the same number because adding a number to 0 does not change the value of a number. The Identity Rule of Multiplication states that when we multiply any number by 1, the value doesn't change.

The next Rule of Arithmetic is the Inverse Rule. This rule states that while working with the operation of addition, we must find its inverse or its opposite and add them together. In the case of addition, the opposite of a positive number, 10 would be its negative, -10 and when we add them, we would get 0. With the multiplication rule, we must do the same, find the inverse of a number and multiply the two to get the result of 1. The inverse in the case of multiplication is the reciprocal, which introduces fractions. For example, if we have the number 6, its reciprocal would be 1/6 and we know this to be true because when multiplied, the product is 1: $1/6\times6=1$. From this rule, we are able to use subtraction and division where necessary.

The final rule that students will learn is the Distributive Rule, which involves a combination of addition and multiplication. A clear understanding of the Distributive Rule is necessary because students often times are unsure of what it means to distribute a number. If this remains the case, as we get to the algebraic expressions, students will become discouraged because they will not always be able to deal with expressions involving multiplying a sum in parentheses, this will throw them off and cause many issues. Students incorrectly want to multiply the specific number with the first term, but not the second, which causes them to come up with an incorrect answer.

The symbolic statement of the Distributive Rule is, for any numbers *a, b,* and *c*, $a\bullet(b+c)=a\bullet b+a\bullet c$. This canbe illustrated with the area model of multiplication in Figure 2. It is visually clear that this area model has 3 rows and the columns are split into two sections, one of 4 units and the other of x units. To find the total number of square units of this model we can find the area, *LW,* of each section. For the first section, we would multiply x and 3 to get $x\bullet3$ or $3\bullet x$ as the area. In the second section of the rectangle, we would multiply 4 and 3 to get $4\bullet3$ or $3\bullet4$. The total area in this case would be found by adding the area of the first section and the area of the second section, $(3\bullet x)+(3\bullet4)$. Finally, through the Distributive Rule, we can factor out the 3 to get $3\bullet(x+4)$ which means that 3 should be multiplied by the whole sum of x+4.

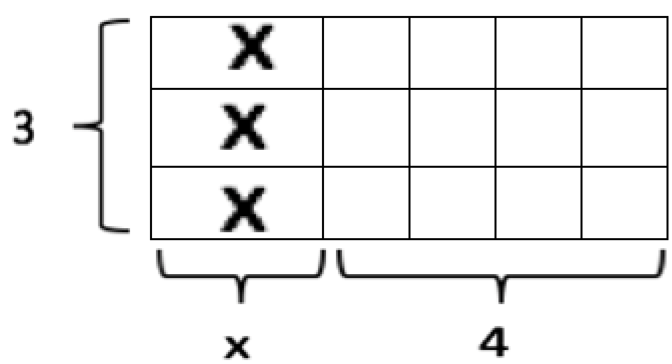

Figure 2

The questioning method in this section will follow the same process as when students were translating expressions. The questions will be tiered for each Rule of Arithmetic. Tier 1 will be only numerical expressions, Tier 2 will be algebraic expressions, and Tier 3 will be a combination of the rules. This allows students to gradually understand why the rules operate the way that they do. The problems for Tiers 1 and 2 can be referred to in Classroom Activities.

Tier 3 is not specific to any one rule. It will include problems that incorporate different combinations of the rules of arithmetic. To build students up to expressions that look more complex, there must be a progression of problems. The table below shows this progression.

Basic	Intermediate	Complex
(u + 5) + u	2(r-4)+7	-2/3(5c+6)+7c

If we look at the problem in the "Basic" column visually it is much simpler than the other two. The question that would be asked of the students would be to name the rules that would be used to transform the given expression to an equivalent expression in standard form, ax + b, using the Rules of Arithmetic and it would be answered in the following way:

(u + 5) + u	Beginning
=u+(5+u)	Associative Rule of Addition
=u+(u+5)	Commutative Rule of Addition
=("u+u)+5	Associative Rule of Addition
=("1u+1u)+5	Identity Rule
=("1+1)u+5	Distributive Rule
=2u+5	Arithmetic: 1 + 1 =2

Beginning with (u + 5) + u, the students should be able to notice that they should get the variables next to each other in order to add them together, but there are parentheses that should be moved first using the Associative Rule of Addition. After moving the parentheses, students will have u+(5+u). The *variable* and 5 should then be switched by using the Commutative Rule of Addition, to get u+(5+u). Once the *variable* and 5 are reversed, they must use the Associative Rule of Addition in order to group the *variables* together, then the Identity Rule and Distributive Rule to obtain(1+1)u . Finally, once they add the coefficients in the parentheses they will get the final expression 2u+5.

This method, of having students show their step-by-step process will allow for them to quickly simplify expressions in the later section because they will have understood how to manipulate complex expressions using the rules. By teaching why we must properly apply these rules to expressions rather than simply teaching our students how to apply these rules, students will be able to catch their mistakes before they make them. This is where the semantics, meaning, take over in the classroom with the syntax, how to solve a problem, backing it up.

This example also illustrates that there are multiple Rules being used even in this relatively simple type of problem. As the expressions get more complicated, it becomes more and more tedious to just use one step at a time. There is a more efficient method that students will be able to use called the Any Which Way Rule. This rule allows students to manipulate expressions that need to be simplified and includes multiple applications of the Associative and Commutative Rules of Addition. If there are one or two sets of parentheses and any number of terms being added together, this rule allows us to reorder the terms and parentheses to have the like terms together to be added at one time.[6] For example in the example above, students would be able to go directly from (u + 5) + u to (u+u)+5 using the Any Which Way Rule. Similarly, they could convert a more complicated expression, such as (3x+5) + (4x – 3) directly to (3x+4x) + (5 – 3).

Simplifying Expressions

As students have received practice on using the rules of arithmetic and understanding how and why they have worked in the previous section, they will begin simplifying these algebraic expressions. Through this process of simplification, students will take any combination of linear algebraic expressions and manipulate them through addition or subtraction to get them down to one equivalent expression in the standard form of "ax + b" where *a* and *b* are specific numbers. In this section, it will be important that I focus on explaining why certain terms can be combined while others cannot. There is a common gap in student understanding during this section of the unit, because many of my students do not know how to apply these rules that they had to memorize when simplifying complex expressions. This is where the "conjoining" will occur and students' thought processes must be heard out so that I can properly allow my students to grow as mathematicians. I will talk them through the reasoning behind why the new expression is equivalent to the compound expression by referring back to the vocabulary that they learned in the beginning and previous lessons. This is crucial in order for them to be able to support their reasoning correctly. This will ensure a clear understanding of the language of mathematics, especially as they build on their ability to speak about math.

An example of a problem they would have to simplify in this section would be -3/4 (4k-8)+ 4/3 which could be done in the following way:

- 3 /4 (4k-8)+ 4 /3	(1) Given
=- 3 /4 (4k)- (- 3 /4)(8)+ 4 /3	(2) Distributive Rule
= (- 3 /4 •4)k- (- 3 /4)(8)+ 4 /3	(3) Associative Rule of Multiplication
=-3k-(-6)+ 4 /3	(4) Arithmetic
=-3k+6+ 4 /3	(5) The negative of a negative is the original number. (Consequence of the Inverse Rule.)
=-3k+6+1 1 /3	(6) Arithmetic
=-3k+7 1 /3	(7) Arithmetic

Students would begin by using the Distributive Rule to multiply 4k and 8 by - ¾ in order to remove the parentheses and be able to combine our numerical terms. After distributing, we end up with an equivalent expression (3) where, through the Associative Rule of Multiplication, we move the parentheses to multiply -3/4 •4 and get -3. We must then multiply -3/4 to 8 which is equal to -6. Since we will be subtracting -6, through the "Definition of Subtraction"[7] when we subtract a negative, it means we will be adding 6. Now that the parentheses were properly removed, we are able to combine our terms that are alike to get to the expression in line (7). To check and ensure that they have gotten the correct answer, students should substitute any number for the x of the original expression and the x of the simplified equivalent expression, and verify that they get the same value.

Teaching Strategies

There are a number of teaching strategies that can be used when teaching the three topics that are being covered in this unit. The strategies that I plan to use in this unit will implement are "think, pair, share", small and whole group discussion, and modeling.

Think, Pair, Share

The teaching strategy, “think, pair, share” allows students time to analyze a specific problem that they are given in order to get to a solution or idea on their own rather than the teacher quickly giving them the answers. Once they have figured out and formed an answer or idea, they share their thoughts with a partner. These partners can be their shoulder partner or strategically picked based on levels of knowledge. In their pairs, it is crucial to monitor that both parties share their ideas and thinking processes because when one speaks about a topic that they may not be sure of, they are able to get a clearer understanding about the subject through fixing any misunderstandings. Through these various combinations of pairs, the students will be able to hear multiple points of views throughout the topics of translating, equivalence, and simplifying of expressions. Not only would it clear up any misunderstandings, but it will expose the students to the various ways that people solve problems. I like to get my students talking as much as possible. It is a great skill to build on from when students are younger in order for them to be comfortable and become more confident in their math skills. It will be beneficial to use the strategy of “think, pair, share” because some students who are more quiet will not feel comfortable sharing whole group just yet. They need to build up to that point.

Whole/Small Group Discussion

Following “think, pair, share” will be whole or small group discussions, which allow the class to have an open discussion about a specific topic in a bigger setting. They work up to this from talking in pairs to ensure that students feel comfortable using the language of math. This is beneficial because with the topics being covered, there is much that we can allow students to talk about. Although the all of the students will not be willing to share, I still can find ways to include them in the whole group conversations. These students who are quiet or shy about answering questions out loud can be asked simple questions, such as to read the problem or what is the *variable* in the expression. This will establish comfort and build their confidence. These discussions must drive the students to a clear understanding of why we can or cannot manipulate expressions in certain ways.

Whole or small group discussions are also helpful because if there are any misconceptions that students may have, they can be cleared up as a whole group. For example, if a student gets the wrong answer when translating or simplifying an expression, I can ask the student to share their answer with the class and then I can lead a discussion as to what the student did correct and incorrect, with the correct parts getting good recognition. After students determine what is correct about the answer, the discussion about what he or she did incorrect will begin in order to understand the student’s thinking before correcting the mistake. By doing this, it allows all of the students in the class to recognize the mistake for the next time that they are solving a similar problem. Through this kind of conversation, I can not only point out the answers that are incorrect, but they can point out all of the various ways that students have solved the same problem. Highlighting student’s work whole class or in small groups, will allow for students to learn from one another. I might like to show my students multiple ways to solve the same problem, but it is often difficult to keep them engaged long enough. But if they see, for example, that one of their peers has simplified an expression in a way that they might be comfortable with, they can use that method of simplification.

Modeling (I Do, We Do, You Do)

A strategy I have seen as being beneficial in my classroom has been modeling, showing students how to approach the problem and the thought process behind it. There are many ways to model problems in the classroom. The specific method of modeling that will be used is the “I Do, We Do, You Do” method. This strategy provides a clear way of getting students comfortable with solving problems on their own through gradual release. In the “I Do” section, students listen to the teacher explain their thought process on why they are following the specific process of solving the problem. Students are able to ask any questions that they have in this part in order to avoid mistakes they might make later. Next is “We Do,” where students work through the problem with the teacher. This is important because I should be helping students through the process by asking leading questions such as “what should I do now” or “what should the next step be”, for students to clarify why they are doing each step or how they got there. While asking these leading questions, I can use this time to build the confidence of the students who are usually struggling in my class. Since the “I Do” should be a clear representation of the process taken to find an answer, the students should have an idea of what the next process should be and will be more willing to participate in the lesson. “You Do” allows students to work on their own or with their peers, similar to small groups, in order to practice the process, they have learned earlier in the lesson. I will allow the students to first work with their peers, in the form of the “think, pair, share” method to transition them into working on their own. This is their final opportunity to ask peers or myself for clarification on anything they did not understand during the “I Do” and “We Do.” To end this type of lesson, the final “You Do” will be in the form of an exit ticket or some sort of small assessment. It is necessary that I end with them solving a problem independently because then it is clear to me which students understand and where other students had misunderstandings.[8]

Classroom Activities

After students have understood the Rules of Arithmetic, they will be able to put what they have learned to use through activities for each tier. As a way to get them moving around the room for the Tier 1 question, students will be given a card labeled A or B with an expression on it and they must find which expression is equivalent to their own on the opposite lettered card.

Using the card given to you, either A or B, find the expression that is your equivalent.

A	B	A	B
(5+3)+7	5+10	(8+22)+3	8+25
2+(5+3)	7+3	(3+4)+5	3+9
(10+9)+31	10+40	(12+6)+8	12+14
(11+4)+5	11+9	9+(11+40)	20+40
4+(44+6)	48+6	1+(13+10)	14+10

The next activity will be used for a Tier 2 question and it will have the students working more closely with one another in pairs and later lead to whole class discussions to talk about how using the rules can lead to equivalent expressions.

With your partner, use the following cards to match the algebraic expressions that are equivalent. Once you find the equivalent expressions, answer the following:

1. *How do you know that the expressions are equivalent? Use complete sentences.*
2. *Use substitution to prove that the expressions are equivalent.*

A	B	A	B

(4x+3)+7	4x+10	4x+(3x+5)	7x+5
1+(2+4x)	3+4x	(20+34x)+32x	20+66x
(10x+4)+13	10x+17	4x+(6x+5)	10x+5
(9+5x)+8x	9+13x	(14x+17)+3	14x+20
(13+14x)+6x	13+20x	11+(2+13x)	13+13x

Appendix 1 – Rules of Arithmetic

Commutative Rule of Addition:	Commutative Rule of Multiplication:
For any numbers *a* and *b*,	
a+b=b+a	a•b=b•a
Associative Rule of Addition:	Associative Rule of Multiplication:
For any numbers *a*, *b,* and *c*,	
(a+b)+c=a+(b+c)	a(a•b)•c=a•(b•c)
Identity Rule of Addition:	Identity Rule of Multiplication:
For any number *a*,	
0+a=a	1•a=a
Inverse Rule of Addition:	Inverse Rule of Multiplication:
For any number *a*,	
-a+a=0	1/a•a=1

Distributive Rule:

For any numbers *a*, *b,* and *c*,

a•(b+c)=a•b+a•c

Any Which Way Rule for Addition:

Given any list of numbers to be added, they may be ordered in any way, and grouped by parentheses in any way, without changing the answer. (This includes subtraction by thinking of subtraction of a number as being the same as adding its negative. When rearrangement is done with subtraction, the minus signs should move with the numbers they are in front of.)

Examples:

1. ((((1 + 2) – 3) + 4) – 5) + 6 = (1 + 2) + (4 + 6) – 3 – 5
2. x + (1 + x) = (x + x) + 1
3. x + (x+ 1) + (x + 2) + (x + 3) = (x + x + x + x) + (1 + 2 + 3)
4. (2x + 3) + (4x + 5) = (2x + 4x) + (3 + 5)
5. 23 + 45 = (20 + 3) + (40 + 5) = (20 + 40) + (3 + 5) = 60 + 8 = 68

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Endnotes

1. See reference 6
2. See reference 6
3. See reference 6
4. See reference 4
5. See reference 5
6. See reference 3
7. See reference 3
8. See reference 2