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Introduction of the Variable by Forming and Interpreting Expressions

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by Jeffrey Rossiter

Overview and Rationale

The students whom I have been teaching have an incomplete understanding of algebraic expressions involving variables. This hinders their effectiveness in creating and manipulating expressions and puts them at a disadvantage when the expressions become more complicated. This can become overwhelming for my students, who have limited English fluency, and come from a variety of backgrounds and varying levels of content knowledge. By 7th grade, students in Chicago are expected to be able to create, combine, and translate expressions into mathematical statements in various forms. (See the list of relevant mathematics standards in the Appendix.) It is very apparent that not enough time is spent on this critical skill. I will also discuss the need to reorder present curriculum to generate further content knowledge. When content is rushed, students are left with partial knowledge whose academic gaps widen as the year progresses. With limited knowledge of operations, my students struggle greatly with interpreting symbolic expressions and do not grasp the rules for transforming them.

In this unit, students will be exposed to a variety of problem sets that require them to translate verbal phrases into symbolic notation. These problems will increase my students' ability to function and participate in a discussion-based classroom, once they practice and become attuned to the ways these phrases differ. The method that I favor in my classroom is the Math Talk. This protocol will be structured by guiding questions that I will keep in mind while teaching this unit. Students can then profit from the powerful idea of taking authority and ownership of their own learning and can then build on this very detailed approach. This will be especially valuable when the expressions and operations get more difficult later on in the school year.

Towards the end of the 2016-2017 school year I had been looking to transition into a new role within Chicago Public Schools. I will be moving schools from Lowell Elementary to another elementary school called Helen C. Peirce School of International Studies. My new school is located in the northern part of the city in the neighborhood of Andersonville. Peirce's mission statement is as follows: **"**Our mission is to guide students to take ownership of their learning through experiential engagement and reflective thinking. We provide a balanced curriculum designed to meet the academic, cultural, and social-emotional needs of our diverse student body. All members of the Peirce community are committed to grow as productive, globally-minded citizens."[1]

Peirce is a neighborhood school composed of a wide demographic with the majority of students having Hispanic descent. Peirce has a low-income population of about two-thirds with students having limited English proficiency hovering around one in five. This is relevant to my unit, because it will primarily focus on how expressions are translated to English. As in every neighborhood school with diverse populations, there is a link between increasing English fluency and overall math achievement. From my experiences, I see students struggle with making expressions from mathematical statements and transforming them into English. Developing a keen eye as to what problems are saying is very challenging for all students in the middle school classroom, and the more so with students not fluent in English. This unit will be a series of discussion-based and exploratory lessons accompanied with Polya’s Problem Solving Method that will help students appreciate the multiple aspects of problem solving. The majority of the Standards for Mathematical Practice of the Common Core Mathematics Standards will be relevant at some point in this unit. However, I will focus on students being able to persevere and attend to precision (Practices 1 and 6).

The majority of my students rely on the latest ‘trick’ or shortcut to finish a problem without any real understanding of the major problem solving techniques. I want to wean them off of these tricks and give my students the foundation that can be applied in other areas. In order for them to engage and reflect critically in a mathematics setting, they need to further their own understanding by building on their own specific understanding of algebraic notation. I want to emphasize to my students that they need to take genuine care when reading word problems. They need to pay special attention to the issue of translating statements, and to analyzing what the problem is asking them to do. I will specifically ask my students to translate expressions into symbolic statements and then back into verbal statements.

The Wrong Path Taken by Past Curricula

There are myriad reasons why my students continue to have difficulties representing phrases symbolically. By sixth grade, which is a benchmark year, most students in the U.S. are required to translate, express and evaluate expressions, not to mention create, justify and apply this information to solving equations. Without careful attention given to the cultivation of this series of skills, students will not be successful. My students do not have a solid method to use when they problem solve. They also have a limited knowledge of the English language. These two characteristics present in my students exacerbate misconceptions and do not allow them easy access points into the curriculum.

Below, I have outlined the progression of the major concepts in this area that I have gathered from the textbooks and curriculum maps that I use. They typically address the Common Core State Standards quite nicely as individual skills and are taught in numerical order. There is a lack in how they are presented coherently together. The progression starts with writing and evaluating expressions with exponents. Students here are already on shaky ground, with limited background knowledge on the meaning and the manipulation of exponents. Simple introduction of exponents at this juncture may not be appropriate.

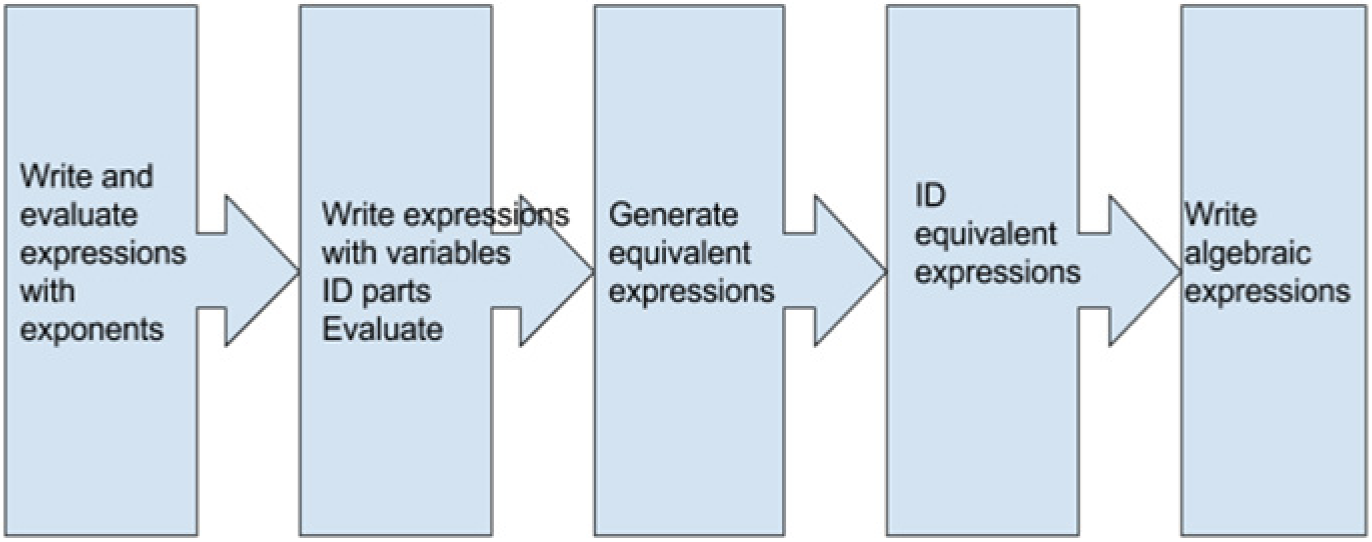

Figure 1. Progression of 6th Grade Standards

Progressing to the next main sections in the diagram, one can see that students are expected to do many things at once. They are writing expressions with variables with very few tools to help them. They are asked to work with variables without really understanding their role in expressions. This is especially problematic for ELL students. In my opinion, most curricula do not include a large variety of terms used, nor enough examples that will lead to the development of fluency. When rote understanding of terms is assessed, students are left with voids in the concepts underlying them. [2] Continuing this progression, students then are asked to create their own versions of algebraic expressions, with at most partial ideas of the roles that variables play. They then are supposed to evaluate these expressions, which requires them to bring to bear a whole new series of skills and background knowledge.

By seventh grade, my students need to be working with ‘higher-level’ problem sets. However, I see myself as a seventh-grade math teacher going back to sixth grade content to help fill in their gaps of learning. The reason why my students continuously struggle is fleshed out above. Maybe the issue is a not thoughtful sixth-grade teacher with a very superficial exposure to material involving expressions, or maybe it is uneven attendance. Following the model above gives students a fragile foundation to build on and has them unprepared to engage in solving equations.

Description of The Multiple Unit Approach and Progression of Learning

By now, I hope, I have convinced you that there must be a better way of sequencing instruction about expressions and equations. The series of student outcomes are split between four units including my own. Each unit will address a particular aspect of the complex process of translating, simplifying, and solving algebraic expressions and equations.

This order will lead to better student fluency and discourse. The four units will begin with Rachelle Soroten’s unit titled “Formulating Algebraic Expressions From Word Problems.” Rachelle will use problems that are relatively simple in nature, to help students learn how to translate words problems into algebra. She will give a variety of examples of translating word problems in to equations involving ‘simple’ expression. Rachelle will teach her students how to appropriately use Singapore bar models and algebra side by side. This gives students an entry point into the curriculum.

Next, my unit will allow students to work on a series of increasingly difficult phrases that are directions to make symbolic expressions. I will rely on Rachelle’s unit for motivation. Since Rachelle is focusing her efforts on conveying foundational understanding of problem structure, my unit presents varied phrases that represent mathematical statements. Since my students are much less familiar with variables, students are not sure how to contextualize or apply variables. During the problem set discussions, students will see that the use of parentheses helps to show the structure of mathematical expressions, and that expressions can be transformed according to the rules of mathematical operations. My students need a reason why they need to be exposed to wide-ranging problems with varying difficulty levels. Multiple representations of verbal expressions will allow students to understand the identification needed for equivalent expressions in a discussion-based classroom.

Immediately following my unit, Xiomara Pacheco’s unit entitled “Simplifying The Issues With Expressions.” She will explain how to manipulate expressions and especially how to reduce first order expressions into standard form. Her problem sets will focus on deepening student knowledge of the distributive property and combining like terms.

Finally, Sally Yoo’s unit “Making Sense of Solving Equations Through Word Problems – The Cornerstone of Algebra” will show how to use all of the above to solve pretty complicated systems of linear equations motivated by word problems. Once Xiomara has reduce the complicated to the more simple, students will be able to focus their efforts primarily on solving two-step equations, or equations that have variables on both sides.

The four units will be taught by all mentioned above in the hopes of creating flexible problem solvers who have built up both their computational fluency and reading capacity. Traditionally, curriculums that I have come across, or even taught for years, have never gone in such detail as the four units that make up this series. This will be new to me as well because I am not satisfied by the end product of my equation/expression units. Starting with word problems as the main motivation for using symbolic algebra makes more sense. Bar models are great tools to use in simple problems, however students need a more sophisticated language to work with. The intent of the series of curriculum units is to replace traditional models with broader investigations into expressions and equations that can take most of the semester to implement.

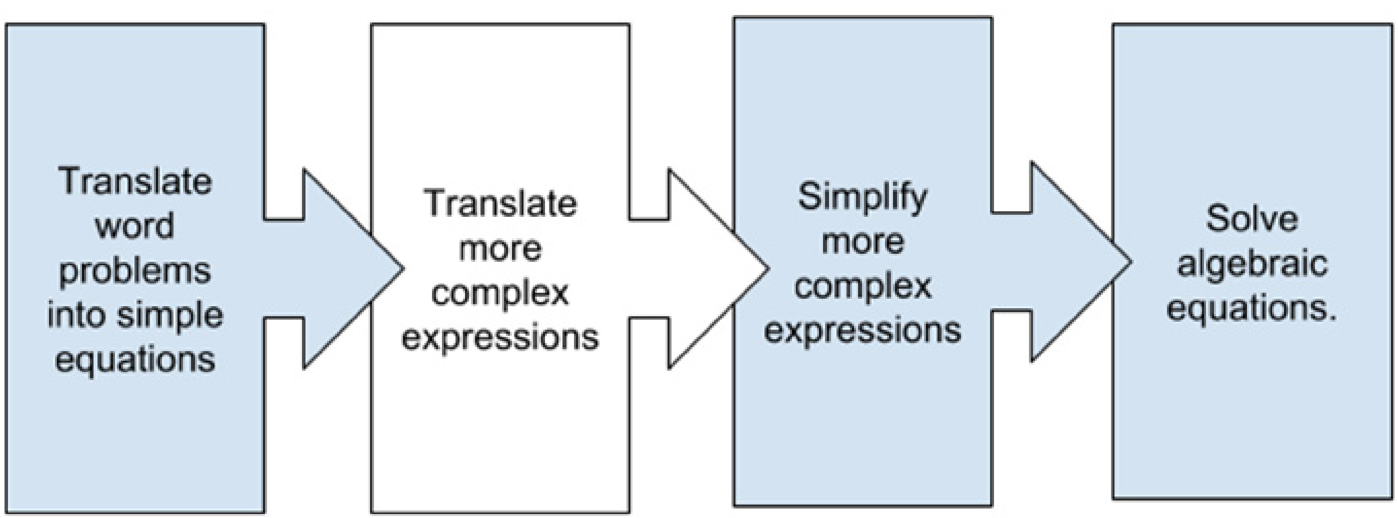

Figure 2. Progression of Combined Unites

Major Implemented Strategies

My students will be able to apply the strategies listed below during classroom discussions as well as later on in their math careers. I will introduce the 3 main strategies that include *Polya's Problem SolvingMethods, Three Reads* and *Math Talks*. I will discuss the effectiveness of all and intertwine them to give students a structure that will help them be successful in a discussion-based environment. I will highlight the advantages of each and describe how they build off of one another to create a culture for rich dialogue in my classroom. I hope that they will be able to apply the discussion techniques in other classrooms at Pierce.

Strategy of Polya's Problem Solving Methods

Students need a tool that can be applied in many junctures in their math careers. This is essential to my unit's success. I will reference the model frequently in classroom discussion and in other units later on in the year. My students need some sort of roadmap that they can apply not only for this unit, but throughout the progression of all units covered. My students are in need of an overall structure to help them organize their problem solving efforts. For this, I will probably eventually want to use Polya's 4-step structure.[3]

The problems sets will require them to read carefully and analyze what is to be done. These can be problems with somewhat complicated wording, but with relatively modest mathematical requirements. *Understand The Problem* can be a serious roadblock that will hinder problem access. It is the first step.

Then, when the problems presented are a little more involved, perhaps two steps rather than one step, they will have to figure out what they should do. This second step is *Devise A Plan*.

When they become skillful at devising a plan, I will present problems for which the calculations are somewhat more demanding with harder numbers (fractions or all variables), so they will have to work harder to *Carry Out The Plan*.

Then, after I have discussed a considerable variety of problems, I will select some to compare, discussing how they are similar and how they are different, and perhaps sorting them into groups of problems that are mathematically similar, although they have different scenarios. That is, I will have them *Reflect,* something that they are not used to doing. This step will allow my students to answer the following questions. *Is the answer reasonable? Can I use another method and get the same answer?* It is in this fourth step that I believe the most learning eventually will take place. I usually will have students keep a journal to help organize their thoughts and reflections post-solving. It is important to have a reading and writing dialogue with students with limited capabilities in the language arts. It helps engage learners who may not be fully motivated to talk in class. This way, students can have discourse in different ways. The written portion is another way in which students can communicate and is an important aspect to a discussion-based classroom.

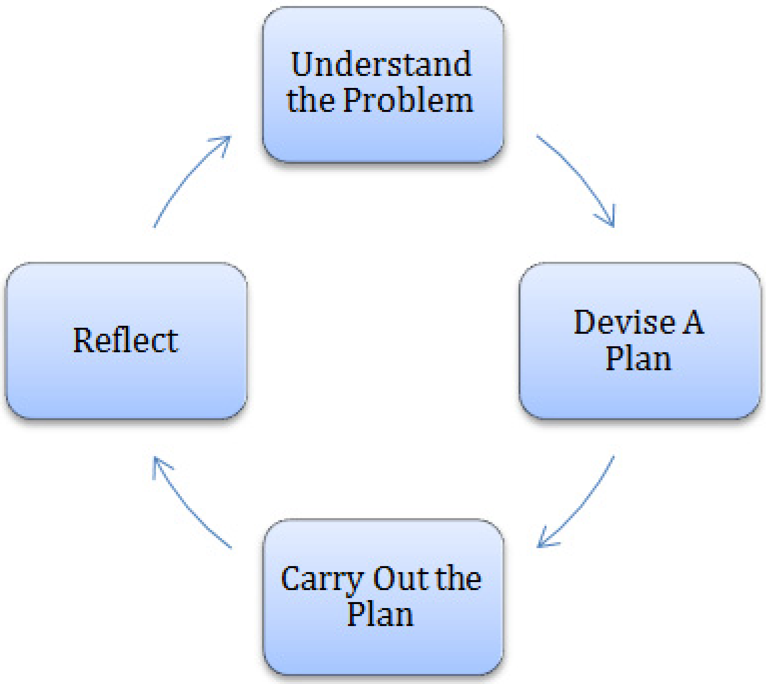

Figure 3. Flowchart of Polya's Problem Solving Steps

This is the perfect strategy to help my students develop a problem solving nature. My students have not yet developed a lot of stamina for persevering through a problem. Polya's method will allow students entry into the problem and give them a roadmap to follow. The most important piece I want my students to gain from method is the area that links reflection to understanding. As my students get more experience with problem solving, reflection can lead to a broader understanding of the topic. Eventually, students will see similarities and differences between problems. Furthermore, this will lead to insight into why certain approaches are more productive than others. Instead of not engaging with the material or question at hand, they will have a jumping off point to get started and go through the steps. Once problems get more complex in format and wording, students will need to figure out a clear direction to go. They will require a patient and gradual practice to overcome the building of complexity. My problem bank has been designed with this in mind.

3 Reads Strategy

A strategy that I will continue to use alongside *Polya's Four-Step Method* will be the *Three Reads Protocol.*[4] The *Three Read Strategy* is another tool that students can rely on once they have been doing problems with me for a while. This can be used alongside Polya. This protocol focuses on SMP 1, where students have to persevere through longer problems. It helps support their sense making and their ability to engage in the solution process. The Three Reads Protocol is a specific approach to Polya's step 1, understand the problem and will continue into step 2, devising a plan.

This strategy involves students reading a text three times and pulling out specific information each read. It is important to introduce this idea with a higher level of cognitive demand task. During the first read of the problem, students are asked to either talk about or write what the problem is. It can be done through a variety of strategies such as: *Think, Pair, Share* or just whole group sharing. This is very similar to Polya's understand the problem. The second read, students are asked to pick out all of the quantities involved in the problem. The students are asked what they mean, how do they relate to each other. Here, I am usually scribing, meaning, writing on the blackboard student responses when they are picking out the numbers and the units. Finally, the last read will revolve around asking the students to combine all the different quantities into mathematical statements. What equations that fit the scenario can you think of? This will allow students to be wrong or partially correct in a positive setting. This will relax the environment because the focus is on understanding rather than the solution.

Math Talks

As in all my classes, I will continue to use the Math Talk principles from *Intentional Talk*.[5] There are four major ones to keep in mind.

Clearly defined goals are a must. Every talk will have a goal, to be accomplished by its end. For each talk, I will need to pay attention to what my students' specific needs are and how the talk will address and ultimately meet that need.

Students should have some sort of universal format when sharing and engaging with other classmates. My favorite one is 'Agree/Disagree with ideas and not the person' because this helps increase a respectful culture. Ideas and thoughts can be very personal and disagreement may lead to hurt feelings that hinder the learning process and slow growth. Additional Resource 1 lists some of the norms that I try to foster in my classroom. I find myself revisiting them frequently during instruction.

The third principle is to set students up for the cultivation of their own ideas. This is not an easy process to develop in my classroom because I have to refrain from over-scaffolding when students struggle.

Lastly, I make clear that all ideas are important to the learning process and that brains matter. I sometimes have told my classes that it is more important to advance the conversation than to be correct, and that their fellow students will be grateful for making mistakes for them. This level of respect and care throughout develops whole citizens. This is aligned with my school mission and vision for our students. Cultural norms help foster critical thinkers who engage in a respectful manner while developing love and compassion for learning.

I use Math Talks year after year, and maintain the same protocol. It makes it easier on me when I loop with the same students the following year. Also, I can have a seamless transition into my new school this upcoming year with the protocols already figured out in advance. Although the style or ideas change from talk to talk, I need to stay consistent in my delivery. Some ideas for a protocol are located in Additional Resources 2. I usually limit talks to 15 minutes. However there is no reason to end a talk early. If there are wonderful ideas on display and there is value in the material being discussed, I firmly believe that talks should organically continue their own pace until they reach a natural ending. I try to make time for talks one or two days a week.

To start a math talk, I give a prompt, and then allow for 1-2 minutes of think time. This is silent in nature and I want to save all the discussion for when it is appropriate. My hope is that during the think time students will be applying all strategies intended to allow for access and develop a structure to organize a classroom dialogue. If a student is combining both *Polya's Method* and the *Three Reads* strategy effectively, then they should have productive think times and be able to find multiple ways to solve the problem presented. When it is time for whole class sharing I act as scribe for my students. I want to make sure that I record the name of the student who contributes each idea. That student owns that idea and it is crucial for students to see where that idea came from. This generates another growth measure for cultural capital and emphasizes that students' ideas are the primary focus.

Below are the problem sets and phrases that will be discussed during our talks. I will outline at the beginning of each operation how talks can be used. Furthermore, I will explain how am I going to use the format of math talks to increase their awareness that many different phrases can be translated to the same expression. Overall, students should be able to carefully read each expression and produce the same end result. I will include in the Additional Resources 3 some questions that I plan to use to help facilitate deeper conversation.

I plan to introduce Math Talks in a very specific manner. I will practice the method, depending on the class familiarity with the discussion technique. I will start off using the Four 4's problem as an initial talk for this unit. The Four 4's problem is to make expressions using four number 4s, with as many different values as possible, using the standard signs for arithmetic expressions in any configuration that is legal. For example, 4 + 4 + 4 + 4 evaluates to 16, while 4 + 4 – 4/4 evaluates to 7. This is particularly useful so that I can start to see where students struggle. I have to keep in mind that it's OK to give hints along the way. I can support student thinking by showing one or two examples. My favorite is 4,444. This solution is reached by pushing the numbers together. I will also give recognition to different ways to obtain the same value.

This takes careful thinking on the part of the students because they have to create as many numbers as they can, using any combination of operations, but only the number four. A total of four of them will be used and the students describe how they ended with their end number. This might show the students that they can be as creative as they want when generating the expressions, as long as they respect the grammar of arithmetic. An important aspect of this problem is to raise sensitivity to and appreciation of the grammar of mathematical formulas, and the use of symbols, and especially, the order of operations. Further comment is located in the activity list.

The next series of talks will include English to algebra problems, and algebra to English problems. The problems below are categorized by operation and will vary in the form in which they are delivered. I envision using the sets as a basis to start fluid discussions.

Problem Sets

The bulk of the problems for this unit will be about translating verbal phrases into symbolic expressions. More precisely, they will present verbal phrases, and the standing prompt will be to translate the given phrase into a symbolic expression. The core of the unit is a collection of such problems that will showcase the subtle nuances of differences in phrases that result in the same expression. Students need multiple representations of each category to ensure future successes. I believe that when students don't have to rely on a formulaic approach to expressions, they will be flexible enough to translate difficult problems. I have included problems with two (or more) steps, some with the same operation, some with different ones, requiring thinking about which is the correct order to do them in, and when it matters and when it doesn't. Instead of labeling these problem sets a,b,c, etc., I will use colors to denote the difficulty level.

This makes planning talks and discussions easier on my students. It is important to classify these problems by difficulty level because of the wide ranges of student abilities in my classroom. This will also allow for ease of implementation. The color meanings are listed below:

- Red: Entry level/basic understanding. This provides an access point to all students. The majority of students will be able to gain confidence to move forward. This is the base level of understanding where I would want all students to understand. Either one variable will be used or variations will be low level.

- Yellow/Gold: Next level. This color code represents the problems that are just above the entry level. They will ramp up the difficulty level. This is designed for struggling students to become more confident in their abilities.
- Green: Grade level problems. These types of problems meet the 7th grade standards provided by the Common Core.
- Blue: High level. These problems are on or above grade level. This will be used for students who have a firm understanding of the lower level problems and need a challenge. They can also be used in a way that students have to work together to generate better discourse.

Each student will get a card with his or her own individual phrase on it. Students will take turns telling the class what expression they've generated based on what was said on their card. Attached underneath each problem set are some ideas that may come up during discussion. Discussion can also be used to compare problem sets to one another.

Addition Problem Sets and Defining a Variable

The first section in a series of problems covering all four operations will be the addition problem sets. As stated above, simple problems should be introduced first. The problems about addition need to be carefully crafted in a way that the commutative and associative properties become clear to the students in an organic way. This can develop from the exposure to the hierarchy of problem sets below.

In the items that have a context, such as Gertrude's salary, my students will need some guidance about defining a variable. This is an additional act of creativity not dictated by the mathematical requirements. Students need to use variables in such a way that leads to the understanding of what role they play. They also need to realize that solving equations will result in a unique value that will make the situation a true statement. That is, the variable will take the value specified by that situation.

Addition: Expression: x+5

- A number plus five
- Five more than a number
- The sum of a number and five
- The total of five and a number – commutative property of addition
- A number increased by five
- Take a number x, and add five to it
- Five added to a number
- Herbert is five years older than Gertrude (who is "x" years old)
- Gertrude's salary is raised by five dollars
- Herbert rode his bike five more minutes than usual

Addition: Expression: (x+5)+7

- A number plus five, and plus seven
- Five more than a number, then seven more
- The sum of a number and five, plus seven
- The total of five and a number, and seven– commutative property of addition
- A number increased by five, then increase it by seven
- Take a number x, and add five to it,then add seven to the result
- Five added to a number, then seven added to that sum
- Gertrude's salary is raised by five dollars, then she got another seven dollar raise
- Herbert rode his bike five more minutes than usual, rested, then rode for seven more minutes

Addition: Expression: (x+5)+(x+7)

- The sum of number plus five, plus that same number plus seven
- (Five more than a number) plus (seven more than a number)
- The sum of a number and five together plus the same number and seven
- A number increased by five and then add it to that same number increased by seven
- Take a number x, and add five to it. Take that same number and add seven to it. Express their sum

Addition: Expressions: (x+5)+(y+7)

- The sum of a number plus five,plus a different number plus seven
- (Five more than a number) plus (seven more than a different number)
- A number increased by five and then add the sum to another number increased by seven
- Take a number x, and add five to it. Take a different number and add seven to it. Express their sum

Figure 4. Addition Problem Sets

The commutative and associative rules will come up during classroom discussions. When talking specifically about the phrases 'a number increased by five' and 'the total of a number and five' students might not feel completely confident that these expressions are the same. Below, I will review the rule involving these expressions. I will replace 'the total of a number and five' with 'x + 5.'

Commutative Rule of Addition Using the Problem Set: $x + 5 = 5 + x$

This rule needs to be sensibly taught and I have used a problem from the set to showcase how to teach the commutative rule. My students might be uncomfortable with this rule and apply it incorrectly. I will tell students to replace x with a simple whole number. When $x = 1$, the left hand side of the equation becomes $(1) + 5 = 6$. The right hand side of the equation becomes $5 + (1)$, which also is equal to 6. They then come to the conclusion that $6 = 6$ and this makes a true statement. The conversation will continue with assigning x = 2, 3 . . . and so on. It is important to review the commutative rule early in the progression of my unit because it shows up again in multiplication, where it is substantially subtler. To further my students understanding of the commutative rule, I will have students generate new phrases that switch the order of the words used. Students then can write down the symbolic notation and compare their results.

Another rule that makes my students uncomfortable is the associative rule. The orange colored problem of $(x + 5) + 7$ might be its own math talk for the whole class. The use of parentheses in addition tells my students what they will be doing first, and then last. It is important to note here that what the students are doing is arithmetically very simple. My goal is that my students will realize that using parentheses can assist in combining like terms and be of aid when the phrases and symbolic notation becomes more complex. I want my students to be able to see a mathematical expression similar to $(x + 5) + 7$ as a set of instructions to follow.

Associative Rule of Addition Using the Problem Set: $(x+5) + 7 = x + (5+7)$.

This expression above can be simplified to $x+12$. A more specific roadmap for simplification and combining like terms will be in Xiomara's unit. It will be acceptable if I choose to expose my students to this idea earlier than the progression of units suggests. I think that my students will already understand the concepts surround how to combine like terms and simplify these expressions. Historically, my students have a harder time when the terms become more complex. A jump in the progression of the units during this point may not be such a bad idea.

The first two columns of problem sets can be used for a comparison-based discussion. It will be useful to highlight the use of parentheses and solidify their meaning when discussing the rules of addition. This is valuable to students because it allows for them to slow down during the less mathematically demanding problem sets. These do not require much cognitive ability, but the true richness is in the comparison aspect to them.

Subtraction Problem Sets

My students struggle with the operation of subtraction a great deal. There are numerous ways that subtraction can be phrased. They often get confused as to what number is first when translating the phrase symbolically. Order did not make a difference with addition, but with subtraction, it is crucial. With practice and exposure to the variety of ways subtraction is phrased students will have an easier time solving word problems with this operation. Again, reading the phrases more than once is a necessity. *Five less than a number* and *a number less than five* do not have the same meaning. If an operation is subtracting five, or taking away five from what you started with, then the phrase *five less than a number* is appropriate. But, it is not as clear as “pick a number and subtract 5 from it.” The other phrase means a number less than five, ie. 4, 3, 2, and so on. Repeated exposure to this during successive math talks will help students make the distinction. Students will have to make sense of how subtle differences in these phrases change the corresponding symbolic expressions.

I think the main take away with these problem sets is to have the discussion surrounding the question: Why doesn’t the commutative property work for subtraction? Within whole numbers, subtraction of a larger from a smaller does not make sense. Within the integers, subtraction one way yields the negative or opposite of subtraction the other way. This can be accomplished at all problem set levels, so it should be able to reach the majority of students. The discussion of subtraction will be quite different, depending on whether negative numbers are part of the discussion or not. I want my students to see that in the realm of whole numbers, one of a – b and b – a does not even make sense. For the purposes of this unit, the focus should be on positive integers. I think that generating new phrases that involve negative integers can be quite useful to further my students’ understanding of subtraction.

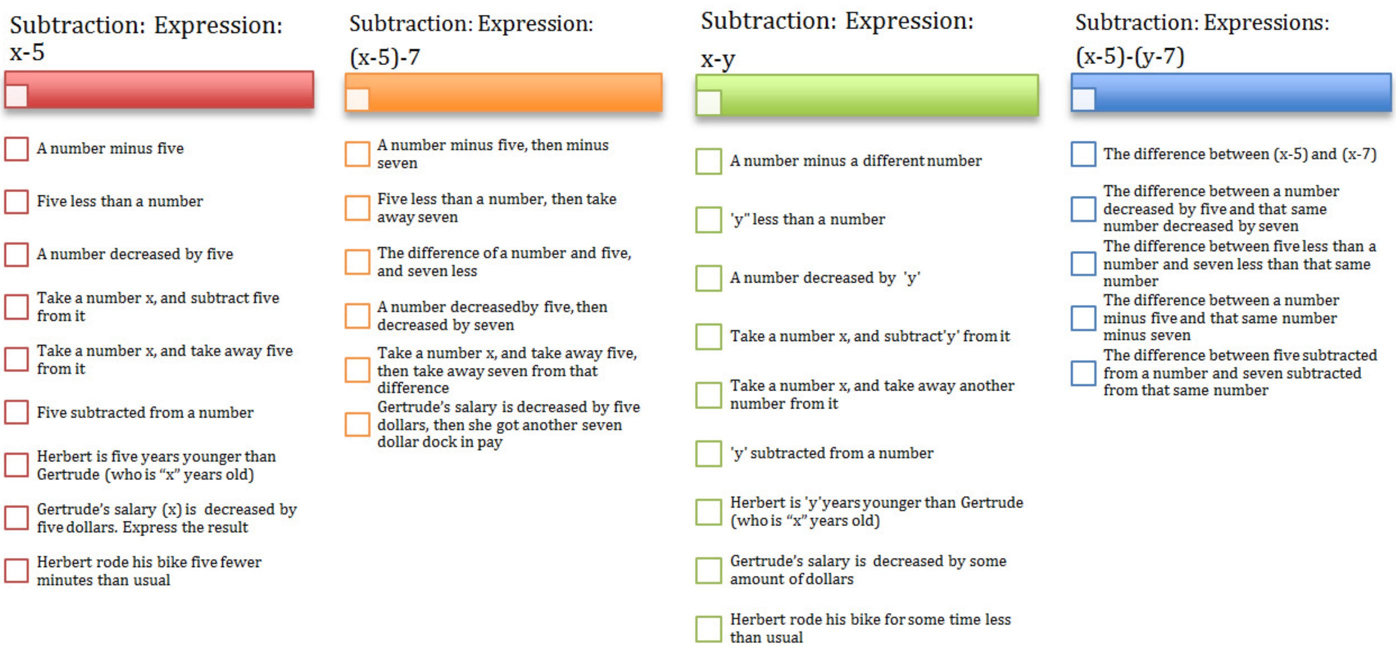

Figure 5. Subtraction Problem Sets

Multiplication Problem Sets

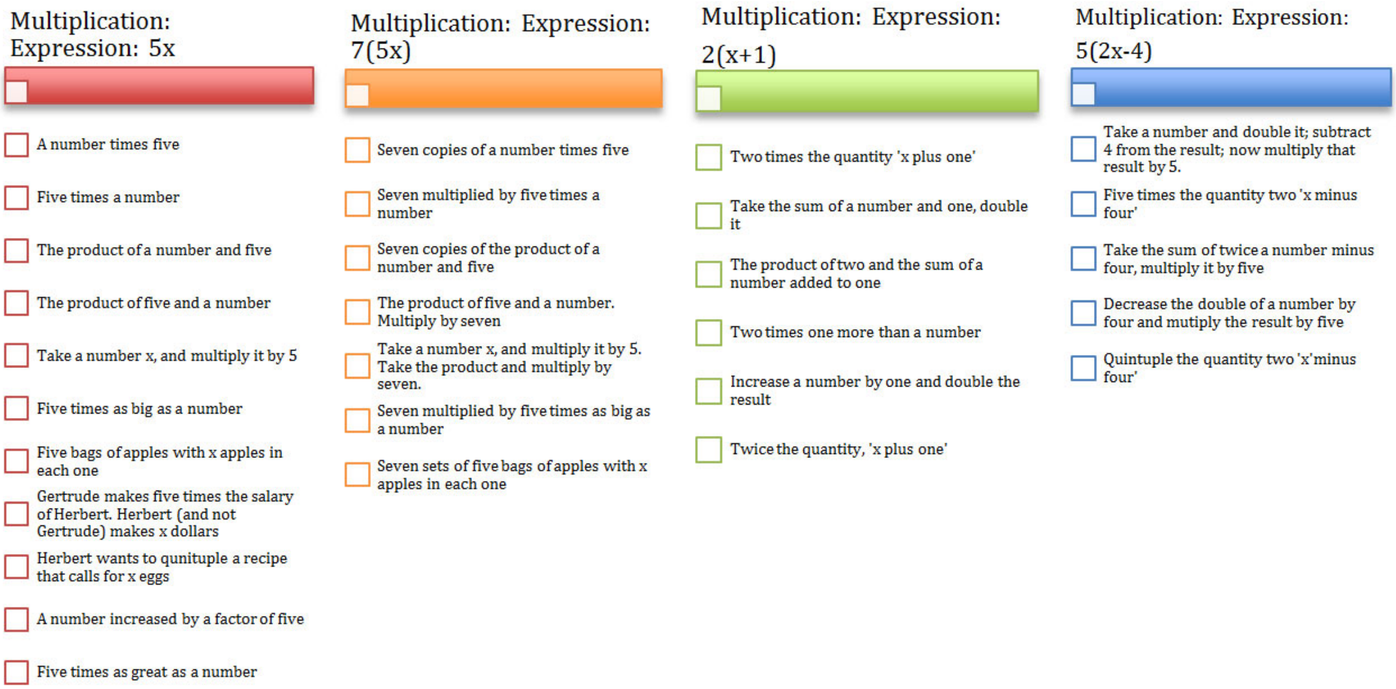

Figure 6. Multiplication Problem Sets

Conversations will continue to focus on the rules of arithmetic. Specifically, students will be exposed to the Commutative Rule and the Associative Rule for multiplication. I want my students to develop a firm meaning of what multiplication and the many ways that this operation is phrased. In my unit “Rational Number Placement on the Number Line,” I talked about students seeing numbers as an oriented distance from zero. I also spoke to the importance of defining a unit and having that unit determine the rest of the whole numbers to the right of zero. They are simply multiples of that unit measure. Assuming that I’ve taught this unit at the beginning of the year, my students will understand that every whole number after one is a multiple of the unit distance from 0 to 1. To place whole number, students simply lay off copies of that unit measure.

Using the example from the problem sets, 5x can mean “the product of five and a number” or “the product of a number and five.” Going forward, I will use the symbol ‘’ for multiplication.

Commutative Rule of Multiplication Using the Problem Set: *5 x = x 5*

This is important to mention that there are many ways that the commutative property will be discussed. To help students understand the order in which numbers are multiplied does not matter will depend on going through the problem set and replacing a set value for the variable. The conversation may look like this: ‘*First, set x=2. What will be 2 times 5. Hopefully, you will get the answer 10! Now take the product of 5 and 2. Again, the answer is 10!’* This can continue to include and exhaustive list of integers substituted into the value for x.

Using the yellow/gold problem set of 7(5x), students can develop understanding of the *Associative Rule.*

Associative Rule of Multiplication Using the Problem Set: 7 • (5•x) = (7•5)•x

Students will see that these two rules are related. More importantly, I want students to again see the use of parentheses as a set of instructions. With the above example, the 5 and the x should be multiplied first, and then multiplied by the 7. I can use the same method to by assigning a value for x and plug that into both sides of the equation. When x = 2 we have the following: *7•(5•2) = (7•5)•2 = 7•(10)=(35)•2 = 70.* More values can replace x to help facilitate conversation. The description of expressions as computations can be used to distinguish between *7 • (5•x)* and *(7•5)•x.* Both of these can be very confusing for students. The operation might not be as clear as it was in previous problems and they may need added supports. The subtleties will become more apparent once this attention to detail becomes common practice. Furthermore, students will get a sneak peak into the *Distributive Rule.* I will use a problem from the set to emphasize the rule. When the expressions start getting a little complicated, it can be valuable to ask students to read them aloud, and show by their intonation where the parentheses are. One student can read, and the others can try to write it, then everyone can discuss.

Distributive Property Rule Using the Problem Set: 2(x+1) = 2 (x) + 2(1) = 2x + 2

The distributive property is something that my students know how to do computationally. However, they do tend to leave out multiplying every term inside the parentheses. To remedy this I anticipate my students leaving out *2(1).* When replacing values for x, when x = 2, my students can see that two different answers arise:

Correctly done: Set x =2 in the problem 2(x+1) = 2 (2) + 2(1) = 4 + 2 = 6

Alternative: 2(x+1) = 2 (2+1) = 2 3 = 6

Anticipated Student Solution: Set x =2 in the problem 2(x+1) = 2 (2) = 4

Anticipated Student Solution: 2(2) + 1 = 5

There is value to writing out all of the steps when first introducing this topic. I think it will force my kids to slow down and be deliberate in their actions when multiplying through the entire expression. There is value to these problem sets now in the progression of the unit. My students will need to be very skilled in applying the algorithm distributive property, more importantly the exact language of this property.

Division Problem Sets

The algorithm of division is a difficult concept for my students to understand. They often interchange the divisor and the dividend. Reviewing these problem sets will help remedy this confusion through studying the phrases involving the operation. In the past, I have always had to review how to do long division and have a discussion with students about '*what number goes under the division symbol*.' However, students adhering to the progression model presented above, they will further their content knowledge and not drain any precious moments we have in the classroom.

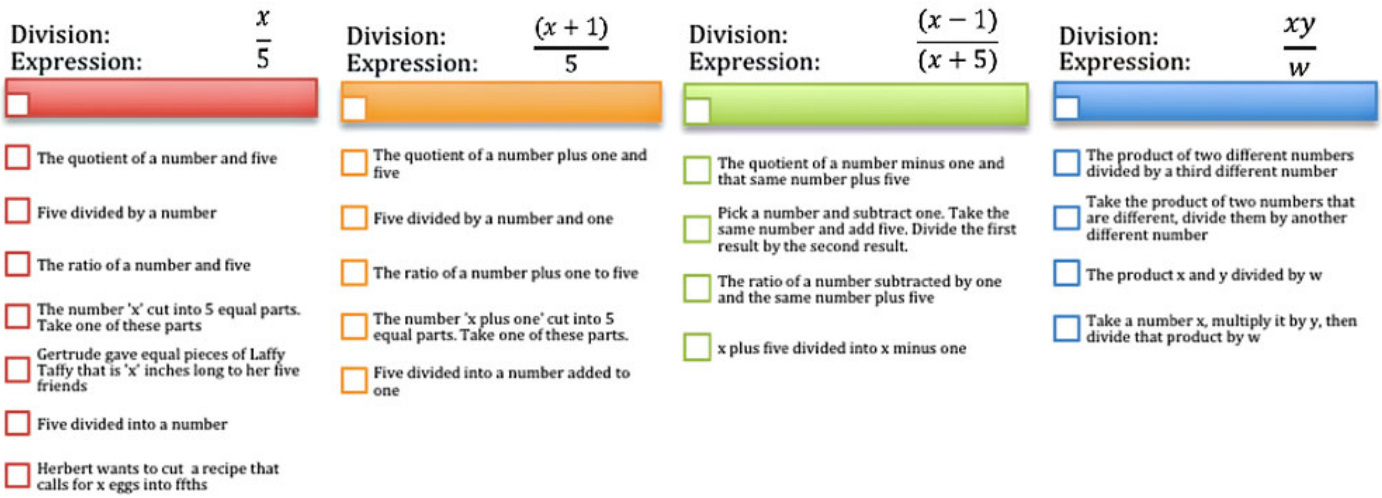

Figure 7. Division Problem Sets

Mixed Problem Sets

This is the cornerstone of my unit. I would want all students to get to this point in the unit with fluid understanding of how expressions fit together and appreciation of the subtle nuances of the language that generates these expressions. Again, the distributive property will dominate this section as it did in the multiplication. I want my students to practice these phrases with more complexity because the process of solving them shows up in later units.

There is a different method to teaching the blue level problem sets at this point in the progression of the unit. Rather than providing many ways of saying the expression 2(x+1) -4(2x+2), I would encourage the kids to apply what they have done in previous problem sets. By this final section of the unit, students should have a starting point so that they can build off of prior successes. Using what they learned in the multiplication section, students will be able to generate expressions and phrases using the distributive property.

This is not only for the higher-level students. I will pair students up and have each group generate one half of the problem. This phrase then will be combined with another group so that every group has two phrases that combine. This will allow students to work and pair with multiple other students in the class. They would combine each phrase into one expression. This scheme should lead to greater buy-in and collegiality in my classroom because students will all play an important role at creating and interpreting these expressions.

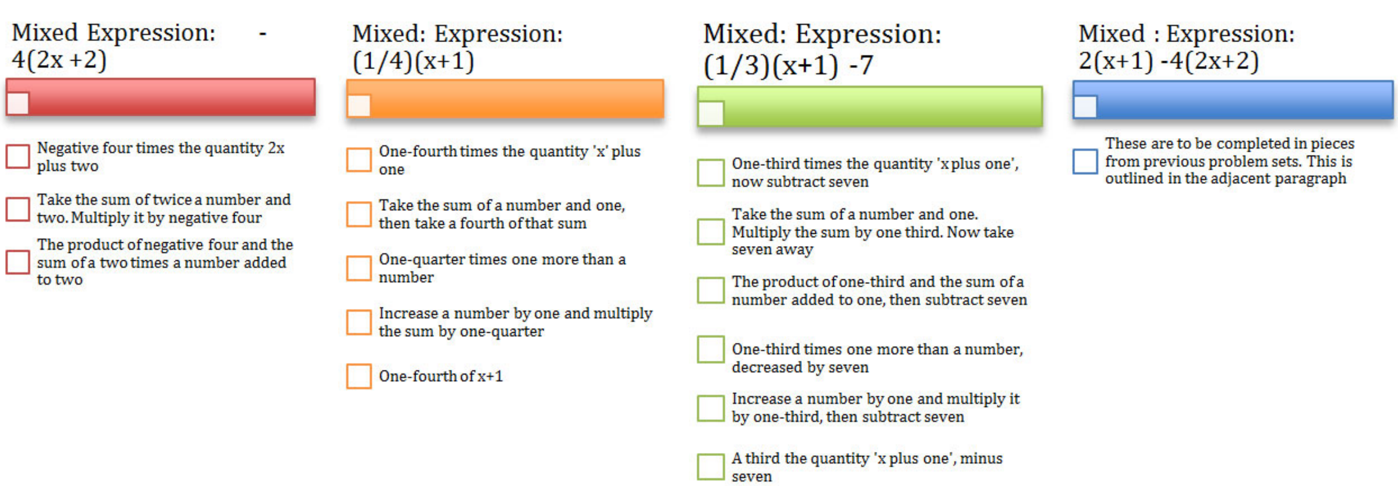

Figure 8. Mixed Problem Sets

Activity List

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1. The "Four 4s" Problem

This problem is made to have students try to create expressions involving four 4s, with as many different values as possible. Then have students explain what calculation each expression describes, in a step-by-step fashion. You can vary it also: 5 fours, 4 fives, and have them see how things change. If you change the number from 4 to 5, some expressions will keep the same value, owing to their structure, although most will change. Some numbers will be easier to produce in one context, and others in the other. This also can lead to valuable discussions. Again, students need to be the focus on the generation of responses. The teacher is the one facilitating the discussion and taking notes using the *Math Talk Protocol.* One way to run this is to write the numbers from 1 to n, with n as large as will fit, at even spaces at the top of the blackboard. Ask students in succession to come up and write under a number an expression that yields that number. Require each new student to produce a different number, until no one can do that. Then you can change to, does anyone have a different expression for some number that we already have. And you can assign them for homework to try to create expressions that produce new numbers.

2. Vocabulary Tri Fold – Maintaining a Running List of Terms

At the beginning of the unit students will be making a tri-folded paper that will house the key vocabulary words for this unit. The paper will be folded into thirds and labeled “What I know”, “What I need to know”, and “What I learned” at the top of each column.

Addition: sum, added to, plus, more than

Subtraction: subtracted from, less than, minus, difference, take away, taken from. Note that “How much greater than” or “How much more” can also be prompts for subtraction as well. The change minus, original quantity unknown type will use a term indicating subtraction, but will require addition for its solution. Similarly, addition language can occur in a problem that requires subtraction. Special care should be taken here.

Multiplication: times, product, multiplied by, groups of

Division: divided by, quotient, divided into Likewise, “How many times greater” can be a prompt for division. It is important to make students conscious of these alternative formulations, and to get them into the habit of reading carefully and analyzing. “Attend to precision” is one of the relevant CCSSM habits.

Many of these terms can be turned into another graphic organizer. This will also be a reference chart in the room when this unit is implemented.

3. Quizlet - Vocabulary Flashcards

This is a technology component where students will be making/completing flashcards with important vocabulary for the unit. Included in this site are a variety of games, independent practice, spelling, and the automatic grouping of students. The site then has groups play against each other in a competitive manor. This can be used with smart phones or iPads. Understanding the basic words and their meanings an essential component to this unit. However, developing the habits of careful reading and analysis, enabled by clear understanding of vocabulary, is the end goal.

4. Think, Pair, Share or a Write, Pair, Share

There are variations of this very common method that I frequently use during instruction. I usually give a prompt and allow for a short amount of think time and the variety is within the next steps and how the information is shared between the students. Sometimes I will have students write, then pass. This is where a partner will write the second portion of a solution or add comments and feedback. The whole group then shares these.

Appendix

Sixth Grade

Students are expected 'apply and extend previous understandings of arithmetic to algebraic expressions.' This leaves the door open for multiple deficiencies in content knowledge.

CCSS.Math.Content.6.EE.A.1 Write and evaluate numerical expressions involving whole-number exponents.

CCSS.Math.Content.6.EE.A.2 Write, read, and evaluate expressions in which letters stand for numbers.

CCSS.Math.Content.6.EE.A.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y*.

CCSS.Math.Content.6.EE.A.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Seventh Grade

CCSS.Math.Content.7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

CCSS.Math.Content.7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

SMP

MP1 - Make sense of problems and persevere in solving themMP6 – Attend to precision

MP7 – Look for and make use of structure

Additional Resources

1 - Math Talk Norms to Facilitate Discussion.

- Agree/Disagree with ideas and not the person.
- Allow people to speak for themselves and allow for appropriate think time.
- It's ok to make mistakes and revise our thinking.
- Make sense of mathematics by asking questions to clarify misconceptions.

2 – Protocol for Math Talks

I want my class to keep in mind that it is not about the speed in which they arrive at the answer. I want them to think deeply about the problem presented and come up with as many unique methods as possible. Once students think they have a solution, they signal to me, with a thumb up close to their chest, that they have one solution. If they have two, another finger goes up and so on. If the problem is more involved and does not require mental math skills, I usually walk around the room to see student progress. I try to sequence student responses by anticipating answers that will lead to overall better understanding of the problem.

3 – Guiding Questions for Teachers During Math Talks

These can also be used as journal entries to further dialogue in my class

What are the similarities and differences between the approaches taken by two students? More specifically, which one is more effective?

Can you explain what happened in a different way?

Ask students to rephrase other solutions

I like to change values mid problem to emphasize a rule or method of solving a problem.

Cold and Hot – Meaning, give an improvement to the presenter's solution and a complement.

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Notes

1. Mission and Vision
2. OMalley
3. Polya – How to Solve It
4. Three Reads
5. Intentional Talk