Curriculum Units by Fellows of the National Initiative
2018 Volume IV: Big Numbers, Small Numbers

Answering Big Questions by Finally Understanding Big Numbers

Curriculum Unit 18.04.01, published September 2018
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Introduction

> "Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers." - Shakuntala Devi

As a middle school math teacher I find a level of joy in mathematics, and more specifically with numbers. I believe that if we understand and work with numbers, we are better able to make sense of our world. Unfortunately, my students in large part do not share my enthusiasm for math and often times demonstrate an aversion to numbers, especially those that are unfamiliar. My overarching motivation for this unit is to revisit the basic structure of our number system and help them understand numbers in a more in depth and conceptual way. This will allow students to access 8th grade content that involves laws of exponents and scientific notation with greater ease and interest. Ultimately, by developing my student's abilities to reason and operate with big numbers, they will be able to ask and answer their own big questions about the world and hopefully enjoy doing it.

Background and Rationale

This next year I will be teaching eighth grade math at Brentano Math and Science Academy in Chicago's Logan Square neighborhood. The school qualifies for title 1 funding and has an enrollment that is predominately Hispanic. After several years of teaching 8th grade math, I have noticed that my students consistently have the same problem year after year with numbers that are out of their range of comfort - that is to say, any number beyond 100. Students rely on calculators to carry out basic operations and have great difficulty reasoning about larger numbers. These deficits in number sense present a significant obstacle whenever students are given a problem that involves large quantities. More acutely, their resistance to large numbers makes it difficult to teach the required unit on scientific notation and laws of exponents. As 8th graders, students are required to master the following standards:

CCSS.MATH.CONTENT.8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

CCSS.MATH.CONTENT.8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

CCSS.MATH.CONTENT.8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities.

In previous treatments of these topics I have explicitly taught the laws of exponents, how to represent numbers with powers of ten via scientific notation, and finally show students how to perform single step operations with scientific notation. My students have typically demonstrated a basic procedural mastery of the above standards. A typical problem from the 8th grade curriculum is shown in Figure 1 below. In general, my 8th graders would be able to address this problem by performing the required operation using laws of exponents, but would not have the ability to check if their answer is reasonable. Also, they would not know why we are using scientific notation in the first place. Most would show very little conceptual understanding of the given quantities and the procedure they were carrying out.

Figure 1

Here are the masses of the so-called inner planets of the solar system.

Mercury: 3.3022×10^{23} kg

Venus: 4.8685×10^{24} kg

Earth: 5.9722×10^{24} kg

Mars: 6.4185×10^{23} kg

What is the average mass of all four inner planets? Write your answer in scientific notation.

(Open-Up Resources, 2017)

Another concern I have with this topic is, that it has always felt isolated in the 8th grade curriculum and does not receive much attention in the midst of linear algebra and functions, the major work of the grade. Once it is addressed, students are not tasked with applying the skills or concepts in any other unit. Instead of the unit giving students a set of tools to understand and operate with numbers of large and small magnitude in general, they are only able to use these skills when explicitly tasked to do so, as in the problem above. By not giving more weight and priority to these standards, I have missed opportunities for students to use laws of exponents and scientific notation when studying geometry, functions, and statistics.

My goal with this unit is to pay greater attention to this strand of standards, and to present this content in a way that leads to a conceptual and transferable mastery of skills for my students. My hope is that this unit helps students answer essential questions like: Why is it helpful to use scientific notation? What is actually happening when we increase by powers of 10? Why can we round numbers off to 2 or 3 digits when calculating big numbers and still have a reasonable answer? To get students to answer these important questions, I will design this unit to progressively teach the utility and rationale of each concept before focusing on procedural fluencies. This will require a careful sequencing of the key concepts and selecting prompts that give students opportunities to grapple with these essential concepts.

Structure of the Unit

The unit will cover the following key concepts:

1. Understanding the Relative Size of Numbers
2. Numbers as Powers of 10 and Laws of Exponents
3. Estimating
4. Scientific Notation
5. Operating with Scientific Notation

Content Objectives

Concept #1- Understanding the Relative Size of Numbers

To lay the foundation for this unit I will address the concepts of the relative size of numbers and orders of magnitude. Any number past the thousands place is difficult for my students to conceptually understand. When calculating large numbers, my students operate in a procedural sense without reasoning about the size of the numbers they are dealing with. When they arrive at their answers, the lack the tools and habits to test for reasonableness. It is my aim for students to more readily reason with the magnitude of quantities and think about the relative size before blindly starting a procedure. In order to achieve this, I will direct students to reconsider powers of 10 and the basic structure of our base 10 system. A useful vehicle to think about this is the number line. Consider the following task.

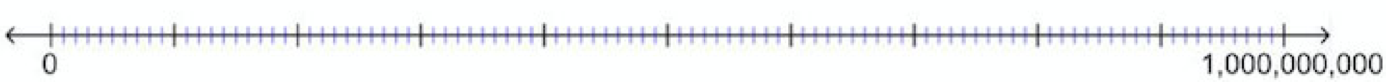
Graph the following populations on the given number line. Be as precise as possible.

Enrollment at Brentano Elementary School: 502

Population of Logan Square Neighborhood: 73,702

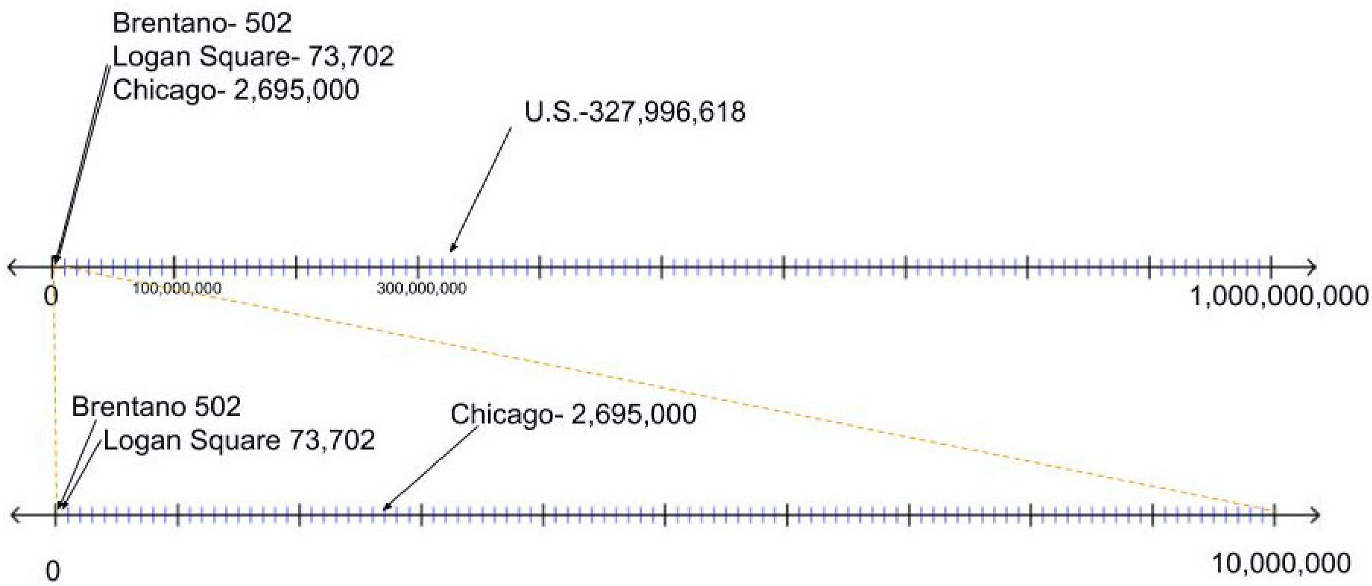
Population of Chicago: 2,695,598

Population of the United States: 327,996,618



Students will initially have a difficult time completing this task because of the wide range in magnitude of the given populations. Students must first be able to identify the intervals that divide 1 billion into tenths which would be 100,000,000. Then consider the intervals that split each tenth by another tenth which would be 10,000,000, or one hundredth of 1 billion. Just this act of labeling the intervals is an important opportunity for students to discuss place value and the number of digits. Every time we divide an interval by 10, we are losing a digit. Instead of writing out the full numbers it will be useful to remind students that these numbers can be represented as a power of 10. One billion can be written as 1×10^9 , and 100 million as 1×10^8 .

Figure 2



Having students then plot these values on a number line provides the opportunity to think about the relative size of the numbers. The first number line labeled from 0 to 1 billion tells us that the difference between the population of Chicago and the U.S. is massive: the U.S. population is more than 100 times that of Chicago, which in turn is more than 40 times (but less than 100 times) the population of Logan Square. Most students wouldn't understand before seeing this number line that, even though in the millions, Chicago's population is almost negligible on a scale of 0 to 1 billion. In order to see where exactly the population of Chicago, Logan Square, and Brentano land on the number line, we need to expand the first of the hundred small intervals and break it up into another hundred intervals, as seen in the second number line in Figure 2. Using the context of a number line will allow students to visualize the degree of difference more concretely than if we were to just discuss the difference by just reading the numbers alone.

Concept #2- Numbers as powers of 10 and laws of exponents

Although my students have spent a great deal of time thinking about place value in earlier grade levels, these basic understandings of our number system are not drawn upon for several years by the time they reach 8^{th} grade and have considerable holes. Consider the question of how many times bigger is the population of the U.S. compared to Chicago. Students may want to take the two numbers and divide them, but if we consider the magnitude of each number in terms of a power of 10, we can reach a reasonable answer without going through the process of division. See the table below.

Population of Chicago	Population of U.S.
≈3,000,000	≈300,000,000

(3×10×10×10×10×10×10) (3×10×10×10×10×10×10×10×10)
(3×10^6) (3×10^8)

Chicago's population is somewhat less than 3 million people (but more than 2 ½ million). The 3 is in the millions place can be seen as 3 multiplied by 10, 6 times. The U.S. is somewhat more than 300 million, which is 3 multiplied by 10, 8 times. Therefore, the U.S. population 10 x 10 or 100 times larger than Chicago's. This is an important realization that students rarely make because they have not been exposed to seeing number in terms of powers of 10. A more refined point can be made by paying attention to how the populations were rounded. We know that we rounded up in the case of Chicago's population and rounded down in the case of the population of the U.S. This means that our determined ratio of 100 is an underestimate. Another prompt to encourage students to pay attention to place value and powers of ten is to give students a prompt like the one below.

Consider the following large number: 1 , 1 1 1 , 1 1 1 , 1 1 1 , 1 1 1

How many times bigger is the 1 in the hundred millions place worth compared to the 1 in the ten thousands place?

We can see that the 1 in the hundred millions place is worth 100,000,000 or (1×10^8) and the 1 in the ten thousands place is worth 10,000 or (1×10^4). Then we can see that the first 1 is multiplied by 10, 4 more times than the second 1. So it is worth 10^4 or 10,000 times more! This will be a new way of thinking about numbers, and will be beneficial in our future study of large numbers and laws of exponents.

How many pennies would it take to make $10,000?

In order to solve this problem, students must complete the calculation of 10,000 × 100. To find the product, my students would line up the factors and work the standard algorithm or remember the shortcut of adding the number of total zeroes seen without being able to provide a mathematical argument as to why it works. To remedy this, I will prompt students to break the numbers down by powers of 10 into what I will call place value pieces.

10,000 × 100

= (1 × 10 × 10 × 10 × 10) × (1 × 10 × 10)

= (1×10^4) × (1×10^2)

Seeing the two factors as a product of a number of 10's allows for a logical justification that the final product being the product of the total number of 10s being multiplied. We can then make the connection to the base ten system. Every time we multiply by ten we move to the next place value, therefore if we are multiplying by 10, 6 times, then the final product will be a 1 with 6 digits to the right, namely 1,000,000. Next, I will direct students to write numbers, starting with the numbers just calculated, as a power of 10 to make the numbers more compact and easier to work with. Although this might seem pedantic, it will allow students to notice some important patterns with powers of tens. After giving students enough iterations of multiplying different powers of ten together, I will prompt students to pay attention to the exponents. Essentially they should notice that when multiplying the powers of 10 we can simply add the exponents. Consider another example:

4,000,000 x 2,000

$= (4 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10) \times (2 \times 10 \times 10 \times 10)$

$= (4 \times 10^6) \times (2 \times 10^3)$

Here, by means of the commutative property we can take this in two parts by first seeing that 4 x 2 is 8 and $10^6 \times 10^3$ is essentially 10 multiplied by itself a total of 9 times or 10^9. The key aspect of showing the numbers broken up by powers of 10 is that it provides a clear justification for adding the exponents. This is an important step in students understanding the rationale behind this rule that is many times reached by students memorizing a procedure rather than being understood conceptually. Students can then also see that this works inversely for division. See the example below.

4,000,000 ÷ 2,000

$= (4 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10) / (2 \times 10 \times 10 \times 10)$

$= (4 \times 10^6) \div (2 \times 10^3)$

$= (4 \div 2) \times 10^{(6-3)})$

= 2,000

By considering the numbers as place value pieces, we can calculate separately 4 ÷ 2 = 2 and then easily see that 10 multiplied by itself 6 times is being divided by 10 multiplied by itself 3 times, leaving the quotient of 10^3. After going through this process several times we will generalize and define these laws of exponents in the following terms.

For a positive whole numbers *x* and when *n* > *m*,

$x^m \bullet x^n = x^{m+n}$

$x^m / x^n = x^{m-n}$

The 8th grade Common Core Standards calls for a broader coverage of the laws of exponents including fractional exponents and negative exponents. Although there are more rules, these are the only ones necessary for this unit where the culminating objective is for student to perform operations with scientific notation for large quantities. The other laws of exponents will be referred to later in the year after students are comfortable using the laws stated above with integer powers of 10.

Concept #3- Estimating

After students have a solid grasp of the relative size of numbers and a fresh understanding of seeing numbers in terms of powers of ten, we will cover the topic of estimation. There are two meaningful justifications for estimating numbers that I want students to understand by the end of the unit. The first is that when considering large numbers that we get from measurement, it is difficult to know numbers beyond a couple of digits, due to human error. The other utility of estimating is that it allows us to execute computations with high relative accuracy by rounding numbers to their first 2 or 3 digits.

To illustrate the point that we do not really know most quantities past the first few digits, I will task students with considering certain measurements. I want my students to understand that there is a certain level of error involved with all measurements. Take the population of Chicago for example. The 2010 census reported that there were 2,695,598 people living in Chicago. I will have my students consider the question: Is it possible to know the exact population of a city? My students will discuss factors such as counting errors, people moving in and out of the city, and people being born and dying. Students should reach the conclusion that it would be safer to say that around 2,700,000 people live in Chicago, because in a fairly short period of time the population could fluctuate by thousands of people. Measurements will always have a degree of error. I will be asking students some questions that will involve the surface area of Lake Michigan. A quick search on the internet will give you several different figures for this measurement. See table A.

Table A

Surface Area of Lake Michigan	Source
22,404 sq. miles	Michigan Department of Environmental Quality
22,300 sq. miles	University of Wisconsin Sea Grant Institute
22,394 sq. miles	NOAA Great Lakes Environmental Research

There could be a variety of reasons why these figures are different ranging from the technology used to the year that the measurement was taken. At this point we will discuss measurement error and why we should only use digits that we are sure of. In this case we could safely round the number to hundreds place and use 22,400 as the surface area for Lake Michigan, because the tens and ones place varies in all three figures. We can not be sure that they are accurate and therefore are not significant.

I also want students to appreciate that approximating large numbers serves the purpose of making calculations easier. Students should eventually see rounding as a function of efficiency.

How much water in total do Chicagoans consume in a year? Assume that the average American consumes 58 gallons of water every year.

It is much less daunting to take a calculation like 2,695,598 x 58 and instead multiply 2,700,000 x 60. I expect that this point will be easily received and will most likely be a review for most of my eighth graders. However, a point that I think they have not grasped yet is that these estimated answers are actually fairly accurate. The justification for why we are allowed to round numbers is not covered in the traditional curriculum. In order for students to realize the relatively small impact of rounding numbers, I will have them plot numbers and their respective approximate values on a number line.

Figure 3

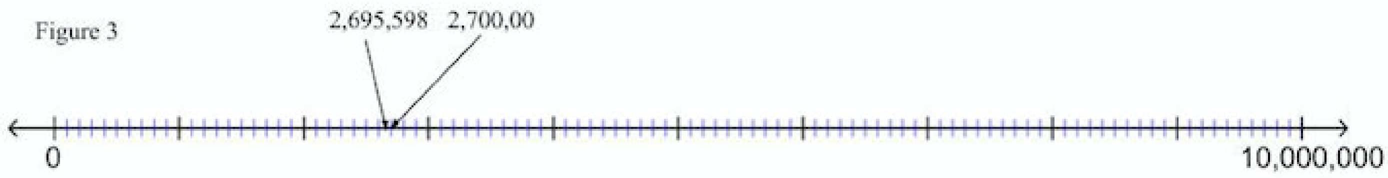


Figure 4

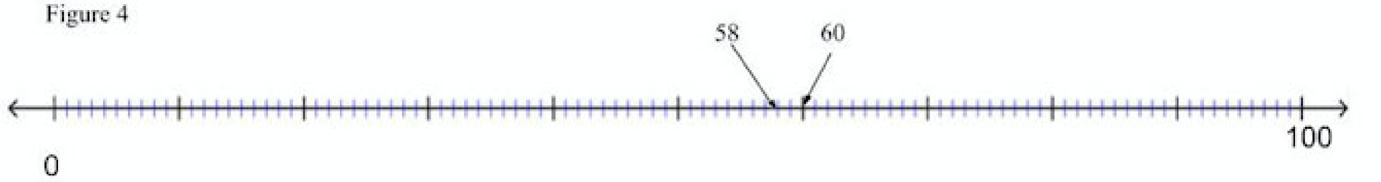


Figure 3 illustrates the point that when the number 2,695,598 is rounded to the first two digits that the change in relative value is hardly noticeable when considering such a large number. Even though we changed the value by 4,402 the impact of its location on the number line is hardly detectable when considering a number in the millions. Further, Figure 4 shows the impact of rounding to one digit is still relatively minimal. It will also be important to show that approximating by rounding to the first 2 or 3 digits roughly the same effect on all sizes of numbers. Throughout the unit we will revisit the number line when rounding numbers to justify why we are able to approximate to the first few digits.

After students are convinced that rounding numbers to the first 2 or 3 digits does not cause a big change in the relative location on a number line, I will push the class to further prove that rounding numbers gives us a very good approximation to work with. To achieve this, we will address the concept of percent error. Consider the following problem:

565,000 is rounded to 570,000 and 565 is rounded to 570. Which rounded number is more off from its original value?

With a prompt such as this one, I expect students to have a discussion about which number changed more as a result of rounding. Most students will only consider the amount changed, and will not compare it to the size of the number being changed. Because 565,000 increased by 5,000 to 570,000 and 565 is only increased by 5 to 570, one could argue that 570,000 is more off. My hope is that some students will employ proportional reasoning and take into account how much was rounded in relation to the whole original number. It is important that students take the time to discuss this nuanced point because it is crucial to understanding the concept of rounding. To illustrate the point of how it makes sense to compare the error with the number being approximated I will ask students, if they lost $100, would they be upset? Then I will ask them to suppose they had a million dollars, and lost $100. Would they be as upset? The points raised from these discussions will allow me to provide the formal language and procedure of finding percent error. The formula to find percent can be defined as:

$|(A-N)/N| \times 100 = \text{Percent Error}$

A = Approximated number

N = Actual Number

In the case above, when 565,000 is rounded to 570,000 the number is rounded up by 5,000 which is less than 1 % error. Students should then see that when 565 is rounded up by 5 to 570 that there is the exact same percent error. To solidify this point, I will have students execute many iterations of rounding numbers to varying place values to realize that if we keep three, or even two, digits when rounding numbers, we introduce a relatively low percent error. By noticing what happens when numbers are rounded to 1, 2, or 3 places we can state the following general rules.

When rounding to 1 digit there will always be less than 50% error and half of the time less than 10%.

When rounding to 2 digits there will always be less than 10% error and half of the time less than 1%.

When rounding to 3 digits there will always be less than 1% error and half of the time less than 0.1%.

By understanding rounding and the impact on percent error, my aim is that students will understand why rounding is an accurate and useful tool rather than an abstract task that is sometimes recommended by their math teacher. Students are often thrown off by large strings of numbers, and by proving that only the first few digits are important, I hope students will be less daunted by large numbers and approach them with greater confidence. It also provides a meaningful framework for understanding scientific notation, the next key concept addressed in this unit.

Concept #4- Scientific notation

After students understand why we can round numbers to the first 2 or 3 digits with an acceptable degree of accuracy, I will introduce the concept of scientific notation. In the past I have taught this concept by simply showing students how to convert numbers into scientific notation and vice versa void of any mathematical rationale beyond scientific notation is easier way to write complicated numbers. Scientific notation is certainly a simple way to write and read numbers with many digits, but there is reason as to why we are allowed to use it. We will consider the following problem:

Chicago residents get their drinking water from Lake Michigan. Assuming that a typical American consumes 58 gallons per year, is there enough water in Lake Michigan for all Americans to drink for a year? How long would Lake Michigan last if everyone in the world drank its water?

Lake Michigan has a volume of 1,186 cubic miles. A useful fact that would be provided to students is that cubic mile can hold 1,101,000,000,000 gallons. This can be computed exactly, by computing the number of cubic inches in a cubic mile. This is a massive number that is confusing to read, with so many digits, and hard to reason with when read in standard form. Tasking students to reason with this number would provide the opportunity to consider an easier way to write this number. It will be important here to have students grapple with the questions: Which digit is most important in this number? How can we quickly know how big this number is? Students should come to the conclusion that the first digit tells us how many trillions of gallons there are in a cubic mile. The 12 digits to the right of the 1 tells us the magnitude of the number or how many powers of ten the 1 is multiplied by, in this case 12. These descriptions lay the framework to understanding that the number can be written in scientific notation as 1.101×10^{12} . Scientific notation expresses a number as a decimal fraction between 1 and 10 multiplied by a power of 10. The factor 1.101 tells us how accurately we know the number, in this case we can be confident up to the billions place. The exponent of 12 indicates the size or magnitude of the number. It will be important to note that it would be sufficient to simplify this number even further and round to just 3 digits, 1.10×10^{12} . This rounded number would only introduce an error of less than 0.1%.

Concept #5- Operating with Scientific Notation

Once students understand the relative size of large numbers, can make approximations by rounding, and express them simply, using scientific notation, they will be ready to perform calculations involving big numbers. This will not be the first time they are attempting to calculate big numbers in this unit but rather the first time they will be expected to demonstrate all of the previously learned concepts to work in an efficient manner. In this section I will discuss the key mechanics of adding, subtracting, dividing and multiplying with scientific notation.

Adding and Subtracting

If you were to stack the 10 tallest buildings in Chicago on top of each other, how high would they reach in inches?

Building	*Height in Inches*
Willis Tower	2.07×10^4
Trump International	1.66×10^4
Hancock Building	1.35×10^4
Aon Center	1.19×10^4
Two Prudential Place	1.15×10^4
311 South Wacker Drive	1.14×10^4
900 North Michigan	1.04×10^4
Chase tower	1.03×10^4
Water Tower Place	9.95×10^3
Aqua Building	9.83×10^3

In the task above we must simply add all of the heights. The sum could be reached in multiple ways. One could efficiently estimate the answer by rounding the heights of Trump International and Willis Tower to 2×10^4 inches and the remaining 8 buildings to 1×10^4 inches. The next step would be to then add all of the heights to get a sum of 12×10^4 inches. Alternatively, students could reach a more exact sum by taking all of the given digits of each height and adding them. A common mistake that students could make here is to add the coefficients of the Water Tower Place and Aqua Building without paying attention to the power of 10. Here, we must first change the placement of the decimal to represent the heights using the same order of 10. See this adjustment below.

Water Tower Place	9.95×10^3	0.995×10^4
Aqua Building	9.83×10^3	0.983×10^4

The sum would come to 11.57×10^4 inches. I will make sure that we take note that this is rather close to the sum reached in the first method which used rounded numbers. Overall, when adding or subtracting with scientific notation the main point that should be highlighted for student is that just as we line up numbers by place value when working with numbers in standard from we must ensure that we are adding digits that multiply the same power of 10.

Multiplying and Dividing

Chicago Public Schools spent a total of 5,460,000,000 dollars in 2017. CPS reports that there were 371,382 *student enrolled last school year. How much money did CPS pay per student?*

This problem will allow students to see the utility of applying all of the previously learned concepts from this unit. First, we can reason with these numbers better if they are rounded, then converted into scientific notation. The amount of money CPS spent in a year can be represented as 5.46×10^9 . Then the number of students enrolled in CPS can be safely rounded to 3.72×10^5 without introducing a significant amount of error. Finally we are left with the calculation $(5.46 \times 10^9) \div (3.72 \times 10^5)$. Before calculating for a final answer, I will ask students to figure how big the number will be. Essentially, I will require them to consider powers of 10 before finding a precise number. Before they deal with the coefficients, they should know that the answer is going to be in the tens of thousands. Ultimate we can divide the coefficients 5.46 and 3.72 and then subtract the exponents for the power of 10 to arrive at an answer of 1.47×10^4 . Another strategy that students might choose is to round the figures to (6 ´ 10^9) and (4 ´ 10^5). The quotient of these two numbers would be 1.5 ´ 10^4 , extremely close to our previous answer.

Teaching Strategies

Structured problems solving

The general form of lessons I will use in this unit will be centered on structured problem solving. This is different from a typical gradual release lesson format where students are taught a strategy explicitly, practice the strategy with heavy teacher guidance, and then eventually apply it through independent practice. In a problem-based format, students are presented a problem to be first worked on independently. Here students will apply their own mathematical knowledge in an effort to develop strategies to reach a solution. Once students have had time to work independently, their different strategies and solutions are shared and discussed in groups or as a class. During this time, students will be able to see multiple approaches, discuss misconceptions, and come to new conclusions about the material. The discussion must be carefully facilitated in order to reach the desired outcomes for the lesson. If all student ideas have been exhausted and the strategies or understandings the lesson set out to achieve have still not been reached, a different task must be presented. Finally, the students are given a set of additional problems to apply and practice what they learned from the problem solving and the discussion. The key idea behind a structured problem solving approach is that students are first given the opportunity to reason and construct their own concepts and strategies, which leads to a deeper understanding of the content covered in a given lesson. In the pursuit of nurturing new ways for students to think about large numbers, this approach will be fundamental.

Structured problem solving is laid out in Akihiko Takahashi's paper titled, "Characteristics of Japanese Mathematics Lessons". Takahashi stresses that in addition to the attention devoted to extensive discussion, the selection of problems and activities needs to be carefully considered as well. (Takahashi 2006) The progression of problems in this unit is designed to bring out concepts and strategies that cohesively build on each other. In general, each lesson will present a new problem context for students to solve. The problems are designed to have a question of high interest or a compelling visual to build student investment in the task. After the structured problem-solving process has taken place, students will be given some decontextualized examples to focus more in on the number concepts. These exercises will give students the opportunity to process, apply, and generalize the previously discussed strategies and understandings.

Some of the tasks I will use in each lesson are detailed in the following section.

Tasks

Concept #1- Understanding the relative size of numbers

+ A base ten block is 1 centimeter long. If we were to line up 1 thousand blocks end to end, how far would the line they make stretch? Would it fit inside our classroom? How about 1 million? Would it fit inside our school? Inside Chicago? Inside Illinois? 1 billion?

+ *Graph the following populations on the given number line. Be as precise as possible.*

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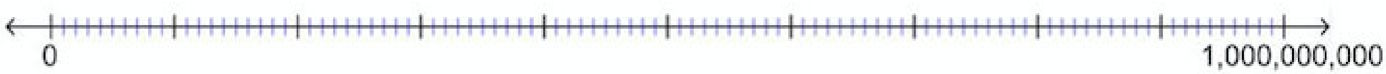
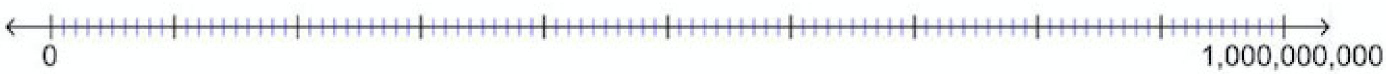


+ Plot each of the numbers in the table below as a point on a number line from 1 to a billion.



The table shows how fast light waves or electricity can travel through different materials.

Material	Speed (meters per second)
Space	300,000,000
Water	$(2.25) \bullet 10^{8}$
Copper wire (electricity)	280,000,000
Diamond	$124 \bullet 10^{6}$
Ice	$(2.3) \bullet 10^{8}$
Olive oil	200,000,000

(Open-Up Resources, 2017)

Concept #2- Numbers as powers of ten and laws of exponents

+ How many times bigger is the population of the United states compared to Chicago?

+ How many pennies would it take to make a million dollars?

+ How many centimeters in 10 kilometers?

+ How many seconds old are you? How long will it take for you to get to a billion seconds old?

Concept #3- Estimating

+ Suppose that the average Chicagoan drinks 58 gallons of water per year. How much drinking water is needed every year for Chicago?

+ How many school lunches are eaten every day in Chicago?

Concept #4- Scientific Notation

+ A conservative estimate of the number of stars in the universe is 6×10^{22} . The average human can see about 3,000 stars at night with his naked eye. About how many times more stars are there in the universe compared to the stars a human can actually see? (Eureka Math, 2015)

+ The average person takes about 30,000 breaths per day. Express this number as a single-digit integer times a power of 10. If the average American lives about 80 years (or about 30,000 days), how many total breaths will a person take in her lifetime? (Eureka Math, 2015)

Concept #5- Operating with Scientific Notation

+ Chicago is known as the alley capitol of the United States. If you were to line up all the alleyways from end to end it would stretch a total of 1.003×10^7 feet. If the average width of an alleyway is 16 feet, how many square feet of pavement would be needed to repave all of Chicago's alleyways?

+ If every person in the world jumped into Lake Michigan, what would happen to the level of water?

+ Consider the cost it took to build Willis Tower. If we stacked that amount in dollar bills, would the tower of cash be as a high as the building?

+ Culminating Project- Design your own big question and show all of your steps in answering it. Explain how you know your answer is reasonable. Plot the values you used and your final answer on a number line.

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