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Curriculum Units by Fellows of the National Initiative
2017 Volume V: From Arithmetic to Algebra: Variables, Word Problems, Fractions and the Rules

Exploring Kinematic Proportional Relationships

Curriculum Unit 17.05.10, published September 2017
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> "And yet it moves."
>
> -- Galileo Galilei

Introduction

For many people, physics is an intimidating mixture of contemplation and critical thinking about everyday phenomena. Students in particular are often overwhelmed with its multifaceted nature and the complexity involved even with simple motion; especially when coupled with integrating mathematical formulas into problem solving. Questions often arise as to which formula to use, or what is the next step, rather than critically examining the question posed. Students view mathematics as a means to an end rather than a language of symbols to express and examine relationships among quantities. The foundational concepts of physics arose from understanding proportional relationships, which is a major deficit for many of my students.

Kinematics offers an approachable platform to connect the inherent relationships between mathematics and physics by strengthening students' understandings of proportional relationships with regard to motion. To truly understand the nature of motion, the rate at which an object travels within space-time (i.e., when will the train arrive at its destination, how far will the ball travel given its initial velocity), students must delve deep into the meaning of algebraic expressions and ratios. This unit seeks to enhance meaning between mathematics and physics by inviting students to explain natural phenomena that involve motion, focusing on rates and proportional relationships. Over a period of 3 – 4 weeks, students will conduct several inquiry investigations, collect and analyze data, and interpret algebraic expressions. The unit will focus on student centered learning and adjust student work in accordance to data gathered from pre-assessment data. Class will be anchored in exploring physical concepts through experiential learning to elevate content comprehension and overall student engagement. The cumulative assessment(s) will consist of a lab report, student demonstration video explaining problem sets verbally, or an infographic that will connect mathematical and physical concepts in a visual medium. This assessment choice will allow students to demonstrate mastery in a variety of formats and offer opportunities for creative expression. It is my hope that this unit will motivate students to think critically about their physical environment, prompt active discussions based on their observations, and elevate their mastery in both mathematics and physical science.

Demographics

I serve as the 11th and 12th academy Physics teacher at Ballou Senior High School (BSHS), where approximately 95% of students are below grade level. BSHS is a low-income, low-performing school with nearly 800 African American students, located in the southeast of Washington DC. Like many urban schools, Ballou suffers from an epidemic of violence where many students experience emotional and physical trauma, often leading to volatile behavior in the classroom. Many students struggle to get to school due to trouble at home or with the law. As a result truancy is high at Ballou High School, with approximately 1 student in 3 absent on a given day. Students have historically tested well below grade level in both Mathematics and English literacy. After two years at Ballou High School I have learned that students respond best to a positive, structured, dynamic classroom, with hands-on activities. This unit will seek to capitalize on the familiarity of motion and engage my students through hands-on demonstrations that will facilitate greater opportunities for mastery.

Objectives

This 3-4 week unit is designed to elevate high school students' conceptual understanding of the physical concepts (i.e., distance, displacement, speed, velocity, and acceleration) introduced during the kinematics portion of the unit by solving problem sets, conducting inquiry activities, and familiarizing students with proportional relationships with regard to motion. Topics will follow NGSS standards (see Appendix) as well as DCPS scope and sequence. The unit is subdivided into three sections (i.e., unit conversion using proportional relationships, position using scalar and vector quantities, rate of motion). The unit has a number of objectives to assess student growth including:

1. Use units to guide the solution of multistep conversion problems.
2. Interpret data describing motion at constant velocity, and develop an algebraic expression that relates velocity to slope.
3. Differentiate between speed and velocity through investigations.

Unit Content

Measurements, and Units

Science is based on the collection and analysis of data, which is often quantitative in nature, represented by a number with an assigned unit. The scientifically accepted unit system (the SI system) recognizes seven base units (i.e., meter, kilogram, second, ampere, kelvin, mole, candela). The system was established in 1960, based on the meter-kilogram-second system of units, with the intention to maintain congruency among the international community. [IV] Prefixes are added to each base unit to denote multiples of the original unit by a power of ten. For example, a millimeter describes a thousandth of a meter and a kilometer indicates 1000 meters. Measurements and units are the first window into the intricate relationship between mathematics and science; they provide the context for analysis and allow repeatability in experimentation. In fact, measurements give rise to numbers through the description of a given quantity as a multiple of the chosen base unit (i.e., kilogram, meter, second).

It is essential to understand the proportional relationships among a variety of measurements to compare quantities. We will examine several problems that ask students to convert between SI and non-SI using appropriate conversion factors.

Unit Conversion with proportional relationships

1. Coach Blais brought 32 L of water to the football game, and she divided the water equally between 8

How many milliliters of water did Coach Blais put in each cooler?

Let x equal the volume of water per water cooler.

Here is the calculation of the total number of milliliters of water provided by Coach Blais.

1 liter = 1000 milliliters

1 L / 1000 milliliters = 32 L / 32000 milliliters

32 (1 L) / 32 (1000 milliliters) = 32 L / 32000 milliliters

Since there are 8 water coolers,

32000 ml / 8 water coolers = x ml/cooler

x = 4000 ml per water cooler

2. A spacecraft travels 7000 meters per second.

What is the spacecraft's speed in miles per hour?

1 *inch* = 2.54 *cm* = 0.0254 *m*   60 *seconds* = 1 *minute*

12 *inches* = 1 *foot* 60 *minutes* = 1 *hour*

1 *foot* = 12 × .0254 *m* = 0.3048 *meters, hence 1 meter = (1/.3048) feet ≈ 3.28 feet.*

1 *mile* = 5280 *feet*

Solution I

This problem asks students to convert between rates of speed. Since the relationship between m/s and mph is a proportional one, a natural approach to this problem would be to find the unit rate: how many mph is 1 m/s?

For this solution we multiply 7000 m by 3.28 feet since we know from the list of conversion factors above that every meter approximates to 3.28 feet.

7000 m/ 1 s = x mile/ 1 hr

Step I. (7000 m/ 1s)(3.28 ft/ 1 m) = 22,960 ft/s ≈ 23000 ft / s.

It should be noted that measurements are limited in their relative degree of accuracy, so we must acknowledge this constraint by rounding products of quantities to the appropriate number of significant figures. In our problem, the 3-digit approximation of 1 meter = 3.28 feet restricts all subsequent calculations to three significant figures thus 22,960 ft/s should be rounded to 23,000 ft/s. Rounding errors can have a compounding effect on calculations and can cause solutions to deviate. We must recognize the limitation of our solutions and use significant figures as a convention in reporting our solutions.

Once we have calculated meters to feet, the next step would be to convert feet to miles. We divide 23,000 feet by 5280 feet since there are 5280 feet for every mile.

Step II. (23000 ft/ 1s)(1 mile / 5280 ft) ≈ 4.3560 miles / s ≈ 4.36 miles/s

Next we convert seconds to hours, by multiplying 60 seconds by 60 minutes. We know that for every minute there are 60 seconds and for every hour there are 60 minutes thus the product of these two ratios will determine the number of seconds in a hour.

Step III. (1 min / 60 sec)(1 hr/ 60 min) = 1 hr / 3600 s

Finally we combine our terms from the distance (meter to miles) and time (seconds to minutes) conversions to determine the speed of the spacecraft in miles per hours.

Step VI.   4.36 mi / 1 s = x mi / 3600 s

3600(4.36 mi) / 3600 (1 s) = 15,696 mi / 1 hr ≈ **15,700 mi / 1 hr**

Solution II

Alternatively, this problem could more accurately be solved using the official definition of an inch which is equivalent to 2.54 cm. Again, three significant digits are retained on all subsequent calculations for this solution since 1 inch = 2.54 cm. This ratio could then derive the ratio of meters to miles, as is discussed below.

If the official definition of an inch is 2.54 cm then we can convert this ratio's equivalency to feet. Since there are twelve inches in a foot, multiplying the ratio of 1 inch to 2.54 cm by 12 would convert inches to feet.

Step I. 1 inch / 2.54 cm = 12 inches / 30.48 cm = 1 ft / 30.48 cm

The next step would convert feet to miles. Since we know there are 5280 feet in a mile we multiply the product from the previous step by 5280 thus for every mile there are 160,934.4 cm.

Step II. 1 ft (5280) / 30.48 (5280) cm = 5280 ft / 160,934.4 cm = 1 mi / 160,934.4 cm

The next step requires the transformation of our ratio from the previous step. Since our recorded speed of the spacecraft is meters per second, we must convert centimeters to meters. There are 100 centimeters for every meter thus we divide 160,934.4 cm by 100 resulting in 1609.344 meters for every mile.

Step III. (160,934.4 cm)(1 m /100 cm) = 1609.344 m

1 mi / 1609.344 m

Lastly we can take determine how many miles are in 7000 meters by using the ratio that was previously determined.

Step IV. (7000 m)(1 mi / 1609.344 m) ≈ 4.35 miles

We utilize the same strategy from the previous solution to convert seconds to hours by multiplying 60 seconds by 60 minutes (see alternative solution).

Step V. (1 min / 60 sec)(1 hr/ 60 min) = 1 hr / 3600 s

Finally we combine our terms from the distance (meter to miles) and time (seconds to minutes) conversions to determine the speed of the spacecraft in miles per hours.

Step VI. 4.36 mi / 1 s = x mi / 3600 s

3600(4.36 mi) / 3600 (1 s) = **15,700 mi / 1 hr**

Solution III

A third approach to this problem looks to convert miles to kilometers using the conversion factors previously established. There are 5280 feet in one mile and 0.3028 feet in a meter, resulting in 160,934.4 meters or approximately 1.61 km.

Step I. 1 mile = 5280 feet = (5280)×(0.3048 m) = 1609.3440 m =

1.6093440 km ≈ 1.61 km

The speed of the spacecraft is traveling at 7,000 m/s or 7 km/s, by multiplying 7km by 3600 seconds we can determine approximately how many km/hr the spacecraft travels. Only 3 significant digits are maintained in our approximation of miles per kilometer

Step II. 7,000 m/s = 7 km/s = (7 km/s)×(3600 s/hr) = 25,200 km/hr

≈ (25,200 km/hr)

Finally we convert km/hr to miles/hr by using the ratio previously determined in step I, again the solution is rounded 3 significant figures.

Step III. (25,200 km/hr)×(1 mi/1.61 km) = (25,200/1.61) m/hr

≈ 15,652.1739 m/hr

≈ 15,700 m/hr

This problem highlights the importance of significant figures with regards to overall accuracy of a solution. We will not touch upon this concept as it is beyond the scope of this unit but should be taken into consideration for future modifications. The systematic process of writing each proportion will allow the students to account for each ratio and solve problems that require multiple conversions. The alternative solutions will offer skills to be practiced and enable rich discussions of ratios to facilitate a deeper understanding of the derivation of measurements that quantify speed, velocity, and acceleration introduced later in the kinematics unit.

Position in the context of vector and scalar quantities

An object’s position is its location within space, defined by a coordinate system as a frame of reference (Figure 1). An object’s starting position is traditionally placed at the origin of the coordinate system, unless otherwise stated.

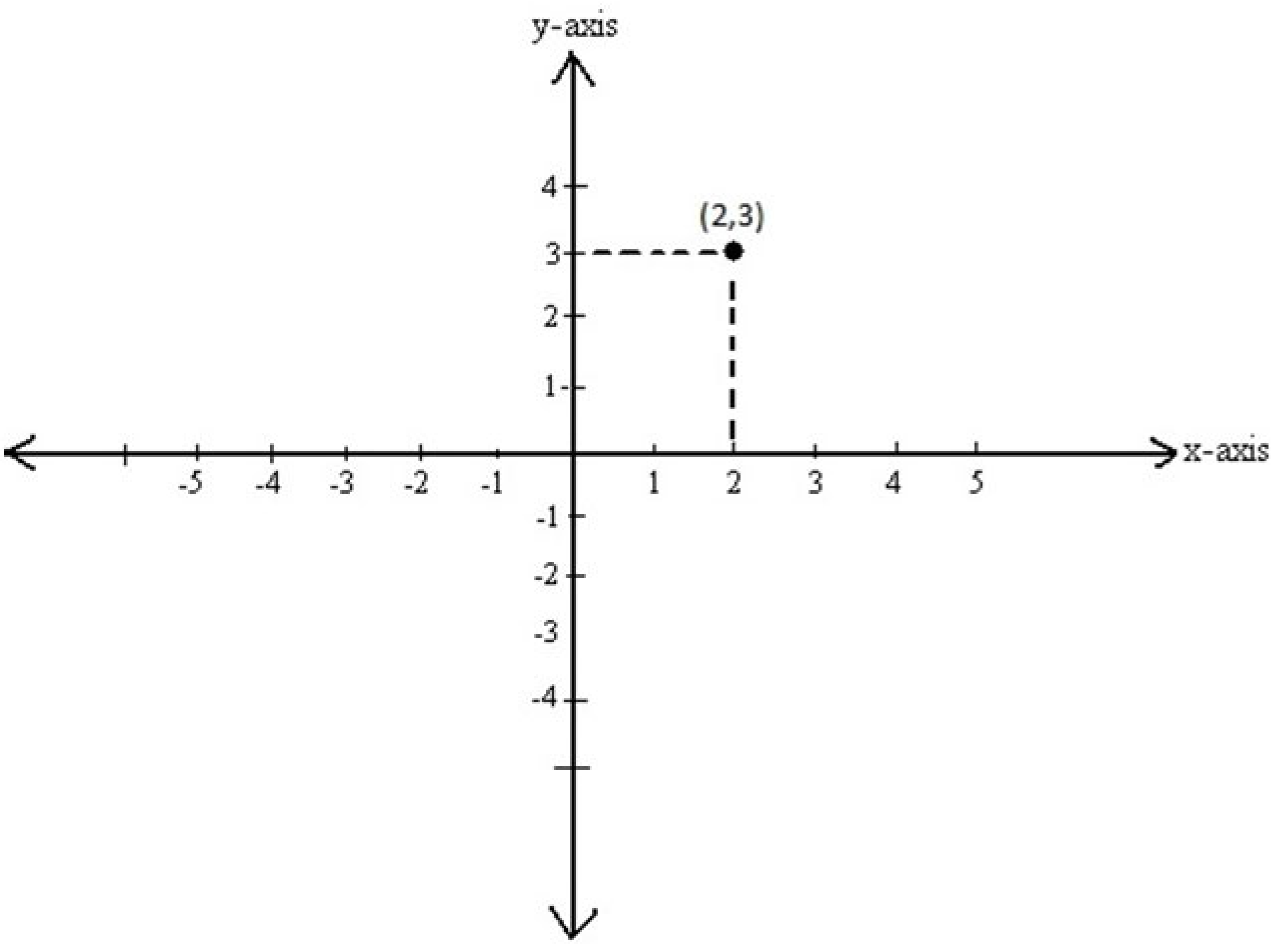


Figure 1. This illustration depicts a common coordinate system used to display the position of an object moving in a plane. We assume that the reader is familiar with the conventions of the Cartesian coordinate system, and will not review them here. However, I will make sure that my students understand these issues.

The displacement of an object is the overall change in spatial location (i.e., final position – initial position), denoted by Δs, which includes change in direction.[ii] By contrast the distance traveled by an object is the length of the total path an object travels along, regardless of direction. Both displacement and distance are measured with the SI unit meters.

Students have trouble understanding displacement, because it is a vector quantity. In general, vectors are harder to understand since both the x and y components are needed to determine direction of an object's motion. Students will participate in inquiry investigations that differentiate scalar and vector quantities prior to the introduction of distance and displacement in the classroom (see Activities). Distance and displacement represent two fundamentally different types of quantities, scalar and vector. Essentially, an object's physical position can be described by two classification schemas [III] . A **scalar** quantity is described by a numerical value that represents a fixed measurement (for example, distance and speed are scalar quantities). To describe a **vector** quantity, such as displacement or velocity, you need to specify a direction as well as a magnitude. Vectors in the plane, which are the main concern in this unit, can also be described by their Cartesian coordinates (x, y). These numbers, describe in this paired number notation, implicitly specify a direction. The first coordinate indicates a displacement in the horizontal direction, and the second coordinate indicates a displacement in the vertical direction. The magnitudes of these two displacements in different directions determines the direction of the vector (x, y), by a somewhat complicated formula involving trigonometry. These will be essential concepts to master prior to the introduction of distance and displacement problem sets. It should be noted that students will be asked to interpret text and diagrams to reinforce mathematic translations. My students will be interpreting and constructing graphs based on data measurements throughout the year and it will be essential for them to refine these skills to develop their conceptual understanding of rates of motion. Let's look at some examples to illustrate the difference between an object's total distance traveled compared to the object's displacement.

1. A car completes five laps along an oval track 5,000 meters long. Find the car's displacement and its distance traveled in meters.

Since the car returns to the same position as the starting position after every lap, the number of laps the car completes is irrelevant when considering its displacement. Thus there is no difference between the car's final position and its initial position equating to a displacement of zero. The total distance the car traveled is 25000 m. This is calculated by multiplying 5000 m by 5 since the track is 5000 m in length and the car completed five laps.

Displacement

$s_{final} – s_{initial} = \Delta s$

$0\ m_{final} – 0\ m_{initial} = \Delta s = 0\ m$

Total Distance Traveled

5000 m = *length of track*

5 = *number of laps*

(5000m) + (5000m) + (5000m) + (5000m) + (5000m) = 25000 m

$(5000 m/lap)\times(5\ laps) = 25000\ m$

2. What is the displacement and total distance traveled in the figure below? Describe the motion of a person walking from point A to point C on the position-time graph. (Note: The vertical axis of the graph indicates the distance an object travels and the horizontal axis represents time)

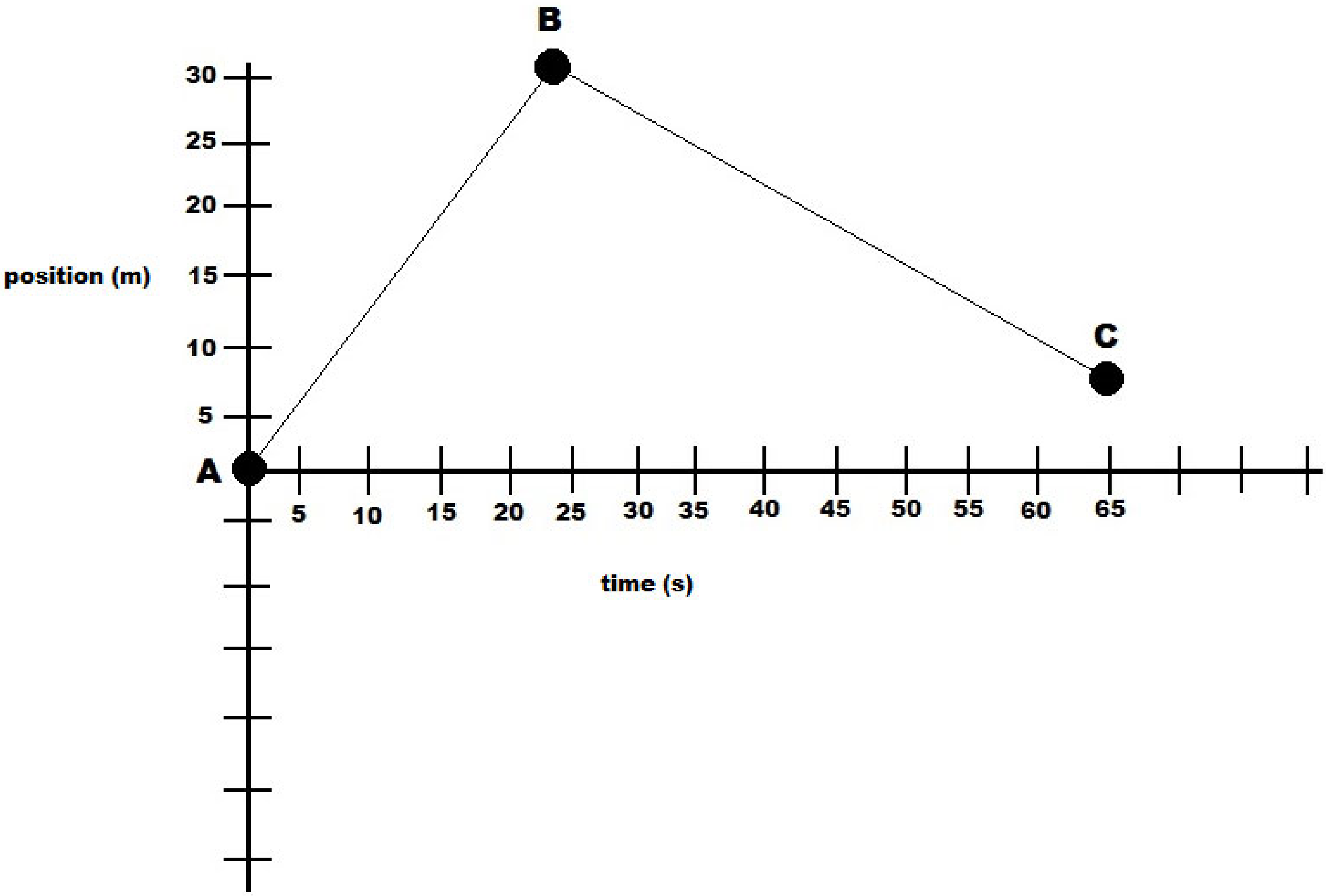


Figure 2. This position-time graph illustrates the motion of an object from position A to C. For this example the points A, B, and C are restricted to space in one direction.

The walker starts at 0 m at time t = 0 seconds, the walker travels 30 m in a positive direction over 25 seconds. The walker then travels 23 m in a negative direction from t = 25 s to t = 65, spanning 40 seconds. The total distance traveled is 53 m, whereas the walker's displacement is 7 m in the positive direction. Another way to phrase the walker's displacement is a net gain of 7 m forwards. Signed numbers have a vector component indicating over net direction along a line (e.g., left – right, up – down, or forwards – backwards). Students often find it difficult to translate graphs into words, which is essential for mastery (see Activities). Thus, decomposing the motion depicted by the graph into its components will be essential to develop translation skills. Over the course of the unit, several problems of this type will be presented, gradually increasing in complexity, with additional path segments and directions.

Displacement

$s_{final} - s_{initial} = \Delta s$

$7\text{ m}_{final} - 0\text{ m}_{initial} = \Delta s = +7\text{ m}$

Total Distance Traveled

$\overline{(AB)} = 30$ m

$\overline{BC} = 23$ m

$\overline{AB} + \overline{BC} = \overline{AC}$

30m + 23m = d

53 m = d

Rates of Motion

Proportional relationships are essential to understanding rates of motion in physics; they compare two equal ratios. This can be written algebraically with the following equation.

$x_2 / x_1 = y_2 / y_1$

This equation expresses the key aspect of proportionality: a relationship between two variables is called *proportional* if, when one quantity changes by some factor, the other quantity changes by the same factor. For example pencils are sold in a set of 10 for 2 dollars (Table 1). If 50 pencils were bought, a proportional relationship could be setup to determine the overall cost. 50/10 = 5, that is, 50 pencils is five times as much as 10 pencils. This is the x_2/x_1 in the equation above. If I buy 5 times as many pencils, I must pay 5 times as much. This is the y_2/y_1 in the equation above. Thus, $y_2/y_1 = 5$ also. Since we are given that y_1, the cost of 10 pencils, is \$2, the cost of 10 pencils is 5×\$2 = \$10.

Proportional relationships can be, and usually are, expressed in terms of a *constant ofproportionality*, also known as a *rate of proportionality*, or a *rate of conversion*. This rate appears if we multiply the equation above that defines a proportional relationship by y_1/x_2. This turns the proportionality relationship into the relation

$y_1 / x_1 = y_2 / x_2$.

This equation says that the ratio of the amount of y to the corresponding amount of x is the same for corresponding the first pair values as for the second. If we fix one of the pairs of values, and let the other ones vary arbitrarily, we can conclude that this ratio is always the same, for any pair of corresponding values. This common ratio is called the constant (or rate) of proportionality. It is important to note that, although we call it a “constant”, which suggests a number, the constant of proportionality is typically not a number, but has units attached to it. For example, in our example of pencils and dollars, as the table below shows, the constant of proportionality, is 5 pencils/dollar.

Cost (*x*)	Pencils (*y*)	Proportionality Constant (*y*/*x*)
1	5	5
2	10	5
3	15	5
4	20	5
5	25	5
6	30	5
7	35	5

Table 1. Describes the cost of pencils at a flat rate of 1 dollar for every 5 pencils.

The calculations below illustrate the derivation of the proportionality constant with regards to the cost per pencil as has previously been described.

$x_2 / x_1 = y_2 / y_1$

2 dollars / 1 dollar = 10 pencils / 5 pencils

The proportionality constant is found by multiplying both sides of the first equation by y_1 / x_2.

$y_1 / x_1 = y_2 / x_2$

5 pencils / 1 dollar = 10 pencils / 2 dollars

r = 5 pencils/dollar.

To determine the cost of 50 pencils we can setup two equal ratios with the quotient equating to the proportionality constant since we have previously determined a proportional relationship between number of pencils and total costs.

$y_1 / x_1 = y_2 / x_2 = 5$

10 pencils / 2 dollars = 50 pencils / y_1 = 5 pencils/1 dollar

where y_1 equals the cost of 50 pencils.

50 = 5

y_1 = 1

y_1 = 10 dollars

Many students understand rate with respect to the slope of the line between two points in a coordinate system. This fundamental concept of mathematics will be reviewed using several real-world applications. Students will describe and provide examples of rates prior to its derivation and introduction through the lens of motion.

In physics this is most often demonstrated in kinematic problems with respect to rate of change in velocity and acceleration of an object. An object that travels from one position to another over a set period of time experiences a physical phenomenon known as motion. [ii] Rates are found in many facets of life including interest rates, currency conversions, measurement conversions from one unit to another of the same type (e.g., units of length, such as feet to meters), prices of items, inflation rates or even the production rates of goods or services, and of course rates of motion.

To further expand upon the idea of proportional relationships we will first discuss the concept of constant motion and its relevance to speed and velocity. An object is in *constant velocity* if the displacement, both in distance and direction, traveled by the object is the same in any two equal time intervals (for example, every second, or every minute). An object is in *constant speed* if it travels the same total distance in any two equal time intervals. Two objects can have the same constant speed but different velocities if, for example, one is traveling around a circle and the other travels in a linear fashion. As the object that is traveling in a circle is constantly changing directions, its velocity will vary, even if it travels at a constant speed. Both velocity and speed are two metrics used to quantify changes in an object's position per unit of time.

A reexamination of scalar and vector quantities within the context of an object's motion will highlight the importance of this classification with regard to speed and velocity. To be ready for this, students must have mastery of basic arithmetic functions, be familiar with algebra, and have experience with ratios. This unit will heavily focus on introducing students to proportional relationships with specific attention placed on rates and ratios related to motion. In addition, students should have the capacity to translate between different schemes of graphical representations of data. We will examine these concepts in detail with respect to the following example.

A bullet train travels 600 miles northward in six hours, and then reverses direction for 3 hours, traveling 300 miles south. What is the train's average speed and average velocity?

The average speed of an object is determined by the total distance traveled in a given amount of time. Thus for every hour the bullet train travels a total distance of 100 miles. Alternatively, the train travels 900 miles for a total trip duration of 9 hours resulting in an average speed of 100 mph. Velocity is derived from the concept of displacement and includes direction in the description to an object's motion. The average velocity is based on the train's displacement per unit of time and would only include the difference between the train's final position and initial position. Since the train reversed directions for 3 hours the train moved a total of 300 miles in 9 hours, resulting in an average velocity of 33 mph (to the nearest whole mph) northward. From the train's original position, it moved a net direction north of 300 miles; thus the average velocity of the train would be written as 33 mph, northward.

Average speed = total distance traveled / time of travel

= (600 mi + 300 mi) / 9 hrs

= (900 mi) / 9 hrs

= 100 mi / hr

Average velocity = displacement / time

= $(s_{final} – s_{initial}) / (t_{final} – t_{initial})$

= (600 mi northwards – 300 mi northwards)/(9 hrs – 0) =(300 mi – 0) / (9 hrs – 0)

= 300 mi / 9 hrs

= 33.33 mi / hr, Northwards

The tables below describe two objects traveling in constant motion at 1 second intervals of time. The object's position, duration of travel, and displacement per unit of time is recorded. Assume that each object starts in the same position and describe each object's motion

Object I

Position (m)	Time (s)	Displacement
0	0	0
3	1	3
6	2	3
9	3	3
12	4	3
15	5	3
18	6	3
21	7	3

Object II

Position (m)	Time (s)	Displacement
0	0	0
5	1	5
10	2	5
28	3	18
17	4	-11
9	5	-8
5	6	-4
0	7	-5

Table 2. Describes two objects traveling in constant motion at 1 second intervals of time. The object's position, duration of travel, and displacement per unit of time is recorded. Assume each object starts at the same position.

When examining objects I and II from the data tables above we can see that two objects start at the same location; however the nature of motion for each object is quite different. For example, object I experiences constant speed over a period of 7 seconds; for every second the object moves 3 meters. The average speed and velocity of object I would be 3 m/s. If the movement of object I were to be graphically illustrated we would observe a linear slope, meaning that every point along the slope would represent the same rate of speed. (See Figure 3 below, left side).

When we examine object II's motion we see an initial rate greater than that of object I. After 3 seconds object II alters its direction of motion and begins to move back towards its place of origin. Upon closer examination we can determine that object II is not traveling at a constant speed. The distances traveled in different one second intervals are different; moreover the direction of travel is also different in different 1 second intervals. The total distance traveled by object II is 28 meters out and 28 meters back, for a total of 56 meters. It does this over a period of 7 seconds. Hence, the average speed of object II is 56m/7s = 8.0 m/s, however, the speed in different one second intervals along the trip may be faster or slower since the speed is not uniform. For example, in the third second object II travels 18 meters, for a speed of 18m/s.

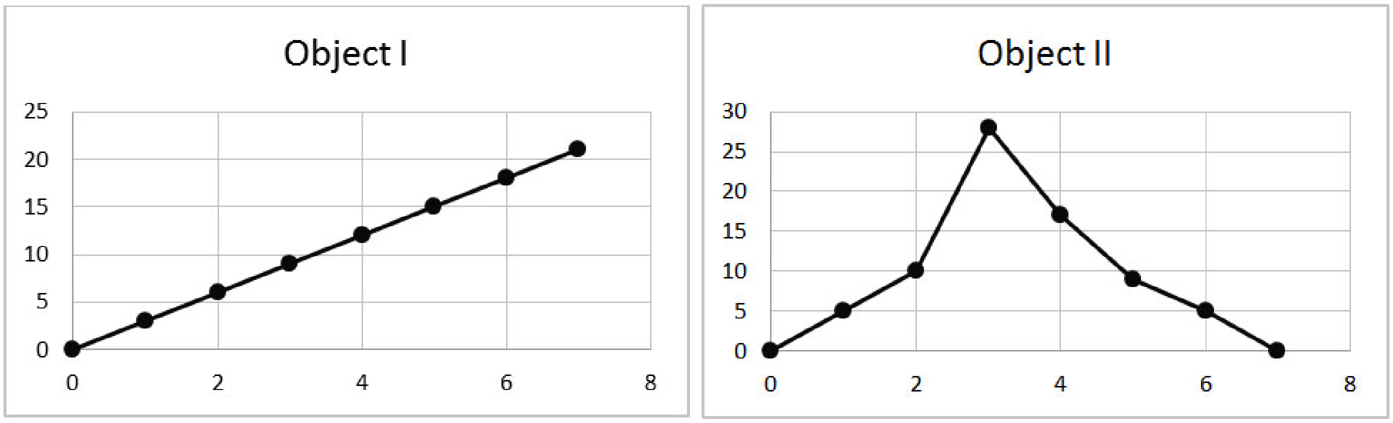


Figure 3. The position time graphs illustrate the motion of two objects. The linear slope of objects I depicts constant motion whereas object II exhibits motion at varying velocities.

Constant velocity occurs when an object travels the same distance and direction in any single unit of time. This means that in two units of time, it has double the displacement of one second, and in three seconds, it has triple the displacement, and so on for longer intervals. This means that the relation between displacement and time traveled is a proportional one. From our discussion above of proportional relationships, we know that this relationship can be described algebraically by the following equation, where velocity of an object is denoted by *v*, total displacement is represented by *d* and time is *t*. For constant velocity motion, the velocity is the constant of proportionality between displacement and time.

v = d/t

If for example an object moves at a constant velocity of 20 m/s for 10 s eastward, the object would be displaced 200 m, Eastward.

v = d/t

d = vt

d = (20m/s)(10s)

d = 200 m, Eastward

The equation could be transformed to calculate the interval of time, as long as the scenario explicitly states the object travels at a constant velocity. This unique relationship between time, displacement, and velocity only applies to objects that are traveling in constant motion and the same direction. If you were to plot a position-time graph that involved a constant rate, a linear slope would result. This demonstrates graphically the proportional relationship between total displacements per unit of time in the case of constant velocity. At every point along the slope the same rate would result meaning that the average velocity and instantaneous velocity of the object would be the same. In order other words, for every unit of time the velocity of the object would be the same.

How long does it take for a runner traveling at a constant speed of 5 meters per second to travel 1500 meters?

s = d/t

$t = d/s$

$t = (1500\ m)(5\ m/s)$

$t = 300\ s$

Teaching Strategies

Station Rotation

Station rotation facilitates the engagements of students by rotating students through several concurrent activities throughout the class period or week, depending on the model. This instructional strategy allows students multiple opportunities to refine conceptual understanding and mastery by participating in activities that target various modes of learning (e.g., kinetic, auditory, visual). Students will spend approximately 20 minutes working independently or in groups on activities. As a class, students will share findings, observations, and misconceptions that persist, once every student has rotated through each station.

The unit will concentrate heavily on a blended learning (i.e., PHeT simulations) and project-based (i.e., designing a soapbox car) approach that incorporates station rotation with inquiry activities, labs, and problem sets. [V] Students will be expected to take Cornell Notes, a system of notetaking that actively engages the students to ask questions, organize information, and summarize key ideas. [VI] Students will apply and practice concepts introduced during the first half of class through station rotations. Student will be grouped based on pre-assessment data, attendance, and behavior. The number of stations may vary based on the number of students and classroom dynamics.

Inquiry Activities

As a science, physics offers opportunities for students to apply a multitude of mathematical concepts and arithmetic skills when describing physical phenomena. This unit will seek to strengthen students' content mastery of motion while simultaneously practicing skills and engaging in discussion with proportions, ratios, and rates. From previous years at Ballou HS, kinesthetic activities have often led to the most successful lessons. This unit will utilize inquiry as an access point for student ingenuity and provide the context for students' to revise their ideas about the concepts being introduced. The activities will vary in duration and rigor, requiring students to work in collaborative groups.

PHeT Simulations

Students will utilize PHeT (Physics Education Technology) simulations throughout the kinematic unit to explore the concepts of distance, displacement, speed and velocity. Research has shown that students are more likely to engage in simulations if provided with minimal guidance and open-ended questioning. [VII] This unit will utilize several PHeT simulations, namely "The Moving Man".[V] The "Moving Man" asks students to explore position-time and velocity-time graphs by manipulating the position of a digital man. These self-governed activities will ask students to record data and develop predictions from their observed patterns. In addition, PHeT simulations will serve as recovery opportunities for students whose attendance is inconsistent or for students with medical issues that require assistance at home.

Progressive Problem Sets

A pre-assessment will be administered prior to the unit to establish baseline data of each student's capability. Questions will consist of a selection from the district's standardized test (PARCC) as well as word problems from physics class. Students will be asked to interpret graphs and data and to solve for unknown quantities using algebra. Data will be compiled from last year's PARCC assessment as well as the physics customized assessment to determine overall academic performance. Students will be assigned problem sets that will gradually challenge their skill level. Each problem set will consist of five to six questions that will begin with simple arithmetic questions and end with problems that require a multi-step solution. These problem sets with serve as informal assessments and facilitate discussion for Think-Pair-Share (see below) sessions. Students should be able to articulate their attempt at a problem and will be required to record every step. See examples of problems sets within each kinematic subsection below.

Think-Pair-Share

To challenge their conceptual understanding of content and foster a collaborative classroom environment students will frequently be asked to engage in discussions. Initially students tend to be resistant in participating. However, through positive reinforcement students will be more willing to share their ideas. This collaborative learning strategy requires students to first think about a question independently before they share their ideas with a classmate and/or whole group. This allows every student to be held accountable for learning that is taking place in class and presents opportunities for content misconceptions to be addressed.

Classroom Activities

Creating a Unit

At the start of the kinematic unit each student will be given a piece of blank card stock. Each student will take multiple measurements of known distances, using his/her card as unit length, and will use these measurements to determine the conversion rate of their unknown base unit to develop ratios. Students will subdivide their card stock to measure more precisely. They can mark their ruler in smaller intervals based upon a new unit interval, like the width of their thumb or the length of a locker key, to construct a more precise measuring instrument. Students will pair up, and using their new conversion factors, estimate the length of a specific object, like a table length. Units are essential to understanding multiplicative comparison problems and this will be especially important in the context of an object's position and movement through time. The standardization of units allows data to be effectively communicated which leads to the refinement of understanding physical phenomena.

Move to the Movement

This lab is an adaptation of a PHeT simulation titled "The Moving Man".[v] Students will record the time at every 5 meter interval as a walker travels at a constant velocity on a simulated football field. Students will plot the data on a distance vs. time graph for three walkers and interpret the meaning of the slope. Each walker will travel at a constant rate, but different rates for different walkers. The data gathered will be used to prompt a discussion about proportional relationships. Student will be asked to predict the distance that each walker will walk in a given time, based on their recorded speed. Students will work in groups of 2-3 to develop and analyze a variety of scenarios, including resting in place (speed = zero) and reversing directions. Each group will present their conclusions to the class and discuss the relationship(s) with respect to time, distance, and speed. To ensure engagement of all students, each will be expected to take notes and ask questions during whole group discussion.

Relay Race

A relay race will be performed in the lab to reinforce the relationships between position, time, and speed. Students will work in groups of three to develop a data table that records three trials of walking, skipping and hopping for distances of 50 and 100 meters. Each group member will rotate roles, participating in turns as a racer, timekeeper, and recorder. Each group will calculate the average speed for all three types of movements (i.e., walking, skipping, and hopping) at both distance intervals. Students will be asked to compare the speeds between 20 m and 40 m for walking skipping, or hopping. In addition students will create position-time graphs for each and compare the slope to determine which movement created the greatest speed? An automatic timer will be setup on the football field prior to the activity to minimize error.

Graphical Translation of Movement

Students will be asked to translate position-time and velocity time graphs into descriptive texts of an object's movement. For example, students comparing object I and II in figure 2 might state that the position time graph illustrates the motion of two objects. The linear slope of objects I depicts constant motion whereas object II demonstrates motion at varying rates of velocity of movement. In addition, students will be given data and complementary text to generate corresponding position-time and velocity-time graphs. This will allow students to practice translating movement from text to visual representation (i.e., graphs) and vice versa.

Sample Problem Sets

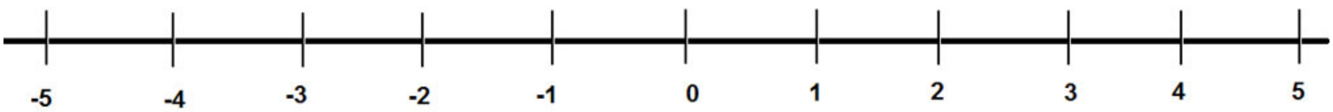
Unit Conversion

1. Jonae is 5 feet tall. How tall is Jonae in inches?
2. Kevin has television that is 24 inches tall. If Kevin set the television on a stand that is one foot tall, how far from the floor will the top of the television be in feet, and in cm?
3. A football field is 100 yards from end to end. What is the length of a football field in feet?
4. Coby started his homework at 1:50 p.m. and finished his homework 96 minutes later. Coby had band practice at 4:00 p.m. How much time did Coby have between finishing his homework and the beginning of band practice?
5. How many seconds did Coby have before band practice?
6. Daquan and Richard each ran 30 minutes on a treadmill. Richard's treadmill said he had run 10,000 feet. Daquan's treadmill said he had run for 2 miles. Who ran farther, and how much farther?
7. How many inches farther?
8. How many centimeters farther?
9. Ballou's science group built a robot that carries a cube with 12 cm long edges. Anacostia's group built a robot that carries a cube with 4 cm long edges. How many times larger is the volume of the larger cube than the smaller one?

10. Suppose your weight on Earth is 166 pounds. The force of gravity on Mars is only 1/3 of the force of gravity on Earth. If you go to Mars and stand on a scale that measures weight in pounds, what would it record as your weight?
11. Meyers' laptop weighs 6.3 pounds. What is the weight of his laptop in ounces? Your answer should have 4 significant digits.
12. A crayon has a mass of 25 grams. How many crayons can be made with 25 kilograms of colored wax? We will ignore accuracy issues here.
13. Brianna took 12 minutes 40 seconds to type a 500-word essay. Kianna took 15 minutes 12 seconds to type the same essay. How much longer did Kianna take to type the essay than Brianna?

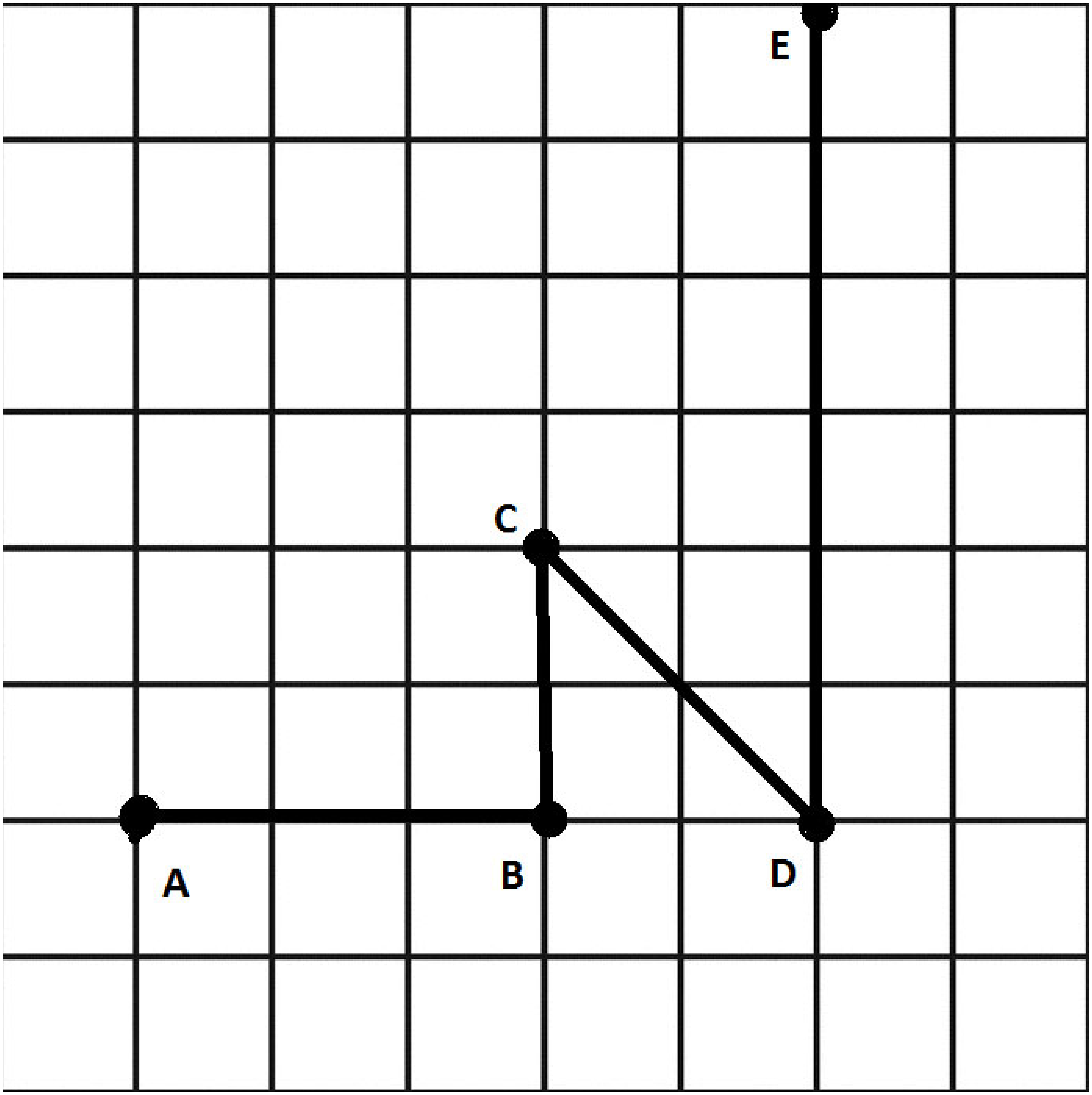
Displacement and Total Distance Travelled

a. An Escalade navigates the neighborhood on MLK Avenue by traveling East for a distance of 5 km, turns around and goes West for 2 km and finally turns around again and heads East for 4 km.
    a. What is the total distance traveled?
    b. What is the displacement?

b. 

    a. Calculate the distance an object travels starting from position 1 to position 4, and then reversing direction and ending at position 2.
    b. Calculate the displacement of an object that starts at position 1, travels to position 5, then reverses to position 4.
    c. Calculate the distance an object travels, starting from position -2, moving to position 3, and then reversing direction to 0.

c. If you run exactly four times around a quarter-mile track, what is your displacement? Why? Calculate the total distance traveled.
d. An object moves along the grid through the points A, B, C, D, and E as shown below.
    a. Find the distance traveled by the moving object.
    b. Find the magnitude of the displacement of the object



Rates of Motion

1. How long will it take a bus traveling 65 km/hr to travel 260 km?
2. How fast, in miles per hour, must a car travel to go 600 miles in 15 hours?
3. Grace is driving her sports car at 20 m/s when a ball rolls out into the street in front of her. Grace slams on the brakes slows her speed to half of her initial speed. What is Grace’s average speed in miles per hour? Grace comes to a complete stop in 3.0 seconds, her average breaking speed is half her initial speed (20 m/s), calculate the total distance, in meters, Grace would traveled while braking.
4. Jay-Juan and Romello live 5 miles apart. The both start walking toward each other at noon, one at a rate of 3 mph, and the other at 4 mph. After how many hours will they meet? To the nearest minute, how long will they walk until they meet?

5. A car travels 7 hours from Washington, DC to New York. Use the table to answer the following questions.
    a. Calculate the average speed of the car
    b. According to the table, is the car moving at a constant speed in its first seven hours of travel? Support you answer.
    c. If the car travels in constant motion, how long would it take to travel 210 miles?

Position (mi)	Time (hrs)
0	0
30	1
60	2
90	3
120	4
150	5
180	6
210	7

Teacher and Student Resources

For supplemental information that will be utilized at varying degrees throughout the unit please review the following. A deactivated link is provided along with a summary of the resource.

Khan Academy

Khan Academy is a non-profit educational organization that provides free lectures in the form of YouTube videos. There are practice exercises along with a personalized dashboard if students register on the website. This resource is helpful for students that require supplemental learning or need to make up work due to absences. (https://www.khanacademy.org)

Physics classroom

Physics classroom is a free online resource for beginning students and teachers. There are a number of animations, problem sets, and tutorials that supplement classroom content. The website provides guidance to targeted misunderstandings and strengthens students' critical thinking skills with multi-tiered word problems. (http://www.physicsclassroom.com)

Ballou HS Physics Classroom Website

The Ballou HS classroom website serves as a depository for notes, problem sets, and external supplemental resources throughout the year. It is designed to highlight content specific to the students of Ballou but also serves as a resource for other teachers that would like to modify the protocol from inquiry activities throughout the year. (http://www.ballouhsphysics.info)

Appendix

Standards

NGSS Standard Integration

The unit will incorporate standards from the Next Generation Science Standards (NGSS) in Unit I and II. The focus will be primarily on the nature of motion and the factors that influence an object's speed, velocity, and acceleration.

Disciplinary Core Ideas

- Newton's second law accurately predicts changes in the motion of macroscopic objects. (HS-PS2-1) I
- If a system interacts with objects outside itself, the total momentum of the system can change; however, any such change is balanced by changes in the momentum of objects outside the system. (HS-PS2-2) (HS-PS2-3) I
- Momentum is defined for a particular frame of reference; it is the mass times the velocity of the object (HS-PS2-2) I

Crosscutting Concepts

- When investigating or describing a system, the boundaries and initial conditions of the system need to be defined and their inputs and outputs analyzed and described using models. (HS-PS3-4) I.

Science & Engineering Practices

- Use mathematical representation of phenomena to describe explanations. (HS-PS2-2) I
- Analyze data using tools, technologies, and/or models (e.g., computational, mathematical) in order to make valid and reliable scientific claims or determine an optimal design solution. (HS-PS2-1)

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