



YALE NATIONAL INITIATIVE

to strengthen teaching in public schools®

Curriculum Units by Fellows of the National Initiative

2017 Volume V: From Arithmetic to Algebra: Variables, Word Problems, Fractions and the Rules

Introduction

by Roger E. Howe, William R. Kenan, Jr. Professor Emeritus of Mathematics

The Yale National Seminar, "From Arithmetic to Algebra", was centered around a study of word problems and their solution. Much discussion focused on the taxonomies, of one-step addition and subtraction problems, and one-step multiplication and division problems, given in Tables 1 and 2 of the Mathematics Glossary of the Common Core State Standards in Mathematics (<http://www.corestandards.org/Math/>). Becoming familiar with these taxonomies, and seeing examples of how multistep problems can be broken down into sequences of single step problems, can give teachers a way in to the very substantial literature of mathematics word problems, and enable them to lead class discussions that help their students learn to read word problems carefully and interpret them correctly.

Another topic that received considerable attention was the rather large range of problems that can be approached either arithmetically, by direct reasoning with the quantities in the problem, or algebraically, by defining variables, setting up equations, and solving them using manipulations governed by the Rules of Arithmetic and the Principles of Equality. Looking at the relationship between the two solutions can help students understand what role algebra is playing in problem solving, and give them greater confidence in tackling harder problems in which the arithmetic approach is quite difficult. In producing algebraic solutions, a point of emphasis was the importance of defining the variable(s) carefully, including specifying the units of all variables.

Reflecting this emphasis on word problems, all the units devote attention to solving word problems. At the elementary level, Gilbert Carter's unit attempts to improve both the reading and the mathematics competence of his students through carefully tailored word problems. Brittany McCann's unit uses literature to engage her students in the problem solving enterprise. Valerie Schwarz uses Singapore Bar Models to help her students organize their thinking about word problems. Jolene Smith's unit features a novel collection of word problems devoted to quantitative aspects of the Diné hooghan, in order to promote awareness of traditional Diné culture as well as mathematics.

A group of 4 Fellows took on the task of helping students to make the transition to symbolic solutions to algebra word problems. Rachele Soroten starts the work by concentrating on translating relatively simple verbal problems into symbolic form, and looking at how to solve the relatively simple equations that result. Jeffrey

Rossiter has created lists of problems that require translating variant phrases into symbolic form. A point of emphasis here is that several different phrases may well have the same symbolic translation. After symbolic expressions have been found, the solution process often requires that they be simplified. Xiomara Pacheco's

unit helps students understand that a large family of expressions, the "first order expressions", can always be converted to the standard form $ax+b$, where x is the variable, and a and b are specific numbers. Finally, Sally Yoo's unit puts it all together and helps students use the techniques of the previous units, as well as the principles of equality, to solve the equations that result from algebra word problems. She also presents both arithmetic and algebraic solutions for several problems.

Irina Alekseeva's unit is closely related to the four described in the previous paragraph. Her students also need practice at translating words into expressions, but must progress much faster, since they are taking higher level mathematics. To facilitate progress, Irina has created multipart problems that look at the translation process from many perspectives.

Finally, there are two units that explore more advanced topics. Lawrence Yee discusses sequences, and explores the use of difference sequences and second difference sequences to analyze sequences described by quadratic functions.

Zachary Meyers develops the fundamental idea of proportional relationship to study simple motion, especially motion at constant velocity, and motion at constant speed. This unit can serve as an introduction to a subsequent discussion of motion with constant acceleration, one of Gallileo's great achievements.

Roger E. Howe

<https://teachers.yale.edu>

©2023 by the Yale-New Haven Teachers Institute, Yale University, All Rights Reserved. Yale National Initiative®, Yale-New Haven Teachers Institute®, On Common Ground®, and League of Teachers Institutes® are registered trademarks of Yale University.

For terms of use visit https://teachers.yale.edu/terms_of_use